Neutrosophic Goal Programming

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Abstract

In this chapter, the goal programming in neutrosophic environment is introduced. The degree of acceptance, indeterminacy and rejection of objectives is considered simultaneous. In the two proposed models to solve Neutrosophic Goal Programming Problem (NGPP), our goal is to minimize the sum of the deviation in the model (I), while in the model (II), the neutrosophic goal programming problem NGPP is transformed into the crisp programming model using truth membership, indeterminacy membership, and falsity membership functions. Finally, the industrial design problem is given to illustrate the efficiency of the proposed models. The obtained results of Model (I) and Model (II) are compared with other methods.

Keywords

Neutrosophic optimization; Goal programming problem.

1 Introduction

Goal programming (GP) Models was originally introduced by Charnes and Cooper in early 1961 for a linear model . Multiple and conflicting goals can be used in goal programming. Also, GP allows the simultaneous solution of a system of Complex objectives, and the solution of the problem requires the establishment among these multiple objectives. In this case, the model must be solved in such a way that each of the objectives to be achieved. Therefore, the sum of the deviations from the ideal should be minimized in the objective function. It is important that measure deviations from the ideal should have a single scale, because deviations with different scales cannot be collected. However, the target value associated with each goal could be neutrosophic in the real-world application. In 1995, Smarandache [17] starting from philosophy (when [8] fretted to distinguish between *absolute truth* and *relative truth* or between *absolute falsehood* and *relative falsehood* in logics, and respectively between *absolute membership* and *relative membership* or *absolute non-membership* and *relative non-membership* in set theory) [12] began to use the non-standard analysis. Also, inspired from the sport games (winning, defeating, or tie scores), from votes (pro, contra, null/black votes), from positive/negative/zero numbers, from yes/no/NA, from decision making and control theory (making a decision, not making, or hesitating), from accepted/rejected/pending, etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics. [12] combined the non-standard analysis with a tri-component logic/set/probability theory and with philosophy. How to deal with all of them at once, is it possible to unity them? [12].

Netrosophic theory means Neutrosophy applied in many fields in order to solve problems related to indeterminacy. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every entity <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. entities supporting neither <A> nor<antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every entity <A> tends to be neutralized and balanced by <antiA> and <nonA> entities - as a state of equilibrium. In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. Hence, in one hand, the Neutrosophic Theory is based on the triad <A>, <neutA>, and <antiA>. In the other hand, Neutrosophic Theory studies the indeterminacy, labeled as I, with In = I for n ≥ 1, and mI + nI = (m+n)I, in neutrosophic structures developed in algebra, geometry, topology etc.

The most developed fields of Netrosophic theory are Neutrosophic Set, Neutrosophic Logic, Neutrosophic Probability, and Neutrosophic Statistics - that started in 1995, and recently Neutrosophic Precalculus and Neutrosophic Calculus, together with their applications in practice. Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T,I,F are standard or non-standard subsets of $]^-0, 1^+[$.

The important method for multi-objective decision making is goal programming approaches in practical decision making in real life. In a standard GP formulation, goals and constraints are defined precisely, but sometimes the system aim and conditions include some vague and undetermined situations. In particular, expressing the decision maker's unclear target levels for the goals mathematically and the need to optimize all goals at the same needs to complicated calculations.

The neutrosophic approach for goal programming tries to solve this kind of unclear difficulties in this chapter.

The organization of the chapter is as follows. The next section introduces a brief some preliminaries. Sections 3 describe the formation of the Problem and develop two models to neutrosophic goal programming. Section 4 presents an industrial design problem is provided to demonstrate how the approach can be applied. Finally, conclusions are provided in section 5.

2 Some Preliminaries

Definition 1. [17]

A real fuzzy number $\tilde{J}$ is a continuous fuzzy subset from the real line *R* whose triangular membership function $\mu_{\tilde{J}}(J)$ is defined by a continuous mapping from *R* to the closed interval [0,1], where

(1) $\mu_{\tilde{J}}(J) = 0$ for all $J \in (-\infty, a_1]$,

(2) $\mu_{\tilde{J}}(J)$ is strictly increasing on $J \in [a_1, m]$,

(3) $\mu_{\tilde{J}}(J) = 1$ for $J = m$,

(4) $\mu_{\tilde{J}}(J)$ is strictly decreasing on $J \in [m, a_2]$,

(5) $\mu_{\tilde{J}}(J) = 0$ for all $J \in [a_2, +\infty)$.

This will be elicited by:

$$\mu_{\tilde{J}}(J) = \begin{cases} \dfrac{J - a_1}{m - a_1}, & a_1 \le J \le m, \\ \dfrac{a_2 - J}{a_2 - m}, & m \le J \le a_2, \\ 0, & otherwise. \end{cases} \quad (1)$$

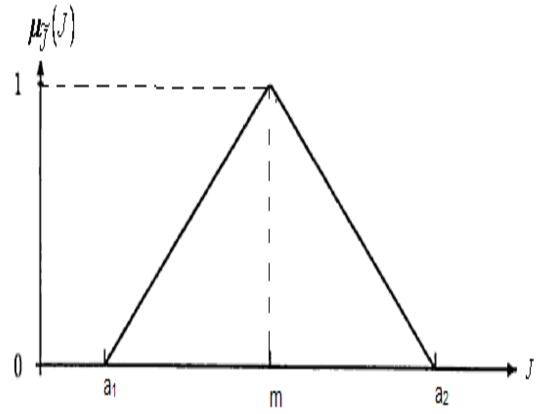

Fig. 1: Membership Function of Fuzzy Number *J*.

Where m is a given value a_1 and a_2 denote the lower and upper bounds. Sometimes, it is more convenient to use the notation explicitly highlighting the membership function parameters. In this case, we obtain

$$\mu(J;a_1,m,a_2) = \text{Max}\left\{\text{Min}\left[\frac{J-a_1}{m-a_1},\frac{a_2-J}{a_2-m}\right],0\right\} \tag{2}$$

In what follows, the definition of the α-level set or α-cut of the fuzzy number $\tilde{J}$ is introduced.

Definition 2. [1]

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed non-empty universe, an intuitionistic fuzzy set IFS *A* in *X* is defined as

$$A = \left\{\left\langle x, \mu_A(x), \upsilon_A(x)\right\rangle | x \in X\right\} \tag{3}$$

which is characterized by a membership function $\mu_A : X \to [0,1]$ and a non-membership function $\upsilon_A : X \to [0,1]$ with the condition $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for all $x \in X$ where μ_A and υ_A represent, respectively, the degree of membership and non-membership of the element *x* to the set *A*. In addition, for each IFS *A* in X, $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ for all $x \in X$ is called the degree of hesitation of the element *x* to the set A. Especially, if $\pi_A(x) = 0$, then the IFS *A* is degraded to a fuzzy set.

Definition 3. [4] The α-level set of the fuzzy parameters $\tilde{J}$ in problem (1) is defined as the ordinary set $L_\alpha\left(\tilde{J}\right)$ for which the degree of membership function exceeds the level, α, $\alpha \in [0,1]$, where:

$$L_\alpha\left(\tilde{J}\right) = \left\{J \in R \,\middle|\, \mu_{\tilde{J}}\left(J\right) \geq \alpha\right\} \tag{4}$$

For certain values α_j^* to be in the unit interval.

Definition 4. [10] Let X be a space of points (objects) and $x∈X$. A neutrosophic set A in X is defined by a truth-membership function (x), an indeterminacy-membership function (x) and a falsity-membership function (x). It has been shown in figure 2. (x), (x) and (x) are real standard or real nonstandard subsets of]0−,1+[. That is $T_A(x)$:X→]0−,1+[, $I_A(x)$:X→]0−,1+[and $F_A(x)$:X→]0−,1+[. There is not restriction on the sum of (x), (x) and (x), so 0−≤sup$T_A(x)$≤sup$I_A(x)$≤$F_A(x)$≤3+.

In the following, we adopt the notations $\mu(x)$, $\sigma_A(x)$ and $v_A(x)$ instead of $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively. Also, we write SVN numbers instead of single valued neutrosophic numbers.

Definition 5. [10] Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

A={⟨x, $\mu_A(x)$, $\sigma_A(x)$,$v_A(x)$⟩:$x∈X$},

where $\mu_A(x)$:X→[0,1], $\sigma_A(x)$:X→[0,1] and $v_A(x)$:X→[0,1] with $0 \leq \mu_A(x)+\sigma_A(x)+v_A(x) \leq 3$ for all $x∈X$. The intervals $\mu(x)$, $\sigma_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

For convenience, a SVN number is denoted by A=(a,b,c), where a,b,c∈[0,1] and $a+b+c \leq 3$.

Definition 6. Let $\tilde{J}$ be a neutrosophic number in the set of real numbers R, then its truth-membership function is defined as

$$T_{\tilde{J}}\left(J\right) = \begin{cases} \dfrac{J-a_1}{a_2-a_1}, & a_1 \leq J \leq a_2, \\ \dfrac{a_2-J}{a_3-a_2}, & a_2 \leq J \leq a_3, \\ 0, & otherwise. \end{cases} \tag{5}$$

its indeterminacy-membership function is defined as

$$I_{\tilde{J}}(J)=\begin{cases}\dfrac{J-b_1}{b_2-b_1}, & b_1 \le J \le b_2,\\ \dfrac{b_2-J}{b_3-b_2}, & b_2 \le J \le b_3,\\ 0, & otherwise.\end{cases} \tag{6}$$

and its falsity-membership function is defined as

$$F_{\tilde{J}}(J)=\begin{cases}\dfrac{J-c_1}{c_2-c_1}, & c_1 \le J \le c_2,\\ \dfrac{c_2-J}{c_3-c_2}, & c_2 \le J \le c_3,\\ 1, & otherwise.\end{cases} \tag{7}$$

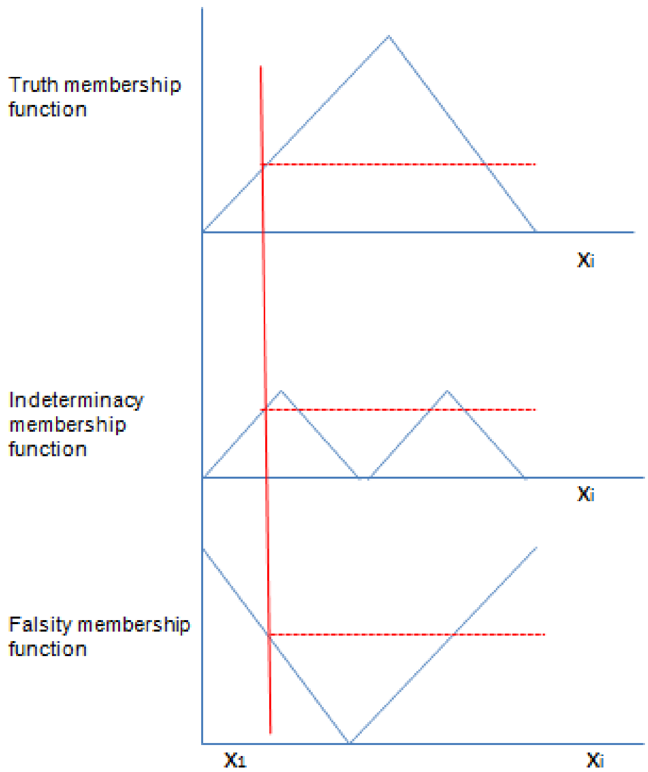

Fig. 2: Neutrosophication process [11]

3 Neutrosophic Goal Programming Problem

Goal programming can be written as:

Find $x = (x_1, x_2, ..., x_n)^T$

To achieve:

$$z_i = t_i, \quad i = 1, 2, ..., k \tag{8}$$

Subject to

$x \in X$

where t_i, are scalars and represent the target achievement levels of the objective functions that the decision maker wishes to attain provided, X is feasible set of the constraints.

The achievement function of the (8) model is the following:

$$Min \sum_{i=1}^{k} (w_{1i} n_i + w_{2i} p_i) \tag{9}$$

Goal and constraints:

$$z_i + n_i - p_i = t_i, \; i \in \{1, 2, ..., k\}$$

$$x \in X, \quad n, p \ge 0, \; n \cdot p = 0$$

n_i, p_i are negative and positive deviations from t_i target.

The NGPP can be written as:

Find $x = (x_1, x_2, ..., x_n)^T$

So as to:

$Minimize \; z_i$ with target value t_i, acceptance tolerance a_i, indeterminacy tolerance d_i, rejection tolerance c_i,

Subject to

$x \in X$

$g_j(x) \le b_j, \; j = 1, 2, ..., m$

$x_i \ge 0, \qquad i = 1, 2, ..., n$

with truth-membership, indeterminacy-membership and falsity-membership functions:

$$\mu_i^I(z_i)=\begin{cases}1, & if \quad z_i \le t_i,\\ 1-\dfrac{z_i-t_i}{a_i}, & if \quad t_i \le z_i \le t_i+a_i,\\ 0, & if \quad z_i \ge t_i+a_r\end{cases} \tag{10}$$

$$\sigma_i^I(z_i)=\begin{cases}0, & if \quad z_i \le t_i,\\ \dfrac{z_i-t_i}{d_i}, & if \quad t_i \le z_i \le t_i+d_i,\\ 1-\dfrac{z_i-t_i}{a_i-d_i}, & if \quad t_i+d_i \le z_i \le t_i+a_i,\\ 0, & if \quad z_i \ge t_i+a_i\end{cases} \tag{11}$$

$$\upsilon_i^I(z_i)=\begin{cases}0, & if \quad z_i \le t_i,\\ \dfrac{z_i-t_i}{C_i}, & if \quad t_i \le z_i \le t_i+C_i,\\ 1, & if \quad z_i \ge t_i+C_i\end{cases} \tag{12}$$

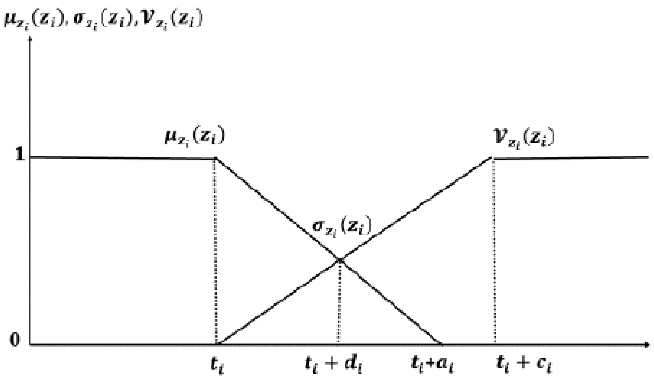

Fig. 3: Truth-membership, indeterminacy-membership and falsity-membership functions for z_i.

To maximize the degree the accptance and indeterminacy of NGP objectives and constriants, also to minimize the dgree of rejection of NGP objectives and constriants

$$\begin{aligned}&Max \quad \mu_{z_i}(z_i), \; i=1,2,...,k\\ &Max \quad \sigma_{z_i}(z_i), \; i=1,2,...,k\\ &Min \quad \upsilon_{z_i}(z_i), \; i=1,2,...,k\end{aligned} \tag{13}$$

Subject to

$$0 \le \mu_{z_i}(z_i) + \sigma_{z_i}(z_i) + \upsilon_{z_i}(z_i) \le 3,\ i = 1,2,...,k$$

$$\upsilon_{z_i}(z_i) \ge 0,\ \ i = 1,2,...,k$$

$$\mu_{z_i}(z_i) \ge \upsilon_{z_i}(z_i),\ \ i = 1,2,...,k$$

$$\mu_{z_i}(z_i) \ge \sigma_{z_i}(z_i),\ \ i = 1,2,...,k$$

$$g_j(x) \le b_j,\ j = 1,2,...,m$$

$$x \in X$$

$$x_j \ge 0, \qquad j = 1,2,...,n$$

where $\mu_{z_i}(z_i),\ \sigma_{z_i}(z_i),\ \upsilon_{z_i}(z_i)$ are truth membership function, indeterminacy membership function, falsity membership function of Neutrosophic decision set respectively.

The highest degree of truth membership function is unity. So, for the defined the truth membership function $\mu_{z_i}(z_i)$, the flexible membership goals having the aspired level unity can be presented as

$$\mu_{z_i}(z_i) + n_{i1} - p_{i1} = 1$$

For case of indeterminacy (indeterminacy membership function), it can be written:

$$\sigma_{z_i}(z_i) + n_{i2} - p_{i2} = 0.5$$

For case of rejection (falsity membership function), it can be written

$$\mu_{z_i}(z_i) + n_{i3} - p_{i3} = 0$$

Here $n_{i1}, p_{i1},\ n_{i2}, p_{i2},\ n_{i3}$ *and* p_{i3} are under-deviational and over-deviational variables.

Our goals are maximize the degree of the accptance and indeterminacy of NGP objectives and constriants, and minimize the dgree of rejection of NGP objectives and constriants.

Model (I). The minimization of the sum of the deviation can be formulated as:

$$Min\ \lambda = \sum_{i=1}^{k} w_{i1} n_{i1} + \sum_{i=1}^{k} w_{i2} n_{i2} + \sum_{i=1}^{k} w_{i3} p_{i3} \tag{14}$$

Subject to

$$\mu_{z_i}(z_i) + n_{i1} \ge 1,\ i = 1,2,...,k$$

$$\sigma_{z_i}(z_i)+n_{i2} \geq 0.5,\ i=1,2,...,k$$

$$\upsilon_{z_i}(z_i)-p_{i3} \leq 0,\ i=1,2,...,k$$

$$\upsilon_{z_i}(z_i) \geq 0,\ i=1,2,...,k$$

$$\mu_{z_i}(z_i) \geq \upsilon_{z_i}(z_i),\ i=1,2,...,k$$

$$\mu_{z_i}(z_i) \geq \sigma_{z_i}(z_i),\ i=1,2,...,k$$

$$0 \leq \mu_{z_i}(z_i)+\sigma_{z_i}(z_i)+\upsilon_{z_i}(z_i) \leq 3,\ i=1,2,...,k$$

$$g_j(x) \leq b_j,\ j=1,2,...,m$$

$$n_{i1}, n_{i2},\ p_{i3} \geq 0, i=1,2,...,k$$

$$x \in X$$

$$x_j \geq 0, \qquad j=1,2,...,n$$

On the other hand, neutrosophic goal programming NGP in Model (13) can be represented by crisp programming model using truth membership, indeterminacy membership, and falsity membership functions as:

$$Max\ \alpha, Max\ \gamma, Min\ \beta \tag{15}$$

$$\mu_{z_i}(z_i) \geq \alpha,\ i=1,2,...,k$$

$$\sigma_{z_i}(z_i) \geq \gamma,\ i=1,2,...,k$$

$$\upsilon_{z_i}(z_i) \leq \beta,\ i=1,2,...,k$$

$$z_i \leq t_i,\ i=1,2,...,k$$

$$0 \leq \alpha+\gamma+\beta \leq 3$$

$$\alpha,\gamma \geq 0,\ \beta \leq 1$$

$$g_j(x) \leq b_j,\ j=1,2,...,m$$

$$x_j \geq 0, \qquad j=1,2,...,n$$

In model (15) the $Max\ \alpha, Max\ \gamma$ are equivalent to $Min(1-\alpha), Min(1-\gamma)$ respectively where $0 \leq \alpha,\ \gamma \leq 1$

$$Min\ \beta(1-\alpha)(1-\gamma) \tag{16}$$

Subject to

$$z_i \leq t_i + a_i(a_i - d_i)\beta(1-\alpha)(1-\gamma),\ i=1,2,...,k$$

$$z_i \leq t_i,\ i=1,2,...,k$$

$$0 \le \alpha + \gamma + \beta \le 3$$

$$\alpha, \gamma \ge 0, \ \ \beta \le 1$$

$$g_j(x) \le b_j, \ j = 1, 2, \ldots, m$$

$$x_j \ge 0, \qquad j = 1, 2, \ldots, n$$

If we take $\beta(1-\alpha)(1-\gamma) = v$ the model (16) becomes:

Model (II).

$$\text{Minimize } v \tag{17}$$

Subject to

$$z_i \le t_i + a_i(a_i - d_i)v, \ i = 1, 2, \ldots, k$$

$$z_i \le t_i, \ i = 1, 2, \ldots, k$$

$$0 \le \alpha + \gamma + \beta \le 3$$

$$\alpha, \gamma \ge 0, \ \ \beta \le 1$$

$$g_j(x) \le b_j, \ j = 1, 2, \ldots, m$$

$$x_j \ge 0, \qquad j = 1, 2, \ldots, n$$

The crisp model (17) is solved by using any mathematical programming technique with v as parameter to get optimal solution of objective functions.

4 Illustrative Example

This industrial application selected from [15]. Let the Decision maker wants to remove about 98.5% biological oxygen demand (BOD) and the tolerances of acceptance, indeterminacy and rejection on this goal are 0.1, 0.2 and 0.3 respectively. Also, Decision maker wants to remove the said amount of $BODS_5$ within 300 (thousand \$) tolerances of acceptance, indeterminacy and rejection 200, 250, 300 (thousand \$) respectively. Then the neutrosophic goal programming problem is:

$$\begin{aligned} \min\ z_1(x_1, x_2, x_3, x_4) &= 19.4x_1^{-1.47} + 16.8x_2^{-1.66} \\ &\quad + 91.5x_3^{-0.3} + 120x_4^{-0.33}, \\ \min\ z_2(x_1, x_2, x_3, x_4) &= x_1 x_2 x_3 x_4, \\ s.t.: & \\ x_i \ge 0,\ i &= 1, 2, 3, 4. \end{aligned}$$

With target 300, acceptance tolerance 200, indeterminacy tolerance 100 , and rejection tolerance 300 for the first objective z_1.

Also, with target 0.015, acceptance tolerance 0.1, indeterminacy tolerance 0.05, and rejection tolerance 0.2 for the second objective z_2.

Where x_i is the percentage BOD5(to remove 5 days BOD) after each step. Then after four processes the remaining percentage of BOD5 will be x_i, $i=1, 2, 3, 4$. The aim is to minimize the remaining percentage of BOD5 with minimum annual cost as much as possible. The annual cost of BOD5 removal by various treatments is primary clarifier, trickling filter, activated sludge, carbon adsorption. z_1 represent the annual cost. While z_2 represent removed from the wastewater.

The truth membership, indeterminacy membership, falsity membership functions were considered to be neutrosophic triangular.

The truth membership functions of the goals are obtained as follows:

$$\mu_1^I(z_1) = \begin{cases} 1, & if \quad z_1 \le 300, \\ 1 - \dfrac{z_1 - 300}{200}, & if \quad 300 \le z_1 \le 500, \\ 0, & if \quad z_1 \ge 500 \end{cases}$$

$$\mu_2^I(z_2) = \begin{cases} 1, & if \quad z_2 \le 0.015, \\ 1 - \dfrac{z_2 - 0.015}{0.1}, & if \quad 0.015 \le z_2 \le 0.115, \\ 0, & if \quad z_2 \ge 0.115. \end{cases}$$

The indeterminacy membership functions of the goals are given:

$$\sigma_1^I(z_1) = \begin{cases} 0, & if \quad z_1 \le 300, \\ \dfrac{z_1 - 300}{100}, & if \quad 300 \le z_1 \le 400, \\ 1 - \dfrac{z_1 - 300}{100}, & if \quad 400 \le z_1 \le 600, \\ 0, & if \quad z_1 \ge 600 \end{cases}$$

$$\sigma_2^I(z_2) = \begin{cases} 0, & if \quad z_2 \le 0.015, \\ \dfrac{z_2 - 0.015}{0.05}, & if \quad 0.015 \le z_2 \le 0.065, \\ 1 - \dfrac{z_2 - 0.015}{0.05}, & if \quad 0.065 \le z_2 \le 0.215, \\ 0, & if \quad z_2 \ge 0.215 \end{cases}$$

The falsity membership functions of the goals are obtained as follows:

$$\upsilon_1^I(z_1) = \begin{cases} 0, & \text{if} \quad z_1 \le 300, \\ \dfrac{z_1 - 300}{300}, & \text{if} \quad 300 \le z_1 \le 600, \\ 1, & \text{if} \quad z_1 \ge 600 \end{cases}$$

$$\upsilon_2^I(z_2) = \begin{cases} 0, & \text{if} \quad z_2 \le 0.015, \\ \dfrac{z_2 - 0.015}{0.2}, & \text{if} \quad 0.015 \le z_2 \le 0.215, \\ 1, & \text{if} \quad z_2 \ge 0.215 \end{cases}$$

The software LINGO 15.0 is used to solve this problem. Table (1) shows the comparison of the obtained results among the proposed models and the others methods.

Table 1: Comparison of optimal solution based on different methods:

Methods	z_1	z_2	x_1	x_2	x_3	x_4
FG^2P^2 Ref[15]	363.8048	0.04692	0.705955	0.7248393	0.1598653	0.5733523
IFG^2P^2 Ref[15]	422.1483	0.01504	0.638019	0.662717	0.09737155	0.3653206
Model (I)	317.666	0.1323	0.774182	0.7865418	0.2512332	0.8647621
Model (II)	417.6666	0.2150	2.628853	3.087266	0.181976E-01	1.455760

It is to be noted that model (I) offers better solutions than other methods.

5 Conclusions and Future Work

The main purpose of this chapter was to introduce goal programming in neutrosophic environment. The degree of acceptance, indeterminacy and rejection of objectives are considered simultaneously. Two proposed models to solve neutrosophic goal programming problem (NGPP), in the first model, our goal is to minimize the sum of the deviation, while the second model, neutrosophic goal programming NGP is transformed into crisp programming model using truth membership, indeterminacy membership, and falsity membership functions.

Finally, a numerical experiment is given to illustrate the efficiency of the proposed methods.

Moreover, the comparative study has been held of the obtained results and has been discussed. In the future studies, the proposed algorithm can be solved by metaheuristic algorithms.

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