



Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application

Muhammad Saqlain¹, Muhammad Saeed², Muhammad Rayees Ahmad³, Florentin Smarandache⁴

¹ Lahore Garrison University, DHA Phase-VI, Sector C, Lahore, 54000, Pakistan. E-mail: msaqlain@lgu.edu.pk

² University of Management and Technology, C-II Johar Town, Lahore, 54000, Pakistan. E-mail: muhammad.saeed@umt.edu.pk

³ University of Management and Technology, C-II Johar Town, Lahore, 54000, Pakistan. E-mail: f20182650031@umt.edu.pk

⁴Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA. E-mail: smarand@unm.edu

Abstract. The purpose of MCDM is to determine the best option amongst all the probable options. Due to linguistic assessments, the traditional crisp techniques are not good to solve MCDM problems. This paper deals with the generalization of TOPSIS for neutrosophic hypersoft set primarily based issues explained in section 3. In section 4, the proposed technique is implemented. The proposed technique is easy to implement, and precise and sensible for fixing the MCDM problem with multiple-valued neutrosophic data. In the end, the applicability of the developed method, the problem of parking on which decision maker has normally vague and imprecise knowledge is used. It seems that the outcomes of these examinations are terrific.

Keywords: Uncertainties, Decision making, FNSS, FNHSS, Linguistic variable, Accuracy Function AF, TOPSIS

1 Introduction

To describe the characteristics people generally use apt values when they come across the decision-making problems. On the other hand, it is observed that in an environment of real decision making we face various complex and alterable factors and for these fuzzy expressions, the decision makers take help from the linguistic evaluations. For instance, the evaluation values are represented with the use of expressions like excellent, v. good, and good by decision makers. Zadeh [15-16] proposed a linguistic variable set to express the evaluation values. The idea of vague linguistic variables and the operational rules were devised by Xu [12]. The level of a linguistic variable just depicts the values of linguistic evaluation of a decision maker, but these can not aptly describe the vague level of decision maker particularly in the environment of linguistic evaluation. This flaw can be taken into account by adjoining the linguistic variables as well as by putting forward its other sets. For example, Ye [13] put forward an interval Neutrosophic linguistic set (INLS) and an interval Neutrosophic linguistic number (INLN); Ye [14] also found a single and multiple valued Neutrosophic linguistic set (SVNLS & MVNLS).

At the primary, soft set theory was planned by a Russian scientist [7] that was used as a standard mathematical mean to come back across the difficulty of hesitant and uncertainty. He additionally argues that however, the same theory of sentimental set is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, and applied mathematics. Neutrosophic set could be a terribly powerful tool to agitate incomplete and indeterminate data planned by F. Smarandache [10] and has attracted the eye of the many students [1], which might offer the credibleness of the given linguistic analysis worth and linguistic set can offer qualitative analysis values. Florentin [11] generalized soft set to hypersoft set by remodeling the function into a multi-attribute function, NHSS (Neutrosophic Hyper Soft Set) is additionally planned in his pioneer work.

[8] applies neutrosophic TOPSIS and AHP to reinforce the normal strategies of personal choice to realize the perfect solutions. To investigate and verify the factors influencing the choice of SCM suppliers, [2] used the neutrosophic set for deciding and analysis technique (DEMATEL). [3] offers a unique approach for estimating the sensible medical devices (SMDs) choice method in an exceedingly cluster deciding (GDM) in an exceedingly obscure call atmosphere. Neutrosophic with TOPSIS approach is applied within the decision-making method to handle the unclearness, incomplete knowledge and therefore the uncertainty, considering the selections criteria within the knowledge collected by the choice manufacturers (DMs) [3]. [4] projected a technique of the ANP method and therefore the VIKOR underneath the neutrosophic atmosphere for managing incomplete info and high order inexactitude. [9] used a neutrosophic soft set to predict FIFA 2018.

The sturdy ranking technique with neutrosophic set [5] to handle practices and performances in green supply chain management (GSCM). [6] projected T2NN, Type 2 neutrosophic number, which might accurately describe real psychological feature info.

In this paper, the generalization of TOPSIS for the neutrosophic hypersoft set is proposed. In the proposed method Fuzzy Neutrosophic Numbers FNNs are converted into crisp by using accuracy function $N(A)$.

2 Preliminaries

Linguistic Set [9]: In a crisp set, an element Y in the universe \aleph is either a member of some crisp set \hat{A} or not. It can be represented mathematically with indicator function: $\mu_{\hat{A}}(Y) = \{1, \text{ if } Y \text{ belongs to } \hat{A} \text{ and } 0, \text{ if } Y \text{ doesn't belong to } \hat{A}\}$.

Fuzzy Set [10]: Fuzzy set μ in a universe \aleph is a mapping $\mu: \aleph \rightarrow [0,1]$ which assigns a degree of membership to each element with symbol $\mu_{\hat{A}}(y)$ such that $\mu_{\hat{A}}(y) \in [0, 1]$.

Fuzzy Neutrosophic set: A Fuzzy Neutrosophic set FNs \mathcal{A} over the universe of discourse \mathcal{X} is defined as

$$\mathcal{A} = \langle x, T_{\mathcal{A}}(x), I_{\mathcal{A}}(x), F_{\mathcal{A}}(x) \rangle, \quad x \in \mathcal{X} \text{ where } T, F, I: \mathcal{X} \rightarrow [0, 1] \text{ \&}$$

$$0 \leq T_{\mathcal{A}}(x) + I_{\mathcal{A}}(x) + F_{\mathcal{A}}(x) \leq 3.$$

Fuzzy Neutrosophic soft set: Let \mathcal{X} be the initial universal set and \bar{E} be a set of parameters. Consider a non-empty set $\mathcal{A}, \mathcal{A} \subset \bar{E}$. Let $\mathcal{P}(\mathcal{X})$ denote the set of all FNs of \mathcal{X} .

Throughout this paper Fuzzy Neutrosophic soft set is denoted by FNS set / FNSS.

3 Algorithm

Let the function be

$$F: P_j \times P_k \times P_l \times \dots \times P_m \rightarrow P(\mathcal{X}), \text{ such that } P_q = P_j, P_k, P_l, \dots, P_m$$

Where

$$P_j = p_1, p_2, p_3, \dots, p_n \quad 1 \leq j \leq n$$

$$P_k = p_1, p_2, p_3, \dots, p_n \quad 1 \leq k \leq n$$

$$P_l = p_1, p_2, p_3, \dots, p_n \quad 1 \leq l \leq n$$

⋮

$$P_m = p_1, p_2, p_3, \dots, p_n \quad 1 \leq m \leq n$$

are multiple valued neutrosophic attributes and \mathcal{X} is a universe of discourse.

Step 1: Construct a matrix of multiple-valued P_q of attributes of order $m \times n$.

$$A = [p_{qr}]_{m \times n}, \quad 1 \leq q \leq m, \quad 1 \leq r \leq n$$

Step 2: Fill the column values with zeros if multiple valued attributes are less than equal to n to form a matrix of order $m \times n$ as defined in the below example.

Step 3: Decision makers will assign fuzzy neutrosophic numbers (FNNs) to each multiple valued linguistic variables.

Step 4: Selection of the subset of NHSS.

Step 5: Conversion of fuzzy neutrosophic values of step: 4 into crisp numbers by using accuracy function $A(N)$.

$$A(N) = \left[\frac{P_{ij}}{3} \right]$$

Step 6: Calculate the relative closeness by using the TOPSIS technique of MCDM.

Step 7: Determine the rank of relative closeness by arranging in ascending order.

Remark 1: In step 2, if all values of each tuple of complete row or complete column are null, then eliminate that respective row or column.

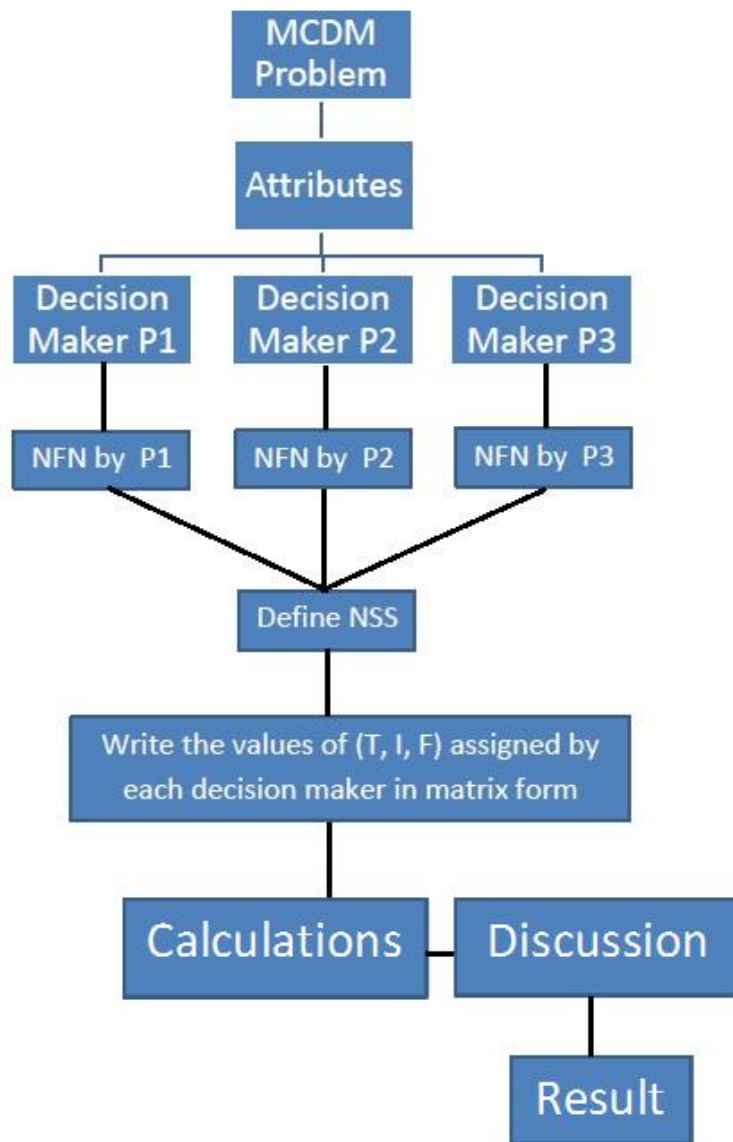


Figure 1: Algorithm design for the proposed technique

We apply the neutrosophic set theory to handle vague data, imprecise knowledge, incomplete information, and linguistic imprecision. The efficiency of the proposed method is evaluated by considering the parking problem as stated below.

The environment of decision making is a multi-criteria decision making surrounded by inconsistency and uncertainty. This paper contributes to supporting the parking problem by integrating a neutrosophic soft set with the technique for order preference by similarity to an ideal solution (TOPSIS) to illustrate an ideal solution amongst different alternatives.

4 Problem Statement

Environmental pollution strongly affects life in cities. The major issue of blockage is due to an excessive number of vehicles in the cities. This causes a major problem in finding a proper place for parking. Therefore, various techniques are implemented to cover this problem. Among them, an application of NHSS (Neutrosophic Hypersoft Set) is used.

In figure 2: there is an elaboration of the trip of a vehicle driver, to his final point. Now he has three numbers of choices to park his vehicle at different distances. So, by Using the NHSS algorithm he will be able to find the nearby spot to stand his vehicle. The driver is intended to go there in the minimum time. This work helped in the Following ways:

- Four Linguistic inputs and an output.
- During his trip how many traffic signals are sensed by the sensor?
- The measure of motor threshold on the way up to final spot is shown by PCU (Parking Car Unit) and
- The Separation between the parking slot and the final point.

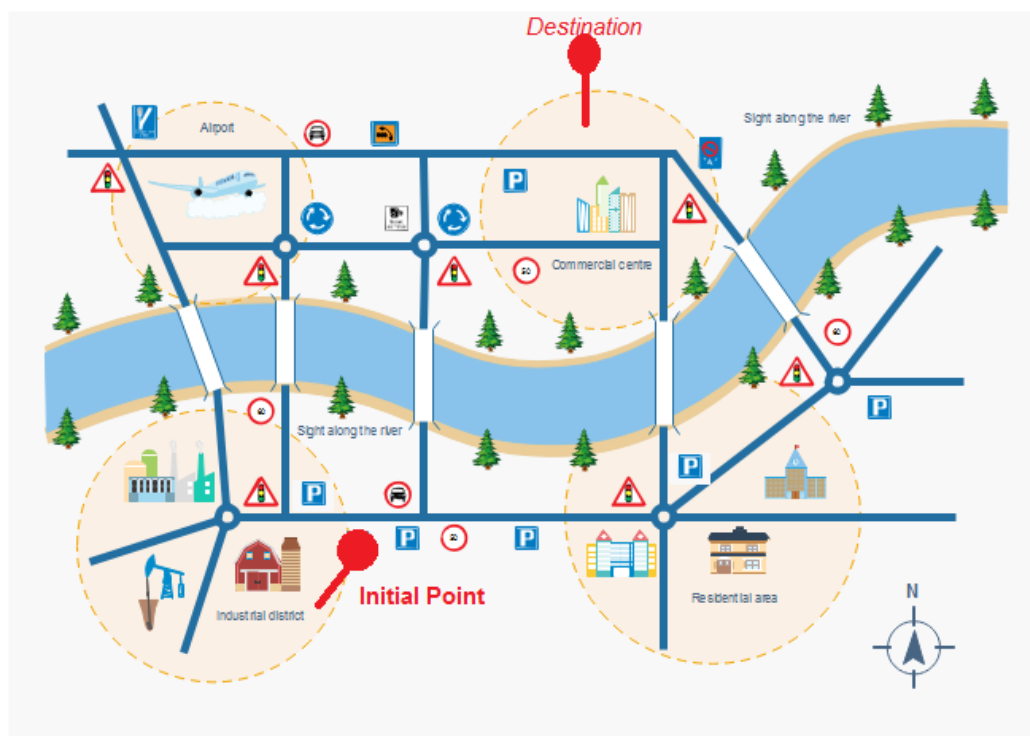


Figure 2: Initial Problem Model

5 Modelling problem into NHSS form

Due to fractional knowledge about the attributes as well as lack of information, mostly the decision makers are observed to be using certain linguistic variables instead of exact values for evaluating characteristics. In such a situation, preference information of alternatives provided by the decision makers may be vague, imprecise, or incomplete.

Sr. #	Linguistic Variable	Code	NFN 1	NFN 2	NFN 3	NFN 4
1	Normal	α	(0.4,0.1,0.0)	(0.3,0.3,0.2)	(0.7,0.2,0.3)	(1.0,1.0,1.0)
2	High	β	(0.3,0.5,0.2)	(0.1,0.1,0.1)	(0.5,0.5,0.3)	(0.5,0.3,0.5)
3	Medium	γ	(0.6,0.6,0.2)	(0.2,0.1,0.1)	(0.6,0.3,0.3)	(0.6,0.4,0.4)
4	Distance i.e., Near	N	(0.2,0.0,0.2)	(0.1,0.2,0.1)	(0.6,0.6,0.1)	(0.4,0.5,0.4)
5	Distance i.e., Far	\bar{N}	(0.4,0.4,0.1)	(0.1,0.3,0.4)	(0.1,0.4,0.1)	(0.4,0.2,0.1)
6	No. of Traffic Signal i.e., one	--	(0.5,0.2,0.4)	(0.5,0.4,0.3)	(0.3,0.3,0.3)	(0.6,0.6,0.2)

7	No. of Traffic Signal i.e., two	=	(0.5,0.2,0.1)	(0.5,0.2,0.1)	(0.6,0.5,0.5)	(1.0,1.0,1.0)
8	No. of Traffic Signal i.e., three	≡	(0.4,0.4,0.2)	(0.2,0.1,0.2)	(0.3,0.3,0.1)	(0.4,0.4,0.6)
9	Parking Space i.e., medium	<i>M</i>	(0.3,0.6,0.2)	(0.3,0.6,0.2)	(0.3,0.6,0.2)	(1.0,1.0,1.0)
10	Parking Space i.e., high	<i>H</i>	(0.3,0.1,0.4)	(0.3,0.3,0.3)	(0.1,0.2,0.5)	(0.6,0.6,0.2)

Table 1: Neutrosophic fuzzy number and corresponding linguistic variable.

6 Numerical calculations of problem

Let $F: P_1 \times P_2 \times P_3 \times P_4 \rightarrow \mathcal{P}(\mathcal{X})$, where \mathcal{X} is the universe of discourse, such that

$$P_1 = \text{Trafic Threshold} = \{\alpha, \beta, \gamma\}$$

$$P_2 = \text{Distance of destination from initial point} = \{N, \mathfrak{N}\}$$

$$P_3 = \text{No. of trafic lights} = \{T_1, T_2, T_3\}$$

$$P_4 = \text{Distance from parking area to destination point} = \{M, h\}$$

Sr. #	PCU	Distance	No. of traffic signals	Parking space
1	A	<i>N</i>	--	M
2	B	\mathfrak{N}	=	h
3	γ		≡	

Table 2: Linguistic variables used in parking problem.

Consider a multiple valued neutrosophic hyper soft set $A = \{P_1, P_3, P_4\}$ such that

$$F(A) = F(\alpha, T_2, M) = \{\alpha(0.4,0.1,0.0), T_2(0.5,0.2,0.1), M(0.3,0.6,0.2), \alpha(0.7,0.3,0.2), T_2(0.6,0.5,0.5), M(0.1,0.5,0.4) \\ \alpha(1.0,1.0,1.0), T_2(1.0,1.0,1.0), M(1.0,1.0,1.0)\}$$

Step 1: Construct a matrix of multiple valued Pq of attributes of order $m \times n$.

$$\begin{bmatrix} \alpha & \beta & \gamma \\ N & \mathfrak{N} & \\ T_1 & T_2 & T_3 \\ M & h & \end{bmatrix}$$

Step 2: Fill the column values with zeros if multiple valued attributes are less than equal to n to form a matrix of order $m \times n$ as defined:

$$\begin{bmatrix} \alpha & \beta & \gamma \\ N & \mathfrak{N} & 0 \\ T_1 & T_2 & T_3 \\ M & h & 0 \end{bmatrix}$$

Step 3: The decision makers gives the values to the selected subset i.e. $F(\alpha, T_2, M)$.

$$\begin{bmatrix} (0.4,0.1,0.0) & (0.5,0.2,0.1) & (0.3,0.6,0.2) \\ (0.7,0.2,0.3) & (0.6,0.5,0.5) & (0.3,0.6,0.2) \\ (1.0,1.0,1.0) & (1.0,1.0,1.0) & (1.0,1.0,1.0) \end{bmatrix}$$

Step 4: Conversion of fuzzy neutrosophic values of step 4 into crisp numbers by using accuracy function $A(N)$.

$$\begin{bmatrix} (0.4 + 0.1 + 0.0)/3 & (0.5 + 0.2 + 0.1)/3 & (0.3 + 0.6 + 0.2)/3 \\ (0.7 + 0.2 + 0.3)/3 & (0.6 + 0.5 + 0.5)/3 & (0.3 + 0.6 + 0.2)/3 \\ (1.0 + 1.0 + 1.0)/3 & (1.0 + 1.0 + 1.0)/3 & (1.0 + 1.0 + 1.0)/3 \end{bmatrix}$$

$$\begin{bmatrix} 0.17 & 0.27 & 0.37 \\ 0.4 & 0.53 & 0.37 \\ 1 & 1 & 1 \end{bmatrix}$$

Step 5: Now we will apply the TOPSIS on the resulting matrix.

	A	T_2	M
P_1	0.17	0.40	1
P_3	0.27	0.53	1
P_4	0.37	0.37	1

Table 3: Decision matrix of the parking problem.

Applying the technique of TOPSIS on the above-mentioned matrix obtained in step 5, the following are the results.

Si+	Si-	ci	Rank
0.177459059	0.015787	0.081693	3
0.081871695	0.117439	0.589226	2
0.084195951	0.163743	0.660417	1

Table 4: Results of calculations done by applying TOPSIS technique of MCDM

Graphical representation of the results obtained by applying the TOPSIS technique of MCDM is shown below in figure 3.

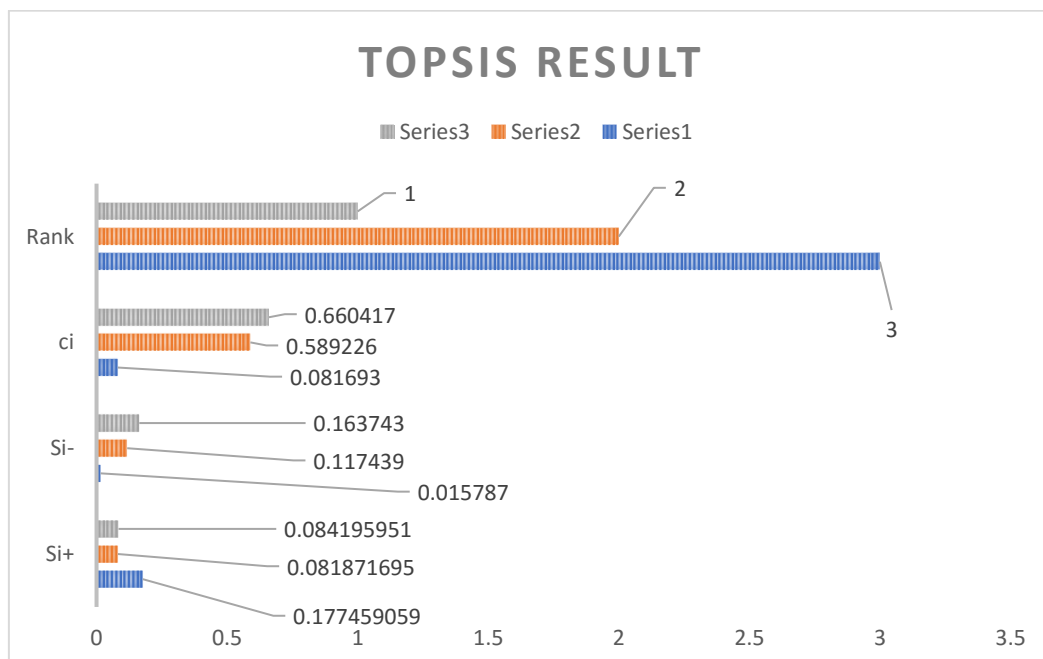


Figure 3: Graphical representation of results done by applying TOPSIS technique of MCDM

In figure 3, $P_1 = \text{series 1}$, $P_3 = \text{series 2}$ and $P_4 = \text{series 3}$ and the result shows that P_4 is the best alternative for the shortest time to reach the destination for the problem discussed above.

Conclusion

This paper introduces the Generalized Fuzzy TOPSIS by using an accuracy function for NHSS given in [4]. The proposed technique is used to solve a parking problem. Results show that the technique can be implemented to solve the MCDM problem with multiple-valued neutrosophic data in a vague and imprecise environment. In the future, the stability of the proposed technique is to be investigated and the proposed algorithm can be used in neutrosophic set (NS) theory to handle vague data, imprecise knowledge, incomplete information, and linguistic imprecision.

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