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Research Article

Induced Choquet Integral Aggregation Operators with Single-Valued Neutrosophic Uncertain Linguistic Numbers and Their Application in Multiple Attribute Group Decision-Making

Shuping Zhao,[1,2] **Dong Wang**,[1,2] **Liang Changyong**,[1,2] **and Wenxing Lu**[1,2]

[1]*School of Management, Hefei University of Technology, Hefei 230009, China*
[2]*Ministry of Education Engineering Research Center for Intelligent Decision-Making and Information Systems Technologies, Hefei 230009, China*

Correspondence should be addressed to Dong Wang; wangd8801@163.com

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For real decision-making problems, aggregating the attributes which have interactive or correlated characteristics by traditional aggregation operators is unsuitable. Thus, applying Choquet integral operator to approximate and simulate human subjective decision-making process, in which independence among the input arguments is not necessarily assumed, would be suitable. Moreover, using single-valued neutrosophic uncertain linguistic sets (SVNULSs) can express the indeterminate, inconsistent, and incomplete information better than FSs and IFSs. In this paper, we studied the MAGDM problems with SVNULSs and proposed two single-valued neutrosophic uncertain linguistic Choquet integrate aggregation operators where the interactions phenomena among the attributes or the experts are considered. First, the definition, operational rules, and comparison method of single-valued neutrosophic uncertain linguistic numbers (SVNULNs) are introduced briefly. Second, induced single-valued neutrosophic uncertain linguistic Choquet ordered averaging (I-SVNULCA) operator and induced single-valued neutrosophic uncertain linguistic Choquet geometric (I-SVNULCG) operator are presented. Moreover, a few of its properties are discussed. Further, the procedure and algorithm of MAGDM based on the above single-valued neutrosophic uncertain linguistic Choquet integral operator are proposed. Finally, in the illustrative example, the practicality and effectiveness of the proposed method would be demonstrated.

1. Introduction

In the real-life, due to the complexity of environment and the limitation of human knowledge, human preference judgments may be difficult to express by crisp numbers. The decision information in real-world situations is uncertain, incomplete, and inconsistent. Therefore, numerous decision methods based on fuzzy information which can be considerably appropriate to express decision makers' preference were developed. Zadeh [1] first proposed fuzzy sets (FSs), which are regarded as important tools to solve decision-making problems. Given that FSs only consider a membership function, Atanassov [2] introduced the intuitionistic fuzzy sets (IFSs), which are an extension of FSs and considered membership $T_A(x)$ and nonmembership $F_A(x)$ and have been extensively applied in solving MAGDM problems [3–11]. However, FSs and IFSs can only express incomplete information rather than indeterminate and inconsistent information. Accordingly, Smarandache [12] introduced neutrosophic sets (NSs), which can substantially express indeterminate information, since it is difficult to apply NSs in the real decision-making process without specific description. Therefore, Wang [13] defined the concept of single-valued neutrosophic sets (SVNS), which is an instance of NSs. Moreover, interval neutrosophic sets (INSs) [14], as a particular extension of an NS, have also been proposed. Subsequently, various aspects of SVNSs and INSs have been studied by more and more scholars and experts [15–23]. Ye [24] presented correlation and weighted correlation coefficients of SVNSs. Liu [21] proposed some aggregation operators by combined

the PA operator and GWA operator to INS and discussed these operators' properties.

NSs are progressing rapidly in theory and application. Shahzadi [25] proposed distance measures and similarity measures of SVNSs which are applied to medical diagnosis. Then, the tangent similarity measure and cosine similarity measure of SVNS were proposed and applied in medical diagnosis [26, 27]. Tan [28] proposed exponential aggregation operator of interval neutrosophic numbers for Typhoon Disaster Evaluation. Abdel-Basset [29] developed a hybrid method with neutrosophic sets and applied it in developing supplier selection criteria. Abdel-Basset [30] developed three-way decisions based on NSs and applied it in supplier selection problem. Ye [31] presented some exponential aggregation operator in INSs and applied it in the selection problem of global suppliers. Yang [32] developed a generalized interval neutrosophic fuzzy correlated averaging operator and a linear assignment method to accommodate the interval neutrosophic sets based on Choquet integral to solve the selection problem of investment companies.

The people's thinking is ambiguous and the objective things are complex, which have occasionally prevented us from using a few real numbers to express several pieces of qualitative information in real MAGDM problems. However, this qualitative information can be easily expressed by linguistic terms. However, there is a downside to using linguistic variables; it cannot handle uncertain and inconsistent information. Therefore, Ye [33] proposed the concepts of single-valued neutrosophic linguistic set (SVNLS) and single-valued neutrosophic linguistic number (SVNLN) and introduced their operational rules. Thereafter, motivated by SVNS and intuitionistic uncertain linguistic set [34], Liu [35] proposed the concepts of single-valued neutrosophic uncertain linguistic sets (SVNULSs) and single-valued neutrosophic uncertain linguistic numbers (SVNULNs). SVNULSs can deal with fuzzy, uncertain, inconsistent, or indeterminate information and are the generalization of SVNLSs. At present, research on SVNULSs is minimal [35, 36].

Information aggregation operators generally play an important part in the process of MAGDM problems and thereby attract the attention of an increasing number of researchers [37–44]. However, these aggregation operators are assembled based on the DM's preference and attributes are independent of each other [45]. For real decision-making problems, mutual influence and interaction among attributes or experts are constantly present and ignored in the decision-making. Therefore, the concept of fuzzy measure introduced by Sugeno [46] is an effective tool for addressing the interaction phenomena among input arguments, has attracted increasing attention from researchers, and has been applied in numerous application domains [9, 45, 47–52]. To date, no research has been conducted on neutrosophic uncertain linguistic decision-making that considers the various interactions between input single-valued neutrosophic uncertain linguistic information and various decision makers. The current study is motivated by the Choquet integral [53] and (1) extends the induced Choquet integral to aggregate the decision variables with SVNULNs, (2) develops two single-valued neutrosophic uncertain linguistic Choquet integral operators, (3) investigates their various properties, and (4) discusses a few of their special cases. Thereafter, we propose one procedure for MAGDM under the environments of SVNULNs based on the proposed operators in this paper.

The rest of this article is organized as follows. First, we simply introduce some basic concepts of SVNLSs and LVs and a few operational laws of SVNULNs in Section 2. Then, Section 3 reviews fuzzy measure, Choquet integral, and I-COA operator. Section 4 proposes some aggregation operators with SVNULNs. This section also discusses their properties of their operators. Section 5 develops a MAGDM method. We propose an illustrative example to demonstrate the application of the proposed aggregation operator and method and analyze the differences compared with other methods in Section 6. The conclusions and further future research are provided in the end of this paper.

2. Preliminaries

In this section, some concepts and definitions of linguistic term sets (LTSs) provided by Zadeh, NSs, simplified neutrosophic sets (SNSs), and SVNULS are introduced, and these concepts and definitions will be used in the remainder of this paper.

2.1. Uncertain Linguistic Term Set. As an effective tool to express qualitative information, linguistic variables are proposed by Zadeh [54]. Suppose that $H = \{h_i \mid 0, 1, \cdots, t-1\}$ is a finite and totally ordered discrete linguistic term set, in which h_i is LVs. Accordingly, h_i and h_j satisfy the following properties [55, 56]:

(1) The set is ordered: $h_i < h_j$ if and only if $i < j$.

(2) If $h_i \le h_j$, $\min(h_i, h_j) = h_i$.

(3) Negation operator: $neg(h_i) = h_{t-1-i}$.

For any LVs h_i and h_j, the operations are defined as follows [55, 56]:

(1) $h_i \oplus h_j = h_{i+j}$.

(2) $\lambda h_i = h_{\lambda i}$, $0 \le \lambda \le 1$.

(3) $h_i \otimes h_j = h_{i \times j}$.

(4) ${h_i}^{\lambda} = h_{i^{\lambda}}$, $0 \le \lambda \le 1$.

For better completeness of the discrete linguistic term set, we extend the discrete term set into a continuous linguistic term set $\overline{H} = \{h_\alpha \mid h_1 \le h_\alpha \le h_q, \ \alpha \in [1, q]\}$, where q is a sufficient large number; h_α would be called the original term when $h_\alpha \in H$; otherwise, h_α is the virtual term [57]. Generally, the original linguistic terms would be utilized to evaluate alternatives by the DM, while only in the operation process the virtual linguistic terms appear.

Definition 1 (see [58]). Let $H = [h_a, h_b]$, where $h_a, h_b \in H$, and $a \le b$. h_a is the lower bound of H and h_b is the upper bound of H. Hence H is called an uncertain linguistic variable (ULV).

Definition 2 (see [55]). For two ULVs $H_1 = [h_{a1}, h_{b1}]$ and $H_2 = [h_{a2}, h_{b2}]$, the operations are defined as follows:

$$H_1 \oplus H_2 = [h_{a1}, h_{b1}] \oplus [h_{a2}, h_{b2}] = [h_{a1+a2}, h_{b1+b2}]; \quad (1)$$

$$H_1 \otimes H_2 = [h_{a1}, h_{b1}] \otimes [h_{a2}, h_{b2}] = [h_{a1*a2}, h_{b1*b2}]; \quad (2)$$

$$\lambda H_1 = \lambda [h_{a1}, h_{b1}] = [h_{\lambda a1}, h_{\lambda b1}], \quad \lambda \geq 0; \quad (3)$$

$$H_1^{\lambda} = [h_{a1}, h_{b1}]^{\lambda} = [h_{a1^{\lambda}}, h_{b1^{\lambda}}], \quad \lambda \geq 0. \quad (4)$$

2.2. NSs and SNSs

Definition 3 (see [12]). Let X be a space of points, and let x denote a generic element in X. A neutrosophic set A in X is characterized by the truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$. Thus, all three neutrosophic components $T_A(x)$, $I_A(x)$, $F_A(x) :\longrightarrow]0^-, 1^+[$ and meet the condition $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

However, NSs were difficult to be applied to practical problems. Then, by changed nonstandard interval numbers of NSs, Ye [59] introduced the concept and operations of SNSs, which is a subclass of NS and can be defined as follows.

Definition 4 (see [59]). Let X be a universal set. A NS A in X is characterized by $T_A(x)$, $I_A(x)$, and $F_A(x)$, which are single subintervals/subsets in the real standard $[0, 1]$. That is, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ and satisfy the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. Thereafter, A can be simply denoted by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$, which is called SNSs.

2.3. SVNULNs and Their Operations.

Liu et al. [35] used uncertain linguistic term sets and SNSs as bases to introduce the concept and operations of SVNULNs and define a method with the proposed score and accuracy functions for comparing two SVNULNs.

Definition 5 (see [35]). Let $[h_{\theta(x)}, h_{\tau(x)}] \in H$, X, and X be the given discourse domains. Accordingly, $A = \{\langle x \mid [h_{\theta(x)}, h_{\tau(x)}], (T_A(x), I_A(x), F_A(x))\rangle \mid x \in X\}$ is called SVNULSs, where $h_{\theta(x)}$, $h_{\tau(x)} \in H$, and $T_A(x)$, $I_A(x)$, and $F_A(x)$ are three sets of certain single value in real unit interval $[0, 1]$ and express the truth-membership, indeterminacy-membership, and falsity-membership function, respectively, of the element x to A. For convenience, $\alpha = \langle |[h_\theta, h_\tau], (T_\alpha, I_\alpha, F_\alpha)\rangle$ is defined SVNULNs. Furthermore, α degenerates to an ULV when $T_A = 1$, $I_A = 0$ and $F_A = 0$.

SVNULSs are extension of an uncertain linguistic term and SVNS. Compared with ULVs, SVNULSs can more accurately reflect uncertainty and fuzziness. Compared with SVNSs, SVNULSs integrate ULVs and SVNSs, and assign truth-membership, indeterminacy-membership, and falsity-membership functions to a specific ULV. Thus, SVNULSs are effective tools to address the problems which are defined by qualitative expression that involve incomplete, indeterminate, and inconsistent information.

The following section introduces the operations of SVNULNs [35].

Definition 6 (see [35]). Suppose $\alpha = \langle |[h_{\theta_1}, h_{\tau_1}], (T_\alpha, I_\alpha, F_\alpha)\rangle$ and $\beta = \langle |[h_{\theta_2}, h_{\tau_2}], (T_\beta, I_\beta, F_\beta)\rangle$ be two SVNULNs, $\lambda > 0$. The algebraic operations between α and β can be defined as follows:

$$\alpha \oplus \beta = \left\langle \left[h_{\theta_1+\theta_2}, h_{\tau_1+\tau_2}\right], \left(T_\alpha + T_\beta - T_\alpha T_\beta, I_\alpha I_\beta, F_\alpha F_\beta\right)\right\rangle; \quad (5)$$

$$\alpha \otimes \beta = \left\langle \left[h_{\theta_1*\theta_2}, h_{\tau_1*\tau_2}\right], \left(T_\alpha T_\beta, I_\alpha + I_\beta - I_\alpha I_\beta, F_\alpha + F_\beta - F_\alpha F_\beta\right)\right\rangle; \quad (6)$$

$$\lambda\alpha = \left\langle \left[h_{\lambda\theta_1}, h_{\lambda\tau_1}\right], 1 - (1 - T_\alpha)^{\lambda}, I_\alpha^{\lambda}, F_\alpha^{\lambda}\right)\right\rangle; \quad (7)$$

$$\alpha^{\lambda} = \left\langle \left[h_{\theta_1^{\lambda}}, h_{\tau_1^{\lambda}}\right], \left(T_\alpha^{\lambda}, 1 - (1 - I_\alpha)^{\lambda}, 1 - (1 - F_\alpha)^{\lambda}\right)\right\rangle; \quad (8)$$

$$neg(\alpha) = \left\langle \left[h_{-\theta_1}, h_{-\tau_1}\right], (F_\alpha, 1 - I_\alpha, T_\alpha)\right\rangle. \quad (9)$$

These operational results remain to be SVNULNs.

Theorem 7. *Suppose that $\alpha = \langle |[h_{\theta_1}, h_{\tau_1}], (T_\alpha, I_\alpha, F_\alpha)\rangle$ and $\beta = \langle |[h_{\theta_2}, h_{\tau_2}], (T_\beta, I_\beta, F_\beta)\rangle$ be two SVNULNs, $\lambda > 0$. The operational laws have the following characteristics:*

(1) $\alpha \oplus \beta = \beta \oplus \alpha$.

(2) $\alpha \otimes \beta = \beta \otimes \alpha$.

(3) $\lambda(\alpha \oplus \beta) = \lambda\alpha \oplus \lambda\beta$.

(4) $\alpha^{\lambda} \otimes \beta^{\lambda} = (\alpha \otimes \beta)^{\lambda}$.

Definition 8 (see [35]). Let $\alpha = \langle |[h_\theta, h_\tau], (T_\alpha, I_\alpha, F_\alpha)\rangle$ be a SVNULN. The score function $E(\alpha)$ and accuracy function $H(\alpha)$ of α can be defined, as follows:

$$S(\alpha) = \frac{1}{3}(1 + T_\alpha + 1 - I_\alpha - F_\alpha) \times (h_{(\theta+\tau)/2}) = h_{(2+T_\alpha-I_\alpha-F_\alpha)(\theta+\tau)/6}, \quad (10)$$

$$H(\alpha) = (T_\alpha + I_\alpha + F_\alpha) \times (h_{(\theta+\tau)/2}) = h_{(T_\alpha+I_\alpha+F_\alpha)*(\theta+\tau)/2}. \quad (11)$$

Definition 9 (see [35]). Let $\alpha = \langle |[h_{\theta 1}, h_{\tau 1}], (T_\alpha, I_\alpha, F_\alpha)\rangle$ and $\beta = \langle |[h_{\theta 2}, h_{\tau 2}], (T_\beta, I_\beta, F_\beta)\rangle$ be any two SVNULNs. The comparison method between α and β can be defined as follows:

(1) If $S(\alpha) < S(\beta)$, then $\alpha \prec \beta$.

(2) If $S(\alpha) = S(\beta)$ and $H(\alpha) < H(\beta)$, then $\alpha \prec \beta$.

(3) If $S(\alpha) = S(\beta)$ and $H(\alpha) = H(\beta)$, then $\alpha = \beta$.

(4) If $S(\alpha) = S(\beta)$ and $H(\alpha) > H(\beta)$, then $\alpha \succ \beta$.

3. Fuzzy Measure and Choquet Integral

In real decision problems, a certain degree of interdependent or interactive characteristics often exist among the attributes or experts [60]. However, measuring the importance of attributes by using additive measures is not suitable because the independence of these attributes is often violated [61]. The concept of fuzzy measure introduced by Sugeno [46] is an effective tool for addressing the interaction phenomena among input arguments [45, 60, 62–65].

Definition 10 (see [66]). Let a universal set $X = \{x_1, x_2, \cdots, x_n\}$, while $P(X)$ is the power set of X. A fuzzy measure on X is a set function $\mu : P(X) \longrightarrow [0, 1]$ that meets the following conditions:

(1) $\mu(\emptyset) = 0$ and $\mu(X) = 1$.

(2) If $A, B \in P(X)$ and $A \subseteq B$, then $\mu(A) \leq \mu(B)$.

(3) $\mu(A \cup B) = \mu(A) + \mu(B) + \rho\mu(A)\mu(B)$, for all $A, B \in P(X)$ and $A \cap B = \emptyset$, where $\rho \in (-1, \infty)$.

In particular, if $\rho = 0$, then condition (3) in Definition 10 is reduced to the axiom of the additive measure $\mu(A \cup B) = \mu(A) + \mu(B)$, thereby indicating a lack of interaction between B and C; if $\rho > 0$, then $\mu(A \cup B) > \mu(A) + \mu(B)$; that is to say, sets A and B have a multiplicative effect. If $\rho < 0$, then $\mu(A \cup B) < \mu(A) + \mu(B)$, thereby expressing that sets A and B have a substitutive effect. The use of parameter ρ can adequately represent the interaction between sets in MAGDM.

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite set, in which $\bigcup_{i=1}^{n} x_i = X$. Sugeno [46] provided the following equation to avoid the computational complexity of fuzzy measure on X:

$$\begin{aligned}\mu(X) &= \mu\left(\bigcup_{i=1}^{n} x_i\right) \\ &= \begin{cases} \dfrac{1}{\rho}\left(\prod_{i=1}^{n}\left(1 + \rho\mu(x_i)\right) - 1\right) & \rho \neq 0 \\ \sum_{i=1}^{n} m(x_i) & \rho = 0 \end{cases}\end{aligned} \tag{12}$$

where $x_i \cap x_j = \emptyset$ for all $i, j = 1, 2, \cdots, n$ and $i \neq j$. Note that $\mu(x_i)$ for a subset with a single x_i is called a fuzzy density and can be denoted as $\mu_i = \mu(x_i)$.

In particular, we have the following equation for every subset $A \subseteq P(X)$:

$$\mu(A) = \begin{cases} \dfrac{1}{\rho}\left(\prod_{x_i \in A}\left(1 + \rho\mu_i\right) - 1\right) & \rho \neq 0 \\ \sum_{x_i \in A} \mu_i & \rho = 0 \end{cases} \tag{13}$$

The value of ρ can be uniquely determined by $\mu(X) = 1$ based on (2) and can be expressed as follows:

$$\prod_{i=1}^{n}\left(1 + \rho\mu_i\right) = 1 + \rho. \tag{14}$$

Definition 11 (see [53, 60]). Let f be a positive real-valued function on X and μ be a fuzzy measure on X. The discrete Choquet integral of f with respective to μ is defined as follows:

$$C_\mu(f) = \sum_{i=1}^{n} f_{(i)}\left[\mu(A_{(i)}) - \mu(A_{(i-1)})\right] \tag{15}$$

where the subscript (.) indicates a permutation on X such that $f_{(1)} \geq f_{(2)} \geq \cdots \geq f_{(n)}$, $f_{(i)}$ is the largest value in the set $\{f_1, f_2, \cdots, f_n\}$, while $A_{(i)} = \{x_{(1)}, \cdots, x_{(i)}\}$ when $i = 1, 2, \cdots, n$, $A_{(0)} = \emptyset$.

The Choquet integral can aggregate the attributes even when the mutual preferential independence assumption is violated [67]. Inspired by the Induced ordered weight averaging (IOWA) operator [68], Yager [69] considered a considerably general policy toward ordering the arguments and formulating the ordered argument vector and defined a considerably general type of Choquet integral operator (i.e., I-COA operator), as follows.

Definition 12 (see [69]). Let f be a positive real-valued function on X and μ be a fuzzy measure on X. An induced Choquet ordered averaging operator of dimension n is a function I-COA: $(R^+ \times \Omega)^n \longrightarrow \Omega$, which is defined to aggregate the set of second arguments of a list of n 2-tuples $\{\langle u_1, f_1\rangle, \langle u_2, f_2\rangle, \cdots, \langle u_n, f_n\rangle\}$ based on the following expression:

$$\begin{aligned}&\text{I-COA}_\mu\left(\langle u_1, f_1\rangle, \langle u_2, f_2\rangle, \cdots, \langle u_n, f_n\rangle\right) \\ &\quad = \sum_{i=1}^{n} f_{(i)}\left[\mu(A_{(i)}) - \mu(A_{(i-1)})\right]\end{aligned} \tag{16}$$

where $(i) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$ is a permutation, such that $u_{(1)} \geq u_{(2)} \geq \cdots \geq u_{(n)}$, $\langle u_{(i)}, f_{(i)}\rangle$ indicates the 2-tuple with $u_{(i)}$ as the *ith* largest value in the set $\{u_1, u_2, \cdots, u_n\}$, $A_{(i)} = \{1, \cdots, i\}$, $i = 1, 2, \cdots, n$ when $i \geq 1$, and $A_{(0)} = \emptyset$.

4. Induced Simplified Neutrosophic Linguistic Choquet Integral Operators

The I-COA operator [69] can only aggregate crisp numbers and has not been used in conditions where the input arguments are SVNULNs. We use Definitions 6, 10, and 12, as base to (1) extend the I-COA operator to accommodate the conditions of the input arguments are SVNULNs, (2) define the I-SVNULCA and I-SVNULCG operators, and (3) analyze a few necessary properties under the SVNUL environments.

Definition 13. Let $\alpha_i = \langle[h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i)\rangle$ $(i = 1, 2, \cdots, n)$ be a collection of SVNULNs on X and μ be a fuzzy measure on $P(X)$. An I-SVNULCA operator of dimension n is a function I-SVNULCA: $(R^+ \times \Omega)^n \longrightarrow \Omega$, which is defined to

aggregate the set of second arguments of a collection of n 2-tuples $\{\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\}$ based on the following expression:

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right)\\&\quad = \bigoplus_{i=1}^{n} \alpha_{(i)}\left(\mu\left(A_{(i)}\right) - \mu\left(A_{(i-1)}\right)\right)\end{aligned} \tag{17}$$

where the subscript $(i) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$ is a permutation, such that $u_{(1)} \geq u_{(2)} \geq \cdots \geq u_{(n)}$. That is, $\langle u_i, \alpha_i\rangle$ is 2-tuple with $u_{(i)}$ being the ith largest value in the set $\{u_1, u_2, \cdots, u_n\}$, $A_{(i)} = \{x_{(1)}, x_{(2)}, \cdots, x_{(i)}\}$, $i = 1, 2, \cdots, n$, and $A_{(0)} = \emptyset$.

Theorem 14. *Let $\alpha_i = \langle[h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i)\rangle$ $(i = 1, 2, \cdots, n)$ be a collection of SVNULNs on X and μ be a fuzzy measure on $P(X)$. Their aggregated value by using the I-SVNULCA operator is also an SVNULN,*

$$\begin{aligned}&I\text{-}SVNULCA\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right)\\&\quad = \left(\left[h_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta_{(i)}}, h_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau_{(i)}}\right],\right.\\&\quad\left(1 - \prod_{i=1}^{n}\left(1 - T_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}, \prod_{i=1}^{n}\left(I_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},\right.\\&\quad\left.\left.\prod_{i=1}^{n}\left(F_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\right)\right)\end{aligned} \tag{18}$$

Proof. (1) For $n = 2$, according to the operational laws of Definition 2, we have

$$\begin{aligned}\alpha_{(1)}\left(\mu\left(A_{(1)}\right) - \mu\left(A_{(0)}\right)\right) &= \left\langle\left[h_{(\mu(A_{(1)})-\mu(A_{(0)}))\theta_{(1)}},\right.\right.\\&\quad\left.h_{(\mu(A_{(1)})-\mu(A_{(0)}))\tau_{(1)}}\right], \left(1 - \left(1 - T_{\alpha_{(1)}}\right)^{\mu(A_{(1)})-\mu(A_{(0)})},\\&\quad\left.\left(I_{\alpha_{(1)}}\right)^{\mu(A_{(1)})-\mu(A_{(0)})}, \left(F_{\alpha_{(1)}}\right)^{\mu(A_{(1)})-\mu(A_{(0)})}\right)\right\rangle\\\alpha_{(2)}\left(\mu\left(A_{(2)}\right) - \mu\left(A_{(1)}\right)\right) &= \left\langle\left[h_{(\mu(A_{(2)})-\mu(A_{(1)}))\theta_{(2)}},\right.\right.\\&\quad\left.h_{(\mu(A_{(2)})-\mu(A_{(1)}))\tau_{(2)}}\right], \left(1 - \left(1 - T_{\alpha_{(2)}}\right)^{\mu(A_{(2)})-\mu(A_{(1)})},\\&\quad\left.\left(I_{\alpha_{(2)}}\right)^{\mu(A_{(2)})-\mu(A_{(1)})}, \left(F_{\alpha_{(2)}}\right)^{\mu(A_{(2)})-\mu(A_{(1)})}\right)\right\rangle\end{aligned} \tag{19}$$

Since $\alpha \oplus \beta = \langle[h_{\theta 1+\theta 2}, h_{\tau 1+\tau 2}], (T_\alpha + T_\beta - T_\alpha T_\beta, I_\alpha I_\beta, F_\alpha F_\beta)\rangle$, then

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle\right) = \alpha_{(1)}\left(\mu\left(A_{(1)}\right)\right.\\&\quad\left.- \mu\left(A_{(0)}\right)\right) + \alpha_{(2)}\left(\mu\left(A_{(2)}\right) - \mu\left(A_{(1)}\right)\right)\\&\quad = \left\langle\left[h_{\sum_{i=1}^{2}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta_{(i)}}, h_{\sum_{i=1}^{2}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau_{(i)}}\right],\right.\\&\quad\left(1 - \left(1 - T_{\alpha_{(1)}}\right)^{\mu(A_{(1)})-\mu(A_{(0)})} + 1 - \left(1\right.\right.\\&\quad\left.- T_{\alpha_{(2)}}\right)^{\mu(A_{(2)})-\mu(A_{(1)})} - \left(1\right.\\&\quad\left.- \left(1 - T_{\alpha_{(1)}}\right)^{\mu(A_{(1)})-\mu(A_{(0)})}\right)\left(1\right.\\&\quad\left.- \left(1 - T_{\alpha_{(2)}}\right)^{\mu(A_{(2)})-\mu(A_{(1)})}\right), \left(I_{\alpha_{(1)}}\right)^{\mu(A_{(1)})-\mu(A_{(0)})}\\&\quad\cdot \left(I_{\alpha_{(2)}}\right)^{\mu(A_{(2)})-\mu(A_{(1)})}, \left(F_{\alpha_{(1)}}\right)^{\mu(A_{(1)})-\mu(A_{(0)})}\\&\quad\left.\cdot \left(F_{\alpha_{(2)}}\right)^{\mu(A_{(2)})-\mu(A_{(1)})}\right)\right\rangle\\&\quad = \left\langle\left[h_{\sum_{i=1}^{2}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta_{(i)}},\right.\right.\\&\quad\left.h_{\sum_{i=1}^{2}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau_{(i)}}\right], \left(\left(1\right.\right.\\&\quad\left.- \prod_{i=1}^{2}\left(1 - T_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}, \prod_{i=1}^{2}\left(I_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},\right.\\&\quad\left.\left.\left.\prod_{i=1}^{2}\left(F_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\right)\right)\right\rangle\end{aligned} \tag{20}$$

(2) When $n = k$, we obtain (21) by using (18).

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_k, \alpha_k\rangle\right)\\&\quad = \left(\left[h_{\sum_{i=1}^{k}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta_{(i)}}, h_{\sum_{i=1}^{k}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau_{(i)}}\right],\right.\\&\quad\left(1 - \prod_{i=1}^{k}\left(1 - T_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}, \prod_{i=1}^{k}\left(I_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},\right.\\&\quad\left.\left.\prod_{i=1}^{k}\left(F_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\right)\right)\end{aligned} \tag{21}$$

(3) When $n = k + 1$, by utilizing (18) and Definition 8, we obtain

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_{k+1}, \alpha_{k+1}\rangle\right)\\&\quad = \left(\left[h_{\sum_{i=1}^{k+1}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta_{(i)}},\right.\right.\\&\quad\left.h_{\sum_{i=1}^{k+1}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau_{(i)}}\right], \left(1 - \prod_{i=1}^{k}\left(1 - T_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\right.\\&\quad + 1 - \left(1 - T_{\alpha_{(k+1)}}\right)^{\mu_{(k+1)}-\mu_{(k)}}\\&\quad\left.- \left(1 - \prod_{i=1}^{k}\left(1 - T_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\right)\right.\end{aligned}$$

$$\cdot\left(1-\left(1-T_{\alpha_{(k+1)}}\right)^{\mu_{(k+1)}-\mu_{(k)}}\right),\prod_{i=1}^{k+1}\left(I_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},$$

$$\prod_{i=1}^{k+1}\left(F_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\Bigg)\Bigg)=\Bigg(\left[h_{\sum_{i=1}^{k+1}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta_{(i)}},\right.$$

$$\left.h_{\sum_{i=1}^{k+1}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau_{(i)}}\right],\Bigg(1$$

$$-\prod_{i=1}^{k+1}\left(1-T_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},\prod_{i=1}^{k+1}\left(I_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},$$

$$\prod_{i=1}^{k+1}\left(F_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\Bigg)\Bigg) \tag{22}$$

Therefore, we obtain (18) for any n based on the previous results. This condition completes the proof. □

Theorem 15 (commutativity). *Let α_i $(i=1,2,\cdots,n)$ be a collection of SVNULNs, while $(\langle u_1',\alpha_1'\rangle,\langle u_2',\alpha_2'\rangle,\cdots,\langle u_n',\alpha_n'\rangle)$ is any permutation of $(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle)$. Thus, we have*

$$\begin{aligned}&I\text{-}SVNULCA\left(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle\right)\\&=I\text{-}SVNULCA\left(\left\langle u_1',\alpha_1'\right\rangle,\left\langle u_2',\alpha_2'\right\rangle,\cdots,\left\langle u_n',\alpha_n'\right\rangle\right).\end{aligned} \tag{23}$$

Proof. Let

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle\right)\\&\quad=\alpha_{(1)}\left(\mu_{(1)}-\mu_{(0)}\right)\oplus\alpha_{(2)}\left(\mu_{(2)}-\mu_{(1)}\right)\oplus\cdots\\&\qquad\oplus\alpha_{(n)}\left(\mu_{(n)}-\mu_{(n-1)}\right)\\&\text{I-SVNULCA}\left(\left\langle u_1',\alpha_1'\right\rangle,\left\langle u_2',\alpha_2'\right\rangle,\cdots,\left\langle u_n',\alpha_n'\right\rangle\right)\\&\quad=\alpha_{(1)}'\left(\mu_{(1)}-\mu_{(0)}\right)\oplus\alpha_{(2)}'\left(\mu_{(2)}-\mu_{(1)}\right)\oplus\cdots\\&\qquad\oplus\alpha_{(n)}'\left(\mu_{(n)}-\mu_{(n-1)}\right)\end{aligned} \tag{24}$$

Since $(\langle u_1',\alpha_1'\rangle,\langle u_2',\alpha_2'\rangle,\cdots,\langle u_n',\alpha_n'\rangle)$ is any permutation of $(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle)$, we have $\alpha_{(i)}=\alpha_{(i)}'$ $(i=1,2,\cdots,n)$, and then

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle\right)\\&=\text{I-SVNULCA}\left(\left\langle u_1',\alpha_1'\right\rangle,\left\langle u_2',\alpha_2'\right\rangle,\cdots,\left\langle u_n',\alpha_n'\right\rangle\right).\end{aligned} \tag{25}$$

□

Theorem 16 (idempotency). *Let $\alpha_i=([h_{\theta_i},h_{\tau_i}],(T_i,I_i,F_i))$ $(i=1,2,\cdots,n)$ be a collection of SVNULNs. If $\alpha_i=([h_{\theta_i},h_{\tau_i}],(T_i,I_i,F_i))=\alpha=([h_\theta,h_\tau],(T,I,F))$ for all i, then*

$$I\text{-}SVNULCA\left(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle\right)=\alpha \tag{26}$$

Proof. Since $\alpha_i=\alpha$ $(i=1,2,\cdots,n)$ for all i, then

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle\right)\\&\quad=\alpha_{(1)}\left(\mu_{(1)}-\mu_{(0)}\right)\oplus\alpha_{(2)}\left(\mu_{(2)}-\mu_{(1)}\right)\oplus\cdots\\&\quad\oplus\alpha_{(n)}\left(\mu_{(n)}-\mu_{(n-1)}\right)\\&\quad=\Bigg(\left[h_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta_{(i)}},h_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau_{(i)}}\right],\\&\quad1-\prod_{i=1}^{n}\left(1-T_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},\prod_{i=1}^{n}\left(I_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},\\&\quad\prod_{i=1}^{n}\left(F_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\Bigg)\\&\quad=\Bigg(\left[h_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta},h_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau}\right],1\\&\quad-\prod_{i=1}^{n}(1-T)^{\mu_{(i)}-\mu_{(i-1)}},\prod_{i=1}^{n}(I)^{\mu_{(i)}-\mu_{(i-1)}},\\&\quad\prod_{i=1}^{n}(F)^{\mu_{(i)}-\mu_{(i-1)}}\Bigg)=\left(\left[h_\theta,h_\tau\right],1-(1-T)^{\mu_{(n)}-\mu_{(0)}},\right.\\&\quad\left.(I)^{\mu_{(n)}-\mu_{(0)}},(1-F)^{\mu_{(n)}-\mu_{(0)}}\right)=\left(\left[h_\theta,h_\tau\right],(T,I,F)\right)=\alpha\end{aligned} \tag{27}$$

□

Theorem 17 (monotonicity). *Let $\alpha_i=([h_{\theta_i},h_{\tau_i}],(T_i,I_i,F_i))$ $(i=1,2,\cdots,n)$ and $\alpha_i'=([h_{\theta_i}',h_{\tau_i}],(T_i',I_i',F_i'))$ $(i=1,2,\cdots,n)$ be a collection of SVNULNs. If $(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle)$ and $(\langle u_1',\alpha_1'\rangle,\langle u_2',\alpha_2'\rangle,\cdots,\langle u_n',\alpha_n'\rangle)$ are two collections of 2-tuples, such that $\alpha_i\le\alpha_i'$, $i=1,2,\cdots,n$, then*

$$\begin{aligned}&I\text{-}SVNULCA\left(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle\right)\\&\le I\text{-}SVNULCA\left(\left\langle u_1,\alpha_1'\right\rangle,\left\langle u_2,\alpha_2'\right\rangle,\cdots,\left\langle u_n,\alpha_n'\right\rangle\right)\end{aligned} \tag{28}$$

Proof. Let

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle\right)\\&\quad=\alpha_{(1)}\left(\mu_{(1)}-\mu_{(0)}\right)\oplus\alpha_{(2)}\left(\mu_{(2)}-\mu_{(1)}\right)\oplus\cdots\\&\qquad\oplus\alpha_{(n)}\left(\mu_{(n)}-\mu_{(n-1)}\right)\\&\text{I-SVNULCA}\left(\left\langle u_1,\alpha_1'\right\rangle,\left\langle u_2,\alpha_2'\right\rangle,\cdots,\left\langle u_n,\alpha_n'\right\rangle\right)\\&\quad=\alpha_{(1)}'\left(\mu_{(1)}-\mu_{(0)}\right)\oplus\alpha_{(2)}'\left(\mu_{(2)}-\mu_{(1)}\right)\oplus\cdots\\&\qquad\oplus\alpha_{(n)}'\left(\mu_{(n)}-\mu_{(n-1)}\right)\end{aligned} \tag{29}$$

Since $\alpha_i\le\alpha_i'$ $(i=1,2,\cdots,n)$, it follows that $\alpha_{(i)}\le\alpha_{(i)}'$ $(i=1,2,\cdots,n)$, then

$$\begin{aligned}&\text{I-SVNULCA}\left(\langle u_1,\alpha_1\rangle,\langle u_2,\alpha_2\rangle,\cdots,\langle u_n,\alpha_n\rangle\right)\\&\le\text{I-SVNULCA}\left(\left\langle u_1,\alpha_1'\right\rangle,\left\langle u_2,\alpha_2'\right\rangle,\cdots,\left\langle u_n,\alpha_n'\right\rangle\right)\end{aligned} \tag{30}$$

□

Theorem 18 (boundedness). *Let* $\alpha_{min} = \min(\alpha_1, \alpha_2, \cdots, \alpha_n)$ *and* $\alpha_{max} = \max(\alpha_1, \alpha_2, \cdots, \alpha_n)$*, then*

$$\begin{aligned}&\alpha_{min}\\&\le \textit{I-SVNULCA}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right)\\&\le \alpha_{max}\end{aligned} \tag{31}$$

where $(i) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$ *is a permutation such that* $\langle u_{(i)}, \alpha_{(i)}\rangle$ *is the 2-tuple with* $u_{(i)}$ *being the* i*th largest value in the set* $\{u_1, u_2, \cdots, u_n\}$*,* $A_{(i)} = \{x_{(1)}, x_{(2)}, \cdots, x_{(i)}\}$ *when* $i \ge 1$ *and* $A_{(0)} = \emptyset$*.*

Proof. Let $(i) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$ be a permutation such that $\langle u_{(i)}, \alpha_{(i)}\rangle$ is the 2-tuple with $u_{(i)}$ the ith largest value in the set $\{u_1, u_2, \cdots, u_n\}$, then $\alpha_{\min} \le \alpha_{(i)} \le \alpha_{\max}$.

So, we have

$$\begin{aligned}&\bigoplus_{i=1}^{n}\alpha_{\min}\left(\mu\left(A_{(i)}\right) - \mu\left(A_{(i-1)}\right)\right)\\&\le \bigoplus_{i=1}^{n}\alpha_{(i)}\left(\mu\left(A_{(i)}\right) - \mu\left(A_{(i-1)}\right)\right)\\&\le \bigoplus_{i=1}^{n}\alpha_{\max}\left(\mu\left(A_{(i)}\right) - \mu\left(A_{(i-1)}\right)\right).\end{aligned} \tag{32}$$

Since $\oplus_{i=1}^{n}(\mu(A_{(i)}) - \mu(A_{(i-1)})) = 1$,

thus $\alpha_{\min} \le$ I-SVNULCA$(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle) \le \alpha_{\max}$.

If the order-inducing variable is the argument variable, then the I-SVNULCA operator is reduced to the SVNULCA operator. That is, if $u_i = \alpha_i$, for all i, then the I-SVNULCA operator (see (18)) is reduced to the single-valued neutrosophic uncertain linguistic Choquet ordered averaging (SVNULCA) operator.

$$\begin{aligned}&\text{SVNULCA}\left(\alpha_1, \alpha_2, \cdots, \alpha_n\right)\\&= \left(\left[h_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i-1)}))\theta_{(i)}}, h_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i-1)}))\tau_{(i)}}\right],\right.\\&\left(1 - \prod_{i=1}^{n}\left(1 - T_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}, \prod_{i=1}^{n}\left(I_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}},\right.\\&\left.\left.\prod_{i=1}^{n}\left(F_{\alpha_{(i)}}\right)^{\mu_{(i)}-\mu_{(i-1)}}\right)\right)\end{aligned} \tag{33}$$

where the subscript $(i) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$ is a permutation, such that $\alpha_{(1)} \ge \alpha_{(2)} \ge \cdots \ge \alpha_{(n)}$, $A_{(i)} = \{x_{(1)}, x_{(2)}, \cdots, x_{(i)}\}$, $i = 1, 2, \cdots, n$, and $A_{(0)} = \emptyset$.

The SVNULCA operator has the same properties as those of the I-SVNULCA operator, such as commutativity, idempotency, and monotonicity. □

In the following, we propose an I-SVNULCG operator based on Definition 13 and the OWG operator.

Definition 19. Let $\alpha_i = \langle[h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i)\rangle$ $(i = 1, 2, \cdots, n)$ be a collection of SVNULNs on X and μ be a fuzzy measure on $P(X)$. An I-SVNULCG operator of dimension n is a function I-SVNULCG: $(R^+ \times \Omega)^n \longrightarrow \Omega$, which is defined to aggregate the set of second arguments of a collection of n 2-tuples $\{\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\}$ based on the following expression:

$$\begin{aligned}&\text{I-SVNULCG}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right)\\&= \bigotimes_{i=1}^{n}\alpha_{(i)}^{\mu(A_{(i)})-\mu(A_{(i-1)})}\end{aligned} \tag{34}$$

where the subscript $(i) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$ is a permutation such that $u_{(1)} \ge u_{(2)} \ge \cdots \ge u_{(n)}$. That is, $\langle u_i, \alpha_i\rangle$ is 2-tuple with $u_{(i)}$ as the ith largest value in the set $\{u_1, u_2, \cdots, u_n\}$, $A_{(i)} = \{x_{(1)}, x_{(2)}, \cdots, x_{(i)}\}$, $i = 1, 2, \cdots, n$, and $A_{(0)} = \emptyset$.

Theorem 20. *Let* $\alpha_i = \langle[h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i)\rangle$ $(i = 1, 2, \cdots, n)$ *be a collection of SVNULNs on* X *and* μ *be a fuzzy measure on* $P(X)$*. Their aggregated value by using the I-SVNULCG operator is also an SVNULN.*

$$\begin{aligned}&\textit{I-SVNULCG}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right)\\&= \bigotimes_{i=1}^{n}\alpha_{(i)}^{(\mu(A_{(i)})-\mu(A_{(i-1)}))}\\&= \left(\left[h_{\prod_{i=1}^{n}\theta_{(i)}^{\mu(A_{(i)})-\mu(A_{(i-1)})}}, h_{\prod_{i=1}^{n}\tau_{(i)}^{\mu(A_{(i)})-\mu(A_{(i-1)})}}\right],\right.\\&\left(\prod_{i=1}^{n}\left(T_{\alpha_{(i)}}\right)^{\mu(A_{(i)})-\mu(A_{(i-1)})}, 1\right.\\&- \prod_{i=1}^{n}\left(1 - I_{\alpha_{(i)}}\right)^{\mu(A_{(i)})-\mu(A_{(i-1)})}, 1\\&\left.\left.- \prod_{i=1}^{n}\left(1 - F_{\alpha_{(i)}}\right)^{\mu(A_{(i)})-\mu(A_{(i-1)})}\right)\right)\end{aligned} \tag{35}$$

The I-SVNULCG operator has the following properties that are similar to those of the I-SVNULCA operator.

Theorem 21 (commutativity). *Let* α_i $(i = 1, 2, \cdots, n)$ *be a collection of SVNULNs;* $(\langle u_1', \alpha_1'\rangle, \langle u_2', \alpha_2'\rangle, \cdots, \langle u_n', \alpha_n'\rangle)$ *is any permutation of* $(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle)$*. Thus, we have the following equation:*

$$\begin{aligned}&\textit{I-SVNULCG}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right)\\&= \textit{I-SVNULCG}\left(\langle u_1', \alpha_1'\rangle, \langle u_2', \alpha_2'\rangle, \cdots, \langle u_n', \alpha_n'\rangle\right).\end{aligned} \tag{36}$$

Theorem 22 (idempotency). *Let* $\alpha_i = ([h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i))$ $(i = 1, 2, \cdots, n)$ *be a collection of SVNULNs. If* $\alpha_i = ([h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i)) = \alpha = ([h_\theta, h_\tau], (T, I, F))$ *for all* i*, then*

$$\textit{I-SVNULCG}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right) = \alpha \tag{37}$$

Theorem 23 (monotonicity). *Let* $\alpha_i = ([h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i))$ $(i = 1, 2, \cdots, n)$ *and* $\alpha_i' = ([h_{\theta_i}', h_{\tau_i}], (T_i', I_i', F_i'))$ $(i = 1, 2, \cdots, n)$ *be a collection of SVNULNs. If* $(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle)$ *and* $(\langle u_1', \alpha_1'\rangle, \langle u_2', \alpha_2'\rangle, \cdots, \langle u_n', \alpha_n'\rangle)$ *are two collections of 2-tuples, such that* $\alpha_i \le \alpha_i'$, $i = 1, 2, \cdots, n$, *then*

$$\begin{aligned} &\text{I-SVNULCG}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right) \\ &\le \text{I-SVNULCG}\left(\left\langle u_1, \alpha_1'\right\rangle, \left\langle u_2, \alpha_2'\right\rangle, \cdots, \left\langle u_n, \alpha_n'\right\rangle\right). \end{aligned} \tag{38}$$

Theorem 24 (boundedness). *Let* $\alpha_{min} = \min(\alpha_1, \alpha_2, \cdots, \alpha_n)$ *and* $\alpha_{max} = \max(\alpha_1, \alpha_2, \cdots, \alpha_n)$, *then*

$$\begin{aligned} &\alpha_{min} \\ &\quad \le \text{I-SVNULCG}\left(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \cdots, \langle u_n, \alpha_n\rangle\right) \\ &\quad \le \alpha_{max} \end{aligned} \tag{39}$$

where $(i) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$ *is a permutation, such that* $\langle u_{(i)}, \alpha_{(i)}\rangle$ *is the 2-tuple with* $u_{(i)}$ *as the ith largest value in the set* $\{u_1, u_2, \cdots, u_n\}$, $A_{(i)} = \{x_{(1)}, x_{(2)}, \cdots, x_{(i)}\}$ *when* $i \ge 1$ *and* $A_{(0)} = \emptyset$.

If the order-inducing variable is the argument variable, then the I-SVNULCG operator is reduced to the SVNULCG operator. That is, if $u_i = \alpha_i$, *for all i, then the I-SVNULCG operator (see (35)) is reduced to the single-valued neutrosophic uncertain linguistic Choquet ordered geometric (SVNULCG) operator.*

$$\begin{aligned} &\text{SVNULCG}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \bigotimes_{i=1}^{n} \alpha_{(i)}^{(\mu(A_{(i)}) - \mu(A_{(i-1)}))} \\ &= \left(\left[h_{\prod_{i=1}^{n} \theta_{(i)}^{\mu(A_{(i)}) - \mu(A_{(i-1)})}}, h_{\prod_{i=1}^{n} \tau_{(i)}^{\mu(A_{(i)}) - \mu(A_{(i-1)})}}\right], \right. \\ &\left(\prod_{i=1}^{n} \left(T_{\alpha_{(i)}}\right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})}, 1\right. \\ &- \prod_{i=1}^{n} \left(1 - I_{\alpha_{(i)}}\right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})}, 1 \\ &\left.\left. - \prod_{i=1}^{n} \left(1 - F_{\alpha_{(i)}}\right)^{\mu(A_{(i)}) - \mu(A_{(i-1)})}\right)\right) \end{aligned} \tag{40}$$

where the subscript $(i) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$ *is a permutation such that* $\alpha_{(1)} \ge \alpha_{(2)} \ge \cdots \ge \alpha_{(n)}$, $A_{(i)} = \{x_{(1)}, x_{(2)}, \cdots, x_{(i)}\}$, $i = 1, 2, \cdots, n$, *and* $A_{(0)} = \emptyset$.

5. MAGDM Method Based on Induced Single-Valued Neutrosophic Uncertain Linguistic Choquet Aggregation Operator

The MAGDM problem is to determine the optimal alternative in the candidate alternatives. The alternatives for the decision-making would be evaluated by the experts from the perspectives of some specific attributes. However, MAGDM problems often include uncertain and inaccurate information and attributes and expert weights are usually relevant. Therefore, this section will investigate an approach to MAGDM problems by using the proposed operators. In particular, the decision information given takes the form of SVNULNs.

For a MAGDM problem, let $A = (a_1, a_2, \cdots, a_m)$ $(i = 1, 2, \cdots, m)$ be a set of alternatives, $C = (c_1, c_2, \cdots, c_n)$ $(j = 1, 2, \cdots, n)$ be a set of attributes, and $D = (d_1, d_2, \cdots, d_t)$ $(k = 1, 2, \cdots, t)$. Assume that $R_k = (\alpha_{ij}^k)_{m\times n}$ is the SVNUL matrix. With respect to attribute c_j $(j = 1, 2, \cdots, n)$ value of the alternative a_i $(i = 1, 2, \cdots, m)$ performance is expressed as the SVNULN $\alpha_{ij}^k = \langle[h_{\theta_{ij}}^k, h_{\tau_{ij}}^k], (T_{ij}^k, I_{ij}^k, F_{ij}^k)\rangle$ by the experts D_k, where α_{ij}^k indicates that the attribute c_j value given by the DM D_k. Thereafter, we apply the I-SVNULCA or I-SVNULCG operator to deal with MAGDM problem with single-valued neutrosophic uncertain linguistic information.

The main steps by utilizing the proposed decision-making method are as follows.

Step 1. Standardize decision matrices. Generally, benefit and cost types are observed in the attributes. The attributes should be converted to the same type before aggregating the information. For the decision matrix $D^k = (d_{ij}^k)_{m\times n}$, we can use the following conversion form:

$$\begin{aligned} &\alpha_{ij}^k = d_{ij}^k = \left\langle\left[h_{\theta_{ij}}^k, h_{\tau_{ij}}^k\right], \left(T_{ij}^k, I_{ij}^k, F_{ij}^k\right)\right\rangle \\ &\qquad\qquad \textit{for benefit attribute } c_j \\ &\alpha_{ij}^k = \left\langle\left[h_{l-1-\tau_{ij}}^k, h_{l-1-\theta_{ij}}^k\right], \left(T_{ij}^k, I_{ij}^k, F_{ij}^k\right)\right\rangle \\ &\qquad\qquad \textit{for cost attribute } c_j \end{aligned} \tag{41}$$

Thereafter, the standardized decision matrix $R^k = (\alpha_{ij}^k)_{m\times n}$ can be obtained.

Step 2. Determine the fuzzy measures of ω_k weighting vector of the DMs D_k $(k = 1, 2, \cdots, t)$ and sets of DMs D.

Step 3. Utilize the decision information given in matrix $R_k = (\alpha_{ij}^k)_{m\times n}$ $(k = 1, 2, \cdots, t)$ and the I-SVNULCA operator or I-SVNULCG operator,

$$\begin{aligned} \alpha_{ij} &= \left\langle\left[h_{\theta_{ij}}, h_{\tau_{ij}}\right], \left(T_{ij}, I_{ij}, F_{ij}\right)\right\rangle \\ &= \text{I-SVNULCA}\left(\left\langle\mu(D_1), \alpha_{ij}^1\right\rangle, \left\langle\mu(D_2), \alpha_{ij}^2\right\rangle, \cdots, \right. \\ &\quad \left.\left\langle\mu(D_t), \alpha_{ij}^t\right\rangle\right) \\ \text{or } \alpha_{ij} &= \left\langle\left[h_{\theta_{ij}}, h_{\tau_{ij}}\right], \left(T_{ij}, I_{ij}, F_{ij}\right)\right\rangle \\ &= \text{I-SVNULCG}\left(\left\langle\mu(D_1), \alpha_{ij}^1\right\rangle, \left\langle\mu(D_2), \alpha_{ij}^2\right\rangle, \cdots, \right. \\ &\quad \left.\left\langle\mu(D_t), \alpha_{ij}^t\right\rangle\right) \end{aligned} \tag{42}$$

to get the collective decision matrix $R = (\alpha_{ij})_{m\times n}$, where $\mu = (\mu(D_1), \mu(D_2), \cdots, \mu(D_t))$ represents the weighting vector of the DMs.

Table 1: Decision matrix R_1.

	C_1	C_2	C_3
A_1	$\langle[s_5, s_5], (0.265, 0.350, 0.385)\rangle$	$\langle[s_2, s_3], (0.330, 0.390, 0.280)\rangle$	$\langle[s_5, s_6], (0.245, 0.275, 0.480)\rangle$
A_2	$\langle[s_4, s_5], (0.345, 0.245, 0.410)\rangle$	$\langle[s_5, s_5], (0.430, 0.290, 0.280)\rangle$	$\langle[s_3, s_4], (0.245, 0.375, 0.380)\rangle$
A_3	$\langle[s_3, s_4], (0.365, 0.300, 0.335)\rangle$	$\langle[s_4, s_4], (0.480, 0.315, 0.295)\rangle$	$\langle[s_4, s_5], (0.340, 0.370, 0.290)\rangle$
A_4	$\langle[s_6, s_6], (0.430, 0.300, 0.270)\rangle$	$\langle[s_2, s_3], (0.460, 0.245, 0.295)\rangle$	$\langle[s_3, s_4], (0.310, 0.520, 0.170)\rangle$

Table 2: Decision matrix R_2.

	C_1	C_2	C_3
A_1	$\langle[s_3, s_4], (0.125, 0.470, 0.405)\rangle$	$\langle[s_3, s_4], (0.220, 0.420, 0.360)\rangle$	$\langle[s_3, s_4], (0.345, 0.490, 0.165)\rangle$
A_2	$\langle[s_5, s_6], (0.355, 0.315, 0.330)\rangle$	$\langle[s_3, s_4], (0.300, 0.370, 0.330)\rangle$	$\langle[s_4, s_5], (0.205, 0.630, 0.165)\rangle$
A_3	$\langle[s_4, s_5], (0.315, 0.380, 0.305)\rangle$	$\langle[s_4, s_4], (0.330, 0.565, 0.105)\rangle$	$\langle[s_2, s_3], (0.280, 0.520, 0.200)\rangle$
A_4	$\langle[s_5, s_5], (0.365, 0.365, 0.270)\rangle$	$\langle[s_4, s_5], (0.335, 0.320, 0.325)\rangle$	$\langle[s_2, s_3], (0.425, 0.485, 0.090)\rangle$

Table 3: Decision matrix R_3.

	C_1	C_2	C_3
A_1	$\langle[s_5, s_5], (0.260, 0.425, 0.315)\rangle$	$\langle[s_3, s_4], (0.220, 0.450, 0.330)\rangle$	$\langle[s_4, s_5], (0.255, 0.500, 0.245)\rangle$
A_2	$\langle[s_4, s_5], (0.270, 0.370, 0.360)\rangle$	$\langle[s_5, s_5], (0.320, 0.215, 0.465)\rangle$	$\langle[s_2, s_3], (0.135, 0.575, 0.290)\rangle$
A_3	$\langle[s_4, s_4], (0.245, 0.465, 0.290)\rangle$	$\langle[s_5, s_5], (0.250, 0.570, 0.180)\rangle$	$\langle[s_1, s_3], (0.175, 0.660, 0.165)\rangle$
A_4	$\langle[s_3, s_4], (0.390, 0.340, 0.270)\rangle$	$\langle[s_3, s_4], (0.305, 0.475, 0.220)\rangle$	$\langle[s_4, s_5], (0.465, 0.485, 0.050)\rangle$

Step 4. Determine the fuzzy measure of attribute of c_j $(j = 1, 2, \cdots, n)$ and attribute sets of C.

Step 5. Utilize the decision information given in matrix R and the I-SVNULCA operator or I-SVNULCG operator

$$\begin{aligned}\alpha_i &= \left\langle \left[h_{\theta_i}, h_{\tau_i}\right], (T_i, I_i, F_i)\right\rangle \\ &= \text{I-SVNULCA}\left(\left\langle \mu(c_1), \alpha_{ij}\right\rangle, \left\langle \mu(c_2), \alpha_{ij}\right\rangle, \cdots, \left\langle \mu(c_m), \alpha_{ij}\right\rangle\right) \\ \text{or } \alpha_i &= \left\langle \left[h_{\theta_i}, h_{\tau_i}\right], (T_i, I_i, F_i)\right\rangle \\ &= \text{I-SVNULCG}\left(\left\langle \mu(c_1), \alpha_{ij}\right\rangle, \left\langle \mu(c_2), \alpha_{ij}\right\rangle, \cdots, \left\langle \mu(c_m), \alpha_{ij}\right\rangle\right)\end{aligned} \tag{43}$$

to get the collective comprehensive value $\alpha_i = \langle[h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i)\rangle$ of the alternative a_i, where $\mu = (\mu(c_1), \mu(c_2), \cdots, \mu(c_m))$ represents the weighting vector of the attributes.

Step 6. Calculate the scores $S(\alpha_i)$ and $H(\alpha_i)$ $(i = 1, 2, \cdots, m)$ of the collective overall values $\alpha_i = \langle[h_{\theta_i}, h_{\tau_i}], (T_i, I_i, F_i)\rangle$.

Step 7. Rank the alternatives according to $S(\alpha_i)$ and $H(\alpha_i)$ $(i = 1, 2, \cdots, m)$. The larger the $S(\alpha_i)$, the better the alternative.

Step 8. End.

6. Illustrative Example

6.1. An Illustration of the Proposed Approach. In this section, an illustrative example adapted from Liu [35] is employed to the application and effectiveness of the proposed MAGDM method under single-valued neutrosophic uncertain linguistic environment.

An investment company wants to select the best investment alternative. Suppose that four candidate companies A_i $(i = 1, 2, 3, 4)$ are available, and three attributes are considered: (1) C_1 is the growth index. (2) C_2 refers to the potential market and market risk index. (3) C_3 indicates the social-political and environmental impact index.

Suppose that linguistic term set $H = (h_0, h_1, h_2, h_3, h_4, h_5, h_6) = \{$*very poor*, *poor*, *slightly poor*, *fair*, *sightly good*, *good*, *very good*$\}$. The assessment value α_{ij}^k of each company A_i under attributes C_j $(j = 1, 2, 3)$ is expressed by the form of SVNULNs by the decision experts E_k $(k = 1, 2, 3)$, and then the three normalized standardized decision matrices $R_k = (\alpha_{ij}^k)_{m\times n}$ $(k = 1, 2, 3)$ are provided in Tables 1, 2, and 3.

Utilize I-SVNULCA operator to solve the problem based on the following steps.

Step 1. We determine the fuzzy density of each DM and its ρ parameter. Suppose that fuzzy measure of the weighting vector of the decision makers D_k $(k = 1, 2, 3)$ and sets of DMs D are as follows:

$$\begin{aligned}\mu(D_1) &= 0.4, \\ \mu(D_2) &= 0.4, \\ \mu(D_3) &= 0.4.\end{aligned} \tag{44}$$

Thereafter, ρ of the experts can be determined: $\rho_1 = -0.44$. We obtain the following equation based on (14):

$$\begin{aligned}
&\mu(D_1, D_2) = \mu(D_1, D_3) = \mu(D_2, D_3) = 0.73,\\
&\mu(D_1, D_3) = 0.65,\\
&\mu(D_2, D_3) = 0.50,\\
&\mu(D_1, D_2, D_3) = 1.
\end{aligned} \tag{45}$$

Thus,

$$\begin{aligned}
\omega_{(1)} &= \mu(D_1) - \mu(\varnothing) = 0.4,\\
\omega_{(2)} &= \mu(D_1, D_2) - \mu(D_1) = 0.33,\\
\omega_{(3)} &= \mu(D_1, D_2, D_3) - \mu(D_1, D_2) = 0.27.
\end{aligned} \tag{46}$$

Step 2. Utilize the given decision-making matrix $R_k = (\alpha_{ij}^k)_{m\times n}$ $(k = 1, 2, 3)$ and the I-SVNULCA operator; we obtain the collective decision matrix $R = (\alpha_{ij})_{4\times 3}$ as follows:

$$R = \begin{bmatrix}
\langle [s_{4.34}, s_{4.67}], (0.220, 0.407, 0.371)\rangle & \langle [s_{2.60}, s_{3.60}], (0.266, 0.415, 0.318)\rangle & \langle [s_{4.07}, s_{5.07}], (0.282, 0.391, 0.281)\rangle\\
\langle [s_{4.33}, s_{5.33}], (0.329, 0.298, 0.368)\rangle & \langle [s_{4.34}, s_{4.67}], (0.360, 0.290, 0.339)\rangle & \langle [s_{3.06}, s_{4.06}], (0.203, 0.499, 0.268)\rangle\\
\langle [s_{3.60}, s_{4.33}], (0.318, 0.365, 0.312)\rangle & \langle [s_{4.27}, s_{4.27}], (0.376, 0.448, 0.184)\rangle & \langle [s_{2.53}, s_{3.80}], (0.279, 0.484, 0.220)\rangle\\
\langle [s_{4.86}, s_{5.13}], (0.398, 0.331, 0.270)\rangle & \langle [s_{2.93}, s_{3.93}], (0.381, 0.320, 0.281)\rangle & \langle [s_{2.94}, s_{3.94}], (0.393, 0.499, 0.099)\rangle
\end{bmatrix} \tag{47}$$

Step 3. Suppose that the fuzzy measure of the weighting vector of attribute C_k $(k = 1, 2, 3)$ and attribute sets of C are as follows:

$$\begin{aligned}
&\mu(C_1) = 0.362,\\
&\mu(C_2) = 0.2,\\
&\mu(C_3) = 0.438.
\end{aligned} \tag{48}$$

Thereafter, ρ of the attributes can be determined: $\rho_2 = 0.856$. We obtain the following equation based on (14); we have

$$\begin{aligned}
&\mu(C_1, C_2) = 0.626,\\
&\mu(C_1, C_3) = 0.713,\\
&\mu(C_2, C_3) = 0.936,\\
&\mu(C_1, C_2, C_3) = 1.
\end{aligned} \tag{49}$$

Thus, $w_{(1)} = \mu(C_3) - \mu(\varnothing) = 0.438$, $w_{(2)} = \mu(D_1, D_3) - \mu(D_3) = 0.275$, and $\omega_{(3)} = 1 - \mu(D_1, D_3) = 0.287$.

Step 4. Utilize the decision information given in matrix $R = (\alpha_{ij})_{m\times n}$ $(i = 1, 2, 3, 4;\ j = 1, 2, 3)$ and the I-SVNULCA operator; we get the collective comprehensive value R_i $(i = 1, 2, 3, 4)$ for each alternative as follows:

$$\begin{aligned}
R_1 &= \langle [s_{3.722}, s_{4.538}], (0.261, 0.402, 0.314)\rangle,\\
R_2 &= \langle [s_{3.777}, s_{4.584}], (0.286, 0.371, 0.313)\rangle\\
R_3 &= \langle [s_{3.324}, s_{4.081}], (0.319, 0.430, 0.230)\rangle,\\
R_4 &= \langle [s_{3.465}, s_{4.264}], (0.391, 0.392, 0.176)\rangle
\end{aligned} \tag{50}$$

Step 5. We calculate the value $S(R_i)$ of R_i $(i = 1, 2, 3, 4)$.

$$\begin{aligned}
&S(R_1) = h_{2.126},\\
&S(R_2) = h_{2.233},\\
&S(R_3) = h_{2.036},\\
&S(R_4) = h_{2.348}.
\end{aligned} \tag{51}$$

Step 6. Based on the comparison method described in Definition 9, $S(R_4) > S(R_2) > S(R_1) > S(R_3)$. Thereafter, $A_4 \succ A_2 \succ A_1 \succ A_3$.

Thus, the most desirable alternative is A_4.

Apply I-SVNULCG operator to solve the problem based on the following steps.

Step 1. By utilizing the given decision-making matrix $R_k = (\alpha_{ij}^k)_{m\times n}$ $(k = 1, 2, 3)$ and the I-SVNULCG operator, we obtain a collective decision matrix $R = (\alpha_{ij})_{4\times 3}$ as follows:

$$R = \begin{bmatrix}
\langle [s_{4.224}, s_{4.645}], (0.206, 0.412, 0.374)\rangle & \langle [s_{2.551}, s_{3.565}], (0.288, 0.417, 0.321)\rangle & \langle [s_{3.977}, s_{4.996}], (0.277, 0.416, 0.328)\rangle\\
\langle [s_{4.306}, s_{5.310}], (0.326, 0.304, 0.371)\rangle & \langle [s_{4.224}, s_{4.645}], (0.353, 0.299, 0.351)\rangle & \langle [s_{2.957}, s_{3.984}], (0.197, 0.526, 0.290)\rangle\\
\langle [s_{3.565}, s_{4.306}], (0.312, 0.375, 0.313)\rangle & \langle [s_{4.248}, s_{4.248}], (0.356, 0.480, 0.205)\rangle & \langle [s_{2.189}, s_{3.680}], (0.267, 0.512, 0.228)\rangle\\
\langle [s_{4.685}, s_{5.064}], (0.397, 0.333, 0.270)\rangle & \langle [s_{2.805}, s_{3.83}], (0.371, 0.339, 0.286)\rangle & \langle [s_{2.836}, s_{3.864}], (0.384, 0.499, 0.113)\rangle
\end{bmatrix} \tag{52}$$

Table 4: Ranking results of different methods.

Methods	Ranking results
Ye's [70]method by the proposed operator:	
INLWAA	$A_4 \succ A_2 \succ A_3 \succ A_1$
INLWGA	$A_4 \succ A_2 \succ A_1 \succ A_3$
Method proposed by Liu et al.'s [35] (p=1,q=1)	$A_4 \succ A_2 \succ A_1 \succ A_3$
The proposed method in this paper	$A_4 \succ A_2 \succ A_1 \succ A_3$

Step 2. Utilize the above decision-making matrix $R = (\alpha_{ij})_{m\times n}$ $(i = 1, 2, 3, 4;\ j = 1, 2, 3)$ and the I-SVNULCG operator, we obtain the collective comprehensive value R_i $(i = 1, 2, 3, 4)$.

$$\begin{aligned} R_1 &= \left\langle \left[s_{3.560}, s_{4.445}\right], (0.258, 0.415, 0.339)\right\rangle, \\ R_2 &= \left\langle \left[s_{3.632}, s_{4.506}\right], (0.267, 0.411, 0.331)\right\rangle \\ R_3 &= \left\langle \left[s_{3.028}, s_{4.004}\right], (0.302, 0.468, 0.246)\right\rangle, \\ R_4 &= \left\langle \left[s_{3.246}, s_{4.154}\right], (0.384, 0.413, 0.210)\right\rangle \end{aligned} \tag{53}$$

Step 3. We calculate the value $S(R_i)$ of R_i $(i = 1, 2, 3, 4)$.

$$\begin{aligned} S(R_1) &= h_{2.007}, \\ S(R_2) &= h_{2.069}, \\ S(R_3) &= h_{1.861}, \\ S(R_4) &= h_{2.171}. \end{aligned} \tag{54}$$

Step 4. Based on the comparison method described in Definition 9, hence,

$S(R_4) > S(R_2) > S(R_1) > S(R_3)$. Thereafter, $A_4 \succ A_2 \succ A_1 \succ A_3$.

Thus, the most desirable investment company is A_4.

As shown above, the ranking orders and most desirable alternative have the same result. That is, $A_4 \succ A_2 \succ A_1 \succ A_3$, by using two MAGDM method based on SVNULCA operator and SVNULCG operator.

6.2. Comparison with Other Existing Methods. In this section, we validate the feasibility and the advantages of the proposed method by comparative analysis, compared to the existing methods which are the interval neutrosophic linguistic weighted arithmetic average (INLWAA) operator and interval neutrosophic linguistic weighted geometric average (INLWGA) operator developed by Ye [70] and the decision-making method proposed by Liu [35]. Table 4 lists the ranking results yielded by those methods.

Table 4 shows that a slight difference is observed in the ranking results. From these three methods, the best alternatives are the same. To compare with the method proposed by Ye [70], the assessment value $\alpha = \langle |[h_{\theta_\alpha}, h_{\tau_\alpha}], (T_\alpha, I_\alpha, F_\alpha)\rangle$ in this illustrative example is transformed into interval neutrosophic linguistic number $\alpha = \langle |h_{s_\alpha}, ([\inf T_\alpha, \sup T_\alpha], [\inf I_\alpha, \sup I_\alpha], [\inf F_\alpha, \sup F_\alpha])\rangle$ by $\alpha = \langle |h_{s_\alpha}, ([\inf T_\alpha, \sup T_\alpha], [\inf I_\alpha, \sup I_\alpha], [\inf F_\alpha, \sup F_\alpha])\rangle = \langle |[h_{(\theta_\alpha+\tau_\alpha)/2}], ([T_\alpha, T_\alpha], [I_\alpha, I_\alpha], [F_\alpha, F_\alpha])\rangle$. Then, the ranking result $A_4 \succ A_2 \succ A_3 \succ A_1$ is obtained by the proposed approach in this paper which are consistent with the result obtained by the INLWGA operator proposed in Ye [70]; the ranking result $A_4 \succ A_2 \succ A_1 \succ A_3$ is obtained using the INLWAG operator proposed by Ye [70]. Although the ranking results are slightly different in a few conditions, the approach proposed in this paper is superior in credibility. The main reason is that we proposed method which considers the induced variance determined by the decision maker's preference and interaction among the attributes to yield a reasonable result. This result is in accordance with the practical decision-making process. Moreover, compared to the linguistic terms, the ULVs perform more outstanding in expressing qualitative information. That is, the proposed approach in this paper is considerably convenient and feasible when SVNULNs are incorporated.

Compared with the proposed method by Liu [35], the ranking results obtained by the method in this paper are consistent with the proposed approach in [35]. This case demonstrates the availability and feasibility of this approach, because it can obtain different results in varying induced preference conditions, whereas the methods in [35] disregard them.

Therefore, the developed method has strength in that it can easily reflect and express the fuzziness nature of decision maker's subjective judgments by SVNULNs, because SVNULNs are fit to represent imprecise, uncertain, and inconsistent information in some decision-making situations. Moreover, the proposed method can consider the order-inducing variables in accordance with decision maker preference and the correlation or interaction among attributes and decision makers under a SVNUL environment. Thus, the method we proposed has superiority in feasibility and practice than other decision-making methods in MAGDM.

7. Conclusion

In real decision-making, incomplete, indeterminate, and inconsistent are the common features in the decision-making information of alternatives provided by DMs. SVNULNs can considerably describe the decision maker's preference. In particular, the Choquet integral operator can determine the interaction of the attributes and experts. Then, we extend the I-COA operator to the SVNUL environment and

defined I-SVNULCA and I-SVNULCG integral operators. We likewise investigate their properties in detail. Moreover, we develop MAGDM methods based on the I-SVNULCA and I-SVNULCG operators under a SVNUL environment. Lastly, the practicality and effectiveness of the proposed method would be demonstrated by an illustrative example; the results have shown that the proposed method is more suitable than the existing method.

The main advantages of this study are as follows. First, the proposed aggregation operator considers the interaction phenomena among the attributes and decision makers under SVNUL environment, and the result can change based on various linguistic scale and induced variables. This condition enables DMs to choose the most appropriate linguistic scale and input induced parameter according to their preferences and interest. Moreover, SVNULNs can reflect the fuzziness nature of decision maker's subjective judgments very properly. Finally, the proposed method based on the I-SVNULCA and I-SVNULCG operators can accommodate situations, in which the input arguments are represented by SVNULNs. That is, the proposed method is quite feasible and practical for real-world applications. In the future research, the developed aggregation operator and method are further applied to many areas such as medical diagnosis and pattern recognition.

Data Availability

The SVNULNs data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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