Generalized Single Valued Triangular Neutrosophic Numbers and Aggregation Operators for Application to Multi-attribute Group Decision Making

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ABSTRACT

In this study we define the generalizing single valued triangular neutrosophic number. In addition, single valued neutrosophic numbers are transformed into single valued triangular neutrosophic numbers according to the values of truth, indeterminacy and falsity. Furthermore, we extended the Hamming distance given for triangular intuitionistic fuzzy numbers to single valued triangular neutrosophic numbers. We have defined new score functions based on the Hamming distance. We then extended some operators given for intuitionistic fuzzy numbers to single valued triangular neutrosophic numbers. Finally, we developed a new solution to multi-attribute group decision making problems for single valued neutrosophic numbers with operators and scoring functions and we checked the suitability of our new method by comparing the results we obtained with previously obtained results. We have also mentioned for the first time that there is a solution for multi-attribute group decision making problems for single valued triangular neutrosophic numbers.

Keywords: Hamming distance, single valued neutrosophic number, generalized single valued neutrosophic number, multi-attribute group decision making

1. INTRODUCTION

There are many uncertainties in daily life. However, classical mathematical logic is insufficient to account for these uncertainties. In order to explain these uncertainties mathematically and to use them in practice, Zadeh (1965) first proposed a fuzzy logic theory. Although fuzzy logic is used in many field applications, the lack of membership is not explained because it is only a membership function. Then Atanassov (1986) introduced the theory of intuitionistic fuzzy logic. In this theory, he states membership, non-membership and indeterminacy, and has been used in many fields and applications. Later, Li (2010) defined triangular intuitionistic fuzzy numbers. However, in the intuitionistic fuzzy logic, membership, non-membership, and indeterminacy are all completely dependent in each other. Finally, Smarandache (1998 and 2016) proposed the neutrosophic set theory, which is the more general form of intuitionistic fuzzy logic. Many studies have been done on this theory and have been used in many field applications. In this theory, the values of truth, indeterminacy and falsity of a situation are considered and these three values are defined completely independently of each other Smarandache, Wang, Zhang, and Sunderraman (2010) defined single valued neutrosophic sets. Subas (2015) defined single valued triangular neutrosophic numbers is a special form of single valued neutrosophic numbers. Many uncertainties and complex situations arise in decision-making applications. It is impossible to come up with these uncertainties and complexities, especially with known numbers. For example, in multi-attribute decision making (MADM), multiple objects are evaluated according to more than one property and there is a choice of the most suitable one. Particularly in multi-attribute group decision making (MAGDM), the most appropriate object selection is made according to the data received from more than one decision maker. Multi - attribute decision making group and multi-attribute decision making problems have been found by many researchers using various methods using intuitionistic fuzzy numbers. For example; Wan and Dong (2015) studied trapezoidal intuitionistic fuzzy numbers and application to multi attribute group decision making. Wan, Wang, Li and Dong (2016) studied triangular intuitionistic fuzzy numbers and application to multi attribute group decision making. Biswas, Pramanik, and Giri (2016) have studied trapezoidal fuzzy neutrosophic numbers and its application to multi-attribute decision making (MADM) and triangular fuzzy neutrosophic set and its application to multi-attribute decision making (MADM).

However, these methods and solutions are not suitable for neutrosophic sets and neutrosophic numbers. Therefore, many researchers have tried to find solutions to multi-attribute group decision making and multi-attribute decision making problems using neutrosophic sets and neutrosophic numbers. Recently, Liu and Luo (2017) have proposed multi-attribute group decision making problems using "power aggregation operators of simplifield neutrosophic sets"; Sahin, Uluçay, Kargın and Ecemiş (2017) studied centroid single valued triangular neutrosophic numbers and their applications in multi-attribute decision making; Sahin and Liu (2017) used multi-criteria decision making problems using exponential operations of simplest neutrosophic numbers; Liu and Li have produced solutions to multi-criteria decision making problems with "some normal neutrosophic Bonferroni mean operators" (2017). Smarandache (2016) have produced neutrosophic overset, neutrosophic underset, and neutrosophic offset. Biswas, Pramanik, and Giri (2016) have studied single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making (MADM). Ye (2015) have studied multi-attribute decision making (MADM).

Subas (2015) defined $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ as a positive single valued triangular neutrosophic number for $a_1, b_1, c_1 \in \mathbb{R}^+$ or a negative single valued triangular neutrosophic number for $a_1, b_1, c_1 \in \mathbb{R}^-$. However, the condition $a_1, b_1, c_1 \in \mathbb{R}$ -{0} has not been defined. This narrows the applications of single valued triangular neutrosophic numbers. In this study we first define the condition of $a_1, b_1, c_1 \in \mathbb{R}$ for single valued triangular neutrosophic numbers and gave basic operations on these conditions. These basic operations we have given also include operations where $a_1, b_1, c_1 \in \mathbb{R}^-$ and $a_1, b_1, c_1 \in \mathbb{R}^+$. Thus, by generalizing single valued triangular neutrosophic numbers, we made it more useful. Then, single valued neutrosophic numbers were converted to single valued triangular neutrosophic numbers. Thus, we made single valued neutrosophic numbers more useful by carrying single valued triangular neutrosophic numbers, which have rich application fields. We then extended the Hamming distance for triangular intuitionistic fuzzy numbers to single valued triangular neutrosophic numbers and showed some properties. Besides, we defined the scoring and certainty functions for the single-valued neutrosophic numbers and for the single valued triangular neutrosophic numbers based on the Hamming distance according to the truth, indeterminacy and falsity values. We compared the results of the score and certainty functions we obtained with the score and certainty functions. We also made some operators for triangular intuitionistic fuzzy numbers available for single valued triangular neutrosophic numbers and showed some properties of these operators. We mentioned similarities and differences with the operators. Finally, we have found a new solution to the multi-attribute group decision making problems by using the transformation of single valued neutrosophic numbers, new scoring functions and using the operators we have obtained. Since the transformations and the scoring functions are separate according to the values of truth, indeterminacy and falsity, we obtained results separately for each of the three values for multi-attribute group decision making problems. We compared our result with the result of a multi-attribute group decision making problem for single valued neutrosophic numbers. We have checked the applicability of the method we have achieved.

In this study, we gave some definitions of triangular intuitional fuzzy numbers and related definitions about neutrosophic sets, single valued neutrosophic sets and numbers, single valued triangular neutrosophic numbers, and some related definitions in section 2. In Section 3, we generalized the single valued triangular neutrosophic numbers to make them more usable and described the basic operations. In Section 3, we gave transformations for single valued neutrosophic numbers based on their truth, indeterminacy and falsity values. In section 4, we made the Hamming distance for triangular intuitionistic fuzzy numbers available for single valued triangular neutrosophic numbers and showed some properties.

In addition, we have separately defined the score and certainty functions according to the values of truth, indeterminacy and falsity depending on the generalized Hamming distance and compared with the score and certainty functions given before. In Section 5, we made some operators for triangular intuitionistic fuzzy numbers available with single valued triangular neutrosophic numbers, and we showed some properties of these operators and discussed the similarities and differences with the previously given operators . In Section 6, we gave a new method for solving multi-attribute group decision making problems for single valued neutrosophic numbers using the transform functions and operators that we have achieved in this work. In Section 7, we looked at the applicability of our method by comparing the result of a previous multi-attribute group decision making problem with the result of our method. Finally, in Section 8 we briefly discussed the results of our work.

2. PRELIMINARIES

Definition 2.1: A triangular intuitionistic fuzzy number $\tilde{a} = \langle(\underline{a}, a, \overline{a});\ w_{\tilde{a}}, u_{\tilde{a}},\rangle$ is a special intuitionistic fuzzy set on the real number set R, whose truth-membership and falsity-membership functions are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-\underline{a})w_{\tilde{a}}/(a-\underline{a}) & (\underline{a} \leq x < a) \\ w_{\tilde{a}} & (x = a) \\ (\overline{a}-x)w_{\tilde{a}}/(\overline{a}-a) & (a < x \leq \overline{a}) \\ 0 & otherwise \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} (a-x+u_{\tilde{a}}(x-\underline{a}))/(a-\underline{a}) & (\underline{a} \leq x < a) \\ u_{\tilde{a}} & (x = a) \\ (x-a+u_{\tilde{a}}(\overline{a}-x))/(\overline{a}-a) & (a < x \leq \overline{a}) \\ 1 & otherwise \end{cases}$$

respectively. (Li, 2010)

Definition 2.2: Let $\tilde{a}_i = \langle(\underline{a_i}, a_i, \overline{a_i});\ w_{\tilde{a}_i}, u_{\tilde{a}_i},\rangle$ (i=1,2) be two triangular intuitionistic fuzzy numbers. The Hamming distance between $\tilde{a}_1$ and $\tilde{a}_2$ is

$$d_n(\tilde{a}_1,\tilde{a}_2)=\frac{1}{16}[\left|(1+w_{\tilde{a}_1}-u_{\tilde{a}_1})\underline{a_1}-(1+w_{\tilde{a}_2}-u_{\tilde{a}_2})\underline{a_1}\right|+ \left|(1+w_{\tilde{a}_1}-u_{\tilde{a}_1})a_1-(1+w_{\tilde{a}_2}-u_{\tilde{a}_2})a_2\right|+\left|(1+w_{\tilde{a}_1}-u_{\tilde{a}_1})\overline{a_1}-(1+w_{\tilde{a}_2}-u_{\tilde{a}_2})\overline{a_2}\right|]$$

(Wan, Wang, Li and Dang, 2016)

Definition 2.3: Let $\tilde{a}_i = \langle(\underline{a_i}, a_i, \overline{a_i});\ w_{\tilde{a}_i}, u_{\tilde{a}_i}\rangle$ (i = 1,2,3,…,n) be a collection of triangular intuitionistic fuzzy numbers. Then triangular intuitionistic fuzzy generalized ordered weighted averaging operator is defined as;

TIFGOWA : $\mho^n \to \mho$, TIFGOWA $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = g^{-1}\left(\sum_{j=1}^{n} w_i g(\tilde{a}_{(i)})\right)$

Where g is a continuous strictly monotone increasing function, $w = (w_1, w_2, \ldots, w_n)^T$ is a weight vector associated with the TIFGOWA operator, with $w_j \geq 0$, j = 1,2,3,…,n and $\sum_{j=1}^{n} w_j = 1$ and ((1),(2), … , (n)) is a permutation of (1,2, …,n) such that $\tilde{a}_{(i)} \geq \tilde{a}_{(i+1)}$ for all i. (Wan, Wang, Li and Dang, 2016)

Definition 2.4: Let $\tilde{a}_i = \langle(\underline{a_i}, a_i, \overline{a_i});\ w_{\tilde{a}_i}, u_{\tilde{a}_i}\rangle$ (i = 1,2,3,…,n) be a collection of triangular intuitionistic fuzzy numbers. Then triangular intuitionistic fuzzy generalized hybrid weighted averaging operator is defined as;

TIFGHWA : $\mho^n \to \mho$, TIFGHWA $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = g^{-1}\left(\sum_{j=1}^{n} w_i g(\tilde{b}_{(i)})\right)$

Where g is a continuous strictly monotone increasing function, $w = (w_1, w_2, \ldots, w_n)^T$ is a weight vector associated with the TIFGHWA operator, with $w_i \geq 0$, i = 1,2,3,…,n $\sum_{i=1}^{n} w_i = 1$, ω = $(\omega_1, \omega_2, \ldots, \omega_n)^T$ is a weight vector of $\tilde{a}_i$ and $\tilde{b}_{(i)} = \omega_1 . \tilde{a}_i$. (Wan, Wang, Li and Dang, 2016)

Definition 2.5: Let U be an universe of discourse then the neutrosophic set A is on object having the farm A={ (x:$T_{A(x)}$, $I_{A(x)}$,$F_{A(x)}$> , x∈U} where the functions T,I,F:U →]$^-$0,1$^+$[respectively the degree of membership, the degree of indeterminacy and degree of non-membership of the element x∈U to the set A with the condition.

$0^- \leq T_{A(x)}+I_{A(x)}+F_{A(x)} \leq 3^+$. (Smarandache, 2016)

Definition2.6: Let U be an universe of discourse then the single valued neutrosophic set A is on object having the form A={ (x:$T_{A(x)}$, $I_{A(x)}$,$F_{A(x)}$> , x∈U} where the functions T,I,F:U →[0,1]respectively the degree of membership, the degree of indeterminacy and degree of non-membership of the element x∈U to the set A with the condition.

$$0 \leq T_{A(x)}+I_{A(x)}+F_{A(x)} \leq 3$$

For convenience, we can simply use x = (*T*, *I*, *F*) to represent an element x in single valued neutrosophic numbers and the element x can be called a single valued neutrosophic number. (Wang, Smarandache, Zhang, Sunderraman, 2010)

Definition 2.7: Let x = (*T*, *I*, *F*) be a single valued triangular neutrosophic number and then

1) sc(x)=T+1-I+1-F;

2) ac(x)=T-F;

Where sc(x) represents the score function of the single valued neutrosophic number and ac(x) represent certainty function of the single valued neutrosophic number. (Liu, Chu, Li and Chen, 2014)

Definition 2.8: Let x = (T_1, I_1, F_1) and y = (T_2, I_2, F_2) be two single valued neutrosophic numbers, the comparison approach can be defined as follows.

1) If sc(x)>sc(y), then x is greater than y and denoted x $\succ$y.

2) If sc(x)=sc(y) and ac(x)>ac(y), then x is greater than y and denoted x $\succ$y.

3) If sc(x)=sc(y) and ac(x)=ac(y), then x is equal to y and denoted by x$\sim$y.

(Liu, Chu, Li and Chen, 2014)

Definition2.9: Let $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0, 1]$. A single valued triangular neutrosophic number $\tilde{a}$ = $\langle(a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ is a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a_1)w_{\tilde{a}}/(b_1-a_1) & (a_1 \le x < b_1) \\ w_{\tilde{a}} & (x = b_1) \\ (c_1-x)w_{\tilde{a}}/(c_1-b_1) & (b_1 < x \le c_1) \\ 0 & otherwise \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} (b_1-x+u_{\tilde{a}}(x-a_1))/(b_1-a_1) & (a_1 \le x < b_1) \\ u_{\tilde{a}} & (x = b_1) \\ (x-b_1+u_{\tilde{a}}(c_1-x))/(c_1-b_1) & (b_1 < x \le c_1) \\ 1 & otherwise \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} (b_1-x+y_{\tilde{a}}(x-a_1))/(b_1-a_1) & (a_1 \le x < b_1) \\ y_{\tilde{a}} & (x = b_1) \\ (x-b_1+y_{\tilde{a}}(c_1-x))/(c_1-b_1) & (b_1 < x \le c_1) \\ 1 & otherwise \end{cases}$$

respectively.

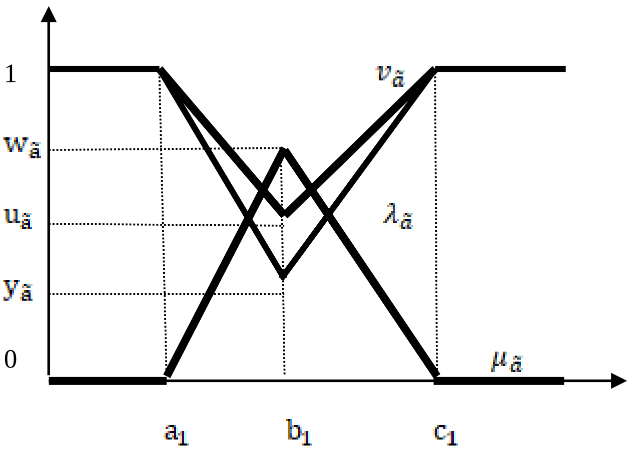


Fig. 1. ($\tilde{a}$ = $\langle(a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ single valued triangular neutrosophic number)

If $a_1 \geq 0$ and at least $c_1 > 0$, then $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a positive triangular neutrosophic number, denoted by $\tilde{a} > 0$. Likewise, If $c_1 \leq 0$ and at least $a_1 < 0$, then $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a negative triangular neutrosophic number, denoted by $\tilde{a} < 0$.

A triangular neutrosophic number $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ may express an ill-known quantity about a, which is approximately equal to a. (Subas, 2017)

Definition 2.10: Let $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (a_2, b_2, c_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$, be two single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any real number. Then,

1. $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

2. $\tilde{a} - \tilde{b} = \langle (a_1 - c_2, b_1 - b_2, c_1 - a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

3. $\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 a_2, b_1 b_2, c_1 c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 > 0, c_2 > 0) \\ \langle (a_1 c_2, b_1 b_2, c_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 > 0) \\ \langle (c_1 c_2, b_1 b_2, a_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 < 0) \end{cases}$

4. $\tilde{a}/\tilde{b} = \begin{cases} \langle (a_1/c_2, b_1/b_2, c_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 > 0, c_2 > 0) \\ \langle (c_1/c_2, b_1/b_2, a_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 > 0) \\ \langle (c_1/a_2, b_1/b_2, a_1/c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & (c_1 < 0, c_2 < 0) \end{cases}$

5. $\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma > 0) \\ \langle (\gamma c_1, \gamma b_1, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & (\gamma < 0) \end{cases}$

6. $\tilde{a}^{-1} = \langle (1/c_1, 1/b_1, 1/a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle \ (\tilde{a} \neq 0)$. (Subas, 2017)

Definition 2.11: We defined a method to compare any two single valued triangular neutrosophic numbers which is based on the score function and the certainty function. Let $\tilde{a}_1 = \langle (a_1, b_1, c_1); w_{\tilde{a}_1}, u_{\tilde{a}_1}, y_{\tilde{a}_1} \rangle$ be any single valued triangular neutrosophic number, then

$$S(\tilde{a}_1) = \frac{1}{8}[a_1 + b_1 + c_1] \mathrm{x} (2 + w_{\tilde{a}_1} - u_{\tilde{a}_1} - y_{\tilde{a}_1})$$

and

$$A(\tilde{a}_1) = \frac{1}{8}[a_1 + b_1 + c_1] \mathrm{x} (2 + w_{\tilde{a}_1} - u_{\tilde{a}_1} + y_{\tilde{a}_1})$$

is called the score and certainty degrees of $\tilde{a}_1$, respectively. (Subas, 2017)

Definition2.12: Let $\tilde{a}_1$ and $\tilde{a}_2$ be two single valued triangular neutrosophic numbers,

1. If $S(\tilde{a}_1) < S(\tilde{a}_2)$, then $\tilde{a}_1$ is smaller then $\tilde{a}_2$, denoted by $\tilde{a}_1 < \tilde{a}_2$.

2. If $S(\tilde{a}_1) = S(\tilde{a}_2)$;

(a) If $A(\tilde{a}_1) < A(\tilde{a}_2)$, then $\tilde{a}_1$ is smaller then $\tilde{a}_2$, denoted by $\tilde{a}_1 < \tilde{a}_2$.

(b) If $A(\tilde{a}_1) = A(\tilde{a}_2)$, then $\tilde{a}_1$ and $\tilde{a}_2$ are the same, denoted by $\tilde{a}_1 = \tilde{a}_2$.

(Subas, 2017)

Definition2.13: Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ (j = 1,2,3,…,n) be a collection of single valued triangular neutrosophic numbers. Then single valued triangular neutrosophic weight averaging operator (SVTNWAO) is defined as;

SVTNWAO: $\overline{N}_R{}^n \to \overline{N}_R$, SVTNWAO$(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{a}_j$

where $w = (w_1, w_2, \ldots, w_n)^T$ is a weight vector associated with the SVTNWAO operator, with $w_j \geq 0$, j = 1,2,3,…,n and $\sum_{j=1}^{n} w_j = 1$. (Subas, 2017)

Definition2.14: Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ (j = 1,2,3,…,n) be a collection of single valued triangular neutrosophic numbers and $w = (w_1, w_2, \ldots, w_n)^T$ is a weight vector associated with $w_j \geq 0$, and $\sum_{j=1}^{n} w_j = 1$.Then single valued triangular neutrosophic ordered averaging operator (SVTNWAO) is defined as;

SVTNOAO: $\overline{N}_R{}^n \to \overline{N}_R$, SVTNOAO$(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{b}_j$

where $\tilde{a}_k = \langle (a_k, b_k, c_k); w_{\tilde{a}_k}, u_{\tilde{a}_k}, y_{\tilde{a}_k} \rangle$, k∈{1,2,3, …,n} is the single valued triangular neutrosophic number obtained by using the score and certainty function and For $\tilde{b}_j$; $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ is the maximum value of K . (Subas, 2017)

3. GENERALIZED SINGLE VALUED TRIANGULAR NEUTROSOPHIC NUMBERS

In this section we will generalize single valued triangular neutrosophic numbers to make them more usable. Because definition 2.9 for a single valued triangular neutrosophic number $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$; The values a_1, b_1, c_1 must either be negative real numbers or positive real numbers. However, some of these values are not defined as negative real numbers of some of them are positive real numbers. This situation narrows the field of use of single valued triangular neutrosophic numbers. We will abolish this limited situation with definitions given in this section.

Definition 3.1: Let $w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \in [0, 1]$ and $a_1, b_1, c_1 \in \mathbb{R}$-{0}. A generalized single valued triangular neutrosophic number $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as follows:

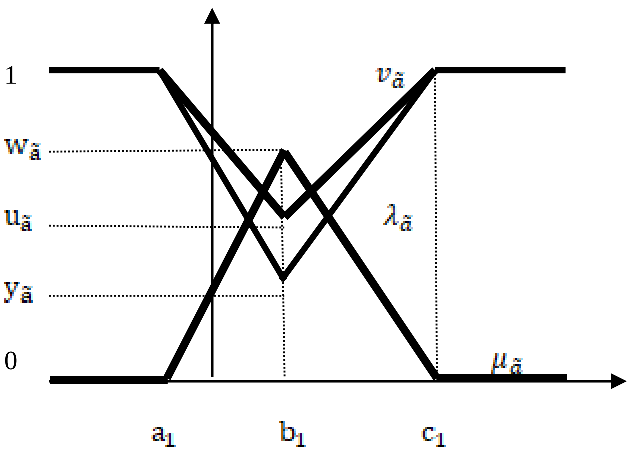
$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a_1)w_{\tilde{a}}/(b_1-a_1) & (a_1 \le x < b_1) \\ w_{\tilde{a}} & (x = b_1) \\ (c_1-x)w_{\tilde{a}}/(c_1-b_1) & (b_1 < x \le c_1) \\ 0 & otherwise \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} (b_1-x+u_{\tilde{a}}(x-a_1))/(b_1-a_1) & (a_1 \le x < b_1) \\ u_{\tilde{a}} & (x = b_1) \\ (x-b_1+u_{\tilde{a}}(c_1-x))/(c_1-b_1) & (b_1 < x \le c_1) \\ 1 & otherwise \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} (b_1-x+y_{\tilde{a}}(x-a_1))/(b_1-a_1) & (a_1 \le x < b_1) \\ y_{\tilde{a}} & (x = b_1) \\ (x-b_1+y_{\tilde{a}}(c_1-x))/(c_1-b_1) & (b_1 < x \le c_1) \\ 1 & otherwise \end{cases}$$

respectively.

The most important and only difference of this definition from definition 2.9 is that $a_1, b_1, c_1 \in \mathbb{R}$-{0}. For example$\langle(-2,-1,3);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$, $\langle(-2,1,3);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$ cannot be single valued triangular neutrosophic numbers according to the previous definition, it is a generalized single valued triangular neutrosophic number according to this new definition. In addition, negative single valued triangular neutrosophic numbers and positive single valued triangular neutrosophic numbers are covered by single valued triangular neutrosophic numbers according to this definition.



$\tilde{a} = \langle(a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$; for $a_1<0$

Fig. 2: ($\tilde{a} = \langle(a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$;for $a_1<0$, generalized single valued triangular neutrosophic number**)**

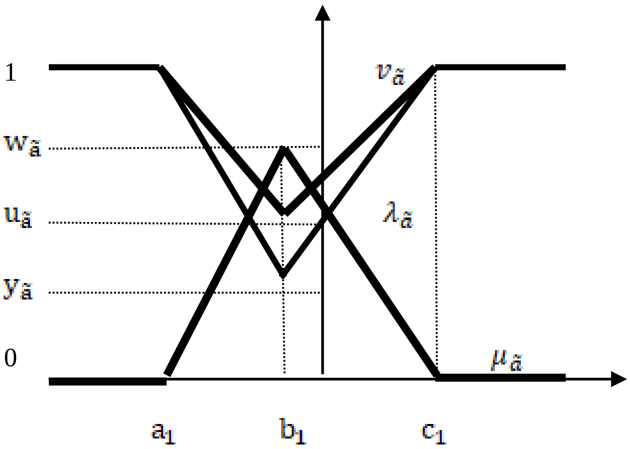


Fig. 3: ($\tilde{a} = \langle(a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$; $a_1, b_1<0$ generalized single valued triangular neutrosophic number**)**

Now let's define the basic operations for generalized single valued triangular neutrosophic numbers.

Degrees of membership / indeterminacy / nonmembership > 1 or < 0 have been proposed by Smarandache since 2007.

Definition 3.2: Let $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (a_2, b_2, c_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$, be two generalized single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any real number. Then,

1. $\tilde{a}+\tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

2. $\tilde{a}-\tilde{b} = \langle (a_1 - c_2, b_1 - b_2, c_1 - a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

3. For the set $\mathcal{L} = \{a_1 a_2, c_1 c_2, a_1 c_2, c_1 a_2\}$;

λ_1 : is the minimum value of $\mathcal{L}$

λ_2: be the largest element of $\mathcal{L}$;

$\tilde{a}\tilde{b} = \langle (\lambda_1,\ b_1 b_2,\ \lambda_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$.

4. For the set $\mathcal{L} = \{a_1/a_2, c_1/c_2, a_1/c_2, c_1/a_2\}$;

λ_1 : is the minimum value of $\mathcal{L}$,

λ_2: be the largest element of $\mathcal{L}$;

$\tilde{a}/\tilde{b} = \langle (\lambda_1,\ b_1/b_2,\ \lambda_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

5. For the set $\mathcal{L} = \{\gamma\, a_1,\ \gamma\, c_1\}$;

λ_1 : is the minimum value of $\mathcal{L}$,
λ_2: be the largest element of $\mathcal{L}$;

$\gamma\tilde{a} = \langle (\lambda_1, \gamma\, b_1,\ \lambda_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

6. For the set $\mathcal{L} = \{1/a_1,\ 1/c_1\}$;

λ_1 : is the minimum value of $\mathcal{L}$,

λ_2: be the largest element of $\mathcal{L}$;

$\tilde{a}^{-1} = \langle (\lambda_1, 1/b_1,\ \lambda_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$

These operations also give the same results as the operations in definition 2.10. Namely, these operations are a generalized description of the operations in Definition 2.10.

4. TRANSFORMED SINGLE VALUED TRIANGULAR NEUTROSOPHIC NUMBERS, HAMMING DISTANCE AND A NEW SCORE FUNCTION BASED ON HAMMING DISTANCE FOR GENERALIZED SINGLE VALUED TRIANGULAR NEUTROSOPHIC NUMBER

In this section, we define single valued triangular neutrosophic numbers by transforming single valued neutrosophic numbers in the definition 2.6. However, since single valued neutrosophic numbers consist of independent truth, falsity, and indeterminacy states, we have defined a separate transformation for each case. However, we have generalized the Hamming distance to single valued triangular neutrosophic numbers in the definition 2.2 for the triangular intuitionistic fuzzy numbers and gave some properties. We then defined new score functions based on the Hamming distance measure. We compared the results obtained with these scoring functions to the results of the scoring functions in definition 2.7 and definition 2.11.

Definition4.1 $\tilde{a}$=($T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}$) conversion to a generalized single valued triangular neutrosophic number according to the truth value for a single valued neutrosophic number;

$a_1 = T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}$

$b_1 = a_1 + (1 + T_{\tilde{a}} - F_{\tilde{a}}) = 1 + 2T_{\tilde{a}} - I_{\tilde{a}} - 2F_{\tilde{a}}$

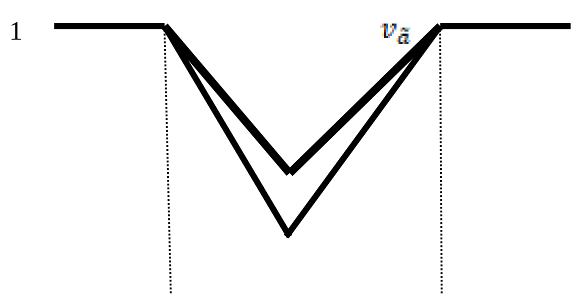
$c_1 = b_1 + (1 + T_{\tilde{a}} - I_{\tilde{a}}) = 2 + 3T_{\tilde{a}} - 2I_{\tilde{a}} - 2F_{\tilde{a}}$ and

$T_{\tilde{a}} = w_{\tilde{a}},\ I_{\tilde{a}} = u_{\tilde{a}},\ F_{\tilde{a}} = y_{\tilde{a}}$;

$$\tilde{a} = (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \xrightarrow{\text{Transformed}} \tilde{a}_T = \langle (a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle.$$ Namely

$$\tilde{a}_T = \langle (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}, 1 + 2T_{\tilde{a}} - I_{\tilde{a}} - 2F_{\tilde{a}}, 2 + 3T_{\tilde{a}} - 2I_{\tilde{a}} - 2F_{\tilde{a}});\ T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \rangle.$$

Thus we obtained the number of $\tilde{a}_T$ generalized single valued triangular neutrosophic number from $\tilde{a}$ single valued neutrosophic number. Hence, $1+T_{\tilde{a}} - F_{\tilde{a}} \geq 0$ and $1+T_{\tilde{a}} - I_{\tilde{a}} \geq 0$ for $a_1 \leq b_1 \leq c_1$. Because of this each $\tilde{a}_T$ number obtained from the definition of single valued neutrosophic number is a generalized single valued triangular neutrosophic number.



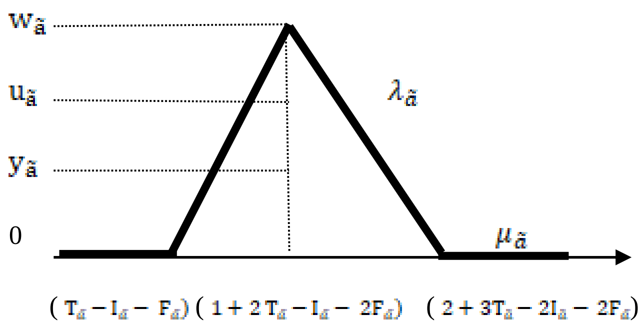


Fig. 4: **(**$\tilde{a}_T = \langle (a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ generalized single valued triangular neutrosophic number**)**

Definition4.2 $\tilde{a} = (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}})$ conversion to a generalized single valued triangular neutrosophic number according to the indeterminacy value for the single valued neutrosophic number;

$a_1 = T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}$

$b_1 = a_1 + (1 + I_{\tilde{a}} - F_{\tilde{a}}) = 1 + T_{\tilde{a}} - 2F_{\tilde{a}}$

$c_1 = b_1 + (1 + I_{\tilde{a}} - T_{\tilde{a}}) = 2 + I_{\tilde{a}} - 2F_{\tilde{a}}$ and

$T_{\tilde{a}} = w_{\tilde{a}},\ I_{\tilde{a}} = u_{\tilde{a}},\ F_{\tilde{a}} = y_{\tilde{a}}$;

$$\tilde{a} = (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \xRightarrow{\text{transformed}} \tilde{a}_I = \langle (a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle.$$ Namely

$\tilde{a}_I = \langle (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}, 1 + T_{\tilde{a}} - 2F_{\tilde{a}}, 2 + I_{\tilde{a}} - 2F_{\tilde{a}});\ T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \rangle$.

Thus we obtained the number of $\tilde{a}_I$ generalized single valued triangular neutrosophic number from $\tilde{a}$ single valued neutrosophic number. Hence $1 + I_{\tilde{a}} - F_{\tilde{a}} \geq 0$ and $1 + I_{\tilde{a}} - T_{\tilde{a}} \geq 0$; $a_1 \leq b_1 \leq c_1$. Because of this each $\tilde{a}_I$ number obtained from the definition of single valued neutrosophic number is a generalized single valued triangular neutrosophic number.

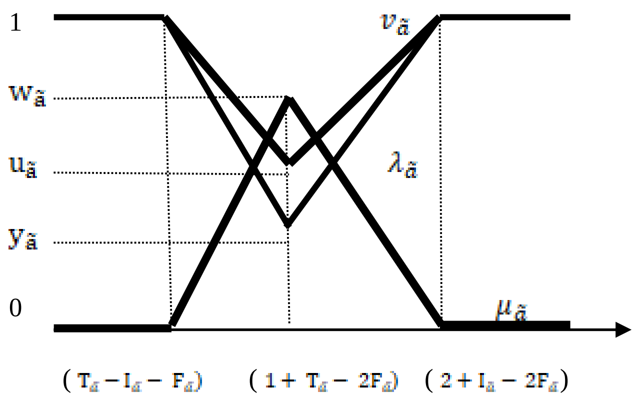


Fig. 5: **(**$\tilde{a}_I = \langle (a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ generalized single valued triangular neutrosophic number**)**

Definition4.3 $\tilde{a}$=($T_{\tilde{a}}$,$I_{\tilde{a}}$,$F_{\tilde{a}}$) conversion to a generalized single valued triangular neutrosophic number according to the falsity value for the single valued neutrosophic number;

$a_1 = T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}$

$b_1 = a_1 + (1+F_{\tilde{a}} - I_{\tilde{a}}) = 1 + T_{\tilde{a}} - 2I_{\tilde{a}}$

$c_1 = b_1 + (1+F_{\tilde{a}} - T_{\tilde{a}}) = 2 + F_{\tilde{a}} - 2I_{\tilde{a}}$ and

$T_{\tilde{a}} = w_{\tilde{a}},\ I_{\tilde{a}} = u_{\tilde{a}},\ F_{\tilde{a}} = y_{\tilde{a}}$;

$$\tilde{a} = (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \xrightarrow{\text{Transformed}} \tilde{a}_F = \langle (a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle.$$ Namely

$\tilde{a}_F = \langle (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}, 1 + T_{\tilde{a}} - 2I_{\tilde{a}}, 2 + F_{\tilde{a}} - 2I_{\tilde{a}});\ T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \rangle$.

Thus we obtained the number of $\tilde{a}_F$ generalized single valued triangular neutrosophic number from $\tilde{a}$ single valued neutrosophic number. Hence, $1+F_{\tilde{a}} - I_{\tilde{a}} \geq 0$ and $1+F_{\tilde{a}} - T_{\tilde{a}} \geq 0$ for $a_1 \leq b_1 \leq c_1$. Because of this each $\tilde{a}_F$ number obtained from the definition of single valued neutrosophic number is a generalized single valued triangular neutrosophic number.

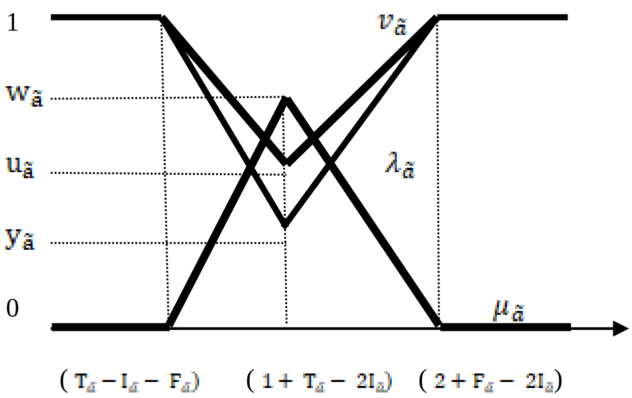


Fig. 6 ($\tilde{a}_F = \langle (a_1, b_1, c_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ generalized single valued triangular neutrosophic number**)**

Definition 4.4:

a) $\tilde{a}$=($T_{\tilde{a}}$,$I_{\tilde{a}}$,$F_{\tilde{a}}$) ideal generalized single valued triangular neutrosophic number according to the truth value for single valued neutrosophic numbers;

$T_{\tilde{a}} = 1$, $I_{\tilde{a}} = 0$ and $F_{\tilde{a}} = 0$; $\tilde{a}_T^i$ =

$\langle (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}, 1 + 2T_{\tilde{a}} - I_{\tilde{a}} - 2F_{\tilde{a}}, 2 + 3T_{\tilde{a}} - 2I_{\tilde{a}} - 2F_{\tilde{a}});\ T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \rangle = \langle (1,3,5);\ 1,0,0 \rangle$.

b) $\tilde{a}$=($T_{\tilde{a}}$,$I_{\tilde{a}}$,$F_{\tilde{a}}$) ideal generalized single valued triangular neutrosophic number according to the indeterminacy value for single valued neutrosophic numbers;

$T_{\tilde{a}}=1$, $I_{\tilde{a}}=0$ and $F_{\tilde{a}}=0$; $\tilde{a}_I^i=$

$\langle (T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}}, 1+T_{\tilde{a}}-2F_{\tilde{a}}, 2+I_{\tilde{a}}-2F_{\tilde{a}}); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle = \langle (1,2,3);\ 1,0,0\rangle.$

c) $\tilde{a}=(T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}})$ ideal generalized single valued triangular neutrosophic number according to the falsity value for single valued neutrosophic numbers;

$T_{\tilde{a}}=1$, $I_{\tilde{a}}=0$ and $F_{\tilde{a}}=0$; $\tilde{a}_F^i=$

$\langle (T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}}, 1+T_{\tilde{a}}-2I_{\tilde{a}}, 2+F_{\tilde{a}}-2I_{\tilde{a}}); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle = \langle (1,2,3);\ 1,0,0\rangle.$

It can be seen from b) and c), $\tilde{a}_I^i=\tilde{a}_F^i$.

Definition 4.5: Let $\tilde{a}_1=\langle (a_1,b_1,c_1);\ w_{\tilde{a}_1}, u_{\tilde{a}_1}, y_{\tilde{a}_1}\rangle$ and $\tilde{a}_2=\langle (a_2,b_2,c_2);\ w_{\tilde{a}_2}, u_{\tilde{a}_2}, y_{\tilde{a}_2}\rangle$ be two generalized single valued triangular neutrosophic numbers. The Hamming distance between $\tilde{a}_1$ and $\tilde{a}_2$ is

$$d_n(\tilde{a}_1,\tilde{a}_2)=\frac{1}{12}[|(2+w_{\tilde{a}_1}-u_{\tilde{a}_1}-y_{\tilde{a}_1})a_1-(2+w_{\tilde{a}_2}-u_{\tilde{a}_2}-y_{\tilde{a}_2})a_2|+$$
$$|(2+w_{\tilde{a}_1}-u_{\tilde{a}_1}-y_{\tilde{a}_1})b_1-(2+w_{\tilde{a}_2}-u_{\tilde{a}_2}-y_{\tilde{a}_2})b_2|+$$
$$|(2+w_{\tilde{a}_1}-u_{\tilde{a}_1}-y_{\tilde{a}_1})c_1-(2+w_{\tilde{a}_2}-u_{\tilde{a}_2}-y_{\tilde{a}_2})c_2|]$$

This definition is the expansion of the Hamming distance given to the triangular intuitionistic fuzzy numbers given in the definition to generalized single valued triangular neutrosophic numbers.

Proposition 4.6: The Hamming distance $d_n(\tilde{a}_1,\tilde{a}_2)$ satisfies the following properties.

1) $d_n(\tilde{a}_1,\tilde{a}_2)\geq 0$

2) $d_n(\tilde{a}_1,\tilde{a}_2)=0$, if $\tilde{a}_1=\tilde{a}_2$, for all $\tilde{a}_1, \tilde{a}_2\in \overline{N}_R$

3) $d_n(\tilde{a}_1,\tilde{a}_2)=d_n(\tilde{a}_2,\tilde{a}_1)$

4) Let $\tilde{a}_j=\langle (a_j,b_j,c_j);\ w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle$, $\tilde{b}_j=\langle (e_j,f_j,g_j);\ w_{\tilde{b}_j}, u_{\tilde{b}_j}, y_{\tilde{b}_j}\rangle$ and $\tilde{c}_j=\langle (h_j,l_j,m_j);\ w_{\tilde{c}_j}, u_{\tilde{c}_j}, y_{\tilde{c}_j}\rangle$ be three single valued triangular neutrosophic numbers.

If $a_j\leq e_j\leq h_j$, $b_j\leq f_j\leq l_j$, $c_j\leq g_j\leq m_j$, $w_{\tilde{a}_j}\leq w_{\tilde{b}_j}\leq w_{\tilde{c}_j}$, $u_{\tilde{a}_j}\leq u_{\tilde{b}_j}\leq u_{\tilde{b}_j}$, $y_{\tilde{a}_j}\geq y_{\tilde{b}_j}\geq y_{\tilde{c}_j}$, then;

$d_n(\tilde{a}_j, \tilde{c}_j)\geq d_n(\tilde{a}_j, \tilde{b}_j)$ and $d_n(\tilde{a}_j, \tilde{c}_j)\geq d_n(\tilde{b}_j, \tilde{c}_j)$

Proof: The proof of 1), 2), 3) can easily be done by the definition 4.5. Now let's prove 4).

Let's show that $d_n(\tilde{a}_j, \tilde{c}_j)\geq d_n(\tilde{a}_j, \tilde{b}_j)$.

$d_n(\tilde{a}_j, \tilde{c}_j)=$

$\frac{1}{12}[\left|\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)a_j-\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)h_j\right|+$
$\left|\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j-\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)l_j\right|+$
$\left|\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)c_j-\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)m_j\right|]$

And $a_j \le e_j \le h_j$, $b_j \le f_j \le l_j$, $c_j \le g_j \le m_j$, $w_{\tilde{a}_j} \le w_{\tilde{b}_j} \le w_{\tilde{c}_j}$, $u_{\tilde{a}_j} \le u_{\tilde{b}_j} \le u_{\tilde{b}_j}$, $y_{\tilde{a}_j} \ge y_{\tilde{b}_j} \ge y_{\tilde{c}_j}$ hence;

$2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j} \le 2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}$ and $a_j \le e_j \le h_j$. Hence;

$\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)a_j \le \left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)h_j$. Similarly;

$\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j \le \left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)l_j$;

$\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)c_j \le \left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)m_j$. From here;

$d_n(\tilde{a}_j, \tilde{c}_j)=$

$\frac{1}{12}[(\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)h_j - \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)a_j)+$

$(\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)l_j - \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j)+$

$(\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)m_j - \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)c_j)]$ ……………………………………………..……(1)

$d_n(\tilde{a}_j, \tilde{b}_j)=$

$\frac{1}{12}[\left|\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)a_j-\left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)e_j\right|+$
$\left|\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j-\left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)f_j\right|+$
$\left|\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)c_j-\left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)g_j\right|]$

and $a_j \le e_j \le h_j$, $b_j \le f_j \le l_j$, $c_j \le g_j \le m_j$,, $w_{\tilde{a}_j} \le w_{\tilde{b}_j} \le w_{\tilde{c}_j}$, $u_{\tilde{a}_j} \le u_{\tilde{b}_j} \le u_{\tilde{b}_j}$, $y_{\tilde{a}_j} \ge y_{\tilde{b}_j} \ge y_{\tilde{c}_j}$ hence;

$2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j} \le 2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}$ and $a_j \le e_j \le h_j$ hence;

$\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)a_j \le \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)e_j$. Similarly,
$\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j \le \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)f_j$;

$\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)c_j \le \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)g_j$. From here;

$d_n(\tilde{a}_j, \tilde{b}_j)=$

$\frac{1}{12}[(\left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)e_j - \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)a_j)+$

$\left(\left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)f_j - \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j\right)+$

$\left(\left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)g_j - \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)c_j\right)]$ ……………………………………….(2)

From (1) and (2) $d_n(\tilde{a}_j, \tilde{c}_j)$- $d_n(\tilde{a}_j, \tilde{b}_j)$=

$\frac{1}{12}[\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)h_j - \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)a_j) - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)e_j + \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)a_j]$

$+\frac{1}{12}[\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j - \left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)l_j - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)f_j + \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j]$

$+\frac{1}{12}[\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)m_j - \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)c_j) - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)g_j + \left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)c_j]$

$=\frac{1}{12}[\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)h_j - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)e_j]$

$+\frac{1}{12}[\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)f_j]$

$+\frac{1}{12}[\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)m_j - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)g_j]$. Here;

$a_j \le e_j \le h_j$, $b_j \le f_j \le l_j$, $c_j \le g_j \le m_j$, $w_{\tilde{a}_j} \le w_{\tilde{b}_j} \le w_{\tilde{c}_j}$, $u_{\tilde{a}_j} \le u_{\tilde{b}_j} \le u_{\tilde{b}_j}$, $y_{\tilde{a}_j} \ge y_{\tilde{b}_j} \ge y_{\tilde{c}_j}$. Hence;

$\frac{1}{12}[\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)h_j - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)e_j] \ge 0$ ……………………………………………….(3)

$\frac{1}{12}[\left(2+w_{\tilde{a}_j}-u_{\tilde{a}_j}-y_{\tilde{a}_j}\right)b_j - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)f_j] \ge 0$……………….……………………..………….(4)

$\frac{1}{12}[\left(2+w_{\tilde{c}_j}-u_{\tilde{c}_j}-y_{\tilde{c}_j}\right)m_j - \left(2+w_{\tilde{b}_j}-u_{\tilde{b}_j}-y_{\tilde{b}_j}\right)g_j] \ge 0$………………...……………………….(5)

From (3), (4) and (5) $d_n(\tilde{a}_j, \tilde{c}_j)$- $d_n(\tilde{a}_j, \tilde{b}_j) \ge 0$. Namely; $d_n(\tilde{a}_j, \tilde{c}_j) \ge d_n(\tilde{a}_j, \tilde{b}_j)$.

$d_n(\tilde{a}_j, \tilde{c}_j) \ge d_n(\tilde{b}_j, \tilde{c}_j)$ can be showed a similar way to the proof of $d_n(\tilde{a}_j, \tilde{c}_j) \ge d_n(\tilde{a}_j, \tilde{b}_j)$.

Definition 4.7: $\tilde{a}$ =($T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}$) single valued neutrosophic number, $\tilde{a}_T$ = $\langle (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}, 1 + 2T_{\tilde{a}} - I_{\tilde{a}} - 2F_{\tilde{a}}, 2 + 3T_{\tilde{a}} - 2I_{\tilde{a}} - 2F_{\tilde{a}}); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ generalized single valued triangular neutrosophic number transformed according to the truth value of $\tilde{a}$, $\tilde{a}_T^i$ = $\langle (1,3,5);\ 1,0,0 \rangle$, ideal generalized single valued triangular neutrosophic number transformed according to the truth value of $\tilde{a}$, and let

d_n be the Hamming distance for generalized single valued triangular neutrosophic number. According to the truth value of single valued neutrosophic numbers certainty and score functions are

$S_T(\tilde{a}) = d_n(\tilde{a}_T, \tilde{a}_T^i)$

$A_T(\tilde{a}) = \min\{|T_{\tilde{a}} - I_{\tilde{a}}|, |T_{\tilde{a}} - F_{\tilde{a}}|\}$ respectively. Here;

$S_T(\tilde{a}) =$

$$\frac{1}{12}[|(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})-(2+1-0-0).1|+ |(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(2T_{\tilde{a}}-I_{\tilde{a}}-2F_{\tilde{a}})-(2+1-0-0).3|+ |(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(2+3T_{\tilde{a}}-2I_{\tilde{a}}-2F_{\tilde{a}})-(2+1-0-0).5|]$$

$$= \frac{1}{12}[|(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})-3|+|(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(2T_{\tilde{a}}-I_{\tilde{a}}-2F_{\tilde{a}})-9|+ |(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(2+3T_{\tilde{a}}-2I_{\tilde{a}}-2F_{\tilde{a}})-15|].$$

Definition 4.8: Let $\tilde{a} = (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}})$ be single valued neutrosophic number, $\tilde{a}_I = \langle(T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}, 1 + T_{\tilde{a}} - 2F_{\tilde{a}}, 2 + I_{\tilde{a}} - 2F_{\tilde{a}}); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle$; be generalized single valued triangular neutrosophic number transformed according to the indeterminacy value of $\tilde{a}$, $\tilde{a}_I^i = \langle(1,2,2); 1,0,0\rangle$, be ideal generalized single valued triangular neutrosophic number transformed according to the indeterminacy value of $\tilde{a}$, and d_n be hamming distance for generalized single valued triangular neutrosophic number. According to the indeterminacy value of single valued neutrosophic numbers certainty and score functions are;

$S_I(\tilde{a}) = d_n(\tilde{a}_I, \tilde{a}_I^i)$

$A_I = \min\{|I_{\tilde{a}} - T_{\tilde{a}}|, |I_{\tilde{a}} - F_{\tilde{a}}|,\}$ respectively. Here;

$S_I(\tilde{a}) =$

$$\frac{1}{12}[|(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})-(2+1-0-0).1|+ |(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(1+T_{\tilde{a}}-2F_{\tilde{a}})-(2+1-0-0).2|+ |(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(2+I_{\tilde{a}}-2F_{\tilde{a}})-(2+1-0-0).2|]$$

$$= \frac{1}{12}[|(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})-2|+|(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(1+T_{\tilde{a}}-2F_{\tilde{a}})-6|+ |(2+T_{\tilde{a}}-I_{\tilde{a}}-F_{\tilde{a}})(2+I_{\tilde{a}}-2F_{\tilde{a}})-6|]$$

Definition 4.9: Let $\tilde{a} = (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}})$ be single valued neutrosophic number, $\tilde{a}_F = \langle(T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}, 1 + T_{\tilde{a}} - 2I_{\tilde{a}}, 2 + F_{\tilde{a}} - 2I_{\tilde{a}}); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}\rangle$; be generalized single valued triangular neutrosophic number transformed according to the falsity value of $\tilde{a}$, $\tilde{a}_F^i = \langle(1,2,2); 1,0,0\rangle$, be ideal generalized single valued triangular neutrosophic number transformed according to the falsity value of $\tilde{a}$, and d_n be Hamming distance for generalized single valued triangular neutrosophic number. According to the falsity value of single valued neutrosophic numbers certainty and score functions are;

$S_F(\tilde{a})= d_n(\tilde{a}_F, \tilde{a}_F^i)$

$A_F=\min\{|F_{\tilde{a}} - I_{\tilde{a}}|, |F_{\tilde{a}} - I_{\tilde{a}}|, \}$ respectively. Here;

$S_F(\tilde{a})$ =

$$\frac{1}{12}[|(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})(T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) - (2 + 1 - 0 - 0).1|+ |(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})(1 + T_{\tilde{a}} - 2I_{\tilde{a}}) - (2 + 1 - 0 - 0).2|+ |(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})(2 + F_{\tilde{a}} - 2I_{\tilde{a}}) - (2 + 1 - 0 - 0).2|]$$

$$= \frac{1}{12}[|(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})(T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}) - 2|+|(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})(1 + T_{\tilde{a}} - 2I_{\tilde{a}}) - 6|+ |(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})(2 + F_{\tilde{a}} - 2I_{\tilde{a}}) - 6|]$$

Definition 4.10: Let $\tilde{a}$=($T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}$) and $\tilde{b}$ =($T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}$) be two single valued neutrosophic numbers and S_T, A_T be score and certainty functions according to truth value.

i) If $S_T(\tilde{a}) > S_T(\tilde{b})$, then $\tilde{a}$ is greater than $\tilde{b}$ and denoted by $\tilde{a} > \tilde{b}$.

ii) If $S_T(\tilde{a}) = S_T(\tilde{b})$ and $A_T(\tilde{a}) > A_T(\tilde{b})$, then $\tilde{a}$ is greater than $\tilde{b}$ and denoted by $\tilde{a} > \tilde{b}$.

iii) If $S_T(\tilde{a}) = S_T(\tilde{b})$ and $A_T(\tilde{a}) = A_T(\tilde{b})$, then $\tilde{a}$ is equal to $\tilde{b}$ and denoted by $\tilde{a} = \tilde{b}$.

This definition can also be done for S_I, A_I score and certainty functions in case of indeterminacy and for S_F, A_F score and certainty functions in case of falsity.

Definition 4.11: Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ and d_n be Hamming distance for the generalized single valued triangular neutrosophic numbers.

i) $\tilde{a}_T^i = \langle (1,3,5); 1,0,0 \rangle$,be ideal generalized single valued triangular neutrosophic number according to the truth value of $\tilde{a}$; depending on the Hamming distance $\tilde{a}_j$ generalized single valued triangular neutrosophic numbers according to the truth value score and certainty functions are;

$S_{TT}(\tilde{a}_j)= d_n(\tilde{a}_j, \tilde{a}_T^i)$

$A_{TT}(\tilde{a}_j)= \min\{|T_{\tilde{a}} - I_{\tilde{a}}|, |T_{\tilde{a}} - F_{\tilde{a}}|\}$ respectively.

ii) $\tilde{a}_I^i = \langle (1,2,2); 1,0,0 \rangle$, be ideal generalized single valued triangular neutrosophic number according to the indeterminacy value of $\tilde{a}$; depending on the hamming distance $\tilde{a}_j$ generalized single valued triangular neutrosophic numbers according to the indeterminacy value score and certainty functions are;

$S_{TI}(\tilde{a}_j)= d_n(\tilde{a}_j, \tilde{a}_I^i)$

$A_{TI}(\tilde{a}_j)$= min{$|I_{\tilde{a}} - T_{\tilde{a}}|$, $|T_{\tilde{a}} - F_{\tilde{a}}|$} respectively.

iii) i) $\tilde{a}^{i}_{F} = \langle(1,2,2);\ 1,0,0\rangle$, be ideal generalized single valued triangular neutrosophic number according to the falsity value of $\tilde{a}$; depending on the Hamming distance $\tilde{a}_j$ generalized single valued triangular neutrosophic numbers according to the falsity value score and certainty functions are;

$S_{TF}(\tilde{a}_j)$= $d_n(\tilde{a}_j, \tilde{a}^{i}_{F})$

$A_{TF}(\tilde{a}_j)$= min{$|F_{\tilde{a}} - T_{\tilde{a}}|$, $|F_{\tilde{a}} - I_{\tilde{a}}|$} respectively.

Thus, for generalized single valued triangular neutrosophic numbers we have also defined a new scoring function based on the Hamming distance.

Definition 4.12: Let $\tilde{a}_j = \langle(a_j,b_j,c_j);\ w_{\tilde{a}_j},u_{\tilde{a}_j},y_{\tilde{a}_j}\rangle$ and $\tilde{b}_j = \langle(d_j,e_j,f_j);\ w_{\tilde{b}_j},u_{\tilde{b}_j},y_{\tilde{b}_j}\rangle$be two generalized single valued triangular neutrosophic numbers andS_{TT}, A_{TT} be score and certainty functions according to truth value.

i) If $S_{TT}(\tilde{a})$> $S_{TT}(\tilde{b})$, then $\tilde{a}$ is greater than $\tilde{b}$ and denoted by $\tilde{a}$ >$\tilde{b}$.

ii) If $S_{TT}(\tilde{a})$= $S_{TT}(\tilde{b})$ and $A_{TT}(\tilde{a})$> $A_{TT}(\tilde{b})$, then $\tilde{a}$ is greater than $\tilde{b}$ and denoted by $\tilde{a}$ >$\tilde{b}$.

iii) if $S_{TT}(\tilde{a})$= $S_{TT}(\tilde{b})$ and $A_{TT}(\tilde{a})$= $A_{TT}(\tilde{b})$, then $\tilde{a}$ is equal to $\tilde{b}$ and denoted by $\tilde{a}$ = $\tilde{b}$.

Example 4.13: Now let's compare the score and certainty function in definition 4.7 with the S_{TT}, A_{TT} score and certainty function according to the truth value, S_I, A_I score and certainty function in definition 4.8 according to the indeterminacy value and S_F, A_F score and certainty function in definition 4.9 according to the falsity value.

Let $\tilde{a}_1$ =(0.9, 0.4,0.3), $\tilde{a}_2$ =(0.8, 0.4,0.2) and $\tilde{a}_3$ =(0.7, 0.4,0.1) be three single valued neutrosophic number.

i) For score and certainty functions in Definition 2.7;

ac($\tilde{a}_1$)= 2.2 sc($\tilde{a}_1$)= 0,6

ac($\tilde{a}_2$)= 2.2 sc($\tilde{a}_2$)= 0,6

ac($\tilde{a}_2$)= 2.2 sc($\tilde{a}_2$)= 0,6 hence; $\tilde{a}_1$ = $\tilde{a}_2$=$\tilde{a}_3$.

ii) For the score function according to the truth value in Definition 4.7;

$S_T(\tilde{a}_1)$ = 1.42 $S_T(\tilde{a}_2)$ = 1.44 $S_T(\tilde{a}_3)$ = 1.46 hence $\tilde{a}_1$ > $\tilde{a}_2$> $\tilde{a}_3$

iii) For the score function according to the indeterminacy value in Definition 4.8;

$S_I(\tilde{a}_1) = 0.51$ $S_I(\tilde{a}_2) = 0.49$ $S_I(\tilde{a}_3) = 0.47$ hence $\tilde{a}_3 > \tilde{a}_2 > \tilde{a}_1$.

iv)For the score function according to the falsity value in Definition 4.9;

$S_F(\tilde{a}_1) = 0.51$ $S_F(\tilde{a}_2) = 0.49$ $S_F(\tilde{a}_3) = 0.47$ hence $\tilde{a}_3 > \tilde{a}_2 > \tilde{a}_1$.

Table 1: **(**Results of scoring functions for single valued triangular neutrosophic numbers**)**

The result of the score and certainty function in Definition 2.7	$\tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3$
The result of the score function according to the truth value in definition 4.7	$\tilde{a}_1 > \tilde{a}_2 > \tilde{a}_3$
The result of the score function according to the indeterminacy value in definition 4.8	$\tilde{a}_3 > \tilde{a}_2 > \tilde{a}_1$
The result of the score function according to the falsity value in Definition 4.9	$\tilde{a}_3 > \tilde{a}_2 > \tilde{a}_1$

Example 4.14: Now let's compare the score and certainty function in definition 4.3 with the S_{TT}, score and certainty function in definition 2.1 according to the truth value, S_{TI}, A_{TI} score and certainty function according to the indeterminacy value and S_{TF}, A_{TF} score and certainty function according to the falsity value.

Let $\tilde{a}_1 = \langle(2,5,6);\ 0.9, 0.6, 0\rangle$, $\tilde{a}_2\ \langle(3,4,6);\ 0.8, 0.5, 0\rangle$ and $\tilde{a}_3 = \langle(1,5,7);\ 0.7, 0.4, 0\rangle$ be three single valued triangular neutrosophic numbers.

i) For the score and certainty functions in Definition 2.11;

$S(\tilde{a}_1) = 3.73$ $A(\tilde{a}_1) = 3.73$

$S(\tilde{a}_2) = 3.73$ $A(\tilde{a}_2) = 3.73$

$S(\tilde{a}_3) = 3.73$ $A(\tilde{a}_3) = 3.73$ hence $\tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3$.

ii) For the score function according to the truth value in Definition 4.11;

$S_{TT}(\tilde{a}_1) = 0{,}458$ $S_{TT}(\tilde{a}_2) = 0{,}450$ $S_{TT}(\tilde{a}_3) = 0{,}358$ hence $\tilde{a}_3 > \tilde{a}_2 > \tilde{a}_1$.

iii) For the score function according to the indeterminacy value in Definition 4.8;

$S_{TI}(\tilde{a}_1) = 1{,}458$ $S_{TI}(\tilde{a}_2) = 1{,}350$ $S_{TI}(\tilde{a}_3) = 1{,}358$ hence $\tilde{a}_2 > \tilde{a}_3 > \tilde{a}_1$.

iv) For the score function according to the falsity value in Definition 4.9;

$S_{TF}(\tilde{a}_1) = 1{,}458$ $S_{TF}(\tilde{a}_2) = 1{,}350$ $S_{TF}(\tilde{a}_3) = 1{,}358$ hence $\tilde{a}_2 > \tilde{a}_3 > \tilde{a}_1$.

Table 2: (Results of scoring functions for single valued triangular neutrosophic numbers)

The result of the score and certainty function in Definition 2.11	$\tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3$
The result of score function according to the truth value in Definition 4.11	$\tilde{a}_3 > \tilde{a}_2 > \tilde{a}_1$
The result of score function according to the indeterminacy value in Definition 4.11	$\tilde{a}_2 > \tilde{a}_3 > \tilde{a}_1$
The result of score function according to the falsity value in Definition 4.11	$\tilde{a}_2 > \tilde{a}_3 > \tilde{a}_1$

5. SOME NEW GENERALIZED AGGREGATION OPERATORS BASED ON GENERALIZED SINGLE VALUED TRIANGULAR NEUTROSOPHIC NUMBERS FOR APPLICATION TO MULTI-ATTRIBUTE GROUP DECISION MAKING

In this section we have generalized some operators given for triangular intuitionistic fuzzy numbers in Definition 2.3 and Definition 2.4 for generalized single valued triangular neutrosophic numbers and showed some properties. We have shown that the new operators we have acquired include operators in definitions 2.13 and 2.14. Additionally, we showed the generalized single valued triangular neutrosophic numbers in this section.

Definition 5.1: Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ (j = 1, 2, 3,…, n) be a collection of generalized single valued triangular neutrosophic numbers. Then generalized single valued triangular neutrosophic generalized weight averaging operator (SVTNGWAO) is defined as;

GSVTNGWAO: $\overline{\mathrm{N}}_{\mathrm{R}}{}^{\mathrm{n}} \to \overline{\mathrm{N}}_{\mathrm{R}}$, GSVTNGWAO($\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$) = $g^{-1}(\sum_{j=1}^{n} w_j g(\tilde{a}_{(j)}))$

where g is a continuous strictly monotone increasing function, w = $(w_1, w_2, \dots, w_n)^T$ is a weight vector associated with the GSVTNGWAO operator, with $w_j \geq 0$, j = 1,2,3,…,n and $\sum_{j=1}^{n} w_j = 1$ (j = 1,2,3,…,n) .

Theorem 5.2: Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ (j = 1, 2, 3, …, n) be a collection of generalized single valued triangular neutrosophic numbers and w = $(w_1, w_2, \dots, w_n)^T$ is a weight vector associated with $w_j \geq 0$, and $\sum_{j=1}^{n} w_j = 1$. Then their aggregated value by using SVTNGWAO operator is also a neutrosophic number and

GSVTNGWAO ($\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$)

$$= \langle [g^{-1}(\sum_{j=1}^{n} w_j g(a_{(j)})), g^{-1}(\sum_{j=1}^{n} w_j g(b_{(j)})), g^{-1}(\sum_{j=1}^{n} w_j g(c_{(j)}))]; \wedge_{j=1}^{n} w_{\tilde{a}_j}\ \vee_{j=1}^{n} u_{\tilde{a}_j}\ \vee_{j=1}^{n} y_{\tilde{a}_j} \rangle$$

Where, g is a continuous strictly monotone increasing function.

Proof: We proof this by using the method of mathematical induction. For this;

i) For n = 2

$\tilde{a}_1 = \langle (a_1, b_1, c_1); w_{\tilde{a}_1}, u_{\tilde{a}_1}, y_{\tilde{a}_1} \rangle$ and $\tilde{a}_2 = \langle (a_2, b_2, c_2); w_{\tilde{a}_2}, u_{\tilde{a}_2}, y_{\tilde{a}_2} \rangle$ be two single valued triangular neutrosophic numbers by definition;

$g^{-1}(w_1 g(\tilde{a}_1)) + g^{-1}(w_2 g(\tilde{a}_2))$ =

$g^{-1}(w_1 g(\langle(a_1,b_1,c_1);\ w_{\tilde{a}_1},u_{\tilde{a}_1},y_{\tilde{a}_1}\rangle))+ g^{-1}(w_2 g(\langle(a_2,b_2,c_2);\ w_{\tilde{a}_2},u_{\tilde{a}_2},y_{\tilde{a}_2}\rangle))$

$=g^{-1}(\langle(w_1 g(a_1),w_1 g(b_1),w_1 g(c_1));\ w_{\tilde{a}_1},u_{\tilde{a}_1},y_{\tilde{a}_1}\rangle)+$

$g^{-1}(\langle(w_2 g(a_2),w_2 g(b_2),w_2 g(c_2));\ w_{\tilde{a}_2},u_{\tilde{a}_2},y_{\tilde{a}_2}\rangle$

$= g^{-1}(\langle(w_1 g(a_1), w_1 g(b_1),w_1 g(c_1));\ w_{\tilde{a}_1},u_{\tilde{a}_1},y_{\tilde{a}_1}\rangle+\langle(w_2 g(a_2),w_2 g(b_2),w_2 g(c_2));\ w_{\tilde{a}_2},u_{\tilde{a}_2},y_{\tilde{a}_2}\rangle)$

$=$

$\langle[g^{-1}(\sum_{j=1}^{2} w_j g(a_{(j)})),g^{-1}(\sum_{j=1}^{2} w_j g(b_{(j)})),g^{-1}(\sum_{j=1}^{2} w_j g(c_{(j)})))];\ \wedge_{j=1}^{2} w_{\tilde{a}_j}\ \vee_{j=1}^{2} u_{\tilde{a}_j}\ \vee_{j=1}^{2} y_{\tilde{a}_j}\rangle$

it's true.

Let it be true for n = k that is we assumed

$g^{-1}(w_1 g(\tilde{a}_1))+ g^{-1}(w_2 g(\tilde{a}_2)) + \ldots + g^{-1}(w_k g(\tilde{a}_k))$

$=$

$\langle[g^{-1}(\sum_{j=1}^{k} w_j g(a_{(j)})),g^{-1}(\sum_{j=1}^{k} w_j g(b_{(j)})),g^{-1}(\sum_{j=1}^{k} w_j g(c_{(j)})))];\ \wedge_{j=1}^{k} w_{\tilde{a}_j}\ \vee_{j=1}^{k} u_{\tilde{a}_j}\ \vee_{j=1}^{k} y_{\tilde{a}_j}\rangle$

equation is true and let show that it is also true for n+1 .then

$g^{-1}(w_1 g(\tilde{a}_1))+ g^{-1}(w_2 g(\tilde{a}_2)) + \ldots + g^{-1}(w_{k+1} g(\tilde{a}_{k+1}))$

$=$

$\langle[g^{-1}(\sum_{j=1}^{k} w_j g(a_{(j)})),g^{-1}(\sum_{j=1}^{k} w_j g(b_{(j)})),g^{-1}(\sum_{j=1}^{k} w_j g(c_{(j)})))];\ \wedge_{j=1}^{k} w_{\tilde{a}_j}\ \vee_{j=1}^{k} u_{\tilde{a}_j}\ \vee_{j=1}^{k} y_{\tilde{a}_j}\rangle$

$+g^{-1}(\langle(w_{k+1} g(a_{k+1}),w_2 g(b_{k+1}),w_2 g(c_{k+1}));\ w_{\tilde{a}_{k+1}},u_{\tilde{a}_{k+1}},y_{\tilde{a}_{k+1}}\rangle$

$\langle[g^{-1}(\sum_{j=1}^{k+1} w_j g(a_{(j)})),g^{-1}(\sum_{j=1}^{k+1} w_j g(b_{(j)})),g^{-1}(\sum_{j=1}^{k+1} w_j g(c_{(j)})))];\ \wedge_{j=1}^{k+1} w_{\tilde{a}_j}\ \vee_{j=1}^{k+1} u_{\tilde{a}_j}\ \vee_{j=1}^{k+1} y_{\tilde{a}_j}\rangle$

Hence the expression is true for n =k+1 as required.

As a result, the proof of the theorem is completed.

Lemma 5.3: Let $\tilde{a}_j= \langle(a_j,b_j,c_j);\ w_{\tilde{a}_j},u_{\tilde{a}_j},y_{\tilde{a}_j}\rangle$ and $\tilde{b}_2 = \langle(d_j,e_j,f_j);\ w_{\tilde{b}_j},u_{\tilde{b}_j},y_{\tilde{b}_j}\rangle$ (j = 1, 2, 3,…,n) be a collection of generalized single valued triangular neutrosophic numbers and $\tilde{a} = \langle(a,b,c);\ w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ be a generalized single valued triangular neutrosophic number. w = $(w_1, w_2, \ldots, w_n)^T$ be a weight vector associated with $w_j \geq 0$, and $\sum_{j=1}^{n} w_j = 1$.

1) If $\tilde{a}_j = \tilde{a}$ (j = 1, 2, 3, …, n), then GSVTNGWAO $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}$

2) If $\tilde{a}_j^{\,-} = \langle(\min\{a_j\},\min\{b_j\},\min\{c_j\});\ \min\{w_{\tilde{a}_j}\},\max\{u_{\tilde{a}_j}\},\max\{y_{\tilde{a}_j}\})$

$\tilde{a}_j^{\,+} = \langle(\max\{a_j\},\max\{b_j\},\max\{c_j\});\ \max\{w_{\tilde{a}_j}\},\min\{u_{\tilde{a}_j}\},\min\{y_{\tilde{a}_j}\})$

Then,

$$\tilde{a}_j^{\,-} \leq \text{GSVTNGWAO}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \tilde{a}_j^{\,+}$$

3) If $a_j \leq d_j, b_j \leq e_j,\ c_j \leq f_j,\ w_{\tilde{a}_j} \leq w_{\tilde{b}_j},\ u_{\tilde{a}_j} \geq u_{\tilde{b}_j},\ y_{\tilde{a}_j} \geq y_{\tilde{b}_j}$ for all j then,

$$\text{GSVTNGWAO}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \text{GSVTNGWAO}(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n)$$

Proof:

1) From theorem 5.2 GSVTNGWAO $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$

$$= \langle [g^{-1}(\textstyle\sum_{j=1}^{n} w_j g(a_{(j)})), g^{-1}(\sum_{j=1}^{n} w_j g(b_{(j)})), g^{-1}(\sum_{j=1}^{n} w_j g(c_{(j)}))]; \bigwedge_{j=1}^{n} w_{\tilde{a}_j} \bigvee_{j=1}^{n} u_{\tilde{a}_j} \bigvee_{j=1}^{n} y_{\tilde{a}_j} \rangle$$

$$= \langle [g^{-1}(\textstyle\sum_{j=1}^{n} w_j g(a)), g^{-1}(\sum_{j=1}^{n} w_j g(b)), g^{-1}(\sum_{j=1}^{n} w_j g(b))]; \bigwedge_{j=1}^{n} w_{\tilde{a}} \bigvee_{j=1}^{n} w_{\tilde{a}} \bigvee_{j=1}^{n} w_{\tilde{a}} \rangle$$

$\sum_{j=1}^{n} w_j = 1$. Hence;

GSVTNGWAO $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \langle (g^{-1}(g(a)), g^{-1}(g(b)), g^{-1}(g(c)); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle = \langle (a, b, c);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$

The proof of 2) and 3) can easily be done from the proposition 4.6 given for the scoring function according to the center of the Hamming distance in the definition 5.1 and section 4.

Definition 5.4: Let $\tilde{\alpha}_j = \langle (a_j, b_j, c_j);\ w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ (j = 1, 2, 3, …, n) be a collection of generalized single valued triangular neutrosophic numbers. Then generalized single valued triangular neutrosophic generalized ordered averaging operator (GSVTNGOAO) is defined as;

GSVTNGOAO: $\bar{N}_R^{\ n} \rightarrow \bar{N}_R$, GSVTNGOAO $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = g^{-1}(\sum_{j=1}^{n} w_j g(\tilde{b}_{(j)}))$

where g is a continuous strictly monotone increasing function, w = $(w_1, w_2, \ldots, w_n)^T$ is a weight vector associated with the GSVTNGOAO operator, with $w_j \geq 0$, j = 1,2,3,…,n and $\sum_{j=1}^{n} w_j = 1$ (j = 1,2,3,…,n) and k is the largest generalized single valued triangular neutrosophic number obtained by using the new score function of $\tilde{b}_{(j)}$; $\tilde{\alpha}_j = \langle (a_j, b_j, c_j);\ w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ for k∈{1,2,3, …,n}.

Theorem5.5 Let $\tilde{\alpha}_j = \langle (a_j, b_j, c_j);\ w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ (j = 1,2,3,…,n) be a collection of single valued triangular neutrosophic numbers and w = $(w_1, w_2, \ldots, w_n)^T$ is a weight vector associated with $w_j \geq 0$, and $\sum_{j=1}^{n} w_j = 1$. Then their aggregated value by using GSVTNGOAO operator is also a neutrosophic number and

GSVTNGOAO $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$

$= g^{-1}(\sum_{j=1}^{n} w_j g(\tilde{b}_{(j)}))$ =

$$\langle [g^{-1}(\sum_{j=1}^{n} w_j g(a_{(j)})), g^{-1}(\sum_{j=1}^{n} w_j g(b_{(j)})), g^{-1}(\sum_{j=1}^{n} w_j g(c_{(j)}))]; \bigwedge_{j=1}^{n} w_{\tilde{a}_j} \bigvee_{j=1}^{n} u_{\tilde{a}_j} \bigvee_{j=1}^{n} y_{\tilde{a}_j} \rangle$$

where g is a continuous strictly monotone increasing function and k is the largest generalized single valued triangular neutrosophic number obtained by using the new score function of $\tilde{b}_{(j)}$; $\tilde{\alpha}_j$ = $\langle (a_j, b_j, c_j);\ w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ for k∈{1,2,3, …,n}.

Proof: Proof is made similar to Theorem 5.2 using Definition 5.4.

Lemma 5.6 Let $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ and $\tilde{b}_2 = \langle (d_j, e_j, f_j); w_{\tilde{b}_j}, u_{\tilde{b}_j}, y_{\tilde{b}_j} \rangle$ (j = 1,2,3,…,n) be collections of generalized single valued triangular neutrosophic numbers and $\tilde{a} = \langle (a, b, c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ be a generalized single valued triangular neutrosophic number. w = $(w_1, w_2, \ldots, w_n)^T$ be a weight vector associated with $w_j \geq 0$, and $\sum_{j=1}^{n} w_j = 1$.

1) If $\tilde{a}_j = \tilde{a}$ (j = 1, 2, 3, …,n), then GSVTNGOAO $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}$

2) If $\tilde{a}_j^{-} = \langle (\min\{a_j\}, \min\{b_j\}, \min\{c_j\}); \min\{w_{\tilde{a}_j}\}, \max\{u_{\tilde{a}_j}\}, \max\{y_{\tilde{a}_j}\} \rangle$

$\tilde{a}_j^{+} = \langle (\max\{a_j\}, \max\{b_j\}, \max\{c_j\}); \max\{w_{\tilde{a}_j}\}, \min\{u_{\tilde{a}_j}\}, \min\{y_{\tilde{a}_j}\} \rangle$

Then,

$$\tilde{a}_j^{-} \leq \text{GSVTNGOAO}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \tilde{a}_j^{+}$$

3) If $a_j \leq d_j, b_j \leq e_j, c_j \leq f_j, w_{\tilde{a}_j} \leq w_{\tilde{b}_j}, u_{\tilde{a}_j} \geq u_{\tilde{b}_j}, y_{\tilde{a}_j} \geq y_{\tilde{b}_j}$ for all j then,

$$\text{GSVTNGOAO}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \text{GSVTNGOAO}(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n)$$

Proof:

The prof of 1) can be done similar to the proof of the theorem 5.3.

The proof of 2) and 3) can easily be done from proposition 4.6 given for the Hamming distance depending on the scoring function in the definition 5.4 and in the section 6.

Corollary 5.7: If g (x) = x (r = 1) is taken in Definition 5.1, the operator in Definition 2.13 is obtained. Similarly, if g (x) = x (r = 1) is taken in 5.2, the operator in Definition 2.14 is obtained.

Note 5.8: If $g(x)=x^r$ is taken in the operators in Definition 5.1 and Definition 5.2; r value should not be taken as an odd number. Indeterminacy emerges when any of the values a_j, b_j, c_j of a generalized single valued triangular neutrosophic number $\tilde{a}_j = \langle (a_j, b_j, c_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle$ takes a negative real number value.

6. MULTI – ATTRIBUTE GROUP DECISION MAKING METHOD BASED ON THE SVTNGWAO OPERATOR

For a multi-attribute group decision making problem, let E = $\{e_1, e_2, \ldots, e_n\}$ be a set of experts (or DMs), A = $\{A_1, A_2, \ldots, A_m\}$ be set of alternatives, X = $\{x_1, x_2, \ldots, x_p\}$ be set of attributes. Assume that the rating of alternative A_i on attribute x_j given by expert e_k is represented by single valued neutrosophic number $\tilde{a}^k{}_{ij}$ = $(a^k{}_{ij}, b^k{}_{ij}, c^k{}_{ij})$ (i = 1,2,…,m; j = 1,2,…,p; k = 1,2,…,n). Additionally, let g be a continuous strictly monotone increasing function. Now let's take the steps we will follow to solve the multi-attribute group decision making problem.

i) The decision matrices obtained by the decision makers are found as $\tilde{D}^k = (\tilde{a}^k{}_{ij})_{mxp}$ (i = 1,2,…,m; j = 1,2,…,p; k = 1,2,…,n).

ii) $\tilde{D}^k$ decision matrices; for $\tilde{a}^k{}_{ij}$ single valued neutrosophic numbers, $\tilde{D}^k{}_T$ matrices are formed that consist of $\widetilde{a_T}^k{}_{ij}$ converted single valued triangular neutrosophic numbers.

iii) Let ω = $(\omega_1, \omega_2, \dots, \omega_n)^T$ f ; be the weight vector of decision makers with $\omega_j \geq 0$, and $\sum_{j=1}^{n} \omega_j = 1$. Accordingly, the weighted decision matrix is $\tilde{D}^k{}_w = (\omega_k \widetilde{a_T}^k{}_{ij})_{mxp}$ (i = 1,2,…,m; j = 1,2,…,p; k = 1,2,…,n).

iv) GSVTNGWAO is the operator in the definition 5.1; the unified decision matrix $\tilde{D}_u$=($\widetilde{d_u}_{ij})_{mxp}$ obtained from the weighted decision matrices. Here;

$\widetilde{d_u}_{i1}$ = GSVTNGWAO($\varphi_k \widetilde{a_T}^1{}_{i1}$, $\varphi_k \widetilde{a_T}^2{}_{i1}$,…, $\varphi_k \widetilde{a_T}^k{}_{i1}$),

$\widetilde{d_u}_{i2}$ = GSVTNGWAO($\varphi_k \widetilde{a_T}^1{}_{i2}$, $\varphi_k \widetilde{a_T}^2{}_{i2}$,…, $\varphi_k \widetilde{a_T}^k{}_{i2}$),

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$\widetilde{d_u}_{ip}$ = GSVTNGWAO($\varphi_k \widetilde{a_T}^1{}_{ip}$, $\varphi_k \widetilde{a_T}^2{}_{ip}$,…, $\varphi_k \widetilde{a_T}^k{}_{ip}$). Where; (i = 1, 2, 3, …, m).

Also here, the weight vector to be used for the GSVTNGWAO operator is φ = $(\varphi_1, \varphi_2, \dots, \varphi_n)^T$ with $\varphi_j \geq 0$, and $\sum_{j=1}^{n} \varphi_j = 1$.

v) $\tilde{D}_u$ =($\widetilde{d_u}_{ij})_{mxp}$ be the unified decision matrix; let w = $(w_1, w_2, \dots, w_p)^T$ weight vector of $\{x_1, x_2, \dots, x_p\}$ with $w_j \geq 0$, and $\sum_{j=1}^{n} w_j = 1$. single valued triangular neutrosophic numbers for the $\{A_1, A_2, \dots, A_m\}$ alternatives is ;

$\widetilde{A_t}$ = GSVTNGWAO$_t$($\widetilde{d_u}_{t1}$, $\widetilde{d_u}_{t2}$,…, $\widetilde{d_u}_{tp}$) (t = 1,2,…,m).

vi) Single valued triangular neutrosophic numbers $\widetilde{A_t}$ (t = 1,2,…,m)for the $\{A_1, A_2, \dots, A_m\}$ alternatives are compared with one of the new score functions in definition 4.7, definition 4.8 or definition 4.9, and the best alternative is found. Here; there is a score function according to the truth value in definition 4.7, according to the indeterminacy value in definition 4.8 and according to the falsity value in definition 4.9.

Corollary 6.1: In this method for single valued neutrosophic numbers, starting directly from the second step, single valued triangular neutrosophic numbers can be taken and processed. Thus the method we have obtained can be used for single valued triangular neutrosophic numbers or generalized single valued triangular neutrosophic numbers.

Example 6.2: A pharmaceutical company wants to choose the most appropriate diabetes drug from four alternatives $\{A_1, A_2, A_3, A_4\}$. For this, a decision committee of three pharmacological specialists $\{e_1, e_2, e_3\}$ was established. This decision commission will review alternative medicines in three qualities. These qualities are; the dose rate of the drug isx_1, suitable for all ages x_2 and its cost is x_3. For the decision committee $\{e_1, e_2, e_3\}$ weight vector ω =$(0.4,0.3,0.3)^T$, (x_1, x_2, x_3). Weight vector for qualities are w = $(0.4,0.3,0.3)^T$ and φ = $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Additionally, let g (x) = x^r is a continuous strictly monotone increasing function. Now let g (x) = x for r = 1 and then perform the steps in section 5.1 according to the truth value of the transformations and scoring function.

i) The table showing single valued neutrosophic numbers for the alternatives evaluated by the decision makers is as follows.

Table 3: (Decision matrix created by e_1 decision maker)

	u_1	u_2	u_3
x_1	(0.8,0.5,0.3)	(0.3,0.8,0.6)	(0.8,0.1,0.3)
x_2	(0.3,0.4,0.5)	(0.8,0.2,0.3)	(0.6,0.2,0.3)
x_3	(0.7,0.2,0.3)	(0.6,0.1,0.3)	(0.4,0.2,0.6)
x_4	(0.4,0.5,0.3)	(0.9,0.1,0.1)	(0.8,0.4,0.1)
x_5	(0.4,0.3,0.2)	(0.7,0.1,0.3)	(0.9,0.1,0.0)

Table 4: (decision matrix created by e_2 decision maker)

	u_1	u_2	u_3
x_1	(0.6,0.3,0.3)	(0.5,0.3,0.3)	(0.7,0.2,0.3)
x_2	(0.6,0.2,0.3)	(0.6,0.3,0.2)	(0.4,0.3,0.4)
x_3	(0.8,0.2,0.1)	(0.6,0.2,0.2)	(0.5,0.3,0.3)
x_4	(0.6,0.2,0.2)	(0.7,0.3,0.1)	(0.8,0.2,0.2)
x_5	(0.7,0.2,0.2)	(0.8,0.1,0.2)	(0.7,0.3,0.1)

Table 5: (decision matrix created by e_3 decision maker**)**

	u_1	u_2	u_3
x_1	(0.7,0.2,0.2)	(0.7,0.2,0.2)	(0.9,0.1,0.1)
x_2	(0.7,0.2,0.3)	(0.7,0.1,0.2)	(0.5,0.3,0.2)
x_3	(0.7,0.1,0.3)	(0.5,0.3,0.3)	(0.6,0.2,0.3)
x_4	(0.5,0.1,0.3)	(0.8,0.1,0.2)	(0.9,0.1,0.2)
x_5	(0.8,0.1,0.1)	(0.6,0.3,0.2)	(0.8,0.2,0.1)

ii) Transformed decision-making matrices created by decision makers;

Table 6: (transformed decision matrix created by e_1 decision maker**)**

	u_1	u_2	u_3
x_1	$\langle(0.0,1.5,2.8); 0.8,0.5,0.3\rangle$	$\langle(-1.1,-0.4,0.1); 0.3,0.8,0.6\rangle$	$\langle(0.4,1.9,3.6); 0.8,0.1,0.3\rangle$
x_2	$\langle(-0.6,0.2,1.1); 0.3,0.4,0.5\rangle$	$\langle(0.3,1.8,3.4); 0.8,0.2,0.3\rangle$	$\langle(0.1,1.4,2.8); 0.6,0.2,0.3\rangle$
x_3	$\langle(0.2,1.6,3.1); 0.7,0.2,0.3\rangle$	$\langle(0.2,1.5,3.0); 0.6,0.1,0.3\rangle$	$\langle(-0.4,0.4,1.6); 0.4,0.2,0.6\rangle$
x_4	$\langle(-0.4,0.7,1.6); 0.4,0.5,0.3\rangle$	$\langle(0.7,2.5,4.3); 0.9,0.1,0.1\rangle$	$\langle(0.3,2.0,3.4); 0.8,0.4,0.1\rangle$
x_5	$\langle(-0.1,1.1,2.2); 0.4,0.3,0.2\rangle$	$\langle(0.3,1.7,3.3); 0.7,0.1,0.3\rangle$	$\langle(0.8,2.7,4.5); 0.9,0.1,0.0\rangle$

Table 7: (transformed decision matrix created by e_2 decision maker)

	u_1	u_2	u_3
x_1	$\langle(0.0,1.3,2.6); 0.6,0.3,0.3\rangle$	$\langle(-0.1,1.1,2.3); 0.5,0.3,0.3\rangle$	$\langle(0.2,1.6,3.1); 0.7,0.2,0.3\rangle$
x_2	$\langle(0.1,1.4,2.8); 0.6,0.2,0.3\rangle$	$\langle(0.1,1.5,2.8); 0.6,0.3,0.2\rangle$	$\langle(-0.3,0.7,1.8); 0.4,0.3,0.4\rangle$
x_3	$\langle(0.5,2.2,3.8); 0.8,0.2,0.1\rangle$	$\langle(0.2,1.6,3.0); 0.6,0.2,0.2\rangle$	$\langle(-0.1,1.1,2.3); 0.5,0.3,0.3\rangle$
x_4	$\langle(0.2,1.6,3.0); 0.6,0.2,0.2\rangle$	$\langle(0.3,1.9,3.3); 0.7,0.3,0.1\rangle$	$\langle(0.4,2.0,3.6); 0.8,0.2,0.2\rangle$
x_5	$\langle(0.3,1.8,3.3); 0.7,0.2,0.2\rangle$	$\langle(0.5,2.1,3.8); 0.8,0.1,0.2\rangle$	$\langle(0.3,1.9,3.3); 0.7,0.3,0.1\rangle$

Table 8: (transformed decision matrix created by e_3 decision maker)

	u_1	u_2	u_3
x_1	$\langle(0.3,1.8,3.3);0.7,0.2,0.2\rangle$	$\langle(0.3,1.8,3.3);0.7,0.2,0.2\rangle$	$\langle(0.7,2.5,4.3);0.9,0.1,0.1\rangle$
x_2	$\langle(0.2,1.6,3.1);0.7,0.2,0.3\rangle$	$\langle(0.4,1.9,3.5);0.7,0.1,0.2\rangle$	$\langle(0.0,1.3,2.5);0.5,0.3,0.2\rangle$
x_3	$\langle(0.3,1.7,3.3);0.7,0.1,.0.3\rangle$	$\langle(-0.1,1.1,2.3);0.5,,0.3,0.3\rangle$	$\langle(0.1,1.4,2.8);0.6,0.2,0.3\rangle$
x_4	$\langle(0.1,1.3,2.7);0.5,0.1,0.3\rangle$	$\langle(0.5,2.1,3.8);0.8,0.1,0.2\rangle$	$\langle(0.6,2.3,4.1);0.9,0.1,0.2\rangle$
x_5	$\langle(0.6,2.3,4.0);0.8,0.1,0.1\rangle$	$\langle(0.1,1.5,2.8);0.6,0.3,0.2\rangle$	$\langle(0.5,2.2,3.8);0.8,0.2,0.1\rangle$

iii) Transformed weighted decision matrices generated by decision makers;

Table 9: (transformed weighted decision matrix created by $e_1$decision maker**)**

	u_1	u_2	u_3
x_1	$\langle(0.00,0.60,1.12);0.8,0.5,0.3\rangle$	$\langle(-0.44,-0.16,0.04);0.3,0.8,0.6\rangle$	$\langle(0.16,0.76,1.44);0.8,0.1,0.3\rangle$
x_2	$\langle(-0.24,0.08,0.44);0.3,0.4,0.5\rangle$	$\langle(0.12,0.72,1.36);0.8,0.2,0.3\rangle$	$\langle(0.04,0.56,1.12);0.6,0.2,0.3\rangle$
x_3	$\langle(0.08,0.64,1.24);0.7,0.2,0.3\rangle$	$\langle(0.08,0.60,1.20);0.6,0.1,0.3\rangle$	$\langle(-0.16,0.16,0.64);0.4,0.2,0.6\rangle$
x_4	$\langle(-0.16,0.28,0.64);0.4,0.5,0.3\rangle$	$\langle(0.28,1.00,1.72);0.9,0.1,0.1\rangle$	$\langle(0.12,0.80,1.36);0.8,0.4,0.1\rangle$
x_5	$\langle(-0.04,0.44,0.88);0.4,0.3,0.2\rangle$	$\langle(0.12,0.68,1.32);0.7,0.1,0.3\rangle$	$\langle(0.32,1.08,1.80);0.9,0.1,0.0\rangle$

Table 10: (transformed weighted decision matrix created by e_2 decision maker**)**

	u_1	u_2	u_3
x_1	$\langle(0.00,0.39,0.78);0.6,0.3,0.3\rangle$	$\langle(-0.03,0.33,0.69);0.5,0.3,0.3\rangle$	$\langle(0.06,0.48,0.93);0.7,0.2,0.3\rangle$
x_2	$\langle(0.03,0.42,0.84);0.6,0.2,0.3\rangle$	$\langle(0.03,0.45,0.84);0.6,0.3,0.2\rangle$	$\langle(-0.09,0.21,0.54);0.4,0.3,0.4\rangle$
x_3	$\langle(0.15,0.66,1.14);0.8,0.2,0.1\rangle$	$\langle(0.06,0.48,0.90);0.6,0.2,0.2\rangle$	$\langle(-0.03,0.33,0.69);0.5,0.3,0.3\rangle$
x_4	$\langle(0.06,0.48,0.90);0.6,0.2,0.2\rangle$	$\langle(0.09,0.57,0.99);0.7,0.3,0.1\rangle$	$\langle(0.12,0.60,1.08);0.8,0.2,0.2\rangle$
x_5	$\langle(0.09,0.54,0.99);0.7,0.2,0.2\rangle$	$\langle(0.15,0.63,1.14);0.8,0.1,0.2\rangle$	$\langle(0.09,0.57,0.99);0.7,0.3,0.1\rangle$

Table 11: (transformed weighted decision matrix created by e_3 decision maker**)**

	u_1	u_2	u_3
x_1	$\langle(0.09,0.54,0.99);0.7,0.2,0.2\rangle$	$\langle(0.09,0.54,0.99);0.7,0.2,0.2\rangle$	$\langle(0.21,0.75,1.29);0.9,0.1,0.1\rangle$
x_2	$\langle(0.06,0.48,0.93);0.7,0.2,0.3\rangle$	$\langle(0.12,0.57,1.05);0.7,0.1,0.2\rangle$	$\langle(0.00,0.39,0.75);0.5,0.3,0.2\rangle$
x_3	$\langle(0.09,0.51,0.99);0.7,0.1,.0.3\rangle$	$\langle(-0.03,0.33,0.69);0.5,,0.3,0.3\rangle$	$\langle(0.03,0.42,0.84);0.6,0.2,0.3\rangle$
x_4	$\langle(0.03,0.39,0.81);0.5,0.1,0.3\rangle$	$\langle(0.15,0.63,1.14);0.8,0.1,0.2\rangle$	$\langle(0.18,0.69,1.23);0.9,0.1,0.2\rangle$
x_5	$\langle(0.18,0.69,1.20);0.8,0.1,0.1\rangle$	$\langle(0.03,0.45,0.84);0.6,0.3,0.2\rangle$	$\langle(0.15,0.66,1.14);0.8,0.2,0.1\rangle$

iv)The resulting unified decision matrix;

Table 12: (unified decision matrix**)**

	u_1	u_2	u_3
x_1	$\langle(0.027,0.459,0.867);0.6,0.5,0.3\rangle$	$\langle(-0.144,0.213,0.516);0.3,0.8,0.6\rangle$	$\langle(0.129,0.597,1.98);0.7,0.2,0.3\rangle$
x_2	$\langle(-0.045,0.294,0.663);0.3,0.4,0.5\rangle$	$\langle(0.081,0.522,0.975);0.6,0.3,0.3\rangle$	$\langle(-0.015,0.348,0.723);0.4,0.3,0.4\rangle$
x_3	$\langle(0.096,0.543,1.011);0.7,0.2,0.3\rangle$	$\langle(0.033,0.423,0.837);0.5,0.3,0.3\rangle$	$\langle(-0.048,0.273,0.651);0.4,0.3,0.6\rangle$
x_4	$\langle(-0.021,0.345,0.705);0.4,0.5,0.3\rangle$	$\langle(0.156,0.660,1.155);0.7,0.3,0.2\rangle$	$\langle(0.126,0.627,1.101);0.8,0.4,0.2\rangle$
x_5	$\langle(0.069,0.501,0.921);0.4,0.3,0.2\rangle$	$\langle(0.009,0.528,0.990);0.6,0.3,0.3\rangle$	$\langle(0.168,0.693,1.179);0.7,0.3,0.1\rangle$

v) Generalized single valued triangular neutrosophic numbers obtained from the unified decision matrix for the alternatives;

$$\widetilde{A_1} = \mathrm{GSVTNGWAO}_{x_1}(u_1, u_2, u_3) = \langle(0.015,0.426,0.831);0.3,0.8,0.6\rangle$$

$$\widetilde{A_2} = \mathrm{GSVTNGWAO}_{x_2}(u_1, u_2, u_3) = \langle(0.001,0.378,,0.774);0.3,0.4,0.5\rangle$$

$$\widetilde{A_3} = \mathrm{GSVTNGWAO}_{x_3}(u_1, u_2, u_3) = \langle(0.033,0.426,0.850);0.4,0.3,0.6\rangle$$

$$\widetilde{A_4} = \mathrm{GSVTNGWAO}_{x_4}(u_1, u_2, u_3) = \langle(0.076,0.524,0.958);0.4,0.5,0.3\rangle$$

$$\widetilde{A_5} = \mathrm{GSVTNGWAO}_{x_5}(u_1, u_2, u_3) = \langle(0.105,0.566,1.019);0.4,0.3,0.3\rangle$$

vi) According to the values in v);

$S_T(\widetilde{A_1}) = 2{,}15$

$S_T(\widetilde{A_2}) = 2{,}11$

$S_T(\widetilde{A_3}) = 2{,}08$

$S_T(\widetilde{A_4}) = 2{,}04$

$S_T(\widetilde{A_5}) = 1{,}99$

Hence $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

If g(x) = x^2 is taken in example 6.2 for r = 2;

$S_T(\widetilde{A_1}) = 2{,}12$

$S_T(\widetilde{A_2}) = 2{,}08$

$S_T(\widetilde{A_3}) = 2{,}06$

$S_T(\widetilde{A_4}) = 2{,}01$

$S_T(\widetilde{A_5}) = 1{,}97$

Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug isx_5.

If g(x)= x^5 is taken in example 6.2 for r = 5;

$S_T(\widetilde{A_1}) = 2{,}14$

$S_T(\widetilde{A_2}) = 2{,}09$

$S_T(\widetilde{A_3}) = 2{,}04$

$S_T(\widetilde{A_4}) = 1{,}97$

$S_T(\widetilde{A_5}) = 1{,}92$

Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug isx_5.

If g(x)= $x^{0.04}$ is taken in example 6.2 for r = 0.04 ;

$S_T(\widetilde{A_1}) = 2{,}246$

$S_T(\widetilde{A_2}) = 2{,}240$

$S_T(\widetilde{A_3})$ = 2,237

$S_T(\widetilde{A_4})$ = 2,234

$S_T(\widetilde{A_5})$ = 2,231

Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

Example 6.3: If the same assumption in Example 6.2 applies to decision making based on indeterminacy;
i) If g(x)= x is taken for r = 1;

$S_I(\widetilde{A_1})$ = 1,185

$S_I(\widetilde{A_2})$ = 1,160

$S_I(\widetilde{A_3})$ = 1,148

$S_I(\widetilde{A_4})$ = 1,113

$S_I(\widetilde{A_5})$ = 1,087
Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

ii) If g(x) = x^2 is taken for r = 2;

$S_I(\widetilde{A_1})$ = 1,169

$S_I(\widetilde{A_2})$ = 1,140

$S_I(\widetilde{A_3})$ = 1,133

$S_I(\widetilde{A_4})$ = 1,094

$S_I(\widetilde{A_5})$= 1,071
Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

iii) If g(x)= x^5 is taken for r = 5;

$S_I(\widetilde{A_1})$ = 1,195

$S_I(\widetilde{A_2})$ = 1,163

$S_I(\widetilde{A_3})$ = 1,127

$S_I(\widetilde{A_4})$ = 1,072

$S_I(\widetilde{A_5})$ = 1,045
Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

iv) If g(x)= $x^{0.04}$ is taken for r = 0.04 ;

$S_I(\widetilde{A_1})$ = 1,245

$S_I(\widetilde{A_2})$ = 1,243

$S_I(\widetilde{A_3})$ = 1,242

$S_I(\widetilde{A_4})$ = 1,241

$S_I(\widetilde{A_5})$ = 1,238
Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

Example 6.4: If the same assumption in Example 6.2 applies to decision making based on falsity;

i) If g(x) = x is taken for r = 1;

$S_F(\widetilde{A_1})$ = 1,173

$S_F(\widetilde{A_2})$ = 1,127

$S_F(\widetilde{A_3})$ = 1,104

$S_F(\widetilde{A_4})$ = 1,094

$S_F(\widetilde{A_5})$ = 1,063
Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

ii) If g(x)= x^2 is taken for r = 2;

$S_F(\widetilde{A_1})$ = 1,154

$S_F(\widetilde{A_2})$ = 1,106

$S_F(\widetilde{A_3})$ = 1,087

$S_F(\widetilde{A_4})$ = 1,073

$S_F(\widetilde{A_5})$ = 1,046
Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

iii) If g(x)= x^5 is taken for r = 5 ;

$S_F(\widetilde{A_1})$ = 1,179

$S_F(\widetilde{A_2})$ = 1,129

$S_F(\widetilde{A_3})$ = 1,076

$S_F(\widetilde{A_4})$ = 1,055

$S_F(\widetilde{A_5}) = 1{,}027$
Hence, $x_1 < x_2 < x_3 < x_4 < x_5$. So the best alternative drug is x_5.

iv) If $g(x) = x^{0.04}$ is taken for r = 0.04;

$S_F(\widetilde{A_1}) = 1{,}246$

$S_F(\widetilde{A_2}) = 1{,}240$

$S_F(\widetilde{A_3}) = 1{,}238$

$S_F(\widetilde{A_4}) = 1{,}239$

$S_F(\widetilde{A_5}) = 1{,}236$ hence $x_1 < x_2 < x_4 < x_3 < x_5$. So the best alternative drug is x_5.

Table 13: (Results obtained according to r in Example 6.2, example6.3 and example 6.4**)**

Value of R	The result according to the value of truth	The result according to the value of indeterminacy	The result according to the value of falsity
r=1	$x_1 < x_2 < x_3 < x_4 < x_5$	$x_1 < x_2 < x_3 < x_4 < x_5$	$x_1 < x_2 < x_3 < x_4 < x_5$
r=2	$x_1 < x_2 < x_3 < x_4 < x_5$	$x_1 < x_2 < x_3 < x_4 < x_5$	$x_1 < x_2 < x_3 < x_4 < x_5$
r=5	$x_1 < x_2 < x_3 < x_4 < x_5$	$x_1 < x_2 < x_3 < x_4 < x_5$	$x_1 < x_2 < x_3 < x_4 < x_5$
r=0.04	$x_1 < x_2 < x_3 < x_4 < x_5$	$x_1 < x_2 < x_3 < x_4 < x_5$	$x_1 < x_2 < x_4 < x_3 < x_5$

7. COMPARISON ANALYSIS AND DISCUSSION

Table 14: (Results obtained from methods**)**

	r=1	r=2	r=5
Method 1	$x_1 < x_2 < x_3 < x_4$	$x_2 < x_1 < x_3 < x_4$	$x_1 < x_2 < x_3 < x_4$
Method 2	$x_2 < x_1 < x_3 < x_4$	$x_2 < x_1 < x_3 < x_4$	$x_2 < x_1 < x_3 < x_4$
Method 3	$x_3 < x_1 < x_2 < x_4$	$x_3 < x_2 < x_1 < x_4$	$x_1 < x_3 < x_2 < x_4$
Method 4	$x_1 < x_2 < x_3 < x_4$	$x_1 < x_2 < x_3 < x_4$	$x_1 < x_2 < x_3 < x_4$

To be able to see the effect of the method given in section 6; we compared the results of the method with those of the method in Section 6. For the same "r" values were comparable a method obtained according to the truth value, indeterminacy value and falsity value in section 6. According to Table 14; the best alternative to the results from all methods is the same and it is x_4. Besides; Hamacher aggregation operators are used for single valued neutrosophic numbers. In the chapter 5, we used generalized single valued triangular neutrosophic numbers obtained by transformed single valued neutrosophic numbers. With these numbers, we used the operators we have generalized to the operators given for intuitionistic fuzzy numbers. These operators include previously given operators for single valued triangular neutrosophic numbers. Thus, in section 6 we used single valued triangular neutrosophic numbers and more general operators used in many decision making methods. We also compared the score and certainty functions used in Table 1 and used in Section 6. In this comparison, the values are not equal according to the scoring functions in Section 6 and therefore we have achieved different results. In addition, we have the possibility to obtain separate results according to the value of truth, falsity and indeterminacy in order to decide on the method in section 6. Thus, we have obtained a more comprehensive result. For this reason, the method in section 6 is effective and applicable.

8. CONCLUSION

In this study, we generalized single valued triangular neutrosophic numbers. Thus, we have defined a new set of numbers that can be more useful and can be very applicable. We have also obtained generalized single valued triangular neutrosophic numbers by converting single valued neutrosophic numbers according to their truth, indeterminacy and falsity values separately. Thus, single valued neutrosophic numbers are transformed into generalized single valued triangular neutrosophic numbers, which are a special case and have a lot of application field. We then defined the Hamming distance for single valued triangular neutrosophic numbers and gave some properties. We have defined the scoring and certainty functions based on this defined distance. We also extended operators for intuitionistic fuzzy numbers to single valued triangular neutrosophic numbers. Finally, we compared multi-attribute group decision making with generalized operators and new score functions, and compared the results with a previous multi-attribute group decision making application. In addition to this, the applied multi-attribute group decision making method can be used in many different scientific researches.

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