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Inspection Plan Based on the Process Capability Index Using the Neutrosophic Statistical Method

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Abstract: The Process Capability Index (PCI) has been widely used in industry to advance the quality of a product. Neutrosophic statistics is the more generalized form of classical statistics and is applied when the data from the production process or a product lot is incomplete, incredible, and indeterminate. In this paper, we will originally propose a variable sampling plan for the PCI using neutrosophic statistics. The neutrosophic operating function will be given. The neutrosophic plan parameters will be determined using the neutrosophic optimization solution. A comparison between plans based on neutrosophic statistics and classical statistics is given. The application of the proposed neutrosophic sampling plan will be given using company data.

1. Introduction

Acceptance sampling is the most widely used tool for the inspection of the raw material, semi-finished product, and finished product. But, the presence of the indeterminacy in the observations or parameters may affect the performance of the sampling plan. A well-designed sampling plan used for the inspection of the product under the uncertainty and determinacy environment is needed at each stage to check that the finished product meets either the customer's upper specification limit (USL) and lower specification limit (LSL) before sending it to market. The quality of interest beyond the LSL and USL creates a non-conforming item. At the time of inspection, a random sample is taken and lot sentencing is made on the basis of this primary information about the lot. Thus, the sample information may mislead the experimenters in making the decision about the submitted product lot. There is a chance of rejecting a good lot and accepting a bad lot on the basis of the sample information. Thus, the sampling schemes are developed with the aim of reducing the cost of the inspection, non-conforming items, and minimizes the risk of the sampling. The acceptance sampling plan has two major types, known as attribute sampling plans and variable sampling plans. Attribute sampling plans are easier to apply but are more costly than the variable sampling plans. On the other hand, the variable sampling plans are more informative than attribute sampling plans [1]. A number of authors designed variable and attribute sampling plans: Jun et al. [2] studied variable sampling plans for sudden death testing; Balamurali and Jun [3] studied skip-lot sampling for the normal distribution; Fallah Nezhad et al. [4] designed a sampling plan using cumulative sums of conforming run-lengths; Pepelyshev et al. [5] applied a variable sampling plan in photovoltaic modules; Gui and Aslam [6] designed a time truncated plan for weighted exponential distribution; and Balamurali et al. [7] designed a mixed variable sampling plan.

The Process Capability Index (PCI) has been widely used in industry for quality improvement purposes and to make a relation between specification limits and process quality. Kane [8] originally proposed the PCI for classical statistics. Boyles [9] provided the bounds on the process yield for the normally distributed process. Kotz and Johnson [10] provided a detailed review of PCIs. More

details on PCIs can be seen in [11]. Pearn et al. [12] discussed an effective decision method for product inspection; Montgomery [1] mentioned the applications of PCIs. Boyles [13] studied PCIs for an asymmetric tolerances case and Ebadi [14] studied a simple linear profile using PCIs. Due to the importance of the PCIs in industry, several authors focused on the development of inspection schemes using classical statistics based on PCIs for various situations including for example, and Chen et al. [15] studied PCIs for entire product inspection. Pearn et al. [12] presented an effective decision method for the inspection. Aslam et al. [16] designed various sampling plans using PCIs. Seifi and Nezhad [17] studied resubmitted sampling using PCI and Arif et al. [18] worked on a sampling plan using PCI for multiple manufacturing lines.

Fuzzy sampling plans have been widely used in the industry when the proportion of the non-conforming product is a fuzzy number [19]. Kanagawa and Ohta [20] introduced an attribute plan using fuzzy sets. Sadeghpour Gildeh et al. [19] designed a single sampling plan using fuzzy parameters. Kahraman et al. [21] designed single and double sampling plans using fuzzy approach. The PCIs using fuzzy logic can be seen in [22–24].

Smarandache [25] defined the neutrosophic logic in 1998 as the generalization of fuzzy logic. Smarandache [26] gave the idea of descriptive neutrosophic statistics. The neutrosophic statistics is the more generalized form of classical statistics and applied when the data from the production process or a product lot is incomplete, incredible, and indeterminate [26]. Chen et al. [27,28] studied the rock joint roughness coefficient using neutrosophic statistics. According to [29] "All observations and measurements of continuous variables are not precise numbers but more or less non-precise. This imprecision is different from variability and errors. Therefore also lifetime data are not precise numbers but more or less fuzzy. The best up-to-date mathematical model for this imprecision is so-called non-precise numbers".

Recently, Aslam [30] introduced the neutrosophic statistics in the area of the acceptance sampling plan. Aslam [30] proposed an acceptance sampling plan using the neutrosophic process loss function. The sampling plan for multiple manufacturing lines using the neutrosophic statistics is proposed by [31]. The sampling plan for the exponential distribution under the uncertainty is proposed by [32]. Some more details about the sampling plan using the neutrosophic plans can be seen in [33–37].

The existing sampling plans using PCIs cannot apply when the data is indeterminate or incomplete. Also, the available sampling plans using the neutrosophic statistics do not consider the PCIs for the inspection of the product. By exploring the literature and best of the author knows there is no work on the sampling plan for PCIs using the neutrosophic statistics. In this paper, we will originally propose a variable sampling plan for the PCIs using the neutrosophic statistics. The neutrosophic operating function will be given. The neutrosophic plan parameters will be determined using the neutrosophic optimization solution. A comparison between plans based on neutrosophic statistics and classical statistics is given. We expect that the proposed plan will be more effective to be applied in an uncertain environment. The application of the proposed sampling plan using neutrosophic statistics will be given using the company data.

2. Design of a Neutrosophic Plan Based on PCI

Let $n_N \epsilon \{n_L, n_U\}$ be a random sample selected from the population having some uncertain observations, where n_L and n_U are the lower and upper sample size of the indeterminacy interval, respectively. Suppose that a neutrosophic quality of interest, X_{Ni}, is expressed in the indeterminacy interval, say, $X_{Ni} \epsilon \{X_L, X_U\}$; $i = 1,2,3, \ldots, n_N$ having indeterminate observations follow the neutrosophic normal distribution, where X_L and X_U are the lower and the upper values, respectively, with the neutrosophic population mean $\mu_N \epsilon \{\mu_L, \mu_U\}$ and neutrosophic population standard deviation (NSD) $\sigma_N \epsilon \{\sigma_L, \sigma_U\}$ (see [26]). The neutrosophic process capability index process (NPCI), say, $\hat{c}_{N_{pk}}$, is defined as:

$$C_{N_{pk}} = Min\left\{ \frac{USL - \mu_N}{3\sigma_N}, \frac{\mu_N - LSL}{3\sigma_N} \right\}; \ \mu_N \epsilon \{\mu_L, \mu_U\}, \ \sigma_N \epsilon \{\sigma_L, \sigma_U\} \tag{1}$$

where *USL* and *LSL* are the upper specification limit and lower specification limit, respectively.

Note that $C_{N_{pk}}$ reduces to PCI for classical statistics when no indeterminate observations are recorded in X_N. Usually, $\mu_N \epsilon \{\mu_L, \mu_U\}$ and $\sigma_N \epsilon \{\sigma_L, \sigma_U\}$ are unknown in practice and the best linear unbiased estimate (BLUE) of $\mu_N \epsilon \{\mu_L, \mu_U\}$ is the neutrosophic sample mean $\overline{X}_N \epsilon \{\overline{X}_L, \overline{X}_U\}$ and a BLUE of $\sigma_N \epsilon \{\sigma_L, \sigma_U\}$ is the neutrosophic sample standard deviation $s_N \epsilon \{s_L, s_U\}$ which can be used to evaluate $C_{N_{pk}}$. The $\hat{C}_{N_{pk}}$ based on sample estimate is given as by:

$$\hat{C}_{N_{pk}} = Min\left\{\frac{USL - \overline{X}_N}{3s_N}, \frac{\overline{X}_N - LSL}{3s_N}\right\}; \ \overline{X}_N \epsilon \{\overline{X}_L, \overline{X}_U\}, \ s_N = \{s_L, s_U\} \tag{2}$$

where $\overline{X}_L = \sum_{i=1}^{n} x_i^L / n_L$,

$$\overline{X}_U = \sum_{i=1}^{n} x_i^U / n_U, \ s_L = \sqrt{\sum_{i=1}^{n} \left(x_i^L - \overline{X}_L\right)^2 / n_L} \text{ and } s_U = \sqrt{\sum_{i=1}^{n} \left(x_i^U - \overline{X}_U\right)^2 / n_U}$$

To design the proposed sampling plan, it is assumed that there is uncertainty in the selection of a random sample from the submitted product lot. Thus, a random sample will be selected from a neutrosophic interval. The proposed sampling plan is stated as follows:

Step 1: Select a random sample of size $n_N \epsilon \{n_L, n_U\}$ from the product lot. Compute the statistic $\hat{C}_{N_{pk}} \epsilon Min\left\{\frac{USL - \overline{X}_N}{3s_N}, \frac{\overline{X}_N - LSL}{3s_N}\right\}; \ \overline{X}_N \epsilon \{\overline{X}_L, \overline{X}_U\}, \ s_N \epsilon \{s_L, s_U\}.$

Step 2: Accept a product lot of $\hat{C}_{N_{pk}} \geq k_N$; $k_N \epsilon \{k_{aL}, k_{aU}\}$, otherwise reject a product lot, where $k_N \epsilon \{k_{aL}, k_{aU}\}$ is the neutrosophic acceptance number. An acceptance number is also called the action number/boundary number. A product lot is rejected if the statistic $\hat{C}_{N_{pk}}$ is smaller than k_N, otherwise, the product lot is accepted.

The evaluation of the proposed sampling plan will be used on two parameters, namely $n_N = \{n_L, n_U\}$ and $k_N \epsilon \{k_{aL}, k_{aU}\}$. The neutrosophic operating characteristic (NOC) for the proposed plan is derived as follows:

$$L(p) = P\left(\hat{C}_{N_{pk}} \geq k_N\right) = P\left\{LSL + 3k_N s_N \leq \overline{X}_N \leq USL - 3k_N s_N\right\} = P\left\{\overline{X}_N + 3k_N s_N \leq USL\right\}$$
$$-P\left\{\overline{X}_N - 3k_N s_N \leq LSL\right\}; \ n_N \epsilon \{n_L, n_U\}, \ \overline{X}_N \epsilon \{\overline{X}_L, \overline{X}_U\} s_N \epsilon \{s_L, s_U\} \text{ and } k_N \epsilon \{k_{aL}, k_{aU}\}. \tag{3}$$

Duncan [38] suggested $\overline{X}_N \pm k_N s_N$; $\overline{X}_N \epsilon \{\overline{X}_L, \overline{X}_U\}$ and $s_N = \{s_L, s_U\}$ is distributed as an approximately neutrosophic normal distribution, that is $\overline{X}_N \pm k_N s_N \sim N_N\left(\mu_N \pm c\sigma_N, \frac{\sigma_N^2}{n_N} + \frac{c^2 \sigma_N^2}{2n_N}\right)$. where $N_N(.)$ shows neutrosophic normal distribution.

Suppose that quality of interest X_N beyond the USL or LSL is labeled as the defective item and this probability is defined as $p_U = P\{X_N > USL | \mu_N\}$ and $p_U = P\{X_N \langle LSL | \mu_N\}$; $\mu_N = \{\mu_L, \mu_U\}$. Thus, the probability of acceptance is given by the following [39]:

$$L(p) = \Phi\left\{\frac{USL - \mu_N - 3k_N \sigma_N}{(\sigma_N / n_N)\sqrt{1 + 9k_N^2/2}}\right\} - \Phi\left\{\frac{LSL - \mu_N + 3k_N \sigma_N}{(\sigma_N / n_N)\sqrt{1 + 9k_N^2/2}}\right\} \tag{4}$$

Let us define the neutrosophic standard normal random variable as:

$$Z_{N_{pU}} = \frac{USL - \mu_N}{\sigma_N} \text{ and } -Z_{N_{pL}} = \frac{LSL - \mu_N}{\sigma_N} \tag{5}$$

Now, the final form of NFOC is given by:

$$L(p) = \Phi\left\{\left(\left(Z_{Np_U} - 3k_N\right)\sqrt{\frac{n_N}{1+\left(9k_N^2/2\right)}}\right)\right\} - \Phi\left\{\left(-\left(Z_{Np_L} - 3k_N\right)\sqrt{\frac{n_N}{1+\left(9k_N^2/2\right)}}\right)\right\} \tag{6}$$

where $\Phi(.)$ is the neutrosophic cumulative standard normal distribution.

Research Methodology

To meet the given producer's risk, say, α, and the custumer's risk, say, β, the plan parameters of the proposed sampling plan will be determined in such a way that NFOC passes through the two points $(p_1, 1-\alpha)$ and (p_2, β), where p_1 is the acceptable quality limit (AQL) and p_2 is the limiting quality limit (LQL). The plan parameters of the proposed sampling plans will be determined through the following non-linear solution under the neutrosophic statistical interval method:

Minimize:

$$n_N \epsilon \{n_L, n_U\} \tag{7}$$

subject to:

$$L_N(p_1) = \Phi\left\{\left(\left(Z_{Np_{U1}} - 3k_N\right)\sqrt{\frac{n_N}{1+\left(9k_N^2/2\right)}}\right)\right\} - \Phi\left\{\left(-\left(Z_{Np_{L1}} - 3k_N\right)\sqrt{\frac{n_N}{1+\left(9k_N^2/2\right)}}\right)\right\} \geq 1- \tag{8}$$
$$\alpha;\ k_N \epsilon \{k_{aL}, k_{aU}\};\ n_N \epsilon \{n_L, n_U\}$$

and:

$$L_N(p_2) = \Phi\left\{\left(\left(Z_{Np_{U2}} - 3k_N\right)\sqrt{\frac{n_N}{1+\left(9k_N^2/2\right)}}\right)\right\} - \Phi\left\{\left(-\left(Z_{Np_{L2}} - 3k_N\right)\sqrt{\frac{n_N}{1+\left(9k_N^2/2\right)}}\right)\right\} \leq \beta; \tag{9}$$
$$k_N \epsilon \{k_{aL}, k_{aU}\};\ n_N \epsilon \{n_L, n_U\}$$

The plan parameters of the proposed plan are determined through Equations (7)–(9) using the search grid method for the various combinations of AQL and LQL. Several combinations of plan parameters in the indeterminacy interval satisfy Equations (7)–(9). The plan parameters having the smallest range in indeterminacy interval are chosen and placed in Table 1. To save the space, we present Table 1 when $\alpha = 0.05$ and $\beta = 0.10$. Similar tables for other values of α and β can be prepared. The neutrosophic lot acceptance probabilities, $L_N(p_1)$ and $L_N(p_2)$ at the consumer's risk and producer's risk are also reported in Table 1.

From Table 1, we note that, for the fixed values of all other parameters, the values of $k_N \epsilon \{k_{aL}, k_{aU}\}$; $n_N \epsilon \{n_L, n_U\}$ decrease as LQL increases. This means the indeterminacy in the sample size and acceptance number reduces. For example, under the uncertainty, when AQL = 0.001 and LQL = 0.02, the sample size will be in the interval [18,20]. This means the industrial engineers should select a sample size between 18 and 20. Furthermore, for the smaller values of AQL and LQL, larger the values of $n_N \epsilon \{n_L, n_U\}$ are required. Note here that the appropriate sample size is decided on the basis of pre-defined parameters, such as AQL, LQL, α, and β. The following algorithm is used to determine the neutrosophic plan parameters:

1. Specify the values of AQL, LQL, α and β.
2. Specify the suitable ranges for $n_N \epsilon \{n_L, n_U\}$ such that $n_L < n_U$ and $k_N \epsilon \{k_{aL}, k_{aU}\}$ such that $k_{aL} < k_{aU}$.
3. Perform the simulation by the grid search method and select those values of the neutrosophic plan parameters where $n_N \epsilon \{n_L, n_U\}$ and satisfy the conditions given in Equations (7)–(9).

Table 1. The plan parameters of the plan when $\alpha = 0.05$, $\beta = 0.10$.

p_1	p_2	n_N	k_N	$L_N(p_1)$	$L_N(p_2)$
0.001	0.002	[602, 643]	[1.093, 1.095]	[0.9500, 0.95033]	[0.0441, 0.0891]
	0.003	[218, 228]	[1.052, 1.054]	[0.9500, 0.9505]	[0.06223, 0.0898]
	0.004	[128, 133]	[1.022, 1.024]	[0.9506, 0.9513]	[0.0700, 0.0914]
	0.006	[69, 71]	[0.978, 0.980]	[0.9513, 0.9517]	[0.0807, 0.0969]
	0.008	[47, 49]	[0.946, 0.948]	[0.9506, 0.9528]	[0.0848, 0.0977]
	0.010	[36, 38]	[0.921, 0.923]	[0.9502, 0.9504]	[0.0849, 0.0958]
	0.015	[24, 28]	[0.874, 0.876]	[0.9541, 0.9675]	[0.0914, 0.0959]
	0.020	[18, 20]	[0.842, 0.844]	[0.9521, 0.9614]	[0.0761, 0.0823]
0.0025	0.030	[21, 23]	[0.793, 0.795]	[0.9529, 0.9606]	[0.0923, 0.0995]
	0.050	[13, 15]	[0.731, 0.735]	[0.9567, 0.9674]	[0.0607, 0.0754]
0.005	0.050	[19, 21]	[0.730, 0.732]	[0.9512, 0.9599]	[0.0897, 0.0967]
	0.100	[9, 11]	[0.631, 0.633]	[0.9575, 0.9740]	[0.0957, 0.0961]
0.01	0.020	[274, 290]	[0.854, 0.856]	[0.9500, 0.9504]	[0.0513, 0.0881]
	0.030	[95, 99]	[0.803, 0.805]	[0.9504, 0.9512]	[0.0696, 0.0918]
0.03	0.060	[165, 174]	[0.718, 0.720]	[0.9503, 0.9509]	[0.0581, 0.0903]
	0.090	[55, 57]	[0.659, 0.661]	[0.9505, 0.9511]	[0.0756, 0.0950]
0.05	0.100	[123, 129]	[0.647, 0.649]	[0.9502, 0.9505]	[0.0690, 0.0986]
	0.150	[41, 43]	0.584, 0.586	0.9509, 0.9530	0.0736, 0.0911

3. Comparison Study

In this section, we will compare the efficiency of the proposed plan with the sampling plan using classical statistics in terms of the sample size required for the inspection of the submitted product lot. For a fair comparison, we will consider the same values of all the specified parameters. The sample size n_N along with range (R = $n_U - n_L$) in the indeterminacy interval of the proposed plan and sample size n using classical statistics when $\alpha = 0.05$, $\beta = 0.10$ are placed in Table 2. From Table 2, it can be noted that the proposed plan provides a smaller indeterminacy interval in the sample size as compared to the plan using classical statistics. For example, when AQL = 0.001 and LQL = 0.002, the proposed plan has $n_N \in [602, 643]$ while the existing plan has $n = 1134$. Therefore, the proposed plan needs a smaller sample size and range in the indeterminacy interval for the inspection of a product lot. From this comparison, it is quite clear that the proposed plan using neutrosophic statistics is more efficient than the existing sampling plan under classical statistics in terms of sample size. In addition, the proposed plan is quite suitable, effective, and informative to be used in uncertainty than the existing plan.

Table 2. The comparison of proposed plan and the plan based on classical statistics.

p_1	p_2	Proposed Plan	Plan Based on Classical Statistics
		n_N	n
0.001	0.002	[602, 643] (R = 41)	1134 (R = 1134)
	0.003	[218, 228] (R = 10)	351 (R = 351)
	0.004	[128, 133] (R = 5)	161 (R = 161)
	0.006	[69, 71] (R = 2)	74 (R = 74)
	0.008	[47, 49] (R = 2)	47 (R = 47)

Table 2. *Cont.*

p₁	p₂	Proposed Plan	Plan Based on Classical Statistics
		n_N	n
0.01	0.020	[274, 290] (R = 16)	449 (R = 449)
	0.030	[95, 99] (R = 4)	132 (R = 132)
0.03	0.060	[165, 174] (R = 9)	240 (R = 240)
	0.090	[55, 57] (R = 2)	68 (R = 68)
0.05	0.100	[123, 129] (R = 6)	167 (R = 167)
	0.150	[41, 43] (R = 2)	46 (R = 46)

4. Application of the Proposed Plan

In this section, we will give the application of the proposed plan using the data of the amplified pressure sensor that came from industry. Viertl [29] commented that the observations obtained from the measurements are not usually precise. According to [40] "For this amplified pressure sensor process, the span is the focused characteristic". As the observations for the quality of interest are measured, some observations in the data may be indeterminate or imprecise. Under the uncertainty, the experimenter is not sure about the sample size for the inspection of a product lot when some indeterminate or imprecise observations are recorded. For this data, LSL = 1.9 V, USL 2.1. Suppose that AQL = 0.001, LQL = 0.04, $\alpha = 0.05$, and $\beta = 0.10$. The neutrosophic plan parameters from Table 1 are $n_N \epsilon \{128, 133\}$. Thus, the experimenter should select a random sample between 128 and 133. Suppose that the industrial engineers decided to select a random sample size of 128 for the inspection of a product lot. The amplified pressure sensor data of $n = 128$ having some indeterminate observations are reported in Table 3. Based on the given data, the neutrosophic average and standard deviation (SD) are computed as follows:

$$\overline{X}_N = \frac{\begin{array}{c}[1.9422, 1.9422] + [1.9651, 1.9651] + [2.0230, 2.0230] + \ldots + [1.9994, 1.9994], \\ [1.9422, 1.9422] + [1.9651, 1.9651] + [2.0435, 2.0435] + \ldots + [2.0512, 2.0512]\end{array}}{128} = [1.9805, 1.9827]$$

and, similarly, $s_N = \{0.0193, 0.0225\}$.

The NPCI is computed as follows: $\hat{C}_{N_{pk}} = Min\left\{\frac{USL - \overline{X}_N}{3s_N}, \frac{\overline{X}_N - LSL}{3s_N}\right\}$, $\hat{C}_{N_{pk}} \epsilon [1.7377, 2.0639]$ for $\overline{X}_N = [1.9805, 1.9827]$ and $s_N = \{0.0193, 0.0225\}$.

The proposed plan will be implemented as follows:

Step 1: Select a random sample of size $n_N = \{128, 133\}$ from a product lot. Compute the statistic $\hat{C}_{N_{pk}} \epsilon [1.7377, 2.0639]$.

Step 2: Accept a product lot as $[1.7377, 2.0639] \geq \{1.022, 1.024\}$.

The application of the proposed sampling plan shows that the proposed sampling plan is quite effective, adequate, and flexible to be used under the uncertainty environment than the plan based on classical statistics which provide the determined values of the plan parameters.

Table 3. Indeterminate data of Amplified Sensors from [40].

[1.9422,1.9422]	[1.9651, 1.9651]	[2.0230, 2.0435]	[1.9712,1.9712]	[1.9975,1.9975]	[2.0164,2.0164]	[1.9927,1.9927]	[1.9566,1.9566]
[1.9738, 1.9738]	[1.9541, 1.9541]	[1.9800, 1.9800]	[1.9596, 1.9596]	[1.9811, 1.9811]	[2.0088, 2.0088]	[1.9858, 1.9858]	[1.9677, 1.9677]
[2.0001, 2.0001]	[1.9659, 1.9659]	[1.9955, 1.9955]	[1.9842, 1.9842]	[1.9909,2.0512]	[1.9829, 1.9829]	[1.9684, 1.9684]	[1.9942, 1.9942]
[1.9897, 1.9897]	[1.9836, 1.9836]	[1.9891, 1.9891]	[1.9608, 1.9608]	[2.0109, 2.0109]	[1.9912, 1.9912]	[2.0077, 2.0077]	[1.9803, 1.9803]
[2.0106, 2.0106]	[1.9885, 1.9885]	[1.9704, 1.9704]	[1.9882, 1.9882]	[1.9689, 1.9689]	[1.9553, 1.9553]	[1.9741, 1.9741]	[1.9825, 1.9825]
[1.9640, 1.9640]	[2.0187, 2.0187]	[1.9616, 1.9616]	[1.9865, 1.9865]	[1.9556, 1.9556]	[1.9817, 1.9817]	[1.9774, 1.9774]	[1.9316, 1.9316]
[1.9841, 1.9841]	[1.9919, 1.9919]	[1.9737, 1.9737]	[1.9958, 1.9958]	[2.0121, 2.0121]	[2.0021, 2.0521]	[1.9665, 1.9665]	[1.9773, 1.9773]
[1.9841, 1.9841]	[1.9570, 1.9875]	[1.9610, 1.9610]	[2.0015, 2.0015]	[1.9750, 1.9750]	[1.9825, 1.9825]	[1.9758, 1.9758]	[1.9682, 1.9682]
[1.9668, 1.9668]	[1.9696, 1.9696]	[2.0334, 2.0334]	[1.9656, 1.9656]	[1.9819, 1.9819]	[2.0116, 2.0116]	[1.9754, 1.9754]	[1.9986, 1.9986]
[2.0114, 2.0114]	[1.9861, 1.9861]	[1.9743, 1.9743]	[1.9594, 1.9594]	[1.9712,1.9914]	[1.9849, 1.9849]	[1.9711, 1.9711]	[1.9486, 1.9486]
[1.9837, 1.9837]	[1.9424, 1.9424]	[1.9744, 1.9744]	[1.9605, 1.9605]	[1.9719, 1.9719]	[1.9656, 1.9656]	[1.9549, 1.9549]	[2.0174, 2.0174]
[1.9779, 1.9779]	[2.0072, 2.0072]	[1.9875, 1.9875]	[1.9781, 1.9781]	[1.9834, 1.9834]	[1.9893, 1.9893]	[1.9276, 1.9276]	[1.9513, 1.9513]
[1.9971, 1.9971]	[1.9963, 1.9963]	[1.9375, 1.9375]	[1.9941, 1.9941]	[1.9763, 1.9763]	[2.0108, 2.0108]	[1.9687, 1.9687]	[1.9559, 1.9559]
[1.9611, 1.9611]	[1.9729, 1.9729]	[1.9992, 1.9992]	[1.9925, 1.9925]	[2.0073, 2.0073]	[1.9742, 1.9742]	[1.9557, 1.9557]	[1.9726, 1.9726]
[1.9964, 1.9964]	[1.9614, 1.9614]	[1.9768, 1.9768]	[1.9991, 1.9991]	[1.9832, 1.9832]	[1.9847, 1.9847]	[1.9849, 1.9849]	[1.9918, 1.9918]
[1.9748, 1.9748]	[1.9664, 1.9664]	[2.0035, 2.0245]	[1.9822, 1.9822]	[1.9882,1.9999]	[1.9809, 1.9809]	[1.9920, 1.9920]	[1.9994,2.0512]

5. Concluding Remarks

In this paper, we originally proposed a variable sampling plan for the PCI under the neutrosophic logic. We presented the NPCI in the paper and used it to design the sampling plan. The proposed plan is the extension of the plan using classical statistics which can be applied where data is indeterminate or unclear. The plan parameters are presented for practical use in industry. A real example from industry is also added to show the application of the proposed sampling plan. The proposed plan is designed under the assumption that the data follow the neutrosophic normal distribution which can be tested using some statistical test or graphical depictions. For non-normal data, a suitable transformation can be applied to transfer non-normal data to normal data. From the comparison study, it is concluded that the proposed plan is more efficient than the plan based on classical statistics in terms of sample size. It is recommended to use the proposed plan in the industry where the data came from the complex situation or where there is a chance of some unclear data in the sampling. The proposed sampling plan using a double sampling scheme will be considered as a future research. The proposed plan using big data can be considered as future research.

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