Annals of Optimization Theory and Practice

Volume 3, Number 1, 47-71
October 2020

DOI: 10.22121/aotp.2020.240887.1034

Geometric operators based on linguistic interval-valued intuitionistic neutrosophic fuzzy number and their application in decision making

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(Received: July 24, 2020 / Accepted: October 8, 2020)

Abstract The paper aims to give some new kinds of operational laws named as neutrality addition and scalar multiplication for the pairs of linguistic interval-valued intuitionistic neutrosophic fuzzy number. The main idea behind these operations is to include the linguistic interval-valued intuitionistic neutrosophic fuzzy number of the decision-maker and score function. We define the linguistic interval-valued intuitionistic neutrosophic fuzzy number and operational laws. We introduce the three geometric operators including, linguistic interval-valued intuitionistic neutrosophic fuzzy weighted geometric operator, linguistic interval-valued intuitionistic neutrosophic fuzzy ordered weighted geometric operator and linguistic interval-valued intuitionistic neutrosophic fuzzy weighted hybrid geometric operatorl. Finally, a multiattribute group decision-making approach based on the proposed operators is presented and investigated with numerous numerical examples.

Keywords Linguistic; Neutrosophic set; Geometric operators, MCDM

1. Introduction

In real life the decision-making problems, the decision information is not enough to determine the information. Smarandache (1999) gave the idea of neutrosophic set which is much better concept to express that kind of information. After that Wang *et al.* (2005), Wang *et al.* (2010) gave the concepts of interval neutrosophic set (INS) and single valued neutrosophic set (SVNS) which are the subclasses of neutrosophic set, and defined the set-theoratic operators and much more properties of SVNSs and INSs. Moreover, Chi and Liu enlarged a TOPSIS method to interval neutrosophioc multiple attribute decision-making problem to rank alternatives. Broumi *et al.* (2015) introduced

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the Hamming and Euclidean distance between INSs and the distances-based similarity measures and apply them to multiple attribute decision-making in interval neutrosophic setting. Ye proposed the idea of a simplified neutrosophic set (SNS), which is also the subclass of neutrosophic set and includes a SVNS and an INS and defined basic operational laws SNSs, and then he explore a simplified neutrosophic weighted arithmetic averaging (SNWAA) operator, simplified neutrosophic weighted geomatric averaging (SNWGA) operator and apply that operator to the multiple attribute decision-making under simplified neutrosophic environment. Sometime the decision makers cannot give a specific numeric or fuzzy value for attributes according to situation. So at that situation linguistic terms are very useful tool to express the thinking of decision maker epically for qualitative data for example the performance of a fabric company. Decision maker can easily express his thinking in linguistic terms for good results. LA Zadeh firstly establish the base for linguistic variables and use it in fuzzy reasoning.

Garg (2020) introduced the neutral characters of the decision-maker towards the preferences of the objects. Garg (2020) introduced the immediate probability-based averaging and geometric aggregation operators for the collection of the single-valued and interval neutrosophic sets.

Garg (2019) presented the some averaging power operators, namely, linguistic single-valued neutrosophic (LSVN) power averaging, weighted average, ordered weighted average, and hybrid averaging AOs along with their desirable properties. Garg (2019) introduced the weighted averaging and geometric aggregation operators (AOs) to collaborate the PLSVNSs into a single one. Further, we present two algorithms based on a complex proportional assessment (COPRAS) method. Garg (2018) presented the different weighted averaging and geometric aggregation operators.

Harish (2020) introduced the some improved score functions to rank the normal intuitionistic and interval-valued intuitionistic sets. Garg (2020) proposed the several weighted averages and geometric aggregating operators to aggregate the linguistic interval-valued Pythagorean fuzzy information.

Li *et al.* (2020) introduced to overcome the shortcomings in previous studies. Mishra *et al.* (2020) introduced the unbalanced fully intuitionistic fuzzy transportation problem. Chiao (2020) introduced the decision making linguistic judgments, the fuzzy real time control systems with linguistic. Jin *et al.* (2020) proposed the construction approach for the multiplicative consistent PHFPRs. Guo *et al.* (2020) introduced the fuzzy formal context of concurrent faults and known faults. Suzan and Yavuzer (2020) introduced the DEMATEL method allows one to identify and analyse significant diseases in internal medicine by considering the cause-and-effect relationship diagram. Liu et al. (2019) introduced the computed to determine the weights of subgroups. Smarandache (2005) introduced the Neutrosophic set-a generalization of the intuitionistic fuzzy set. Ali and Smarandache (2017) introduced the Complex neutrosophic set. Abdel-Basset et al. (2020) introduced the deterministic project scheduling and time-cost trade-offs conflict with the real situation. Khatter (2020) introduced the score and accuracy functions for the proposed interval valued trapezoidal neutrosophic number. Rashno *et al.* (2020)

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presented the two conditions based on distance from cluster centers and value of indeterminacy, are considered for each data point.

Zadeh give the concept of linguistic variable and applied it to the fuzzy reasoning. After that Herrera et al. establish a model of accord in group decision-making under linguistic analysis. Later on, Herrera and Herrera and viedma (1996) extended a linguistic decision survey for solving decision-making problems with linguistic information. Moreover, Xu (2006) present a linguistic hybrid arithmetic averaging operator for multiple attribute group decision-making problem with linguistic information. Wang and Li (2009) combine the linguistic variable with IFS, and give the concept of intuitionistic linguistic fuzzy number (ILFN). Wang and Li (2009) present the operational laws, expected value, score function and accuracy function of ILFNs and elaborate the intuitionistic linguistic weighted arithmetic average (ILWAA) operator and intuitionistic linguistic weighted geometric average (ILWGA) operator and then they applied that operator to multiple attribute group decision-making problem (MADM) with ILFNs. After that Jun Ye defined interval neutrosophic linguistic number and also defined some aggregation operator and then he apply that operator to the multiple attribute group decision-making problem. By taking the motivation from the above information in this paper, we discuss some linguistic interval-value intuitionistic neutrosopic fuzzy sets (LIVINFSs) and their applications to multi-criteria decision making (MCDM) problems. A linguistic interval-value intuitionistic fuzzy set is the extension of linguistic intuitionistic fuzzy set. IFS deals with membership association and non-membership association but an intuitionistic neutrosophic set (INS) deals with truth association, false association and uncertain association. So it is better to use INS as compare to IFS. That is the reason in this paper we use intuitionistic neutrosophic set (INS) as compare to IFS to get more accurate information.

After defining the LIVINFS we define some geometric operators on it as, LIVINF weighted geometric operator, LIVINF ordered weighted geometric operator, LIVINF hybrid geometric operator. Moreover, we discuss MCDM problem in this paper. We also calculate the score function to determine the ranking. Then we define an example on these operators by using the given information. Further we gave a comparison of our proposed method with the existing method to understand which one is better.

In this section 2, we introduce the basic concept. In section 3, we define the LIVINFNs and operational laws. In section 4, we introduce three aggregation operators. In section 5, we develop the MCDM method. In section 6, we introduce the numerical application. In section 7, we introduce the comparison analysis. In section 8, we given in conclusion.

2.Basic Concepts

Definition.2.1. Suppose X is a nonempty set. Then, the formula $\gamma = \{\langle \mu_{\gamma(x)} >: x \in X\rangle\}$ defines a fuzzy set, where $\mu_{\gamma(x)}$ represents a function from X to$[0,1]$ that defines membership of an element x in X.

Definition.2.2. Suppose X represents a universal set and consider an element $x \in X$ An

IVNS Z in X is $Z = \{x(A_A(x), B_A(x), C_A(x))\ x \in X\}$, where$A_A(x), B_A(x), C_A(x)$ are the falsity membership, indeterminacy-membership and the truth-membership function separately, for all x in X we have that $0 \leq \sup A_A(x) + \sup B_A(x) + \sup C_A(x) \leq 3$

Definition.2.3. Let$S_{[0,h]}$ be a continuous linguistic term set and X be a fixed set. Then a LIFS Z is defined as $Z = \{(u, S_{\alpha(u)}, S_{\beta(u)}) | u \in X\}$ where$S_\alpha, S_\beta \in S_{[0,h]}$ such that $0 \leq \alpha + \beta \leq h$ and$S_\alpha(S_\beta)$ represents the linguistic membership (non-membership) degree. The linguistic indeterminacy$S_{\bar{\varpi}}$ is defined as$S_{\bar{\varpi}} = S_{h-\alpha-\beta}$. The pair$(S_\alpha, S_\beta)$ usually, is represented by γ and called LIFN, where$S_\alpha, S_\beta \in S_{[0,h]}$ such that .The LIFN is called original if$S_\alpha, S_\beta \in S$ and actual otherwise.

3. Linguistic interval-valued intuitionistic neutrosophic fuzzy numbers and operational laws Motivation

Therefore, motivated by these problems of the current Linguistic interval-valued intuitionistic neutrosophic fuzzy numbers and operational laws, there is a need to modify the existing operational laws and their based linguistic interval-valued intuitionistic neutrosophic fuzzy numbers and operational laws by adding the neutrality feature of the decision-makers to all DMPs. As the linguistic interval-valued intuitionistic neutrosophic fuzzy numbers and operational laws considers more uncertainties than the other current theories, there is a mean to discuss the linguistic interval-valued intuitionistic neutrosophic fuzzy numbers and operational laws in the environment. Keeping these points in view, we study some new operational laws and the linguistic interval-valued intuitionistic neutrosophic fuzzy numbers and operational laws to explore the DMPs. The objective of the paper is as follows:
(1) to present some neutral operational laws for linguistic interval-valued intuitionistic neutrosophic fuzzy numbers and operational laws by considering decision towards the process;
(2) to present some new geometric operator for Linguistic interval-valued intuitionistic neutrosophic fuzzy numbers and operational laws;
(3) to present a novel MAGDM method and illustrate with some illustrative examples to explore the study.

Definition.3.1. Let $A = \left\{\begin{pmatrix} \left[S_{w_1^l}, S_{w_1^u}\right], \\ \left[S_{\Gamma_1^l}, S_{\Gamma_1^u}\right], \\ \left[S_{\alpha_1^l}, S_{\alpha_1^u}\right], \\ \left[S_{\beta_1^l}, S_{\beta_1^u}\right], \\ \left[S_{c_1^l}, S_{c_1^u}\right], \\ S_{d_1^l}, S_{d_1^u} \end{pmatrix}\right\}$ and $B = \left\{\begin{pmatrix} \left[S_{w_2^l}, S_{w_2^u}\right], \\ \left[S_{\Gamma_2^l}, S_{\Gamma_2^u}\right], \\ \left[S_{\alpha_2^l}, S_{\alpha_2^u}\right], \\ \left[S_{\beta_2^l}, S_{\beta_2^u}\right], \\ \left[S_{c_2^l}, S_{c_2^u}\right], \\ S_{d_2^l}, S_{d_2^u} \end{pmatrix}\right\}$ be two LIVINFNs and λ be any real number. Then we have

$$A \oplus B = \left\{ \begin{array}{l} \left[S_{w_1^l + w_2^l - \frac{w_1^l + w_2^l}{g}}, S_{w_1^u + w_2^u - \frac{w_1^u + w_2^u}{g}} \right], \\ \left[S_{\Gamma_1^l + \Gamma_2^l - \frac{\Gamma_1^l + \Gamma_2^l}{g}}, S_{\Gamma_2^u + \Gamma_2^u - \frac{\Gamma_1^u + \Gamma_2^u}{g}} \right], \\ \left[\left[S_{\frac{\alpha_1^l \alpha_2^l}{g}}, S_{\frac{\alpha_1^u \alpha_2^u}{g}} \right], \left[S_{\frac{\beta_1^l \beta_2^l}{g}}, S_{\frac{\beta_1^u \beta_2^u}{g}} \right] \right], \\ \left[\left[S_{\frac{c_1^l c_2^l}{g}}, S_{\frac{c_1^u c_2^u}{g}} \right], \left[S_{\frac{d_1^l d_2^l}{g}}, S_{\frac{d_1^u d_2^u}{g}} \right] \right] \end{array} \right\}$$

$$A \otimes B = \left\{ \begin{array}{l} \left[\left[S_{\frac{w_1^l w_2^l}{g}}, S_{\frac{w_1^u w_2^u}{g}} \right], \left[S_{\frac{\Gamma_1^l \Gamma_2^l}{g}}, S_{\frac{\Gamma_1^u \Gamma_2^u}{g}} \right] \right], \\ \left[S_{\alpha_1^l + \alpha_2^l - \frac{\alpha_1^l + \alpha_2^l}{g}}, S_{\alpha_1^u + \alpha_2^u - \frac{\alpha_1^u + \alpha_2^u}{g}} \right], \\ \left[S_{\beta_1^l + \beta_2^l - \frac{\beta_1^l + \beta_2^l}{g}}, S_{\beta_1^u + \beta_2^u - \frac{\beta_1^u + \beta_2^u}{g}} \right], \\ \left[S_{c_1^l + c_2^l - \frac{c_1^l + c_2^l}{g}}, S_{c_1^u + c_2^u - \frac{c_1^u + c_2^u}{g}} \right], \\ S_{d_1^l + d_2^l - \frac{d_1^l + d_2^l}{g}}, S_{d_1^u + d_2^u - \frac{d_1^u + d_2^u}{g}} \end{array} \right\}$$

$$\lambda A = \left\{ \begin{array}{l} \left[S_{g - g(1 - \frac{w_1^l}{g})^\lambda}, S_{g - g(1 - \frac{w_1^u}{g})^\lambda} \right], \\ \left[S_{g - g(1 - \frac{\Gamma_1^l}{g})^\lambda}, S_{g - g(1 - \frac{\Gamma_1^u}{g})^\lambda} \right], \\ \left[S_{g(\frac{\alpha_1^l}{g})^\lambda}, S_{g(\frac{\alpha_1^u}{g})^\lambda} \right], \left[S_{g(\frac{\beta_1^l}{g})^\lambda}, S_{g(\frac{\beta_1^u}{g})^\lambda} \right], \\ \left[S_{g(\frac{c_1^l}{g})^\lambda}, S_{g(\frac{c_1^u}{g})^\lambda} \right], \left[S_{g(\frac{d_1^l}{g})^\lambda}, S_{g(\frac{d_1^u}{g})^\lambda} \right] \end{array} \right\}$$

$$A^{\lambda}=\left\{\begin{array}{c}\left[S_{g(\frac{w_1^l}{g})^{\lambda}},S_{g(\frac{w_1^u}{g})^{\lambda}}\right],\left[S_{g(\frac{\Gamma_1^l}{g})^{\lambda}},S_{g(\frac{\Gamma_1^u}{g})^{\lambda}}\right],\\ \left[S_{g-g(1-\frac{\alpha_1^l}{g})^{\lambda}},S_{g-g(1-\frac{\alpha_1^u}{g})^{\lambda}}\right],\\ \left[S_{g-g(1-\frac{\beta_1^l}{g})^{\lambda}},S_{g-g(1-\frac{\beta_1^u}{g})^{\lambda}}\right],\\ \left[S_{g-g(1-\frac{c_1^l}{g})^{\lambda}},S_{g-g(1-\frac{c_1^u}{g})^{\lambda}}\right],\\ \left[S_{g-g(1-\frac{d_1^l}{g})^{\lambda}},S_{g-g(1-\frac{d_1^u}{g})^{\lambda}}\right]\end{array}\right\}$$

Definition.3.2. Let $\alpha_j=\left\{\begin{array}{c}\left[S_{w_1^l},S_{w_1^u}\right],\\ \left[S_{\Gamma_1^l},S_{\Gamma_1^u}\right]\\ \left[S_{\alpha_1^l},S_{\alpha_1^u}\right]\\ \left[S_{\beta_1^l},S_{\beta_1^u}\right]\\ \left[S_{c_1^l},S_{c_1^u}\right]\\ \left[S_{d_1^l},S_{d_1^u}\right]\end{array}\right\}$ be the collection linguistic interval-valued intuitionistic neutrosophic fuzzy numbers. Then the score function is defined as

$$S(a)=s_{\frac{\left\{g+\left[s_{w_1^l}-s_{w_1^u}\right]+\left[s_{\Gamma_1^l}-s_{\Gamma_1^u}\right]+\left[s_{\alpha_1^l}-s_{\alpha_1^u}\right]+\left[s_{\beta_1^l}-s_{\beta_1^u}\right]+\left[s_{c_1^l}-s_{c_1^u}\right]+\left[s_{d_1^l}-s_{d_1^u}\right]\right\}}{12}}$$

Example.3.3. Let $\alpha_1=\left\{\begin{array}{c}[s_2,s_3],[s_3,s_4]\\ [s_3,s_5],[s_1,s_3]\\ [s_2,s_5],[s_1,s_5]\end{array}\right\}$, $\alpha_2=\left\{\begin{array}{c}[s_2,s_3],[s_2,s_4]\\ [s_1,s_3],[s_3,s_4]\\ [s_1,s_2],[s_2,s_5]\end{array}\right\}$ and

$\alpha_3=\left\{\begin{array}{c}[s_2,s_3],[s_1,s_5]\\ [s_2,s_5],[s_2,s_4]\\ [s_1,s_4],[s_4,s_5]\end{array}\right\}$ be the collection of LIVINFNs. Then the score function is defined

$$s_1(a)=\frac{s_{\{12+[2-3]+[3-4]+[3-5]+[1-3]+[2-5]+[1-5]\}}}{12}$$
$$s_2(a)=\frac{s_{\{12+[2-3]+[2-4]+[1-3]+[3-4]+[1-2]+[2-5]\}}}{12},$$
$$s_3(a)=\frac{s_{\{12+[2-3]+[1-5]+[2-4]+[1-4]+[1-4]+[4-5]\}}}{12},$$
$$s_1(a)=-s_{0.0833},s_2(a)=s_{0.1666},s_3(a)=-s_{0.1666}.$$

Definition.3.4. Let $\alpha_j = \left\{\begin{matrix} [S_{w^l}, S_{w^u}], \\ [S_{\Gamma^l}, S_{\Gamma^u}], \\ [S_{\alpha^l}, S_{\alpha^u}], \\ [S_{\beta^l}, S_{\beta^u}], \\ [S_{c^l}, S_{c^u}], \\ [S_{d^l}, S_{d^u}] \end{matrix}\right\}$ be the collection of LIVINFNs. Then the accuracy function is defined as

$$H(a) = \frac{\{s_g + [s_{w^l} + s_{w^u}] + [s_{\Gamma^l} + s_{\Gamma^u}] + [s_{\alpha^l} + s_{\alpha^u}] + [s_{\beta^l} + s_{\beta^u}] + [s_{c^l} + s_{c^u}] + [s_{d^l} + s_{d^u}]\}}{12}$$

Example.3.5. Let $\alpha_1 = \left\{\begin{matrix} [s_2, s_3], [s_3, s_4] \\ [s_3, s_5], [s_1, s_3] \\ [s_2, s_3], [s_1, s_5] \end{matrix}\right\}, \alpha_2 = \left\{\begin{matrix} [s_2, s_3], [s_2, s_4] \\ [s_2, s_5], [s_3, s_4] \\ [s_3, s_5], [s_2, s_3] \end{matrix}\right\}$ and $\alpha_3 = \left\{\begin{matrix} [s_2, s_3], [s_2, s_5] \\ [s_3, s_4], [s_2, s_3] \\ [s_3, s_5], [s_1, s_3] \end{matrix}\right\}$ be the collection of LIVINFNs. Then the accuracy function is defined as

$$H_1(a) = \frac{\{[s_{12+2+3+3+4+3+5+2+5+1+3+1+5}]\}}{12},$$
$$H_2(a) = \frac{\{[s_{12+2+3+2+4+2+5+3+4+3+5+2+3}]\}}{12},$$
$$H_3(a) = \frac{\{[s_{12+2+3+2+5+3+4+2+3+3+5+1+4}]\}}{12},$$
$$H_1(a) = s_{4.0833}, H_1(a) = s_{4.166}, H_1(a) = s_{4.000}$$

4. Aggregation operators based on the Linguistic interval-valued intuitionistic neutrosophic fuzzy numbers

In this paper, we define the three aggregation operators based on Linguistic interval-valued intuitionistic neutrosophic fuzzy weighted geometric operator, Linguistic interval-valued intuitionistic neutrosophic fuzzy weighted ordered geometric operator and Linguistic interval-valued intuitionistic neutrosophic fuzzy hybrid weighted geometric operator.

4.1. Linguistic interval-valued intuitionistic neutrosophic fuzzy weighted geometric operator

Definition.4.1.1. Let $\alpha_t = \begin{Bmatrix} [S_{w^l}, S_{w^u}], \\ [S_{\Gamma^l}, S_{\Gamma^u}], \\ [S_{\alpha^l}, S_{\alpha^u}], \\ [S_{\beta^l}, S_{\beta^u}], \\ [S_{c^l}, S_{c^u}], \\ [S_{d^l}, S_{d^u}] \end{Bmatrix}$ $t = 1,2,,\ldots,n$ be the collection of LIVINFNs, $w = (w_1, w_2, \ldots, w_n)^t$be the weight vector of α_t $t = 1,2,,\ldots,n$ such that $\sum_{t=1}^{n} w_t = 1$, $w_t > 0$ The linguistic interval-valued intuitionistic neutrosophic fuzzy weighted geometric operator is a map linguistic interval-valued intuitionistic neutrosophic fuzzy weighted geometric operator:$\Omega^n -> \Omega$ define by $\text{LIVINFWG}_W(\alpha_1, \alpha_2, \ldots, \alpha_n) = \otimes_{j=1}^{n} {\alpha_t}^{w_t}$ if $w = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^t$, the LIVINFWG operator reduced to LIVINFG operator expressed as

$$\text{LIVINFG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = (\alpha_1 \otimes \alpha_2 \otimes \ldots \otimes \alpha_n)^{\frac{1}{n}}$$

Theorem.4.1.2 The collection of LIVINFNs$\alpha_t = \begin{Bmatrix} [S_{w^l}, S_{w^u}], \\ [S_{\Gamma^l}, S_{\Gamma^u}], \\ [S_{\alpha^l}, S_{\alpha^u}], \\ [S_{\beta^l}, S_{\beta^u}], \\ [S_{c^l}, S_{c^u}], \\ [S_{d^l}, S_{d^u}] \end{Bmatrix}$ $j = 1,2,,\ldots,n$ by using the LIVINFWG also a LIVINFN, and

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$$\mathrm{LIVINFWG}_W(\alpha_1,\alpha_2,\dots,\alpha_n)=\left\{\begin{array}{c}\left[s_{(\prod_{j=1}^{n}(\frac{w_j^l}{g}))^{w_j}},s_{(\prod_{j=1}^{n}(\frac{w_j^u}{g}))^{w_j}}\right]\\ \left[s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^l}{g}))^{w_j}},s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^u}{g}))^{w_j}}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^u}{g})^{w_j})}\right]\end{array}\right\}$$

Proof we will proof the theorem by mathematical induction. Let it is true for $n=2$.

$$\left\{\begin{array}{c}\left[s_{(\frac{w_1^l+w_2^l}{g})^{w_j}},s_{(\frac{w_1^u+w_2^u}{g})^{w_j}}\right],\\ \left[s_{(\frac{\Gamma_j^l+\Gamma_j^l}{g})^{w_j}},s_{(\frac{\Gamma_j^u+\Gamma_j^u}{g})^{w_j}}\right],\\ \left[s_{\left(1-\left\{\left(1-\frac{\alpha_j^l}{g}\right)+(1-\frac{\alpha_j^l}{g})\right\}^{w_j}\right)},s_{\left(1-\left\{\left(1-\frac{\alpha_j^u}{g}\right)+(1-\frac{\alpha_j^u}{g})\right\}^{w_j}\right)}\right]\\ \left[s_{\left(1-\left\{\left(1-\frac{\beta_j^l}{g}\right)+(1-\frac{\beta_j^l}{g})\right\}^{w_j}\right)},s_{\left(1-\left\{\left(1-\frac{\beta_j^u}{g}\right)+(1-\frac{\beta_j^u}{g})\right\}^{w_j}\right)}\right]\\ \left[s_{\left(1-\left\{\left(1-\frac{c_j^l}{g}\right)+(1-\frac{c_j^l}{g})\right\}^{w_j}\right)},s_{\left(1-\left\{\left(1-\frac{c_j^u}{g}\right)+(1-\frac{c_j^u}{g})\right\}^{w_j}\right)}\right]\\ \left[s_{\left(1-\left\{\left(1-\frac{d_j^l}{g}\right)+(1-\frac{d_j^l}{g})\right\}^{w_j}\right)},s_{\left(1-\left\{\left(1-\frac{d_j^u}{g}\right)+(1-\frac{d_j^u}{g})\right\}^{w_j}\right)}\right]\end{array}\right\}$$

Holds for $n = 2$
Let it is true for $n = k$

$$\left\{\begin{array}{c} \left[s_{(\prod_{j=1}^{k}(\frac{w_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{k}(\frac{w_j^u}{g}))^{w_j}} \right] \\ \left[s_{(\prod_{j=1}^{k}(\frac{\Gamma_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{k}(\frac{\Gamma_j^u}{g}))^{w_j}} \right] \\ \left[s_{(1-\prod_{j=1}^{k}(1-\frac{\alpha_j^l}{g})^{w_j})}, s_{(1-\prod_{j=1}^{k}(1-\frac{\alpha_j^u}{g})^{w_j})} \right] \\ \left[s_{(1-\prod_{j=1}^{k}(1-\frac{\beta_j^l}{g})^{w_j})}, s_{(1-\prod_{j=1}^{k}(1-\frac{\beta_j^u}{g})^{w_j})} \right] \\ \left[s_{(1-\prod_{j=1}^{k}(1-\frac{c_j^l}{g})^{w_j})}, s_{(1-\prod_{j=1}^{k}(1-\frac{c_j^u}{g})^{w_j})} \right] \\ \left[s_{(1-\prod_{j=1}^{k}(1-\frac{d_j^l}{g})^{w_j})}, s_{(1-\prod_{j=1}^{k}(1-\frac{d_j^u}{g})^{w_j})} \right] \end{array}\right\}$$

Holds for $n = k$
Let it is true for $n = k + 1$

$$\left\{\begin{matrix} \left[S_{(\prod_{j=1}^{k+1}(\frac{w_j^l}{g}))^{w_j}}, S_{(\prod_{j=1}^{k+1}(\frac{w_j^u}{g}))^{w_j}}\right] \\ \left[S_{(\prod_{j=1}^{k+1}(\frac{\Gamma_j^l}{g}))^{w_j}}, S_{(\prod_{j=1}^{k+1}(\frac{\Gamma_j^u}{g}))^{w_j}}\right] \\ \left[S_{g(1-\prod_{j=1}^{k+1}(1-\frac{\alpha_j^l}{g})^{w_j})}, S_{g(1-\prod_{j=1}^{k+1}(1-\frac{\alpha_j^u}{g})^{w_j})}\right] \\ \left[S_{g(1-\prod_{j=1}^{k+1}(1-\frac{\beta_j^l}{g})^{w_j})}, S_{g(1-\prod_{j=1}^{k+1}(1-\frac{\beta_j^u}{g})^{w_j})}\right] \\ \left[S_{g(1-\prod_{j=1}^{k+1}(1-\frac{c_j^l}{g})^{w_j})}, S_{g(1-\prod_{j=1}^{k+1}(1-\frac{c_j^u}{g})^{w_j})}\right] \\ \left[S_{g(1-\prod_{j=1}^{k+1}(1-\frac{d_j^l}{g})^{w_j})}, S_{(1-\prod_{j=1}^{k+1}(1-\frac{d_j^u}{g})^{w_j})}\right] \end{matrix}\right\} \otimes w_t\alpha_t$$

$$\left\{\begin{matrix} \left[S_{(\prod_{j=1}^{k}(\frac{w_j^l}{g}))^{w_j}}, S_{(\prod_{j=1}^{k}(\frac{w_j^u}{g}))^{w_j}}\right] \\ \left[S_{(\prod_{j=1}^{k}(\frac{\Gamma_j^l}{g}))^{w_j}}, S_{(\prod_{j=1}^{k}(\frac{\Gamma_j^u}{g}))^{w_j}}\right] \\ \left[S_{g(1-\prod_{j=1}^{k}(1-\frac{\alpha_j^l}{g})^{w_j})}, S_{g(1-\prod_{j=1}^{k}(1-\frac{\alpha_j^u}{g})^{w_j})}\right] \\ \left[S_{g(1-\prod_{j=1}^{k}(1-\frac{\beta_j^l}{g})^{w_j})}, S_{g(1-\prod_{j=1}^{k}(1-\frac{\beta_j^u}{g})^{w_j})}\right] \\ \left[S_{g(1-\prod_{j=1}^{k}(1-\frac{c_j^l}{g})^{w_j})}, S_{g(1-\prod_{j=1}^{k}(1-\frac{c_j^u}{g})^{w_j})}\right] \\ \left[S_{g(1-\prod_{j=1}^{k}(1-\frac{d_j^l}{g})^{w_j})}, S_{(1-\prod_{j=1}^{k}(1-\frac{d_j^u}{g})^{w_j})}\right] \end{matrix}\right\} \otimes$$

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$$\left\{\begin{matrix} \left[s_{(\prod_{j=1}^{k+1}(\frac{w_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{k+1}(\frac{w_j^u}{g}))^{w_j}}\right] \\ \left[s_{(\prod_{j=1}^{k+1}(\frac{\Gamma_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{k+1}(\frac{\Gamma_j^u}{g}))^{w_j}}\right] \\ \left[s_{g(1-\prod_{j=1}^{k+1}(1-\frac{\alpha_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{k+1}(1-\frac{\alpha_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{k+1}(1-\frac{\beta_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{k+1}(1-\frac{\beta_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{k+1}(1-\frac{c_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{k+1}(1-\frac{c_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{k+1}(1-\frac{d_j^l}{g})^{w_j})}, s_{(1-\prod_{j=1}^{k+1}(1-\frac{d_j^u}{g})^{w_j})}\right] \end{matrix}\right\}$$

$$\begin{matrix} \left[s_{(\prod_{j=1}^{k+1}(\frac{w_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{k+1}(\frac{w_j^u}{g}))^{w_j}}\right] \\ \left[s_{(\prod_{j=1}^{k+1}(\frac{\Gamma_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{k+1}(\frac{\Gamma_j^u}{g}))^{w_j}}\right] \\ \left[s_{g(1-\prod_{j=1}^{k+1}(1-\frac{\alpha_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{k+1}(1-\frac{\alpha_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{k+1}(1-\frac{\beta_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{k+1}(1-\frac{\beta_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{k+1}(1-\frac{c_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{k+1}(1-\frac{c_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{k+1}(1-\frac{d_j^l}{g})^{w_j})}, s_{(1-\prod_{j=1}^{k+1}(1-\frac{d_j^u}{g})^{w_j})}\right] \end{matrix}$$

It holds for $n = k + 1$, so by mathematical induction it is true for all $n \in Z^+$.

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4.2. Linguistic interval-valued intuitionistic neutrosophic fuzzy ordered weighted geometric operator

Definition.4.2.1. Let $\alpha_t = \left\{\begin{pmatrix} \left[S_{w_1^l}, S_{w_1^u}\right], \\ \left[S_{\Gamma_1^l}, S_{\Gamma_1^u}\right], \\ \left[S_{\alpha_1^l}, S_{\alpha_1^u}\right], \\ \left[S_{\beta_1^l}, S_{\beta_1^u}\right], \\ \left[S_{c_1^l}, S_{c_1^u}\right], \\ \left[S_{d_1^l}, S_{d_1^u}\right] \end{pmatrix}\right\}, t = 1,2,,\ldots,n$ be the collection of LIVINFNs, $w = (w_1, w_2, \ldots, w_n)^t$ be the weight vector of α_t $(t = 1,2,\ldots,n)$ such that $\sum_{t=1}^{n} w_t = 1$, $w_t > 0$ The linguistic interval-valued intuitionistic neutrosophic fuzzy ordered weighted geometric operator is a map linguistic interval-valued intuitionistic neutrosophic fuzzy ordered weighted geometric operator:$\Omega^n -> \Omega$ define by $LIVINFOWG_W(\alpha_1, \alpha_2, \ldots, \alpha_n) = \otimes_{j=1}^{n} {\alpha_t}^{w_t}$ if $w = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^t$, the LIVINFOWG operator reduced to LIVINFG operator expressed as $LIVINOFG(\alpha_1, \alpha_2, \ldots, \alpha_n) = (\alpha_1 \otimes \alpha_2 \otimes \ldots \otimes \alpha_n)^{\frac{1}{n}}$

Theorem.4.1.2 The collection of LIVINFNs $\alpha_t = \left\{\begin{pmatrix} \left[S_{w^l}, S_{w^u}\right], \\ \left[S_{\Gamma^l}, S_{\Gamma^u}\right], \\ \left[S_{\alpha^l}, S_{\alpha^u}\right], \\ \left[S_{\beta^l}, S_{\beta^u}\right], \\ \left[S_{c^l}, S_{c^u}\right], \\ \left[S_{d^l}, S_{d^u}\right] \end{pmatrix}\right\}$ $j = 1,2,,\ldots,n$ by using the LIVINFOWG also a LIVINFN, and

$$\mathrm{LIVINFWG_W}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\left\{\begin{array}{c}\left[s_{(\prod_{j=1}^{n}(\frac{w_j^l}{g}))^{w_j}},s_{(\prod_{j=1}^{n}(\frac{w_j^u}{g}))^{w_j}}\right]\\ \left[s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^l}{g}))^{w_j}},s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^u}{g}))^{w_j}}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^u}{g})^{w_j})}\right]\end{array}\right\}$$

4.3. Linguistic interval-valued intuitionistic neutrosophic fuzzy hybrid weighted geometric operator

Definition.4.3.1. Let $\alpha_t=\left\{\begin{array}{c}[S_{w^l},S_{w^u}],\\ [S_{\Gamma^l},S_{\Gamma^u}],\\ [S_{\alpha^l},S_{\alpha^u}],\\ [S_{\beta^l},S_{\beta^u}],\\ [S_{c^l},S_{c^u}],\\ [S_{d^l},S_{d^u}]\end{array}\right\}$ $t=1,2,,\ldots,n$ be the collection of LIVINFNs, $w=(w_1,w_2,\ldots,w_n)^t$ be the weight vector ofα_t $(t=1,2,\ldots,n)$ such that $\sum_{t=1}^{n}w_t=1$, $w_t>0$.The linguistic interval-valued intuitionistic neutrosophic fuzzy hybrid weighted geometric operator is a map linguistic interval-valued intuitionistic neutrosophic fuzzy hybrid weighted geometric operator:$\Omega^n\rightarrow\Omega$ define by $\mathrm{LIVINFWG_W}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\otimes_{j=1}^{n}\alpha_t{}^{w_t}$ if $w=(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n})^t$, the LIVINFHWG operator reduced to LIVINFG operator expressed as $\mathrm{LIVINFHG}(\alpha_1,\alpha_2,\ldots,\alpha_n)=(\alpha_1\otimes\alpha_2\otimes\ldots\otimes\alpha_n)^{\frac{1}{n}}$

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Theorem.4.3.2. The collection of LIVINFNs $\alpha_t = \left\{\begin{matrix}\left[S_{w_1^l}, S_{w_1^u}\right], \\ \left[S_{\Gamma_1^l}, S_{\Gamma_1^u}\right], \\ \left[S_{\alpha_1^l}, S_{\alpha_1^u}\right], \\ \left[S_{\beta_1^l}, S_{\beta_1^u}\right], \\ \left[S_{c_1^l}, S_{c_1^u}\right], \\ \left[S_{d_1^l}, S_{d_1^u}\right]\end{matrix}\right\}$, $j = 1,2,,\dots,n$ by using the LIVINFHWG also a LIVINFN, and

$$\mathrm{LIVINFHWG}_W(\alpha_1, \alpha_2, \dots, \alpha_n) = \left\{\begin{matrix}\left[s_{(\prod_{j=1}^{n}(\frac{w_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{n}(\frac{w_j^u}{g}))^{w_j}}\right] \\ \left[s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^u}{g}))^{w_j}}\right] \\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^u}{g})^{w_j})}\right]\end{matrix}\right\}$$

5. Multiple attribute group decision making approach of LIVINF data

In this section, we present a decision-making approach based on the proposed operator for solving the MAGDM problem under the LIVINF environment.

Consider a GDM problem in which there are m alternatives $A_1, A_2, \dots, A_m$ and n atterbuates $G_1, G_2, \dots, G_n$ whose weight vector are w_t $t = 1,2,\dots,n$ such that $w_t > 0$ and $\sum_{j=1}^{n} w_j = 1$. Let $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_t)$ be the set of decision-makers and $w = (w_1, w_2, \dots, w_n)^T$be the weight vector of λ_t $(t = 1,2,\dots,n)$ with $w_t > 0$ and $\sum_{t=1}^{n} w_t = 1$. Suppose that the characteristic information of the alternatives A_k $(k = 1,2,\dots,m)$ over the attributes G_t $(t = 1,2,\dots,n)$ is emulated by decision-maker λ_t $(t = 1,2,\dots,n)$ and gives the preference in the form of LIVINFNs

$\alpha_{kt} = \left\{\begin{matrix} \left[S_{w_1^l}, S_{w_2^u}\right], \left[S_{\Gamma_1^l}, S_{\Gamma_2^u}\right], \left[S_{\alpha_1^l}, S_{\alpha_2^u}\right], \\ \left[S_{\beta_1^l}, S_{\beta_2^u}\right], \left[S_{c_1^l}, S_{c_2^u}\right], \left[S_{d_1^l}, S_{d_2^u}\right] \end{matrix}\right\}$, and hence formulated the LIVINF decision matrices.

Based on these the following steps have been summarized for describing the GDM approach based on the proposed operation as;
Step 1: Calculate the LIVINF decision matrix
Step 2: Calculate the LIVINFWG and $w = (w_1, w_2, \dots, w_n)^T$

$$\mathrm{LIVINFW}G(\alpha_1, \alpha_2, \dots, \alpha_n) = \left\{\begin{matrix} \left[s_{(\prod_{j=1}^{n}(\frac{w_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{n}(\frac{w_j^u}{g}))^{w_j}}\right] \\ \left[s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^l}{g}))^{w_j}}, s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^u}{g}))^{w_j}}\right] \\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^u}{g})^{w_j})}\right] \\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^l}{g})^{w_j})}, s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^u}{g})^{w_j})}\right] \end{matrix}\right\}$$

Step 3: Calculate the LIVINFWG and $w = (w_1, w_2, \dots, w_n)^T$
$\mathrm{LIVINFWG_W}(\alpha_1, \alpha_2, \dots, \alpha_n) = \otimes_{j=1}^{n} \alpha_{\vartheta(t)}{}^{w_t}$

$$\mathrm{LIVINFW}G(\alpha_1,\alpha_2,\dots,\alpha_n)=\left\{\begin{array}{c}\left[s_{(\prod_{j=1}^{n}(\frac{w_j^l}{g}))^{w_j}},s_{(\prod_{j=1}^{n}(\frac{w_j^u}{g}))^{w_j}}\right]\\ \left[s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^l}{g}))^{w_j}},s_{(\prod_{j=1}^{n}(\frac{\Gamma_j^u}{g}))^{w_j}}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{\alpha_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{\beta_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{c_j^u}{g})^{w_j})}\right]\\ \left[s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^l}{g})^{w_j})},s_{g(1-\prod_{j=1}^{n}(1-\frac{d_j^u}{g})^{w_j})}\right]\end{array}\right\}$$

Step 4: Calculate the score function

$$S(a)=s_{\frac{\left\{g+\left[s_{w_1^l}-s_{w_1^u}\right]+\left[s_{\Gamma_1^l}-s_{\Gamma_1^u}\right]+\left[s_{\alpha_1^l}-s_{\alpha_1^u}\right]+\left[s_{\beta_1^l}-s_{\beta_1^u}\right]+\left[s_{c_1^l}-s_{c_1^u}\right]+\left[s_{d_1^l}-s_{d_1^u}\right]\right\}}{12}}$$

Step 5: Find the ranking

6. Numerical application

An investment company selects four mines, A_1, A_2 and A_3 as alternatives and considers three factors as the evaluation criteria: (i) C_1 is the geology factor; (ii) C_2 is the mineral reserve risk; (iii) C_3 is the development level of the market. The DMs, Dh. $h=1,2,3,$ gives the evaluation values of alternatives A_i $i=1,2,3$ on the criteria C_j $j=1,2,3$ in the form of linguistic interval-valued intuitionistic neutrosophic fuzzy numbers. The linguistic interval-valued intuitionistic neutrosophic fuzzy decision matrices are constructed and listed in Tables 1-3.
Step 1: Calculate the bipolar neutrosophic fuzzy decision table 1, 2 and 3.

Table 1. Linguistic interval-valued intuitionistic neutrosophic fuzzy decision.

	C_1	C_2	C_3
A_1	$\left\{\begin{matrix}[s_3,s_4],[s_2,s_3],\\ [s_2,s_5],[s_3,s_5],\\ [s_3,s_5],[s_2,s_4]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_2,s_4],[s_1,s_4],\\ [s_2,s_5],[s_3,s_4],\\ [s_2,s_3],[s_3,s_5]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_3,s_4],[s_2,s_3],\\ [s_2,s_4],[s_2,s_3],\\ [s_2,s_5],[s_3,s_4]\end{matrix}\right\}$
A_2	$\left\{\begin{matrix}[s_3,s_4],[s_1,s_3],\\ [s_2,s_3],[s_3,s_5],\\ [s_2,s_5],[s_4,s_5]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_3],[s_2,s_4],\\ [s_2,s_3],[s_3,s_4],\\ [s_2,s_5],[s_2,s_5]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_3],[s_1,s_4],\\ [s_3,s_4],[s_2,s_3],\\ [s_3,s_4],[s_2,s_3]\end{matrix}\right\}$
A_3	$\left\{\begin{matrix}[s_1,s_4],[s_1,s_3],\\ [s_2,s_3],[s_3,s_4],\\ [s_3,s_5],[s_2,s_5]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_3],[s_2,s_3],\\ [s_3,s_4],[s_2,s_4],\\ [s_2,s_5],[s_3,s_5]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_2],[s_1,s_3],\\ [s_3,s_4],[s_2,s_3],\\ [s_2,s_5],[s_3,s_5]\end{matrix}\right\}$

Table 2. Linguistic interval-valued intuitionistic neutrosophic fuzzy decision.

	C_1	C_2	C_3
A_1	$\left\{\begin{matrix}[s_1,s_3],[s_2,s_4],\\ [s_3,s_4],[s_2,s_3],\\ [s_2,s_5],[s_2,s_4]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_2],[s_1,s_3],\\ [s_3,s_4],[s_2,s_4],\\ [s_2,s_5],[s_3,s_5]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_3],[s_1,s_4],\\ [s_2,s_3],[s_2,s_5],\\ [s_3,s_5],[s_2,s_5]\end{matrix}\right\}$
A_2	$\left\{\begin{matrix}[s_1,s_5],[s_1,s_4],\\ [s_2,s_4],[s_2,s_5],\\ [s_3,s_5],[s_3,s_4]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_5],[s_1,s_4],\\ [s_4,s_5],[s_3,s_5],\\ [s_2,s_4],[s_2,s_5]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_3],[s_1,s_5],\\ [s_2,s_4],[s_3,s_5],\\ [s_2,s_5],[s_2,s_3]\end{matrix}\right\}$
A_3	$\left\{\begin{matrix}[s_1,s_5],[s_1,s_2],\\ [s_3,s_4],[s_2,s_4],\\ [s_2,s_5],[s_3,s_5]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_4],[s_1,s_3],\\ [s_3,s_4],[s_2,s_5],\\ [s_4,s_5],[s_2,s_4]\end{matrix}\right\}$	$\left\{\begin{matrix}[s_1,s_2],[s_1,s_3],\\ [s_2,s_3],[s_2,s_5],\\ [s_3,s_5],[s_2,s_3]\end{matrix}\right\}$

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Table 3. Linguistic interval-valued intuitionistic neutrosophic fuzzy decision.

	C_1	C_2	C_3
A_1	$\begin{Bmatrix}[s_1,s_5],[s_1,s_4], \\ [s_2,s_5],[s_3,s_5], \\ [s_4,s_5],[s_3,s_4]\end{Bmatrix}$	$\begin{Bmatrix}[s_1,s_3],[s_1,s_4], \\ [s_3,s_4],[s_3,s_5], \\ [s_2,s_4],[s_3,s_5]\end{Bmatrix}$	$\begin{Bmatrix}[s_1,s_4],[s_1,s_5], \\ [s_2,s_3],[s_3,s_5], \\ [s_3,s_4],[s_2,s_4]\end{Bmatrix}$
A_2	$\begin{Bmatrix}[s_1,s_4],[s_1,s_2], \\ [s_3,s_5],[s_4,s_5], \\ [s_2,s_5],[s_3,s_5]\end{Bmatrix}$	$\begin{Bmatrix}[s_1,s_2],[s_1,s_5], \\ [s_2,s_5],[s_4,s_5], \\ [s_3,s_5],[s_2,s_4]\end{Bmatrix}$	$\begin{Bmatrix}[s_1,s_3],[s_2,s_4], \\ [s_3,s_5],[s_4,s_5], \\ [s_2,s_4],[s_2,s_3]\end{Bmatrix}$
A_3	$\begin{Bmatrix}[s_1,s_5],[s_1,s_4], \\ [s_3,s_5],[s_2,s_5], \\ [s_2,s_4],[s_3,s_4]\end{Bmatrix}$	$\begin{Bmatrix}[s_1,s_4],[s_1,s_5], \\ [s_3,s_4],[s_3,s_5], \\ [s_2,s_5],[s_2,s_5]\end{Bmatrix}$	$\begin{Bmatrix}[s_1,s_4],[s_2,s_5], \\ [s_3,s_5],[s_2,s_4], \\ [s_3,s_5],[s_3,s_4]\end{Bmatrix}$

Step 2: Calculate the LIVINFGA operator and $w = (0.3,0.42,0.28)^T$.

Table 4. Calculations.

	C_1	C_2	C_3
A_1	$\begin{Bmatrix}[s_{0.1382},s_{0.3395}] \\ [s_{0.1506},s_{0.3175}] \\ [s_{0.1369},s_{0.3308}] \\ [s_{0.1875},s_{0.8187}] \\ [s_{0.2127},s_{0.3223}] \\ [s_{0.1639},s_{0.28170}]\end{Bmatrix}$	$\begin{Bmatrix}[s_{0.1223},s_{0.2579}] \\ [s_{0.0974},s_{0.6854}] \\ [s_{0.1874},s_{0.3066}] \\ [s_{0.1874},s_{0.8001}] \\ [s_{0.1359},s_{0.2844}] \\ [s_{0.2103},s_{0.2527}]\end{Bmatrix}$	$\begin{Bmatrix}[s_{0.1382},s_{0.3175}] \\ [s_{0.1223},s_{0.3395}] \\ [s_{0.6113},s_{0.2348}] \\ [s_{0.5013},s_{0.6755}] \\ [s_{0.1874},s_{0.3307}] \\ [s_{0.1638},s_{0.2817}]\end{Bmatrix}$
A_2	$\begin{Bmatrix}[s_{0.0626},s_{0.2487}] \\ [s_{0.0394},s_{0.1500}] \\ [s_{0.2216},s_{0.3710}] \\ [s_{0.2845},s_{0.4043}] \\ [s_{0.2216},s_{0.4575}] \\ [s_{0.3125},s_{0.4300}]\end{Bmatrix}$	$\begin{Bmatrix}[s_{0.0394},s_{0.1647}] \\ [s_{0.0525},s_{0.2487}] \\ [s_{0.2552},s_{0.4043}] \\ [s_{0.3125},s_{0.5003}] \\ [s_{0.2216},s_{0.4300}] \\ [s_{0.1898},s_{0.4300}]\end{Bmatrix}$	$\begin{Bmatrix}[s_{0.0394},s_{0.1576}] \\ [s_{0.0528},s_{0.2847}] \\ [s_{0.2521},s_{0.4011}] \\ [s_{0.2845},s_{0.4043}] \\ [s_{0.2216},s_{0.4011}] \\ [s_{0.1898},s_{0.2814}]\end{Bmatrix}$
A_3	$\begin{Bmatrix}[s_{0.1159},s_{0.4210}] \\ [s_{0.1169},s_{0.2813}] \\ [s_{0.1760},s_{0.2682}] \\ [s_{0.1760},s_{0.2682}] \\ [s_{0.1538},s_{0.2895}] \\ [s_{0.1760},s_{0.2682}]\end{Bmatrix}$	$\begin{Bmatrix}[s_{0.1159},s_{0.3427}] \\ [s_{0.1407},s_{0.3366}] \\ [s_{0.1977},s_{0.2657}] \\ [s_{0.1784},s_{0.3126}] \\ [s_{0.2000},s_{0.3349}] \\ [s_{0.1538},s_{0.3126}]\end{Bmatrix}$	$\begin{Bmatrix}[s_{0.1159},s_{0.3257}] \\ [s_{0.1407},s_{0.3366}] \\ [s_{0.1760},s_{0.2682}] \\ [s_{0.1309},s_{0.2632}] \\ [s_{0.1538},s_{0.3349}] \\ [s_{0.1760},s_{0.2682}]\end{Bmatrix}$

Step 3: Calculate the LIVINFHWG and $w = (0.3\ ,0.42\ ,0.28)^T$

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Table 5. Calculations.

	C_1
A_1	$\left\{\begin{matrix}[s_{0.01614}, s_{0.03392}], [s_{0.01491}, s_{0.04549}] \\ [s_{0.02174}, s_{0.02015}], [s_{0.02030}, s_{0.05311}] \\ [s_{0.01238}, s_{0.01242}], [s_{0.01265}, s_{0.01885}]\end{matrix}\right\}$
A_2	$\left\{\begin{matrix}[s_{0.00815}, s_{0.004748}], [s_{0.0008581}, s_{0.005883}] \\ [s_{0.12994}, s_{0.03785}], [s_{0.02581}, s_{0.04210}] \\ [s_{0.02143}, s_{0.04145}], [s_{0.02232}, s_{0.03675}]\end{matrix}\right\}$
A_3	$\left\{\begin{matrix}[s_{0.01897}, s_{0.04925}], [s_{0.02119}, s_{0.04418}] \\ [s_{0.01185}, s_{0.01730}], [s_{0.04542}, s_{0.01867}] \\ [s_{0.01094}, s_{0.02120}], [s_{0.01090}, s_{0.01831}]\end{matrix}\right\}$

Calculate the score function

$s_1(a) = s_{0.99284}$, $s_2(a) = s_{1.08603}$, $s_3(a) = s_{1.079196}$

7. Comparison Analysis

7.1. LIVNFG with existing method

Now, we compare our result with other related methods for LIVNFG, which is LIVAIFS in Garg and Kumar (2019).

Table 6. Linguistic Interval- valued Atnossive Intuitionistic fuzzy set.

	C_1	C_2	C_3
A_1	$\{[s_3, s_4], [s_2, s_5]\}$	$\{[s_4, s_5], [s_2, s_3]\}$	$\{[s_3, s_5], [s_2, s_4]\}$
A_2	$\{[s_3, s_4], [s_4, s_5]\}$	$\{[s_2, s_3], [s_2, s_4]\}$	$\{[s_2, s_5], [s_2, s_3]\}$
A_3	$\{[s_2, s_4], [s_3, s_4]\}$	$\{[s_4, s_5], [s_3, s_5]\}$	$\{[s_2, s_3], [s_3, s_4]\}$

Method: 1
Calculate the LIVAIFWG operator

Table 7. Calculations.

	LIVNFG table 7
A_1	$\{[s_{0.3395}, s_{0.3957}], [s_{0.1396}, s_{0.2844}]\}$
A_2	$\{[s_{0.1121}, s_{0.2204}], [s_{0.2525}, s_{0.3740}]\}$
A_3	$\{[s_{0.2520}, s_{0.3648}], [s_{0.1977}, s_{0.2895}]\}$

Calculate the score functions$s_1(a) = s_{6.5778}$,$s_2(a) = s_{6.4265}$,$s_3(a) = s_{6.5324}$

Method: 2
Calculate the LIVAIFWG operator

Table 8. Calculaations.

	the LIVAIFWG operator table 8
A_1	$\{[s_{0.5331}, s_{0.3307}], [s_{0.1855}, s_{0.3395}]\}$
A_2	$\{[s_{0.2216}, s_{0.3740}], [s_{0.1265}, s_{0.2204}]\}$
A_3	$\{[s_{0.1784}, s_{0.2682}], [s_{0.2917}, s_{0.3953}]\}$

Calculate the score function
$s_1(a) = s_{6.5847}$,$s_2(a) = s_{6.5621}$,$s_3(a) = s_{6.4398}$

Table 9. The ranking order by utilizing two different method.

Method	Result	Ranking order	Best alternative
Geometric method 1	$\begin{bmatrix} s_1(a) = s_{6.5778} \\ s_2(a) = s_{6.4265} \\ s_3(a) = s_{6.5324} \end{bmatrix}$	$\begin{bmatrix} s_1(a) \\ > s_2(a) \\ > s_3(a) \end{bmatrix}$	$[s_1(a)]$
Geometric method 2	$\begin{bmatrix} s_1(a) = s_{6.5847} \\ s_2(a) = s_{6.5621} \\ s_3(a) = s_{6.4398} \end{bmatrix}$	$\begin{bmatrix} s_1(a) \\ > s_2(a) \\ > s_3(a) \end{bmatrix}$	$[s_1(a)]$

7.2. Advantages of the proposed approach

In this subsection, we define the different advantages of proposed method and written below

Table 10. Different advantages of proposed method.

Alternatives	Ranking	Ranking
LIVINFWG	$s_1(a) > s_2(a) > s_3(a)$	$s_1(a)$
LIVINFOWG	$s_1(a) > s_2(a) > s_3(a)$	$s_1(a)$
LIVINFHWG	$s_1(a) > s_2(a) > s_3(a)$	$s_1(a)$
score function of geometric	$s_1(a) > s_2(a) > s_3(a)$	$s_1(a)$
Accuracy function of geometric	$s_1(a) > s_2(a) > s_3(a)$	$s_1(a)$
SVNNs (Garg (2020)	$s_1(a) > s_2(a) > s_3(a)$	$s_1(a)$
SV and INS (Garg (2020)	$s_1(a) > s_2(a) > s_3(a)$	$s_1(a)$
LSVSP aggregation (Garg (2019)	$s_1(a) > s_2(a) > s_3(a)$	$s_1(a)$

7.3. Proposed results with existing aggregation

This modification is owing to different expressing plans and operations in the aggregation operators. Then, the closing optimal decision vestiges the identical. This singularity signifies the cogency, flexibility, authenticity, and regularity of proposed operators.

In the existing approach Garg (2020), the authors' utilized multi-stage multi-attribute decision-making method based on the prospect theory and linguistic interval-valued Pythagorean Fuzzy Sets to rank the alternatives. By applying these approaches to DM problems, we need an ideal extreme alternative which increases the complexity and computational overhead. These shortcomings are addressed with the proposed approach where we don't need any ideal determination. Thus, the proposed method is more suitable for solving DM problems.

The existing approaches Xu (2006), utilized the linguistic hybrid arithmetic averaging operator in multiple attribute group decision making with linguistic information based on algebraic and Einstein t-norm and t-conorm. These operators can capture the interrelationship between all the input arguments but it is able to give any result for the cases in which a particular number of arguments is to be considered.

Table 11. Results obtained via the existing aggregation operators.

Alternatives	Ranking	Ranking
NS(Smarandache, 1999)	$s_2(a) > s_1(a) > s_3(a)$	$s_2(a)$
LHAA operator (Xu 2006))	$s_2(a) > s_1(a) > s_3(a)$	$s_2(a)$
Single valued neutrosophic sets (Wang et al. 2010)	$s_2(a) > s_1(a) > s_3(a)$	$s_2(a)$

Therefore, the proposed approach can be applied to decision-making problems under LIVINF environment. Hence, the proposed operators are more generalized than any other existing operator to deal with problems having LIVINF information.

8. Conclusion

In this paper, we have proposed some new geometric aggregation operators by including the boldness appearances of the decision-makers into the investigation under the LIVINF setting. This feature was handled by proposing some new neutral addition and scalar multiplication operational laws to aggregated LIVINF information. The geometric representation of these operations is defined also on linguistic interval-valued intuitionistic neutrosophic fuzzy weighted averaging geometric operator, linguistic interval-valued intuitionistic neutrosophic fuzzy ordered weighted geometric averaging operator and linguistic interval-valued intuitionistic neutrosophic fuzzy hybrid weighted geometric averaging operator, for collection of data. We define the MCDM technique. At, last a numerical example is used to illustrate the validity of the exhibit approach in group decision-making problems.

In future work, we will study some more new operational laws under a diverse fuzzy environment to better represent uncertain information and apply them on to different sectors such as differential equations, harmonic series, means series and integral fuzzy equation

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