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γ-OPEN SETS IN N_{NC}-TOPOLOGICAL SPACES

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ABSTRACT. In this paper, a new types of γ-open sets and γ-closed sets are introduced in N-neutrosophic crisp topological spaces and also we discuss their basic properties in $N_{nc}ts$.

1. INTRODUCTION

The concepts of neutrosophy and neutrosophic set was introduced Smarandache [6, 7]. In 2014, the concept of neutrosophic crisp topological space presented by Salama, Smarandache and Kroumov [4]. Al-Omeri [1] also investigated neutrosophic crisp sets in the context of neutrosophic crisp topological Spaces. Lellis Thivagar et al. [8] introduced the notion of N_n-open (closed) sets and N-neutrosophic topological spaces. In 1996, Andrijevic [2] introduced γ (or) b-open sets in general topology.

2. PRELIMINARIES

Definition 2.1. *[5] For any non-empty fixed set Y, a neutrosophic crisp set (briefly, ncs) K, is an object having the form $K = \langle K_1, K_2, K_3\rangle$ where K_1, K_2 and K_3 are subsets of Y satisfying any one of the types:*

(T1) $K_a \cap K_b = \phi$, $a \neq b$ *and* $\bigcup_{a=1}^{3} K_a \subset Y$, $\forall a, b = 1, 2, 3$.
(T2) $K_a \cap K_b = \phi$, $a \neq b$ *and* $\bigcup_{a=1}^{3} K_a = Y$, $\forall a, b = 1, 2, 3$.

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(T3) $\bigcap_{a=1}^{3} K_a = \phi$ *and* $\bigcup_{a=1}^{3} K_a = Y$, $\forall a = 1, 2, 3$.

Definition 2.2. *[5] Types of ncs's $\emptyset_N$ and Y_N in Y are as:*

(i) $\emptyset_N$ *may be defined as* $\emptyset_N = \langle \emptyset, \emptyset, Y\rangle$ *or* $\langle \emptyset, Y, Y\rangle$ *or* $\langle \emptyset, Y, \emptyset\rangle$ *or* $\langle \emptyset, \emptyset, \emptyset\rangle$;
(ii) Y_N *may be defined as* $Y_N = \langle Y, \emptyset, \emptyset\rangle$ *or* $\langle Y, Y, \emptyset\rangle$ *or* $\langle Y, \emptyset, Y\rangle$ *or* $\langle Y, Y, Y\rangle$.

Definition 2.3. *[5] Let Y be a non-empty set and the ncs's K and M in the form $K = \langle K_{11}, K_{22}, K_{33}\rangle$, $M = \langle M_{11}, M_{22}, M_{33}\rangle$, then:*

(i) $K \subseteq M \Leftrightarrow K_{11} \subseteq M_{11}$, $K_{22} \subseteq M_{22}$ *and* $K_{33} \supseteq M_{33}$ *or* $K_{11} \subseteq M_{11}$, $K_{22} \supseteq M_{22}$ *and* $K_{33} \supseteq M_{33}$;
(ii) $K \cap M = \langle K_{11} \cap M_{11}, K_{22} \cap M_{22}, K_{33} \cup M_{33}\rangle$ *or* $\langle K_{11} \cap M_{11}, K_{22} \cup M_{22}, K_{33} \cup M_{33}\rangle$;
(iii) $K \cup M = \langle K_{11} \cup M_{11}, K_{22} \cup M_{22}, K_{33} \cap M_{33}\rangle$ *or* $\langle K_{11} \cup M_{11}, K_{22} \cap M_{22}, K_{33} \cap M_{33}\rangle$;
(iv) $K^c = \langle {K_{11}}^c, {K_{22}}^c, {K_{33}}^c\rangle$ *or* $\langle K_{33}, K_{22}, K_{11}\rangle$ *or* $\langle K_{33}, {K_{22}}^c, K_{11}\rangle$.

Definition 2.4. *[3] Let Y be a non-empty set. Then ${}_{nc}\Gamma_1$, ${}_{nc}\Gamma_2$, $\cdots$, ${}_{nc}\Gamma_N$ are N-arbitrary crisp topologies defined on Y and the collection*

$N_{nc}\Gamma = \{A \subseteq Y : A = (\bigcup_{j=1}^{N} K_j) \cup (\bigcap_{j=1}^{N} L_j),\ K_j, L_j \in {}_{nc}\Gamma_j\}$ *is called N_{nc}-topology on Y if the axioms are satisfied:*

(i) $\emptyset_N$, $Y_N \in N_{nc}\Gamma$;
(ii) $\bigcup_{j=1}^{\infty} A_j \in N_{nc}\Gamma\ \forall\ \{A_j\}_{j=1}^{\infty} \in N_{nc}\Gamma$;
(iii) $\bigcap_{j=1}^{n} A_j \in N_{nc}\Gamma\ \forall\ \{A_j\}_{j=1}^{n} \in N_{nc}\Gamma$.

Then $(Y, N_{nc}\Gamma)$ is called a N_{nc}-topological space (briefly, $N_{nc}ts$) on Y. The $N_{nc}\Gamma$ elements are called N_{nc}-open sets ($N_{nc}os$) on Y and its complement is called N_{nc}-closed sets ($N_{nc}cs$) on Y. The elements of Y are known as N_{nc}-sets ($N_{nc}s$) on Y.

Definition 2.5. *[3] Let $(Y, N_{nc}\Gamma)$ be $N_{nc}ts$ on Y and K be an $N_{nc}s$ on Y, then the N_{nc} interior of K (briefly, $N_{nc}int(K)$) and N_{nc} closure of K (briefly, $N_{nc}cl(K)$) are defined as:*

(i) $N_{nc}int(K) = \cup\{A : A \subseteq K$ *and* A *is a* $N_{nc}os$ *in* $Y\}$;
(ii) $N_{nc}cl(K) = \cap\{C : K \subseteq C$ *and* C *is a* $N_{nc}cs$ *in* $Y\}$.

Definition 2.6. *[3] Let $(Y, N_{nc}\Gamma)$ be any $N_{nc}ts$. Let K be an $N_{nc}s$ in $(Y, N_{nc}\Gamma)$. Then K is said to be a N_{nc}-pre (resp. N_{nc}-semi and N_{nc}-α) open set (briefly, $N_{nc}\mathcal{P}os$ (resp. $N_{nc}\mathcal{S}os$ and $N_{nc}\alpha os$)) if $K \subseteq N_{nc}int(N_{nc}cl(K))$ (resp. $K \subseteq N_{nc}cl(N_{nc}int(K))$ and $K \subseteq N_{nc}int(N_{nc}cl(N_{nc}int(K)))$).*

The complement of an $N_{nc}\mathcal{P}os$ (resp. $N_{nc}\mathcal{S}os$ and $N_{nc}\alpha os$) is called an N_{nc}-pre (resp. N_{nc}-semi and N_{nc}-α) closed set (briefly, $N_{nc}\mathcal{P}cs$ (resp. $N_{nc}\mathcal{S}cs$ and $N_{nc}\alpha cs$)) in Y. The family of all $N_{nc}\mathcal{P}os$ (resp. $N_{nc}\mathcal{P}cs$, $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{S}cs$, $N_{nc}\alpha os$ and $N_{nc}\alpha cs$) of Y is denoted by $N_{nc}\mathcal{P}OS(Y)$ (resp. $N_{nc}\mathcal{P}CS(Y)$, $N_{nc}\mathcal{S}OS(Y)$, $N_{nc}\mathcal{S}CS(Y)$, $N_{nc}\alpha OS(Y)$ and $N_{nc}\alpha CS(Y)$).

3. γ-OPEN SETS IN N_{nc}-TOPOLOGICAL SPACES

Throughout this section, let $(Y, \mathcal{N}_{nc}\Gamma)$ be any $\mathcal{N}_{nc}ts$. Let K and L be an $\mathcal{N}_{nc}s$'s in $(Y, \mathcal{N}_{nc}\Gamma)$.

Definition 3.1. *The K is said to be a*

(i) *N_{nc}-γ-open (or) N_{nc}-b-open (briefly, $N_{nc}\gamma o$ (or) $N_{nc}bo$) set if $K \subseteq N_{nc}cl(N_{nc}int(K)) \cup N_{nc}int(N_{nc}cl(K))$. The complement of an $N_{nc}\gamma o$ set is called an N_{nc}-γ-closed (briefly, $N_{nc}\gamma c$) set in Y. The family of all $N_{nc}\gamma o$ (resp. $N_{nc}\gamma c$) set of Y is denoted by $N_{nc}\gamma OS(Y)$ (resp. $N_{nc}\gamma CS(Y)$).*

(ii) *N_{nc}-regular open (briefly, $N_{nc}ro$) set if $K = N_{nc}int(N_{nc}cl(K))$. The complement of an $N_{nc}ro$ set is called a N_{nc}-regular closed (briefly, $N_{nc}rc$) set in Y.*

Definition 3.2.

(i) $N_{nc}\gamma int(K)$(*resp.* $N_{nc}rint(K)) = \cup\{A : A \subseteq K$ *and* A *is a* $N_{nc}\gamma o$ (*resp.* $N_{nc}ro$) *set in* $Y\}$.

(ii) $N_{nc}\gamma cl(K)$(*resp.* $N_{nc}rcl(K)) = \cap\{C : K \subseteq CandC$ *is a* $N_{nc}\gamma c$ (*resp.* $N_{nc}rc$) *set in* $Y\}$.

Proposition 3.1. *The $N_{nc}\gamma$-closure and $N_{nc}\gamma$-interior operator satisfies properties:*

(i) $K \subseteq N_{nc}\gamma cl(K)$.
(ii) $N_{nc}\gamma int(K) \subseteq K$.
(iii) $K \subseteq L \Rightarrow N_{nc}\gamma cl(K) \subseteq N_{nc}\gamma cl(L)$.
(iv) $K \subseteq L \Rightarrow N_{nc}\gamma int(K) \subseteq N_{nc}\gamma int(L)$.
(v) $N_{nc}\gamma cl(K \cup L) = N_{nc}\gamma cl(K) \cup N_{nc}\gamma cl(L)$.

(vi) $N_{nc}\gamma int(K \cap L) = N_{nc}\gamma int(K) \cap N_{nc}\gamma int(L)$.

(vii) $(N_{nc}\gamma cl(K))^c = N_{nc}\gamma int(K^c)$.

(viii) $(N_{nc}\gamma int(K))^c = N_{nc}\gamma cl(K^c)$.

(ix) $N_{nc}\gamma cl(K) = L$ *iff* L *is an* $N_{nc}\gamma c$ *set.*

(x) $N_{nc}\gamma int(K) = L$ *iff* L *is an* $N_{nc}\gamma o$ *set.*

(xi) $N_{nc}\gamma cl(K)$ *is the smallest* $N_{nc}\gamma c$ *set containing* K.

(xii) $N_{nc}\gamma int(K)$ *is the largest* $N_{nc}\gamma o$ *set containing* K.

Proposition 3.2. *The statements are hold but the equality does not true.*

(i) *Every* $N_{nc}ros$ *(resp.* $N_{nc}rcs$*) is a* $N_{nc}os$ *(resp.* $N_{nc}cs$*).*

(ii) *Every* $N_{nc}os$ *(resp.* $N_{nc}cs$*) is a* $N_{nc}\alpha os$ *(resp.* $N_{nc}\alpha cs$*).*

(iii) *Every* $N_{nc}\alpha os$ *(resp.* $N_{nc}\alpha cs$*) is a* $N_{nc}\mathcal{S}os$ *(resp.* $N_{nc}\mathcal{S}cs$*).*

(iv) *Every* $N_{nc}\alpha os$ *(resp.* $N_{nc}\alpha cs$*) is a* $N_{nc}\mathcal{P}os$ *(resp.* $N_{nc}\mathcal{P}cs$*).*

(v) *Every* $N_{nc}\mathcal{S}os$ *(resp.* $N_{nc}\mathcal{S}cs$*) is a* $N_{nc}\gamma os$ *(resp.* $N_{nc}\gamma cs$*).*

(vi) *Every* $N_{nc}\mathcal{P}os$ *(resp.* $N_{nc}\mathcal{P}cs$*) is a* $N_{nc}\gamma os$ *(resp.* $N_{nc}\gamma cs$*).*

Proof. (i) K is a $N_{nc}ros$, then $K = N_{nc}int(N_{nc}cl(K))$ and so $K = N_{nc}int(K)$. K is a $N_{nc}os$. The other cases are similar. It is also true for their respective closed sets. □

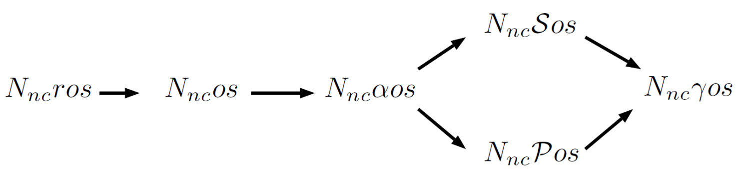

FIGURE 1. Different types N_{nc} open sets.

Proposition 3.3. *If* K *is an* $N_{nc}\gamma os$ *(resp.* $N_{nc}\gamma cs$*) iff* K *is a* $N_{nc}\mathcal{S}os$ *(resp.* $N_{nc}\mathcal{S}cs$*) and* $N_{nc}\mathcal{P}os$ *(resp.* $N_{nc}\mathcal{P}cs$*).*

Proposition 3.4. *The union (resp. intersection) of any family of* $N_{nc}\gamma OS(Y)$ *(resp.* $N_{nc}\gamma CS(Y)$*) is a* $N_{nc}\gamma OS(Y)$ *(resp.* $N_{nc}\gamma CS(Y)$*).*

Remark 3.1. *The intersection of two* $N_{nc}\gamma os$*'s need not be* $N_{nc}\gamma os$*.*

Example 1. *Let* $Y = \{l, m, n\}$, ${}_{nc}\Gamma_1 = \{\phi_N, Y_N, L\}$, ${}_{nc}\Gamma_2 = \{\phi_N, Y_N, M, N, O\}$,
$L = \langle\{m\}, \{l\}, \{n\}\rangle$, $M = \langle\{\phi\}, \{l, n\}, \{m\}\rangle$,

$$N = \langle \{m\}, \{l, n\}, \{\phi\} \rangle, \; O = \langle \{\phi\}, \{l\}, \{m, n\} \rangle$$

then we have $2_{nc}\Gamma = \{\phi_N, Y_N, L, M, N, O\}$. *The sets*

$$\langle \{l, m\}, \{\phi\}, \{n\} \rangle \text{ and } \langle \{l, n\}, \{m\}, \{\phi\} \rangle$$

are $N_{nc}\gamma os$ *but the intersection* $\langle \{l\}, \{\phi\}, \{n\} \rangle$ *is not* $N_{nc}\gamma os$.

Proposition 3.5. *The statements*

(i) K *is a* $N_{nc}\gamma os$.
(ii) $K = N_{nc}\mathcal{P}int(K) \cup N_{nc}\mathcal{S}int(K)$.
(iii) $K \subset N_{nc}\mathcal{P}cl(N_{nc}\mathcal{P}int(K))$

are equivalent.

Proposition 3.6. *If* K *be a* $N_{nc}\gamma os$ *such that* $N_{nc}int(K) = \phi$, *then* K *is a* $N_{nc}\mathcal{P}os$.

Example 2. *Let* $Y = \{l, m, n, o, p\}$, ${}_{nc}\Gamma_1 = \{\phi_N, Y_N, L\}$, ${}_{nc}\Gamma_2 = \{\phi_N, Y_N, M, N, O\}$. $L = \langle \{l\}, \{m, o, p\}, \{n\} \rangle$, $M = \langle \{l, n, p\}, \{m, o\}, \{l, n\} \rangle$, $N = \langle \{l, n, p\}, \{m, o\}, \{n\} \rangle$, $O = \langle \{l\}, \{m, o, p\}, \{l, n\} \rangle$, *then we have* $2_{nc}\Gamma = \{\phi_N, Y_N, L, M, N, O\}$.

(i) L *is a* $N_{nc}os$ *but not a* $N_{nc}ros$.
(ii) $P = \langle \{l, m\}, \{n\}, \{o\} \rangle$ *is a* $N_{nc}\mathcal{P}os$ *but not* $N_{nc}\alpha os$.
(iii) $Q = \langle \{l\}, \{m\}, \{n\} \rangle$ *is a* $N_{nc}\alpha os$ *but not* $N_{nc}os$.
(iv) $R = \langle \{l, o\}, \{m\}, \{n, p\} \rangle$ *is a* $N_{nc}\gamma os$ *but not* $N_{nc}\mathcal{S}os$.
(v) $S = \langle \{l, n\}, \{m, o, p\}, \{l\} \rangle$ *is a* $N_{nc}\mathcal{S}os$ *but not* $N_{nc}\alpha os$.
(vi) S *is a* $N_{nc}\gamma os$ *but not* $N_{nc}\mathcal{P}os$.

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