# Hybrid vector similarity measure of single valued refined neutrosophic sets to multi-attribute decision making problems

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## ABSTRACT

*This paper proposes hybrid vector similarity measures under single valued refined neutrosophic sets and proves some of its basic properties. The proposed similarity measure is then applied for solving multiple attribute decision making problems. Lastly, a numerical example of medical diagnosis is given on the basis of the proposed hybrid similarity measures and the results are compared with the results of other existing methods to validate the applicability, simplicity and effectiveness of the proposed method.*

KEYWORDS: **Single valued neutrosophic sets; Single valued refined neutrosophic sets; Hybrid vector similarity measures; Multi-attribute decision making.**

## 1. INTRODUCTION

Smarandache (1998) initiated the theory of neutrosophic sets (NSs) which is characterized by a truth membership $T_A$ $(x)$, an indeterminacy membership $I_A$ $(x)$ and a falsity membership $F_A$ $(x)$ to cope with indeterminate, incomplete and inconsistent information. However, single valued neutrosophic sets (SVNSs) defined by Wang et al. (2010) is useful tool for practical decision making purposes. Multi attribute decision making (MADM) under SVNSs attracted many researchers and many methods have been proposed for MADM problems such as TOPSIS (Zhang & Wu, 2014, Biswas et al., 2016a), grey relational analysis (Biswas et al., 2014a; Biswas et al., 2014b; Mondal & Pramanik, 2015a; Mondal & Pramanik, 2015c), outranking approach (Peng et al., 2014), maximizing deviation method (Şahin & Liu, 2016), hybrid vector similarity measure (Pramanik et al., 2017), etc. Further theoretical development and applications of SVNS can be found in the studies (Biswas et al. 2016a, 22016b, 2016c, 2016d, 2016e, 2017a, 2017b; Pramanik & Roy, 2104; Sodenkamp, 2102).

Hanafy et al. (2013) proposed a method to determine the correlation coefficient of NSs by using centroid method. Ye (2013a) defined correlation of SVNSs, correlation coefficient of SVNSs, and weighted correlation coefficient of SVNSs. In the same study, Ye (2013a) developed a multi-criteria decision making method (MCDM) based on weighted correlation coefficient and the weighted cosine similarity measure. Ye (2013b) proposed another form of correlation coefficient between SVNSs and presented a MADM method. Broumi and Smarandache (2013) proposed a new method called extended Hausdroff distance for SVNSs and a new series of similarity measures were developed to find the similarity of SVNSs. Majumdar and Samanta (2014) presented some similarity measures between SVNSs based on distance, a matching function, membership grades and defined the notion of entropy measure for

SVNSs. Ye (2014a) proposed cross entropy of SVNSs and solved a MCDM based on the cross entropy of SVNSs. Ye and Zhang (2014) formulated three similarity measures between SVNSs by utilizing maximum and minimum operators and investigated their characteristics. In the same study, Ye and Zhang (2014) developed weighted similarity measures for solving MADM problems under single valued neutrosophic setting. Ye (2014b) suggested three similarity measures between simplified NSs as an extension of the Jaccard, Dice and cosine similarity measures in vector space for solving MCDM problems. Ye (2015) proposed an improved cosine similarity measure for SVNSs and employed the concept for medical diagnosis. Mondal and Pramanik (2015b) defined tangent similarity measure due to Pramanik and Mondal (2015) and Mondal and Pramanik (2015f) and proved its basic properties. In the same study, Mondal and Pramanik (2015b) developed a new MADM method based on tangent similarity measure and presented two illustrative MADM problems. Ye and Fu (2016) presented a multi-period medical diagnosis method using tangent similarity measure and the weighted aggregation of multi-period information for solving multi-period medical diagnosis problems under single valued neutrosophic environment. Pramanik et al. (2017) investigated a new hybrid vector similarity measure under both single valued neutrosophic and interval neutrosophic assessments by extending the notion of variation coefficient similarity method (Xu et al., 2012) with neutrosophic information and proved some of their fundamental properties.

Smarandache (2013) generalized the conventional neutrosophic logic and defined the most $n$- symbol or numerical valued refined neutrosophic logic. Each neutrosophic element $T$, $I$, $F$ can be refined into $T_1$, $T_2$, …, $T_m$, and $I_1$, $I_2$, …, $I_p$, and $F_1$, $F_2$, …, $F_q$, respectively, where $m, p, q$ ($\geq 1$) are integers and $m + p + q = n$. Broumi and Smarandache (2014) proposed cosine similarity measure for refined neutrosophic sets due to Bhattacharya's distance (Bhattacharya, 1946). Ye and Ye (2014) introduced the idea of single valued neutrosophic multi sets (SVNMSs) (refined sets) by combining SVNSs along with the theory of multisets (Yager, 1986) and presented several operational relations of SVNMSs. In the same study, Ye and Ye (2014) proposed Dice similarity measure and weighted Dice similarity measure for SVNMSs and investigated their properties. Chatterjee et al. (2015) slightly modified the definition of SVNMSs (Ye & Ye, 2014) and incorporated few new set-theoretic operators of SVNMSs and their properties. Broumi and Deli (2014) defined correlation measure of neutrosophic refined sets and applied the proposed model to medical diagnosis and pattern recognition problems. Ye et al. (2015) further defined generalized distance and its two similarity measures between SVNMSs and applied the concept to medical diagnosis problem. Mondal and Pramanik (2015e) developed a new multi attribute decision making method in refined neutrosophic set environment based on tangent function due to Mondal and Pramanik (2015b). Mondal and Pramanik (2015d) proposed neutrosophic refined similarity measure based on cotangent function and presented an application to suitable educational stream selection problem. Deli et al. (2015) studied several operators of neutrosophic refined sets such as union, intersection, convex, strongly convex in order to deal with indeterminate and inconsistent information. In their paper, Deli et al. (2015) also examined several results of neutrosophic refined sets using the proposed operators and defined distance measure of neutrosophic refined sets with properties. Karaaslan (2015) developed Jaccard, Dice and cosine similarity based MCDM methods in single valued refined neutrosophic set and interval neutrosophic refined set environment. Broumi and Smarandache (2015) proposed a new similarity measure between refined neutrosophic sets based on extended Housdorff distance of SVNSs and proved some of their basic properties. Mondal and Pramanik (2015e) discussed refined tangent similarity measure for SVNSs and they applied the proposed similarity measure to medical diagnosis problems. Juan-juan and Jian-qiang (2015) defined several multi-valued neutrosophic aggregation operators and established a MCDM method based on the proposed operators. Ye and Smarandache (2016) presented a MCDM method with single valued refined neutrosophic information by extending the concept of similarity method with single valued neutrosophic information of Majumdar and Samanta (2014).

In this paper, we propose another form of cosine similarity measures under SVRNSs by extending the concept given in (Broumi & Smarandache, 2014a; Rajarajeswari & Uma, 2014) and prove some of its basic properties. We propose hybrid vector similarity measure with single valued refined neutrosophic information by extending hybrid vector similarity measure of SVNSs (Pramanik et al., 2017) and prove some of its basic properties. The proposed similarity measure is a hybridization of Dice and cosine similarity measures under single valued refined neutrosophic information. Moreover, we establish weighted hybrid vector similarity measure under single valued refined neutrosophic environment and prove its basic properties. The article is structured in the following way. Section 2 presents some mathematical preliminaries which are required for the construction of the paper. In Section 3 defines hybrid similarity and weighted hybrid similarity measures of SVRNSs and proves some of their properties. Section 4 is devoted to develop two algorithms for solving MADM problems involving single valued refined neutrosophic information. An illustrative example of medical diagnosis is solved to demonstrate the applicability of the proposed procedure in Section 5. Conclusions and future scope of research are presented in Section 6.

## 2. MATHEMATICAL PRELIMINARIES

In this Section, we recall some basic definitions concerning neutrosophic sets, single valued neutrosophic sets, single valued refined neutrosophic sets.

**2.1 Neutrosophic set** (Smarandache, 1998)

Let $U$ be a universal space of objects with a generic element of $U$ denoted by $z$. Then, a neutrosophic set $P$ on $U$ is defined as given below.

$$P = \{z, \langle T_P(z), I_P(z), F_P(z) \rangle \mid z \in U\}$$

where, $T_P(z)$, $I_P(z)$, $F_P(z)$: $U \to\ ]^-0, 1^+[$ stand for the degree of membership, the degree of indeterminacy, and the degree of falsity-membership respectively of a point $z \in U$ to the set $P$ satisfying the condition $^-0 \le T_P(z) + I_P(z) + F_P(z) \le 3^+$.

**2.2 Single valued neutrosophic sets** (Wang et al., 2010)

Consider $U$ be a space of points with a generic element of $U$ denoted by $z$, then a SVNS $Q$ is defined as follows:

$$Q = \{z, \langle T_Q(z), I_Q(z), F_Q(z) \rangle \mid z \in U\}$$

where, $T_Q(x)$, $I_Q(x)$, $F_Q(x)$: $U \to [0, 1]$ denote the degree of membership, the degree of indeterminacy, and the degree of falsity-membership respectively of a point $z \in U$ to the set $Q$ satisfying the condition and $0 \le T_Q(x) + I_Q(x) + F_Q(x) \le 3$ for each point $z \in U$.

**2.3 Single valued neutrosophic refined sets** (Ye & Ye, 2014)

A SVNRS $R$ in the universe $U = \{z_1, z_2, \ldots, z_n\}$ is defined as follows:

$R = \{\langle z, (T_{1R}(z), T_{2R}(z), \ldots, T_{sR}(z)), (I_{1R}(z), I_{2R}(z), \ldots, I_{sR}(z)), (F_{1R}(z), F_{2R}(z), \ldots, F_{sR}(z)) \rangle \mid z \in U\}$

where $T_{1R}(z), T_{2R}(z), \ldots, T_{sR}(z): U \to [0, 1]$, $I_{1R}(z), I_{2R}(z), \ldots, I_{sR}(z): U \to [0, 1]$, $F_{1R}(z), F_{2R}(z), \ldots, F_{sR}(z): U \to [0, 1]$ such that $0 \le T_{iR}(z) + I_{iR}(z) + F_{iR}(z) \le 3$ for $i = 1, 2, \ldots, s$. where, $s$ is said to be the dimension of $R$.

**Definition 2.1** (Ye & Ye, 2014): Let $R_1$ and $R_2$ be two SVRNSs in $U$, where

$R_1 = \{\langle z, (T_{1R_1}(z), T_{2R_1}(z), \ldots, T_{sR_1}(z)), (I_{1R_1}(z), I_{2R_1}(z), \ldots, I_{sR_1}(z)), (F_{1R_1}(z), F_{2R_1}(z), \ldots, F_{sR_1}(z)) \rangle \mid z \in U\}$,

$R_2 = \{\langle z, (T_{1R_2}(z), T_{2R_2}(z), \ldots, T_{sR_2}(z)), (I_{1R_2}(z), I_{2R_2}(z), \ldots, I_{sR_2}(z)), (F_{1R_2}(z), F_{2R_2}(z), \ldots, F_{sR_2}(z)) \rangle \mid z \in U\}$, then the relations between $R_1$ and $R_2$ are presented as follows:

(1). Containment:

$R_1 \subseteq R_2$, if and only if $T_{iR_1}(z) \le T_{iR_2}(z)$, $I_{iR_1}(z) \ge I_{iR_2}(z)$, $F_{iR_1}(z) \ge F_{iR_2}(z)$ for i = 1, 2, …, *s*.

(2). Equality:

$R_1 = R_2$, if and only if $T_{iR_1}(z) = T_{iR_2}(z)$, $I_{iR_1}(z) = I_{iR_2}(z)$, $F_{iR_1}(z) = F_{iR_2}(z)$ for i = 1, 2, …, *s*.

(3). Union:

$R_1 \cup R_2 = \{\langle z, (T_{iR_1}(z) \vee T_{iR_2}(z)), (I_{iR_1}(z) \wedge I_{iR_2}(z)), (F_{iR_1}(z) \wedge F_{iR_2}(z)) \rangle \mid z \in U\}$ for i = 1, 2, …, *s*.

(4). Intersection:

$R_1 \cap R_2 = \{\langle z, (T_{iR_1}(z) \wedge T_{iR_2}(z)), (I_{iR_1}(z) \vee I_{iR_2}(z)), (F_{iR_1}(z) \vee F_{iR_2}(z)) \rangle \mid z \in U\}$ for i = 1, 2, …, *s*.

## 3. HYBRID VECTOR SIMILARITY MEASURES OF SVRNSS

**Definition 3.1** (Ye, 2014c): Let $P = \{z, \langle T_P(z), I_P(z), F_P(z) \rangle \mid z \in U\}$ and $Q = \{z, \langle T_Q(z), I_Q(z), F_Q(z) \rangle \mid z \in U\}$ be two SVNSs (non-refined) in the universe of discourse *U*. Then, the Dice similarity measure of SVNSs is defined as follows.

$$Dice\ (P,\ Q) = \frac{1}{n}\sum_{i=1}^{n} \frac{2(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2 + (T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2\right]} \tag{1}$$

and if $w_i \in [0, 1]$ be the weight of $z_i$ for i = 1, 2, …, *n* such that $\sum_{i=1}^{n} w_i = 1$, then the weighted Dice similarity measure of SVNSs can be defined as follows.

$$Dice_w\ (P,\ Q) = \sum_{i=1}^{n} w_i \frac{2(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2 + (T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2\right]} \tag{2}$$

**Definition 3.2** (Broumi & Smarandache, 2014b): Let $P = \{z, \langle T_P(z), I_P(z), F_P(z) \rangle \mid z \in U\}$ and $Q = \{z, \langle T_Q(z), I_Q(z), F_Q(z) \rangle \mid z \in U\}$ be two SVNSs (non-refined) in the universe of discourse $U = \{z_1, z_2, \ldots, z_n\}$. Then, the cosine similarity measure of SVNSs is defined as given below.

$$Cos\ (P,\ Q) = \frac{1}{n}\sum_{i=1}^{n} \frac{(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}\right]} \tag{3}$$

and if $w_i \in [0, 1]$ be the weight of $z_i$ for i = 1, 2, …, *n* satisfying $\sum_{i=1}^{n} w_i = 1$, then the weighted cosine similarity measure of SVNSs can be defined as follows.

$$Cos_w\ (P,\ Q) = \sum_{i=1}^{n} w_i \frac{(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}\right]} \tag{4}$$

**Definition 3.3** (Pramanik et al., 2017): Hybrid vector similarity measure of SVNSs

Consider $Q_1 = \{z, \langle T_{Q_1}(z), I_{Q_1}(z), F_{Q_1}(z) \rangle \mid z \in U\}$ and $Q_2 = \{z, \langle T_{Q_2}(z), I_{Q_2}(z), F_{Q_2}(z) \rangle \mid z \in U\}$ be two SVNSs in *U*. Then, the hybrid vector similarity measure of $Q_1$ and $Q_2$ is defined as follows:

$$Hyb\ (Q_1,\ Q_2) = \frac{1}{n}\left[\begin{array}{l} \alpha \sum_{i=1}^{n} \dfrac{2(T_{Q_1}(z_i).T_{Q_2}(z_i) + I_{Q_1}(z_i).I_{Q_2}(z_i) + F_{Q_1}(z_i).F_{Q_2}(z_i))}{\left[(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2 + (T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2\right]} \\ + (1-\alpha)\sum_{i=1}^{n} \dfrac{(T_{Q_1}(z_i).T_{Q_2}(z_i) + I_{Q_1}(z_i).I_{Q_2}(z_i) + F_{Q_1}(z_i).F_{Q_2}(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}\right]} \end{array}\right] \tag{5}$$

where $\alpha \in [0, 1]$.

**Definition 3.4** (Pramanik et al., 2017): Weighted hybrid vector similarity measure of SVNSs

The weighted hybrid vector similarity measure of $Q_1$ = $\{z, \langle T_{Q_1}(z), I_{Q_1}(z), F_{Q_1}(z) \rangle \mid z \in U\}$ and $Q_2$ = $\{z, \langle T_{Q_2}(z), I_{Q_2}(z), F_{Q_2}(z) \rangle \mid z \in U\}$ can be defined as follows:

$$WHyb\ (Q_1,\ Q_2) = \left[ \begin{array}{l} \alpha \sum_{i=1}^{n} w_i \dfrac{2(T_{Q_1}(z_i).T_{Q_2}(z_i) + I_{Q_1}(z_i).I_{Q_2}(z_i) + F_{Q_1}(z_i).F_{Q_2}(z_i))}{\left[(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2 + (T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2\right]} \\ + (1-\alpha) \sum_{i=1}^{n} w_i \dfrac{(T_{Q_1}(z_i).T_{Q_2}(z_i) + I_{Q_1}(z_i).I_{Q_2}(z_i) + F_{Q_1}(z_i).F_{Q_2}(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}\right]} \end{array} \right]$$

(6)

where $w_i \in [0, 1]$ be the weight of $z_i$ for i = 1, 2, …, $n$ such that $\sum_{i=1}^{n} w_i = 1$, and $\alpha \in [0, 1]$.

**Definition 3.5** (Ye & Ye, 2014): Dice similarity measure between two SVNRSs $Q_1, Q_2$ is defined as follows.

$Dice_{SVRNS}(Q_1, Q_2)$

$$= \frac{1}{p} \sum_{j=1}^{p} \left\langle \frac{1}{n} \left[ \sum_{i=1}^{n} \frac{2(T^j_{Q_1}(z_i).T^j_{Q_2}(z_i) + I^j_{Q_1}(z_i).I^j_{Q_2}(z_i) + F^j_{Q_1}(z_i).F^j_{Q_2}(z_i))}{\left[((T^j_{Q_1}(z_i))^2 + (I^j_{Q_1}(z_i))^2 + (F^j_{Q_1}(z_i))^2) + ((T^j_{Q_2}(z_i))^2 + (I^j_{Q_2}(z_i))^2 + (F^j_{Q_2}(z_i))^2)\right]} \right] \right\rangle$$

(7)

**Definition 3.6** (Ye & Ye, 2014): Weighted Dice similarity measure between two SVNRSs $Q_1, Q_2$ is presented as follows.

$WDice_{SVRNS}(Q_1, Q_2)$

$$= \frac{1}{p} \sum_{j=1}^{p} \left\langle \left[ \sum_{i=1}^{n} w_i \frac{2(T^j_{Q_1}(z_i).T^j_{Q_2}(z_i) + I^j_{Q_1}(z_i).I^j_{Q_2}(z_i) + F^j_{Q_1}(z_i).F^j_{Q_2}(z_i))}{\left[((T^j_{Q_1}(z_i))^2 + (I^j_{Q_1}(z_i))^2 + (F^j_{Q_1}(z_i))^2) + ((T^j_{Q_2}(z_i))^2 + (I^j_{Q_2}(z_i))^2 + (F^j_{Q_2}(z_i))^2)\right]} \right] \right\rangle$$

(8)

**Definition 3.7:** Cosine similarity measure between two SVNRSs $Q_1, Q_2$ can be defined in the following way:

$Cos_{SVRNS}(Q_1, Q_2)$

$$= \frac{1}{p} \sum_{j=1}^{p} \left\langle \frac{1}{n} \left[ \sum_{i=1}^{n} \frac{(T^j_{Q_1}(z_i).T^j_{Q_2}(z_i) + I^j_{Q_1}(z_i).I^j_{Q_2}(z_i) + F^j_{Q_1}(z_i).F^j_{Q_2}(z_i))}{\left[\sqrt{((T^j_{Q_1}(z_i))^2 + (I^j_{Q_1}(z_i))^2 + (F^j_{Q_1}(z_i))^2)}.\sqrt{(T^j_{Q_2}(z_i))^2 + (I^j_{Q_2}(z_i))^2 + (F^j_{Q_2}(z_i))^2)}\right]} \right] \right\rangle.$$

(9)

**Proposition 3.1** The defined cosine similarity measure $Cos_{SVNRS}(Q_1, Q_2)$ between SVRNSs $Q_1$ and $Q_2$ satisfies the following properties:

- $P_1$. $0 \le Cos_{SVRNS}(Q_1, Q_2) \le 1$
- $P_2$. $Cos_{SVRNS}(Q_1, Q_2) = 1$, if and only if $Q_1 = Q_2$
- $P_3$. $Cos_{SVRNS}(Q_1, Q_2) = Cos_{SVRNS}(Q_2, Q_1)$.

**Proof.**

$P_1$: According to Cauchy-Schwarz inequality:

$(\mu_1.\nu_1 + \mu_2.\nu_2 + \ldots + \mu_n.\nu_n)^2 \le (\mu_1^2 + \mu_2^2 + \ldots + \mu_n^2).(\nu_1^2 + \nu_2^2 + \ldots + \nu_n^2)$, where $(\mu_1, \mu_2, \ldots, \mu_n) \in \Re^n$ and $(\nu_1, \nu_2, \ldots, \nu_n) \in \Re^n$, we have

$$(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i)) \le \sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}$$

Therefore, $\frac{1}{n} \sum_{i=1}^{n} \frac{(T_P(z_i).T_Q(z_i) + I_P(z_i).I_Q(z_i) + F_P(z_i).F_Q(z_i))}{\left[\sqrt{(T_P(z_i))^2 + (I_P(z_i))^2 + (F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2 + (I_Q(z_i))^2 + (F_Q(z_i))^2}\right]} \le 1$, So, $Cos_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}\right]}\right]\right\rangle \le 1,$$

Obviously, $Cos_{SVRNS}(Q_1, Q_2) \ge 0$, thus $0 \le Cos_{SVRNS}(Q_1, Q_2) \le 1$

$P_2$: If $Q_1 = Q_2$, then, $T_{Q_1}^j(z_i) = T_{Q_2}^j(z_i)$, $I_{Q_1}^j(z_i) = I_{Q_2}^j(z_i)$ and $F_{Q_1}^j(z_i) = F_{Q_2}^j(z_i)$ for i = 1, 2, …, $n$; j = 1, 2, …, $p$.

Therefore, $Cos_{SVRNS}(Q_1, Q_1) =$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_1}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_1}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_1}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}\right]}\right]\right\rangle =1.$$

$P_3$: $Cos_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}\right]}\right]\right\rangle =$$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\sum_{i=1}^{n}\frac{(T_{Q_2}^j(z_i).T_{Q_1}^j(z_i)+I_{Q_2}^j(z_i).I_{Q_1}^j(z_i)+F_{Q_2}^j(z_i).F_{Q_1}^j(z_i))}{\left[\sqrt{((T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}.\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}\right]}\right]\right\rangle = Cos_{SVRNS}(Q_2, Q_1).$$

**Definition 3.8:** Weighted cosine similarity measure between SVNRSs $Q_1, Q_2$ can be defined as follows:

$WCos_{SVRNS}(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n}w_i\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}\right]}\right]\right\rangle$$

(10)

**Proposition 3.2** The defined weighted cosine similarity measure $WCos_{SVNRS}(Q_1, Q_2)$ between SVRNSs $Q_1$ and $Q_2$ satisfies the following properties:

$P_1$. $0 \le WCos_{SVRNS}(Q_1, Q_2) \le 1$

$P_2$. $WCos_{SVRNS}(Q_1, Q_2) = 1$, if and only if $Q_1 = Q_2$

$P_3$. $WCos_{SVRNS}(Q_1, Q_2) = Cos_{SVRNS}(Q_2, Q_1)$

**Proof.**

$P_1$: From Cauchy-Schwarz inequality, we have

$$(T_P(z_i).T_Q(z_i)+I_P(z_i).I_Q(z_i)+F_P(z_i).F_Q(z_i)) \le$$
$$\sqrt{(T_P(z_i))^2+(I_P(z_i))^2+(F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2+(I_Q(z_i))^2+(F_Q(z_i))^2}$$

So, $\sum_{i=1}^{n}w_i\frac{(T_P(z_i).T_Q(z_i)+I_P(z_i).I_Q(z_i)+F_P(z_i).F_Q(z_i))}{\left[\sqrt{(T_P(z_i))^2+(I_P(z_i))^2+(F_P(z_i))^2}.\sqrt{(T_Q(z_i))^2+(I_Q(z_i))^2+(F_Q(z_i))^2}\right]} \le 1$, $w_i \in [0, 1]$ and $\sum_{i=1}^{n}w_i = 1$.

$WCos_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n}w_i\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}\right]}\right]\right\rangle \le 1,$$

where $w_i \in [0, 1]$ be the weight of $z_i$ for i = 1, 2, …, $n$ such that $\sum_{i=1}^{n}w_i = 1$. Obviously, $WCos_{SVRNS}(Q_1, Q_2) \ge 0$, and therefore $0 \le WCos_{SVRNS}(Q_1, Q_2) \le 1$

$P_2$: If $Q_1 = Q_2$, then, $T_{Q_1}^j(z_i) = T_{Q_2}^j(z_i)$, $I_{Q_1}^j(z_i) = I_{Q_2}^j(z_i)$ and $F_{Q_1}^j(z_i) = F_{Q_2}^j(z_i)$ for i = 1, 2, …, $n$; j = 1, 2,

…, $p$.
$WCos_{SVRNS}(Q_1, Q_1) =$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n} w_i \frac{(T_{Q_1}^j(z_i).T_{Q_1}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_1}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_1}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}\right]}\right]\right\rangle=1.$$

P$_3$: $WCos_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n} w_i \frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}\right]}\right]\right\rangle=$$

$$\frac{1}{p}\sum_{j=1}^{p}\left\langle\left[\sum_{i=1}^{n} w_i \frac{(T_{Q_2}^j(z_i).T_{Q_1}^j(z_i)+I_{Q_2}^j(z_i).I_{Q_1}^j(z_i)+F_{Q_2}^j(z_i).F_{Q_1}^j(z_i))}{\left[\sqrt{((T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}.\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}\right]}\right]\right\rangle= WCos_{SVRNS}(Q_2, Q_1).$$

Next, we have defined hybrid vector similarity methods between SVRNSs by extending the concept of Pramanik et al. (2017) as given below.

**Definition 3.9:** Hybrid vector similarity measure between SVNRSs $Q_1$, $Q_2$ can be defined as follows:
$Hyb_{SVRNS}(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\frac{1}{n}\left[\begin{array}{l}\alpha\sum_{i=1}^{n}\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)+((T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)\right]}\\+(1-\alpha)\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}\right]}\end{array}\right]\right\rangle$$

(11)
where $\alpha \in [0, 1]$.

**Proposition 3.3** The defined single valued refined hybrid vector similarity measure $Hyb_{SVNRS}(Q_1, Q_2)$ between two SVRNSs $Q_1$ and $Q_2$ satisfies the following properties:

P$_1$. $0 \le Hyb_{SVRNS}(Q_1, Q_2) \le 1$

P$_2$. $Hyb_{SVRNS}(Q_1, Q_2) = 1$, if and only if $Q_1 = Q_2$.

P$_3$. $Hyb_{SVRNS}(Q_1, Q_2) = \mathrm{Hyb}_{\mathrm{SVRNS}}(Q_2, Q_1)$.

**Proof.**

P$_1$. From Dice and cosine measures of SVRNSs defined in Equation (7) and Equation (9), we have $0 \le Dice_{SVRNS}(Q_1, Q_2) \le 1$, $0 \le Cos_{SVRNS}(Q_1, Q_2) \le 1$.
Therefore, we have, $Dice_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}\le 1, \text{ for } j = 1, 2, \ldots, p,$$

$Cos_{SVRNS}(Q_1, Q_2) =$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{((T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2)}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2)}\right]}\le 1, \text{ for } j = 1, 2, \ldots, p,$$

Here, $(\alpha)\, Dice_{SVRNS}(Q_1, Q_2) + (1-\alpha)\, Cos_{SVRNS}(Q_1, Q_2) =$

$$(\alpha)\frac{1}{n}\left(\sum_{i=1}^{n}\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}\right)+(1-\alpha)\frac{1}{n}\left(\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}\right)\le \alpha + (1-\alpha) = 1,$$

for j = 1, 2, …, $p$.

Therefore, $Hyb_{SVRNS}(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle \frac{1}{n}\left[\begin{array}{l}\alpha\sum_{i=1}^{n}\frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2+(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2\right]}\\ +(1-\alpha)\sum_{i=1}^{n}\frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}.\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}\right]}\end{array}\right]\right\rangle \le 1.$$

Also, $Dice_{SVRNS}(Q_1, Q_2)$, $Cos_{SVRNS}(Q_1, Q_2) \ge 0$, for j = 1, 2, …, $p$ .

Obviously, $Hyb_{SVRNS}(Q_1, Q_2) \ge 0$.

This proves that $0 \le Hyb_{SVNRS}(Q_1, Q_2) \le 1$.

**P₂.** For any two SVNRSs $Q_1$and $Q_2$, if $Q_1 = Q_2$, this implies $T_{Q_1}^{j}(z_i)=T_{Q_2}^{j}(z_i)$, $I_{Q_1}^{j}(z_i)=I_{Q_2}^{j}(z_i)$, $F_{Q_1}^{j}(z_i)=F_{Q_2}^{j}(z_i)$, for i = 1, 2, …, $n$ and j = 1, 2, …, $p$.

$$Dice_{SVRNS}(Q_1, Q_2) = \frac{1}{n}\sum_{i=1}^{n}\frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2+(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2\right]}=1,$$

for j = 1, 2, …, $p$, and

$$Cos_{SVRNS}(Q_1, Q_2) = \sum_{i=1}^{n}\frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}.\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}\right]}=1,$$

for j = 1, 2, …, p. Hence, $Hyb_{SVRNS}(Q_1, Q_2) =1$.

**P₃.** $Hyb_{SVRNS}(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle \frac{1}{n}\left[\begin{array}{l}\alpha\sum_{i=1}^{n}\frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2+(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2\right]}\\ +(1-\alpha)\sum_{i=1}^{n}\frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}.\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}\right]}\end{array}\right]\right\rangle$$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle \frac{1}{n}\left[\begin{array}{l}\alpha\sum_{i=1}^{n}\frac{2(T_{Q_2}^{j}(z_i).T_{Q_1}^{j}(z_i)+I_{Q_2}^{j}(z_i).I_{Q_1}^{j}(z_i)+F_{Q_2}^{j}(z_i).F_{Q_1}^{j}(z_i))}{\left[(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2+(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2\right]}\\ +(1-\alpha)\sum_{i=1}^{n}\frac{(T_{Q_2}^{j}(z_i).T_{Q_1}^{j}(z_i)+I_{Q_2}^{j}(z_i).I_{Q_1}^{j}(z_i)+F_{Q_2}^{j}(z_i).F_{Q_1}^{j}(z_i))}{\left[\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}.\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}\right]}\end{array}\right]\right\rangle$$

$= Hyb_{SVRNS}(Q_2, Q_1)$.

**Definition 10:** Weighted hybrid vector similarity measure between SVRNSs can be defined as follows.

$WHyb_w(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle \left[\begin{array}{l}\alpha\sum_{i=1}^{n}w_i\frac{2(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2+(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2\right]}\\ +(1-\alpha)\sum_{i=1}^{n}w_i\frac{(T_{Q_1}^{j}(z_i).T_{Q_2}^{j}(z_i)+I_{Q_1}^{j}(z_i).I_{Q_2}^{j}(z_i)+F_{Q_1}^{j}(z_i).F_{Q_2}^{j}(z_i))}{\left[\sqrt{(T_{Q_1}^{j}(z_i))^2+(I_{Q_1}^{j}(z_i))^2+(F_{Q_1}^{j}(z_i))^2}.\sqrt{(T_{Q_2}^{j}(z_i))^2+(I_{Q_2}^{j}(z_i))^2+(F_{Q_2}^{j}(z_i))^2}\right]}\end{array}\right]\right\rangle$$

(12)

Here, $w_i \in [0, 1]$ represents the weight of $z_i$ for i = 1, 2, …, $n$ such that $\sum_{i=1}^{n} w_i = 1$, where $\alpha \in [0, 1]$, and $WHyb_w(Q_1, Q_2)$ should satisfy the following properties.

**Proposition 3.4**

P₁. $0 \le WHyb_w(Q_1, Q_2) \le 1$.

P₂. $WHyb_w(Q_1, Q_2) = 1$, if and only if $Q_1 = Q_2$.

P₃. $WHyb_w(Q_1, Q_2) = WHyb_w(Q_2, Q_1)$.

**Proof.**

$P_1$. Using Dice and cosine measures of SVRNSs, we have $0 \le Dice_{SVRNS}(Q_1, Q_2) \le 1$, $0 \le Cos_{SVRNS}(Q_1, Q_2) \le 1$.

$(\alpha)\, Dice_{SVRNS}(Q_1, Q_2) + (1-\alpha)\, Cos_{SVRNS}(Q_1, Q_2) =$

$$(\alpha)\frac{1}{n}\left(\sum_{i=1}^{n}\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}\right) + (1-\alpha)\frac{1}{n}\left(\sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}\right) \le \alpha + (1-\alpha) = 1,$$

for j = 1, 2, …, $p$.

Therefore, $WHyb_w(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\begin{bmatrix}\alpha\sum_{i=1}^{n}w_i\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}\\ +(1-\alpha)\sum_{i=1}^{n}w_i\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}\end{bmatrix}\right\rangle \le 1.$$

$Dice_{SVRNS}(Q_1, Q_2)$, $Cos_{SVRNS}(Q_1, Q_2) \ge 0$, for j = 1, 2, …, $p$.

Obviously, $WHyb_w(Q_1, Q_2) \ge 0$, therefore $0 \le WHyb_w(Q_1, Q_2) \le 1$.

**$P_2$.** If $Q_1 = Q_2$, then $T_{Q_1}^j(z_i) = T_{Q_2}^j(z_i)$, $I_{Q_1}^j(z_i) = I_{Q_2}^j(z_i)$, $F_{Q_1}^j(z_i) = F_{Q_2}^j(z_i)$, for i = 1, 2, …, $n$ and j = 1, 2, …, $p$.

$$Dice_{SVRNS}(Q_1, Q_2) = \frac{1}{n}\sum_{i=1}^{n}\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}=1,$$

for j = 1, 2, …, $p$, and

$$Cos_{SVRNS}(Q_1, Q_2) = \sum_{i=1}^{n}\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}=1,$$

for j = 1, 2, …, p. Hence, $WHyb_w(Q_1, Q_2) = 1$.

**$P_3$.** $WHyb_w(Q_1, Q_2)$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\begin{bmatrix}\alpha\sum_{i=1}^{n}w_i\frac{2(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2+(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2\right]}\\ +(1-\alpha)\sum_{i=1}^{n}w_i\frac{(T_{Q_1}^j(z_i).T_{Q_2}^j(z_i)+I_{Q_1}^j(z_i).I_{Q_2}^j(z_i)+F_{Q_1}^j(z_i).F_{Q_2}^j(z_i))}{\left[\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}.\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}\right]}\end{bmatrix}\right\rangle$$

$$=\frac{1}{p}\sum_{j=1}^{p}\left\langle\begin{bmatrix}\alpha\sum_{i=1}^{n}w_i\frac{2(T_{Q_2}^j(z_i).T_{Q_1}^j(z_i)+I_{Q_2}^j(z_i).I_{Q_1}^j(z_i)+F_{Q_2}^j(z_i).F_{Q_1}^j(z_i))}{\left[(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2+(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2\right]}\\ +(1-\alpha)\sum_{i=1}^{n}w_i\frac{(T_{Q_2}^j(z_i).T_{Q_1}^j(z_i)+I_{Q_2}^j(z_i).I_{Q_1}^j(z_i)+F_{Q_2}^j(z_i).F_{Q_1}^j(z_i))}{\left[\sqrt{(T_{Q_2}^j(z_i))^2+(I_{Q_2}^j(z_i))^2+(F_{Q_2}^j(z_i))^2}.\sqrt{(T_{Q_1}^j(z_i))^2+(I_{Q_1}^j(z_i))^2+(F_{Q_1}^j(z_i))^2}\right]}\end{bmatrix}\right\rangle$$

$= WHyb_w(Q_2, Q_1)$.

## 4. MADM WITH SINGLE VALUED REFINED NEUTROSOPHIC INFORMATION BASED ON HYBRID SIMILARITY MEASURE

Assume that $P = \{P_1, P_2, \ldots, P_m\}$ $(m \ge 2)$ be a discrete set of $m$ candidates, $C = \{C_1, C_2, \ldots, C_n\}$, $(n \ge 2)$ be the set of attributes of each candidates, and $A = \{A_1, A_2, \ldots, A_k\}$, $(k \ge 2)$ be the set of alternatives of each candidate. The decision maker or expert presents the ranking of alternatives with regard to each candidate. The ranking represents the performances of $P_i$, i = 1, 2, …, $m$ against the attributes $C_j$, j = 1, 2, …, $n$ and $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of the attributes $C_j$, j = 1, 2, …, $n$ with $0 \le w_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$. The relation between candidates and attributes, and the relation between attributes and alternatives can be presented as follows (see Table 1 and Table 2 respectively).

**Table 1. The relation between candidates and pre-defined attributes**

$$\begin{pmatrix} & C_1 & C_2 & \ldots & C_n \\ P_1 & \beta_{11}^{t} & \beta_{12}^{t} & \ldots & \beta_{1n}^{t} \\ P_2 & \beta_{21}^{t} & \beta_{22}^{t} & \ldots & \beta_{2n}^{t} \\ . & . & . & \ldots & . \\ . & . & . & \ldots & . \\ P_m & \beta_{m1}^{t} & \beta_{m2}^{t} & \ldots & \beta_{mn}^{t} \end{pmatrix}$$

where $\beta_{11}^{t} = \left\langle T_{ij}^{t}, I_{ij}^{t}, F_{ij}^{t} \right\rangle$ represents single valued neutrosophic numbers (SVNNs), i = 1, 2, …, $m$; j = 1, 2, …, $n$; t = 1, 2, …, $s$.

**Table 2. The relation between attributes and alternatives**

$$\begin{pmatrix} & A_1 & A_2 & \ldots & A_k \\ C_1 & \gamma_{11} & \gamma_{12} & \ldots & \gamma_{1k} \\ C_2 & \gamma_{21} & \gamma_{22} & \ldots & \gamma_{2k} \\ . & . & . & \ldots & . \\ . & . & . & \ldots & . \\ C_n & \gamma_{n1} & \gamma_{n2} & \ldots & \gamma_{nk} \end{pmatrix}$$

Here, $\gamma_{j\ell} = \left\langle T_{j\ell}, I_{j\ell}, F_{j\ell} \right\rangle$ denotes SVNNs, j = 1, 2, …, $n$; $\ell$ = 1, 2, …, $k$.

We now develop two algorithms for MADM problems based on hybrid similarity measure with single valued refined neutrosophic information as given below.

**Algorithm 1**

**Step 1.** Calculate the single valued refined hybrid similarity measures between Table 1, and 2 by using Equation 11.

**Step 2.** Rank the alternatives based on the descending order of hybrid similarity measures. The biggest value reflects the best alternative.

**Step 3.** Stop.

**Algorithm 2**

**Step 1.** Compute the single valued refined weighted hybrid similarity measure between Table 1 and 2 by means of Equation 12.

**Step 2.** The alternatives are ranked in descending order of the refined weighted hybrid similarity measures. The highest value of refined weighted hybrid similarity measures indicates the best alternative.

**Step 3.** Stop.

## 5. APPLICATION OF THE PROPOSED METHOD TO MEDICAL DIAGNOSIS PROBLEM

We consider the illustrative example of medical diagnosis with single valued refined neutrosophic information studied in (Mondal & Pramanik, 2015e). Medical diagnosis has to deal with a large amount of uncertainties and huge amount of information available to the medical practitioners using new and advanced technologies. The procedure of classifying dissimilar set of symptoms under a single name of diseases is not easy (Broumi & Smarandache, 2014). Also, it is possible that every object has different truth, indeterminate and false membership functions and the proposed similarity measures among the patients versus symptoms and symptoms versus diseases will provide the appropriate medical diagnosis. In practical situation, there may occur errors in diagnosis if we consider data from single (one time) observation and therefore multi time inspection, by considering the samples of same patient at different times will provide best medical diagnosis (Rajarajeswari & Uma, 2014).

Consider $P = \{P_1, P_2, P_3, P_4\}$ be the set of four patients, C = {viral fever, malaria, typhoid, stomach problem, chest problem} be the set of five diseases, A = {temperature, headache, stomach pain, cough, chest pain} be the set of six symptoms. Now our objective is to examine the patient at different time intervals and we will obtain different truth, indeterminate and false membership functions for every patient. Let three observations are taken in a day: 7 am, 1 pm and 6 pm (see Table 3) (Mondal & Pramanik, 2015e).

**Table 3. The relation between patients and symptoms**

| | Temperature | Headache | Stomach pain | Cough | Chest pain |
|---|---|---|---|---|---|
| $P_1$ | (0.8, 0.1, 0.1)<br>(0.6, 0.3, 0.3)<br>(0.6, 0.3, 0.1) | (0.6, 0.1, 0.3)<br>(0.5, 0.2, 0.4)<br>(0.5, 0.1, 0.2) | (0.2, 0.8, 0.0)<br>(0.3, 0.5, 0.2)<br>(0.2, 0.3, 0.4) | (0.6, 0.1, 0.3)<br>(0.4, 0.4, 0.4)<br>(0.4, 0.3, 0.3) | (0.1, 0.6, 0.3)<br>(0.3, 0.4, 0.5)<br>(0.2, 0.5, 0.4) |
| $P_2$ | (0.0, 0.8, 0.2)<br>(0.2, 0.6, 0.4)<br>(0.1, 0.6, 0.4) | (0.4, 0.4, 0.2)<br>(0.5, 0.4, 0.1)<br>(0.4, 0.6, 0.3) | (0.6, 0.1, 0.3)<br>(0.4, 0.2, 0.5)<br>(0.3, 0.2, 0.4) | (0.1, 0.7, 0.2)<br>(0.2, 0.7, 0.5)<br>(0.3, 0.5, 0.4) | (0.1, 0.8, 0.1)<br>(0.3, 0.6, 0.4)<br>(0.3, 0.6, 0.3) |
| $P_3$ | (0.8, 0.1, 0.1)<br>(0.6, 0.4, 0.1)<br>(0.5, 0.3, 0.3) | (0.8, 0.1, 0.1)<br>(0.6, 0.2, 0.4)<br>(0.6, 0.1, 0.3) | (0.0, 0.6, 0.4)<br>(0.2, 0.5, 0.5)<br>(0.3, 0.4, 0.6) | (0.2, 0.7, 0.1)<br>(0.2, 0.5, 0.5)<br>(0.1, 0.6, 0.3) | (0.0, 0.5, 0.5)<br>(0.2, 0.5, 0.3)<br>(0.3, 0.3, 0.4) |
| $P_4$ | (0.6, 0.1, 0.3)<br>(0.4, 0.3, 0.2)<br>(0.5, 0.2, 0.3) | (0.5, 0.4, 0.1)<br>(0.4, 0.4, 0.4)<br>(0.5, 0.2, 0.4) | (0.3, 0.4, 0.3)<br>(0.2, 0.4, 0.5)<br>(0.1, 0.5, 0.4) | (0.7, 0.2, 0.1)<br>(0.5, 0.2, 0.4)<br>(0.6, 0.4, 0.1) | (0.3, 0.4, 0.3)<br>(0.4, 0.3, 0.4)<br>(0.3, 0.5, 0.5) |

The relation between symptoms and diseases in the form single valued neutrosophic assessments is given in Table 4 below.

**Table 4. The relation between symptoms and diseases**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| Temperature | (0.6, 0.3, 0.3) | (0.2, 0.5, 0.3) | (0.2, 0.6, 0.4) | (0.1, 0.6, 0.6) | (0.1, 0.6, 0.4) |
| Headache | (0.4, 0.5, 0.3) | (0.2, 0.6, 0.4) | (0.1, 0.5, 0.4) | (0.2, 0.4, 0.6) | (0.1, 0.6, 0.4) |
| Stomach pain | (0.1, 0.6, 0.3) | (0.0, 0.6, 0.4) | (0.2, 0.5, 0.5) | (0.8, 0.2, 0.2) | (0.1, 0.7, 0.1) |
| Cough | (0.4, 0.4, 0.4) | (0.4, 0.1, 0.5) | (0.2, 0.5, 0.5) | (0.1, 0.7, 0.4) | (0.4, 0.5, 0.4) |
| Chest pain | (0.1, 0.7, 0.4) | (0.1, 0.6, 0.3) | (0.1, 0.6, 0.4) | (0.1, 0.7, 0.4) | (0.8, 0.2, 0.2) |

Now using Equation (11), Hybrid vector refined similarity measures (HVRSM) by considering $\alpha = 0.5$ between Relation 1, and 2 are presented as given below (see Table 5).

**Table 5. HVRSM between Relation 1 and Relation 2**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | 0.9033 | 0.7953 | 0.7676 | 0.6809 | 0.6809 |
| $P_2$ | 0.8135 | 0.7981 | 0.8892 | 0.8880 | 0.7446 |
| $P_3$ | 0.8846 | 0.7418 | 0.7959 | 0.7074 | 0.6535 |
| $P_4$ | 0.9116 | 0.8231 | 0.8031 | 0.6898 | 0.7526 |

The maximal HVRSM from Table 5 determines the proper medical diagnosis. Therefore, from Table 5, we observe that $P_1$, $P_3$, $P_4$ suffer from viral fever, and $P_2$ suffers from typhoid.

Also, using Equation (12), weighted hybrid vector refined similarity measures (WHVRSM) with known weight information $w = (0.3, 0.2, 0.15, 0.2, 0.15)$ and $\alpha = 0.5$ between Relation 1, and 2 are presented as given below (see the Table 6).

**Table 6. Weighted hybrid vector refined similarity measure (WHVRSM) between Relation 1 and Relation 2**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9078** | 0.7721 | 0.7383 | 0.6533 | 0.6607 |
| $P_2$ | 0.7994 | 0.8165 | **0.8989** | 0.8919 | 0.7909 |
| $P_3$ | **0.8879** | 0.7189 | 0.7664 | 0.6886 | 0.6423 |
| $P_4$ | **0.9189** | 0.8030 | 0.7814 | 0.6788 | 0.7326 |

Here, we also see that $P_1$, $P_3$, $P_4$ suffer from viral fever, and $P_2$ suffers from typhoid. By using Equation. 11, and 12, HVRSMs and WHVRSMs with different values of $\alpha$ between Relation 1, 2 are presented in the following Tables 7, 8, 9, 10, 11, 12, 13, and 14 and which patient suffers from which disease is indicated by $\rightarrow$ mark below the Tables.

**Table 7. HVRSM between Relation 1 and Relation 2 when $\alpha = 0.1$**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9059** | 0.7987 | 0.7706 | 0.6904 | 0.6849 |
| $P_2$ | 0.8156 | 0.8033 | 0.8917 | **0.8931** | 0.7467 |
| $P_3$ | **0.8880** | 0.7434 | 0.7976 | 0.7118 | 0.6562 |
| $P_4$ | **0.9157** | 0.8301 | 0.8066 | 0.6979 | 0.7571 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Stomach problem, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

**Table 8. HVRSM between Relation 1 and Relation 2 when $\alpha = 0.25$**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9049** | 0.7974 | 0.7695 | 0.6868 | 0.6834 |
| $P_2$ | 0.8148 | 0.8014 | 0.8908 | **0.8912** | 0.7459 |
| $P_3$ | **0.8867** | 0.7428 | 0.7970 | 0.7102 | 0.6552 |
| $P_4$ | **0.9142** | 0.8274 | 0.8053 | 0.6949 | 0.7554 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Stomach problem, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

**Table 9. HVRSM between Relation 1 and Relation 2 when $\alpha = 0.75$**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9016** | 0.7931 | 0.7658 | 0.6750 | 0.6784 |
| $P_2$ | 0.8122 | 0.7948 | **0.8876** | 0.8848 | 0.7434 |
| $P_3$ | **0.8825** | 0.7408 | 0.7949 | 0.7047 | 0.6517 |
| $P_4$ | **0.9090** | 0.8187 | 0.8009 | 0.6847 | 0.7498 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

**Table 10. HVRSM between Relation 1 and Relation 2 when $\alpha = 0.90$**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9006** | 0.7918 | 0.7647 | 0.6714 | 0.6769 |
| $P_2$ | 0.8114 | 0.7928 | **0.8867** | 0.8829 | 0.7426 |
| $P_3$ | **0.8813** | 0.7401 | 0.7942 | 0.7030 | 0.6507 |
| $P_4$ | **0.9075** | 0.8161 | 0.7996 | 0.6816 | 0.7482 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

**Table 11. WHVRSM between Relation 1 and Relation 2 when $\alpha = 0.1$**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9136** | 0.7756 | 0.7409 | 0.6616 | 0.6641 |
| $P_2$ | 0.8014 | 0.8224 | **0.9012** | 0.8966 | 0.7890 |
| $P_3$ | **0.8907** | 0.7208 | 0.7679 | 0.6926 | 0.6448 |
| $P_4$ | **0.9233** | 0.8170 | 0.7852 | 0.6875 | 0.7408 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

**Table 12. WHVRSM between Relation 1 and Relation 2 when $\alpha = 0.25$**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9114** | 0.7743 | 0.7399 | 0.6585 | 0.6628 |
| $P_2$ | 0.8006 | 0.8202 | **0.9003** | 0.8948 | 0.7920 |
| $P_3$ | **0.8397** | 0.7201 | 0.7673 | 0.6911 | 0.6438 |
| $P_4$ | **0.9217** | 0.8162 | 0.7838 | 0.6842 | 0.7378 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

**Table 13. WHVRSM between Relation 1, and 2 when $\alpha = 0.75$**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9041** | 0.7698 | 0.7366 | 0.6482 | 0.6585 |
| $P_2$ | 0.7981 | 0.8128 | **0.8975** | 0.8890 | 0.7897 |
| $P_3$ | **0.8695** | 0.7178 | 0.7655 | 0.6861 | 0.6408 |
| $P_4$ | **0.9162** | 0.8138 | 0.7790 | 0.6734 | 0.7274 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

**Table 14. WHVRSM between Relation 1, and 2 when $\alpha = 0.90$**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.9019** | 0.7685 | 0.7356 | 0.6451 | 0.6572 |
| $P_2$ | 0.7974 | 0.8106 | **0.8967** | 0.8873 | 0.7890 |
| $P_3$ | **0.8785** | 0.7171 | 0.7649 | 0.6846 | 0.6400 |
| $P_4$ | **0.9145** | 0.8130 | 0.7775 | 0.6702 | 0.7243 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

**Note 1.** Using neutrosophic refined tangent similarity measure, Mondal and Pramanik (2015e) obtained the results as shown in Table 15.

**Table 15. The tangent refined similarity measure between Relation 1, and 2 (Mandal &Pramanik, 2015e)**

| | Viral fever | Malaria | Typhoid | Stomach problem | Chest problem |
|---|---|---|---|---|---|
| $P_1$ | **0.8963** | 0.8312 | 0.8237 | 0.8015 | 0.7778 |
| $P_2$ | 0.8404 | 0.8386 | **0.8877** | 0.8768 | 0.8049 |
| $P_3$ | **0.8643** | 0.8091 | 0.8393 | 0.7620 | 0.7540 |
| $P_4$ | **0.8893** | 0.8465 | 0.8335 | 0.7565 | 0.7959 |

$P_1 \rightarrow$ Viral fever, $P_2 \rightarrow$ Typhoid, $P_3 \rightarrow$ Viral fever, $P_4 \rightarrow$ Viral fever

From the Table 15, we observe that $P_1, P_3, P_4$ suffer from viral fever, and $P_2$ suffers from typhoid.

## 6. CONCLUSION

We have investigated hybrid vector similarity and weighted hybrid vector similarity measures with single valued refined neutrosophic assessments and proved some of their basic properties. Then, the proposed hybrid similarity measures have been used to solve a medical diagnosis problem. We have compared the obtained results with different values of the parameter $\alpha$ and with the results of other existing method in order to verify the effectiveness of the proposed method. We hope that the proposed hybrid vector similarity measure can be applied to solve decision making problems in refined neutrosophic environment such as fault diagnosis, cluster analysis, data mining, investment, etc.

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