



# Generalized closed sets via neutrosophic topological spaces

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## Abstract

In this paper, we have introduced the notion of generalized closed sets in Neutrosophic topological spaces and studied some of their basic properties.

## Keywords

Neutrosophic Generalized Closed Sets.

## AMS Subject Classification

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## Contents

1	Introduction .....	50
2	Preliminaries .....	50
3	Neutrosophic generalized closed sets .....	52
	References .....	53

## 1. Introduction

In 1970, Levine [9] introduced the concept of generalized closed sets as a weaker form of closed sets in topological spaces. Zadeh [15] introduced the notion of fuzzy sets in the year 1965. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between 0 and 1. The concept of fuzzy topological spaces have been introduced and developed by Chang [2]. In 1983, Atanassov [1] introduced the concept of intuitionistic fuzzy set which was generalization of fuzzy set. In intuitionistic fuzzy set theory, the elements have the degree membership and non-membership value between 0 and 1. Later, In 1997 Coker [4] introduced the concept of intuitionistic fuzzy topological spaces, by using the notion of the intuitionistic fuzzy set.

Floretin Smarandache [5] introduced the concept of Neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminacy function and non membership function that are independently related. In 2012, Salama, Alblowi [11] introduced the concept of Neutrosophic topology. Neutrosophic topological spaces

are very natural generalizations of fuzzy topological spaces, allow more general functions to be members of fuzzy topology. In 2014, Salama, Smarandache and Valeri [10] introduced the concept of Neutrosophic closed sets and Neutrosophic continuous functions. Salama, Alblowi [11] introduced the concept of generalized Neutrosophic set and generalized Neutrosophic topological spaces. A generalized Neutrosophic set  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  can be identified as an ordered triple  $\langle \mu_A, \sigma_A, \gamma_A \rangle$  in  $]^{-}0, 1^{+}[$  on  $X$ , where the triple function satisfy the condition  $\mu_A(x) \cap \sigma_A(x) \cap \gamma_A(x) \leq 0.5$ .

Wadel and Smarandache [14] introduced the Neutrosophic open sets via Neutrosophic topological spaces. Ishwarya and Bageerathi[8]introduced the concept of Neutrosophic semi-open sets in Neutrosophic topological spaces. Dhavaseelan, Saied Jafari [3] introduced generalized Neutrosophic closed sets. In 2018, Shanthi, Chandrasekar and Safina [12] introduced the Neutrosophic generalized semi closed sets in Neutrosophic topological spaces. In this paper, we introduce the concept of generalized closed sets [9] in Neutrosophic topological spaces and studied some of their properties.

## 2. Preliminaries

In this section, we recollect some relevant basic preliminaries about Neutrosophic sets and its operations.

**Definition 2.1.** [10] Let  $X$  be a non-empty fixed set. A Neutrosophic set [NS for short]  $A$  is an object having the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  where  $\mu_A(x)$ ,  $\sigma_A(x)$  and

$\gamma_A(x)$  which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element  $x \in X$  to the set  $A$ .

**Remark 2.2.** [10] A Neutrosophic set

$A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  can be identified to an ordered triple

$$A = \langle \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle \text{ in } ]^{-}0, 1^{+}[ \text{ on } X.$$

**Remark 2.3.** [10] For the sake of simplicity, we shall use the symbol  $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$  for the neutrosophic set  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ .

**Example 2.4.** [10] Every intuitionistic fuzzy set  $A$  is a non-empty set in  $X$  is obviously on NS having the form  $A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) + \gamma_A(x), \gamma_A(x) \rangle : x \in X \}$ . Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic sets  $0_N$  and  $1_N$  in  $X$  as follows:

$0_N$  may be defined as:

- (0<sub>1</sub>)  $0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$
- (0<sub>2</sub>)  $0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$
- (0<sub>3</sub>)  $0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$
- (0<sub>4</sub>)  $0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$

$1_N$  may be defined as:

- (1<sub>1</sub>)  $1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$
- (1<sub>2</sub>)  $1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$
- (1<sub>3</sub>)  $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$
- (1<sub>4</sub>)  $1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$

**Definition 2.5.** [10] Let  $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$  be a NS on  $X$ , then the complement of the set  $A$  [ $C(A)$  for short] may be defined as three kinds of complements:

- (C<sub>1</sub>)  $C(A) = \{ \langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x) \rangle : x \in X \}$
- (C<sub>2</sub>)  $C(A) = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$
- (C<sub>3</sub>)  $C(A) = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$

**Definition 2.6.** [10] Let  $X$  be a non-empty set, and neutrosophic sets  $A$  and  $B$  in the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$ . Then we may consider two possible definitions for subsets ( $A \subseteq B$ ).

( $A \subseteq B$ ) may be defined as:

- ( $A \subseteq B$ )  $\Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x) \forall x \in X$
- ( $A \subseteq B$ )  $\Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x) \forall x \in X$

**Proposition 2.7.** [10] For any Neutrosophic set  $A$ , the following conditions holds:

- $0_N \subseteq A, 0_N \subseteq 0_N$
- $A \subseteq 1_N, 1_N \subseteq 1_N$

**Definition 2.8.** [10] Let  $X$  be a non-empty set, and

- $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ ,
- $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$  are NSs. Then  $A \cap B$  may be defined as:
- (I<sub>1</sub>)  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$

(I<sub>2</sub>)  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$   
 $A \cup B$  may be defined as:

- (U<sub>1</sub>)  $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle$
- (U<sub>2</sub>)  $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle$

We can easily generalize the operations of intersection and union in Definition 2.8 to arbitrary family of NSs as follows:

**Definition 2.9.** [10] Let  $\{A_j : j \in J\}$  be a arbitrary family of NSs in  $X$ , then

I.  $\cap A_j$  may be defined as:

- $\cap A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \wedge_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \gamma_{A_j}(x) \rangle$
- $\cap A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \vee_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \gamma_{A_j}(x) \rangle \cup A_j$  may be defined as:
- $\cup A_j = \langle x, \vee, \vee, \wedge \rangle \cup A_j = \langle x, \vee, \wedge, \wedge \rangle$

**Proposition 2.10.** [10] For all  $A$  and  $B$  are two neutrosophic sets then the following conditions are true:

$$C(A \cap B) = C(A) \cup C(B); C(A \cup B) = C(A) \cap C(B).$$

**Definition 2.11.** [10] A Neutrosophic topology [NT for short] is a non-empty set  $X$  is a family  $\tau_N$  of neutrosophic subsets in  $X$  satisfying the following axioms:

- (NT<sub>1</sub>)  $0_N, 1_N \in \tau_N$ ,
- (NT<sub>2</sub>)  $G_1 \cap G_2 \in \tau_N$  for any  $G_1, G_2 \in \tau_N$ ,
- (NT<sub>3</sub>)  $\cup G_i \in \tau_N$  for every  $\{G_i : i \in J\} \subseteq \tau_N$

Throughout this paper, the pair of  $(X, \tau_N)$  is called a neutrosophic topological space [NTS for short]. The elements of  $\tau_N$  are called neutrosophic open set [NOS for short].

A Neutrosophic set  $F$  is Neutrosophic closed if and only if  $C(F)$  is neutrosophic open.

**Example 2.12.** [10] Any fuzzy topological space  $(X, \tau_0)$  in the sense of Chang is obviously a NTS in the form  $\tau_N = \{A : \mu_A \in \tau_0\}$  wherever we identify a fuzzy set in  $X$  whose membership function is  $\mu_A$  with its counterpart.

The following is an example of Neutrosophic topological space.

**Example 2.13.** [10] Let  $x = \{X\}$  and  $A = \{ \langle x, 0.5, 0.5, 0.4 \rangle : x \in X \}$   $B = \{ \langle x, 0.4, 0.6, 0.8 \rangle : x \in X \}$   $C = \{ \langle x, 0.5, 0.6, 0.4 \rangle : x \in X \}$   $D = \{ \langle x, 0.4, 0.5, 0.8 \rangle : x \in X \}$  Then the family  $\tau_N = \{0_N, 1_N, A, B, C, D\}$  is called a neutrosophic topological space on  $X$ .

**Definition 2.14.** [10] The complement of  $A$  [ $C(A)$  for short] of NOS is called a neutrosophic closed set [NCS for short] in  $X$ .

Now, we define Neutrosophic closure and Neutrosophic interior operations in Neutrosophic topological spaces:

**Definition 2.15.** [10] Let  $(X, \tau_N)$  be NTS and  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$  be a NS in  $X$ . Then the neutrosophic closure and neutrosophic interior of  $A$  are defined by  $NCl(A) = \cap \{K : K \text{ is a NCS in } X \text{ and } A \subseteq K$   
 $NInt(A) = \{G : G \text{ is a NOS in } X \text{ and } G \subseteq A\}$  It can be also shown that  $NCl(A)$  is NCS and  $NInt(A)$  is a NOS in  $X$ .  
 $A$  is NOS if and only if  $A = NInt(A)$   
 $A$  is NCS if and only if  $A = NCl(A)$



**Proposition 2.16.** [10] For any Neutrosophic set  $A$  in  $(X, \tau_N)$  we have

- a.  $NCl(C(A)) = C(NInt(A))$
- b.  $NInt(C(A)) = C(NCl(A))$

**Proposition 2.17.** [10] Let  $(X, \tau_N)$  be a NTS and  $A, B$  be two neutrosophic sets in  $X$ . Then the following properties are holds:

- a)  $NInt(A) \subseteq A$
- b)  $A \subseteq NCl(A)$
- c)  $A \subseteq B \Rightarrow NInt(A) \subseteq NInt(B)$
- d)  $A \subseteq B \Rightarrow NCl(A) \subseteq NCl(B)$
- e)  $NInt(NInt(A)) = NInt(A)$
- f)  $NCl(NCl(A)) = NCl(A)$
- g)  $NInt(A \cap B) = NInt(A) \cap NInt(B)$
- h)  $NCl(A \cup B) = NCl(A) \cup NCl(B)$
- i)  $NInt(0_N) = 0_N$
- j)  $NInt(1_N) = 1_N$
- k)  $NCl(0_N) = 0_N$
- l)  $NCl(1_N) = 1_N$
- m)  $A \subseteq B \Rightarrow C(A) \subseteq C(B)$
- n)  $NCl(A \cap B) \subseteq NCl(A) \cap NCl(B)$
- o)  $NInt(A \cup B) \subseteq NInt(A) \cup NInt(B)$

**Definition 2.18.** [14] Let  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  be a neutrosophic open sets and  $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$  be a neutrosophic set on a neutrosophic topological space  $(X, \tau_N)$  then

- a.  $A$  is called neutrosophic regular open iff  $A = NInt(NCl(A))$ .
- b. If  $B \in NCS(X)$  then  $B$  in called neutrosophic regular close iff  $A = NCl(NInt(A))$ .

**Definition 2.19.** [14] A neutrosophic set  $A$  in a neutrosophic topological space  $(X, \tau_N)$  is called

1. Neutrosophic semi-open set (NSOS) if  $A \subseteq NCl(NInt(A))$ .
2. Neutrosophic pre-open set (NPOS) if  $A \subseteq NInt(NCl(A))$ .
3. Neutrosophic  $\alpha$ -open set ( $N\alpha OS$ ) if  $A \subseteq NInt(NCl(NInt(A)))$ .
4. Neutrosophic  $\beta$ -open set ( $N\beta OS$ ) if  $A \subseteq NCl(NInt(NCl(A)))$ .

An (NSs)  $A$  is called neutrosophic semi-closed set, neutrosophic  $\alpha$ -closed set, Neutrosophic pre-closed set and Neutrosophic regular closed set respectively (NSCS,  $N\alpha CS$ , NPCS and NRCS, resp.), if the complement of  $A$  is a NSOS,  $N\alpha OS$ , NPOS and NROS respectively.

**Definition 2.20.** [8] Let  $A$  be a subset of a neutrosophic spaces  $(X, \tau_N)$  is called neutrosophic generalized semi closed (Ngs-closed) if neutrosophic semi-closed  $A \subseteq G$ , whenever  $A \subseteq G$  and  $G$  is NOS.

### 3. Neutrosophic generalized closed sets

In this section, we introduce the new concept namely Neutrosophic generalized closed sets in Neutrosophic topological spaces.

**Definition 3.1.** Let  $(X, \tau_N)$  be a neutrosophic topological space. A subset  $A$  of  $(X, \tau_N)$  is called Neutrosophic generalized closed set (neutrosophic-g-closed) if  $Ncl(A) \subseteq G$  whenever  $A \subseteq G$  and  $G$  is neutrosophic open set (NOS). Complement of neutrosophic-g-closed set is called the neutrosophic-g-open set.

**Example 3.2.** Let  $X = \{a, b, c\}$  with  $\tau_N = \{0_N, 1_N, A, B\}$  where  $A = \langle (0.5, 0.5, 0.4), (0.7, 0.5, 0.5), (0.4, 0.5, 0.5) \rangle$ ,  $B = \langle (0.3, 0.4, 0.4), (0.4, 0.5, 0.5), (0.3, 0.4, 0.6) \rangle$ . Then  $(X, \tau_N)$  is a neutrosophic topological space. The closed sets of  $(X, \tau_N)$  are  $A' = \langle (0.4, 0.5, 0.5), (0.5, 0.5, 0.7), (0.5, 0.5, 0.4) \rangle$ ,  $B' = \langle (0.4, 0.6, 0.3), (0.5, 0.5, 0.4), (0.6, 0.6, 0.3) \rangle$ . Consider the Neutrosophic set  $C = \langle (0.4, 0.6, 0.5), (0.4, 0.3, 0.5), (0.5, 0.6, 0.4) \rangle$  in  $(X, \tau_N)$ . Here  $C$  is neutrosophic-g-closed set in  $(X, \tau_N)$ .

**Theorem 3.3.** Every Neutrosophic closed set is a Neutrosophic generalized closed set in  $(X, \tau_N)$ .

*Proof.* Let  $A \subseteq G$ , where  $G$  neutrosophic open set in  $(X, \tau_N)$ . Since  $A$  is neutrosophic closed set,  $NCl(A) \subseteq A$  [Since  $A = NCl(A)$ ]. Therefore  $NCl(A) \subseteq A \subseteq G$ . Hence  $A$  is a neutrosophic-g-closed set in  $(X, \tau_N)$ .  $\square$

**Remark 3.4.** The converse of the above theorem need not be true as seen in the following example.

**Example 3.5.** Let  $X = \{a, b\}$  with  $\tau_N = \{0_N, 1_N, A, B\}$  and where  $A = \langle (0.4, 0.5, 0.5), (0.2, 0.4, 0.6) \rangle$ ,  $B = \langle (0.7, 0.5, 0.3), (0.3, 0.4, 0.5) \rangle$ .

Then  $(X, \tau_N)$  is a neutrosophic topological space. The closed sets of  $(X, \tau_N)$  are  $A' = \langle (0.5, 0.5, 0.4), (0.6, 0.6, 0.2) \rangle$ ,  $B' = \langle (0.3, 0.5, 0.7), (0.5, 0.6, 0.3) \rangle$ . Consider the Neutrosophic set  $C = \langle 0.6, 0.5, 0.6 \rangle, \langle 0.4, 0.3, 0.7 \rangle$ .  $C$  is neutrosophic-g-closed set, but  $C$  is not NCS, (Since  $NCl(C) \neq C$ ).

**Theorem 3.6.** If  $A$  and  $B$  are neutrosophic-g-closed sets in  $(X, \tau_N)$  then  $A \cup B$  is neutrosophic-g-closed set in  $(X, \tau_N)$ .

*Proof.* Let  $A$  and  $B$  are neutrosophic-g-closed sets in  $(X, \tau_N)$ . Then  $NCl(A) \subseteq G$  whenever  $A \subseteq G$  and  $G$  is NOS in  $(X, \tau_N)$  and  $NCl(B) \subseteq G$  whenever  $B \subseteq G$  and  $G$  is NOS in  $X$ . Since  $A$  and  $B$  are subsets of  $G$ ,  $A \cup B$  is a subset of  $G$  and  $G$  is neutrosophic open set. Then  $NCl(A \cup B) = NCl(A) \cup NCl(B)$  [by proposition 2.17(h)],  $NCl(A \cup B) \subseteq G$ . Therefore  $A \cup B$  is neutrosophic-g-closed set in  $(X, \tau_N)$ .  $\square$

**Theorem 3.7.** If  $A$  and  $B$  are neutrosophic-g-closed sets in  $(X, \tau_N)$ , then  $Ncl(A \cap B) \subseteq Ncl(A) \cap Ncl(B)$ .



*Proof.* Let  $A$  and  $B$  are neutrosophic- $g$ -closed sets in  $(X, \tau_N)$ . Then  $NCl(A) \subseteq G$  whenever  $A \subseteq G$  and  $G$  is  $NOS$  in  $(X, \tau_N)$  and  $NCl(B) \subseteq G$  whenever  $B \subseteq G$  and  $G$  is  $NOS$  in  $(X, \tau_N)$ . Since  $A$  and  $B$  are subsets of  $G$ ,  $A \cap B$  is a subset of  $G$  and  $G$  is  $NOS$ . Since  $A \cap B \subseteq A$  and  $A \cap B \subseteq B$ , we know that, if  $A \subseteq B$  then  $Ncl(A) \subseteq Ncl(B)$  [10]. Therefore  $Ncl(A \cap B) \subseteq Ncl(A)$  and  $Ncl(A \cap B) \subseteq Ncl(B)$ , which implies that  $Ncl(A \cap B) \subseteq Ncl(A) \cap Ncl(B)$ . Hence proved.  $\square$

**Remark 3.8.** The intersection of two neutrosophic- $g$ -closed sets need not be a neutrosophic- $g$ -closed set as seen from the following example.

**Example 3.9.** Let  $X = \{a, b, c\}$  with  $\tau_N = \{0_N, 1_N, A, B, C\}$  where

$$A = \langle (0.4, 0.5, 0.4), (0.5, 0.5, 0.6), (0.7, 0.4, 0.3) \rangle,$$

$$B = \langle (0.3, 0.4, 0.5), (0.5, 0.4, 0.8), (0.6, 0.3, 0.4) \rangle,$$

$$C = \langle (0.4, 0.5, 0.4), (0.5, 0.5, 0.8), (0.7, 0.5, 0.3) \rangle.$$

Then  $(X, \tau_N)$  is a neutrosophic topological space. The closed sets are  $A' = \langle (0.4, 0.5, 0.4), (0.6, 0.5, 0.5), (0.3, 0.6, 0.7) \rangle,$

$$B' = \langle (0.5, 0.6, 0.3), (0.8, 0.6, 0.8), (0.4, 0.7, 0.6) \rangle,$$

$$C' = \langle (0.4, 0.5, 0.4), (0.8, 0.5, 0.5), (0.3, 0.5, 0.7) \rangle.$$

Consider the neutrosophic- $g$ -closed sets

$$D = \langle (0.5, 0.6, 0.7), (0.5, 0.5, 0.5), (0.6, 0.4, 0.6) \rangle,$$

$$\text{and } E = \langle (0.4, 0.3, 0.8), (0.2, 0.6, 0.7), (0.5, 0.4, 0.7) \rangle,$$

$$\text{then } D \cap E = \langle (0.4, 0.3, 0.8), (0.2, 0.5, 0.7), (0.5, 0.4, 0.7) \rangle,$$

is not a neutrosophic- $g$ -closed set.

**Theorem 3.10.** If  $A$  is neutrosophic- $g$ -closed set in  $(X, \tau_N)$  and  $A \subseteq B \subseteq NCl(A)$ , then  $B$  is neutrosophic- $g$ -closed set in  $(X, \tau_N)$ .

*Proof.* Let  $B \subseteq G$  where  $G$  is  $NOS$  in  $(X, \tau_N)$ . Then  $A \subseteq B$  implies  $A \subseteq G$ . Since  $A$  is neutrosophic- $g$ -closed,  $NCl(A) \subseteq G$ . Also  $A \subseteq NCl(B)$  implies  $NCl(B) \subseteq NCl(A)$ . Thus  $NCl(B) \subseteq G$  and so  $B$  is neutrosophic- $g$ -closed set in  $(X, \tau_N)$ .  $\square$

**Theorem 3.11.** An neutrosophic- $g$ -closed set  $A$  is neutrosophic closed set iff  $NCl(A) - A$  is neutrosophic closed set.

*Proof.* Assume that,  $A$  is  $NCS$ , then  $NCl(A) = A$  and so  $NCl(A) - A = 0_N$  which is  $NCS[x]$ . Conversely, suppose  $NCl(A) - A$  is  $NCS$ . Then  $NCl(A) - A = 0_N$ , that is  $NCl(A) = A$ . Therefore  $A$  is  $NCS$ . Hence proved.  $\square$

**Theorem 3.12.** Suppose that  $A \subseteq B \subseteq X$ ,  $B$  is an neutrosophic- $g$ -closed set relative to  $A$  and that  $A$  is an neutrosophic- $g$ -closed subset of  $X$ . Then  $B$  is neutrosophic- $g$ -closed set relative to  $X$ .

*Proof.* Let  $B \subseteq G$  and suppose that  $G$  is  $NOS$  in  $X$ . Then  $B \subseteq A \cap G$ . Therefore  $NCl(B) \subseteq A \cap G$ . It follows that  $A \cap NCl(B) \subseteq A \cap G$  and  $A \subseteq G \cup NCl(B)$ . Since  $A$  is neutrosophic- $g$ -closed in  $X$ , we have  $NCl(A) \subseteq G \cup NCl(B)$ . Therefore  $NCl(B) \subseteq NCl(A) \subseteq G \cup NCl(B)$  and  $NCl(B) \subseteq G$ . Then  $B$  is neutrosophic- $g$ -closed relative to  $B$  is an neutrosophic- $g$ -closed set relative to  $G$ .  $\square$

**Corollary 3.13.** Let  $A$  be a neutrosophic- $g$ -closed set and suppose that  $F$  is a  $NCS$ . Then  $A \cap F$  is an neutrosophic- $g$ -closed set.

**Theorem 3.14.** If  $A$  is neutrosophic- $g$ -closed set in  $X$ , then  $A$  is neutrosophic- $gs$ -closed set in  $X$ .

*Proof.* Let  $A$  be a neutrosophic- $g$ -closed set in  $X$ . Therefore  $NCl(A) \subseteq G$  and  $A \subseteq G$  whenever  $G$  is  $NOS$  in  $X$ .  $NSCl(A) \subseteq G$ . Then  $NSCl(A) \subseteq NCl(A)$ ,  $NSCl(A) \subseteq G$ . Therefore  $A$  is neutrosophic- $gs$ -closed in  $X$ .  $\square$

**Remark 3.15.** The converse of the above theorem need not be true as seen from the following example.

**Example 3.16.** Let  $X = \{a\}$  with  $\tau_N = \{0_N, 1_N, A, B, C, D\}$  where  $A = \langle 0.6, 0.7, 0.9 \rangle$

$$B = \langle 0.5, 0.4, 0.7 \rangle$$

$$C = \langle 0.6, 0.7, 0.7 \rangle$$

$$D = \langle 0.5, 0.4, 0.9 \rangle$$

Then  $(X, \tau_N)$  is a neutrosophic topological space. Consider the neutrosophic set  $E = \langle 0.4, 0.3, 0.7 \rangle$ .  $E$  is neutrosophic- $gs$ -closed set in  $(X, \tau_N)$ , but  $E$  is not a neutrosophic- $g$ -closed set  $(X, \tau_N)$ , since  $NCl(E) \not\subseteq G$ .

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