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Research Article

Integrated Weighted Distance Measure for Single-Valued Neutrosophic Linguistic Sets and Its Application in Supplier Selection

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The purpose of this study is to propose an integrated distance-based methodology for multiple attribute group decision making (MAGDM) within single-valued neutrosophic linguistic (SVNL) environments. A new SVNL distance measure, namely the SVNL integrated weighted distance (SVNLIWD) measure, is first developed for achieving the aim. The remarkable feature of the SVNLIWD is that it integrates both merits of ordered weighting and average weighting into aggregating SVNL distances; therefore, it can account for both the importance of aggregated deviations as well as ordered positions. Thus, it can highlight the decision makers' subjective risk attitudes and combine the importance of objective decision information. Some distinctive characteristics and special forms of the presented distance framework are then specifically studied. Moreover, a MAGDM model on the basis of the proposed SVNLIWD form is formulated. Finally, an illustrative numerical case regarding selecting low-carbon supplier is used to test the performance of the designed method.

1. Introduction

With the increasing vagueness and uncertainties of objects in multiple attribute group decision making (MAGDM) problems, people may find it more and more difficult to express accurate evaluation on the attributes during decision process. Therefore, it has become a hot issue in decision making areas to research a scientific and reasonable tool for handling such vague and uncertain information. Linguistic term sets [1, 2], intuitionistic fuzzy sets (FSs) [3], hesitant FSs [4], single-valued neutrosophic sets (SVNSs) [5], Pythagorean FSs [6], and spherical FSs [7] emerge at a historic moment in recent years, which have been widely used to express uncertainties or vagueness in various complex decision making situations. The emergence of these methods greatly reduces the pressure of decision makers' depiction of the fuzziness of evaluation objects in the process of decision making.

Generally speaking, due to the complexity of people's judgement and the fuzziness of objective things, people tend

to use language terms instead of actual values or fuzzy values. However, the use of linguistic variables usually means that the truth degree of a linguistic term is 1, while the degrees of indeterminacy and falsity cannot be described. This defect hinders its application in decision making problems. To improve this limitation, a new powerful fuzzy tool introduced by Ye [8], called the single-valued neutrosophic linguistic set (SVNLS), has attracted growing concerns from worldwide authors. The key feature of the SVNLS is that it takes advantage of both the linguistic terms and SVNSs, and thus, it can successfully describe the uncertain information comprehensively and reasonably. In addition, it can eliminate the limitations of intuitionistic linguistic set [9] and the Pythagorean linguistic set [10] as it has three membership (i.e., truth, indeterminacy, and falsity) elements, which makes it more suitable to handle a higher degree of imprecise evaluations.

From the latest research trends, it can be seen that the SVNLS is widely used to deal with MAGDM problems in uncertain and complex environments. Guo and Sun [11]

gave a SVNL decision making using prospect theory. Zhao et al. [12] developed some induced Choquet integral weighted operators for SVNLS and explored their application in MAGDM. Ye [8] extended the classic TOPSIS to handle SVNL information and investigated its application in selecting investment context. Ye [13] introduced several neutrosophic linguistic aggregation methods and used them to select the flexible operating system supplier. Wang et al. [14] studied the usefulness of Maclaurin symmetric mean technique in aggregating SVNL preferences. Chen et al. [15] presented a new aggregated SVNL distance framework by utilizing the ordered weight technique. Based on the results obtained by Chen et al. [15], Cao et al. [16] introduced a combined SVNL distance measure. Kazimieras et al. [17] constructed a WASPAS model to solve SVNL MAGDM problems. Garg and Nancy [18] introduced some prioritized weighted methods to aggregate SVNL information with priority among the attributes.

In MAGDM problems, it is often necessary to measure the deviations between the alternatives and certain ideal schemes, wherein the construction of the distance measure plays a decisive role. Until now, the weighted distance (WD) and the ordered weighted averaging (OWAD) measures [19] are two most widely used tools for reflecting deviations in practical application. In general, the WD measure can account for the importance of the attributes, while the OWAD measure is helpful to highlight decision makers' risk attitude through the weight designing schemes in the aggregation process. At present, numerous OWAD's extensions and their corresponding usefulness in MAGDM problems have shown an increasing trend in recent research, such as the induced OWAD [20, 21], probabilistic OWAD [22], continuous OWAD [23], intuitionistic fuzzy OWAD [24], hesitant fuzzy OWAD [25, 26], intuitionistic fuzzy weighted induced OWAD [27], and Pythagorean OWAD measures [28, 29]. In particular, Chen et al. [15] defined the single-valued neutrosophic linguistic OWAD (SVNLOWAD) measure and explored its extension with the TOPSIS model for handling MAGDM with SVNL information.

Following the previous literature analysis, one can see that the SVNLS is regarded as a popularized tool, while the OWAD measure is of great strategic significance measurement tool and has shown its advantages in actual use. Therefore, it is a very interesting topic to study the theoretical development and application of OWAD framework in the SVNL context. For doing so, this paper tries to further explore the usefulness of the OWAD in solving SVNL decision making problems. To achieve this aim, we first develop a new distance measure for SVNLSs, named the SVNL integrated weighted distance (SVNLIWD) measure, which is a useful extension of the existing SVNLOWAD measure. Moreover, the SVNLIWD measure can overcome the defects of the SVNLOWAD measure as it unifies the superiority of the weighted distance and ordered weighted distance. Several properties and main families of the proposed distance measures are then explored. A MAGDM framework based on the SVNLIWD measure is constructed and its application is verified.

The remainder of this research is carried out as follows: Section 2 reviews some concepts of SVNLS and the OWAD measure. Section 3 proposes the SVNLIWD measure and explores some of its properties and families. Section 4 mainly describes the usefulness of the proposed SVNLIWD in MAGDM field. In Section 5, feasibility and effectiveness of the presented method are discussed through comparing with existing methods. Finally, Section 6 makes a systematic summary of this paper.

2. Preliminaries

Some important concepts concerning the definitions of the SVNLS, the OWAD, and the SVNLOWAD measures are briefly reviewed in this section.

2.1. Single-Valued Neutrosophic Set (SVNS). To improve the computational efficiency of the neutrosophic set [30], Ye [5] gave the definition of SVNS.

Definition 1 (see [5]). A single-valued neutrosophic set (SVNS) Z in finite set X is denoted by a mathematical form as follows:

$$Z = \{ \langle x, T_Z(x), I_Z(x), F_Z(x) \rangle \mid, \quad x \in X \}, \tag{1}$$

where $T_Z(x)$, $I_Z(x)$, and $F_Z(x)$, respectively, denote the truth, the indeterminacy, and the falsity-membership functions, and they must satisfy the following conditions:

$$0 \le T_Z(x), I_Z(x), F_Z(x) \le 1,
0 \le T_Z(x) + I_Z(x) + F_Z(x) \le 3.$$
(2)

The triplet $(T_Z(x), I_Z(x), F_Z(x))$ is named SVN number (SVNN) and simply described as $Z = (T_Z, I_Z, F_Z)$. Let $y = (T_y, I_y, F_y)$ and $z = (T_z, I_z, F_z)$ be two SVNNs; some mathematical operational rules are given as follows [30]:

(1)
$$y \oplus z = (T_y + T_z - T_y * T_z, I_y * T_z, F_y * F_z)$$

(2)
$$\lambda y = (1 - (1 - T_y)^{\lambda}, (I_y)^{\lambda}, (F_y)^{\lambda}), \lambda > 0$$

(3)
$$y^{\lambda} = ((T_y)^{\lambda}, 1 - (1 - I_y)^{\lambda}, 1 - (1 - F_y)^{\lambda}), \lambda > 0$$

2.2. Linguistic Set. A linguistic term set S is generally defined as a finitely ordered discrete set $S = \{s_{\alpha} \mid \alpha = 1, \dots, l\}$, where l is an odd number and s_{α} is a possible linguistic term. Let l = 7; then, S shall be specified $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\} = \{extremely poor, very poor, poor, fair, good, very good, extremely good\}$. Let s_i and s_j be two linguistic terms in S, and they should meet the following rules [1]:

(1)
$$s_i \le s_j \iff i \le j$$

(2)
$$\text{Neg}(s_i) = s_{-i}$$

In practical application, discrete set S shall be extended into a continuous set $\overline{S} = \{s_{\alpha} \mid \alpha \in R\}$ for minimizing information loss. In this case, for $s_{\alpha}, s_{\beta} \in \overline{S}$, they shall meet the following operational laws [31]:

- (1) $s_{\alpha} \oplus s_{\beta} = s_{\alpha+\beta}$
- (2) $\mu s_{\alpha} = s_{\mu\alpha}, \ \mu \ge 0$
- (3) $(s_{\alpha}/s_{\beta}) = s_{(\alpha/\beta)}$

2.3. Single-Valued Neutrosophic Linguistic Set (SVNLS)

Definition 2 (see [8]). The mathematical form of a SVNLS in X is described as in

$$P = \left\{ \langle x, \left[s_{\theta(x)}, \left(T_P(x), I_P(x), F_P(x) \right) \right] \rangle \, \middle| \,, \quad x \in X \right\}, \tag{3}$$

where $s_{\theta(x)} \in \overline{S}$, while $T_p(x)$, $I_p(x)$, and $F_p(x)$ have the following constraints:

$$0 \le T_{p}(x), I_{p}(x), F_{p}(x) \le 1,$$

$$0 \le T_{p}(x) + I_{p}(x) + F_{p}(x) \le 3.$$
(4)

For a SVNLS P in X, the SVNL number (SVNLN) $\langle s_{\theta(x)}, (T_P(x), I_P(x), F_P(x)) \rangle$ is simply formulated as $x = \langle s_{\theta(x)}, (T_x, I_x, F_x) \rangle$ for the convenience of application. Let $x_i = \langle s_{\theta(x_i)}, (T_{x_i}, I_{x_i}, F_{x_i}) \rangle$ (i = 1, 2) be two SVNLNs; then, the following are considered:

(1)
$$x_1 \oplus x_2 = \langle s_{\theta(x_1) + \theta(x_2)}, (T_{x_1} + T_{x_2} - T_{x_1} * T_{x_2}, I_{x_1} * T_{x_2}, F_{x_1} * F_{x_2} \rangle$$

(2)
$$\lambda x_1 = \langle s_{\lambda \theta(x_1)}, (1 - (1 - T_{x_1})^{\lambda}, (I_{x_1})^{\lambda}, (F_{x_1})^{\lambda}) \rangle, \lambda > 0$$

(3)
$$x_1^{\lambda} = \langle s_{\theta^{\lambda}(x_1)}, ((T_{x_1})^{\lambda}, 1 - (1 - I_{x_1})^{\lambda}, 1 - (1 - I_{x_1})^$$

Definition 3 (see [8]). Let $\lambda > 0$; then, the distance measure between SVNLNs $x_i = \langle s_{\theta(x_i)}, (T_{x_i}, I_{x_i}, F_{x_i}) \rangle$ (i = 1, 2) is defined as follows:

$$d_{\text{SVNL}}(x_1, x_2) = \left[\left| \theta(x_1) T_{x_1} - \theta(x_2) T_{x_2} \right|^{\lambda} + \left| \theta(x_1) I_{x_1} - \theta(x_2) I_{x_2} \right|^{\lambda} + \left| \theta(x_1) F_{x_1} - \theta(x_2) F_{x_2} \right|^{\lambda} \right]^{(1/\lambda)}. \tag{5}$$

On the basis of Definition 3, the SVNL weighted distance (SVNLWD) measure is formulated in equation (6) if we consider different importance for the individual deviation:

SVNLWD
$$((x_1, x_1'), ..., (x_n, x_n')) = \sum_{j=1}^{n} w_j d_{SVNL}(x_j, x_j'),$$
(6

where the relative weight vector W satisfies $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

2.4. OWAD Measure. The OWAD measure introduced by Merigó and Gil-Lafuente [19] is used to characterize individual distances on the basis of the ordered weighted averaging method [32]. Let $\eta = \{\eta_1, \eta_2, \dots, \eta_n\}$ and $\gamma = \{\gamma_1, \gamma_2, \dots, \gamma_n\}$ be two crisp sets and $d_i = |\eta_i - \gamma_i|$ be the distance between the crisp numbers η_i and γ_i ; then, we can define the OWAD measure as follows.

Definition 4 (see [19]). An OWAD measure with the weighting vector $W = \{w_j \mid \sum_{i=1}^n w_j = 1, \quad 0 \le w_j \le 1\}$ is defined as

$$OWAD(A, B) = OWAD(d_1, \dots, d_n) = \sum_{j=1}^{n} w_j d_{\sigma(j)}, \quad (7)$$

where $d_{\sigma(j)}$ (j = 1, ..., n) is the reorder values of d_j (j = 1, ..., n), such that $d_{\sigma(1)} \ge ... \ge d_{\sigma(n)}$.

The OWAD measure is generally effective for crisp sets. In order to adapt the OWAD measure to deal with SVNL information, Chen et al. [15] developed the SVNLOWAD measure.

Definition 5 (see [15]). Let $d_{SVNL}(x_j, x_j')$ be the deviation between two SVNLNs x_j, x_j' (j = 1, ..., n) defined in equation (5); then, SVNLOWAD measure is defined as

SVNLOWAD
$$((x_1, x_1'), \dots, (x_n, x_n')) = \sum_{j=1}^n w_j d_{SVNL}(x_{\sigma(j)}, x_{(j)}'),$$
(8)

where $d_{\text{SVNL}}(x_{\sigma(j)}, x_{\sigma(j)}')$ (j = 1, ..., n) is the reorder values of $d_{\text{SVNL}}(x_j, x_j')$ (j = 1, ..., n) such that $d_{\text{SVNL}}(x_{\sigma(1)}, x_{\sigma(1)}') \ge ... \ge d_{\text{SVNL}}(x_{\sigma(n)}, x_{\sigma(n)}')$. $w = (w_1, ..., w^n)^T$ is the associated weighting vector of the SVNLOWAD measure, satisfying $\sum_{j=1}^n w_j = 1$ and $w_i \in [0, 1]$.

Chen et al. [15] explored some characteristics of the SVNLOWAD measure, such as commutativity, boundedness, idempotency, and monotonicity. Moreover, they verified its desired performance in solving SVNL MAGDM problems by constructing a new TOPSIS model. However, the SVNLOWAD measure has some shortcomings; that is, it can only integrate the decision makers' special interests but fails to account for the weights of attributes in aggregation outcomes, which goes against its further application. So we shall present a new SVNL distance measure in the next section.

3. SVNL Integrated Weighted Distance (SVNLIWD) Measure

The SVNL integrated weighted distance (SVNLIWD) is a new extension of SVNL distance that unifies both the merits of the SVNLOWAD and the the SVNLWD measures. Therefore, it can highlight the decision makers' attitudes

through the ordered weighted arguments and combine the importance of attributes' weights in decision making. Moreover, it enables decision makers the chance to flexibly change the weight ratio of the SVNLWD and the SVNLOWAD according to the demands for the specific problem or actual preferences.

Definition 6. Let $d_{SVNL}(x_j, x_j')$ be the distance between two SVNLNs $x_i, x_i' (j = 1, ..., n)$ described as in equation (5); if

SVNLIWD
$$((x_1, x_1'), \dots, (x_n, x_n')) = \sum_{j=1}^n \overline{w}_j d_{\text{SVNL}}(x_{\sigma(j)}, x_{(j)}),$$

$$(9)$$

then the SVNLIWD is called the SVNL integrated weighted distance measure, where $d_{\text{SVNL}}(x_{\sigma(j)}, x_{\sigma(j)})$ $(j=1,\ldots,n)$ is the reorder values of $d_{\text{SVNL}}(x_j, x_j)$ $(j=1,\ldots,n)$ such that $d_{\text{SVNL}}(x_{\sigma(1)}, x_{\sigma(1)}) \geq \ldots \geq d_{\text{SVNL}}(x_{\sigma(n)}, x_{\sigma(n)})$. The integrated weight \overline{w}_j is determined by two weight values: one is the weight w_j for the OWA satisfying $\sum_{j=1}^n w_j = 1$ and $w_j \in [0,1]$, and the other is the weight w_j for weighted average with $\sum_{j=1}^n w_j = 1$ and $w_j \in [0,1]$. The unified weight \overline{w}_j $(j=1,\ldots,n)$ is defined as

$$\overline{w}_{j} = \frac{w_{j}^{\theta} \omega_{\sigma(j)}^{1-\theta}}{\sum_{j=1}^{n} \left(w_{j}^{\theta} \omega_{\sigma(j)}^{1-\theta}\right)},\tag{10}$$

with $\theta \in [0,1]$ and $\omega_{\sigma(j)}$ is the reordered element of the weight ω_j .

Following the Definition 6, one can see that the SVNLIWD is generalized to the SVNLWD and SVNLOWAD measures when $\theta = 0$ and $\theta = 1$, respectively. Thus, the SVNLIWD measure is a generalized model that unifies the SVNLWD, SVNLOWAD, and many other existing distance measures. A mathematical example is utilized to illustrate the computational process of the SVNLIWD measure.

Example 1. Let $X = (x_1, x_2, x_3, x_4, x_5) = (\langle s_3, (0.6, 0.3, 0.1) \rangle, \langle s_5, (0.2, 0.5, 0.5) \rangle, \langle s_6, (0.7, 0.1, 0.1) \rangle, \langle s_1, (0.6, 0.1, 0.6) \rangle, \langle s_4, (0.3, 0.1, 0.9) \rangle)$ and $X' = (x'_1, x'_2, x'_3, x'_4, x'_5) = (\langle s_5, (0.2, 0.9, 0) \rangle, \langle s_4, (0.5, 0.7, 0.2) \rangle, \langle s_5, (0.4, 0.4, 0.5) \rangle, \langle s_3, (0.5, 0.7, 0.2) \rangle, \langle s_3, (0.4, 0.2, 0.6) \rangle)$ be two SVNLSs defined in set $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$. The weighting vector of SVNLIWD measure is supposed to be $w = (0.3, 0.15, 0.25, 0.2, 0.1)^T$. Then, the computational process through the SVNLUWD can be performed as follows:

(1) Calculate distances $d_{\text{SVNL}}(x_i, x_i')$ (i = 1, 2, ..., 5) according to equation (5) (let $\lambda = 1$):

$$\begin{split} d_{\text{SVNL}}\left(x_{1}, x_{1}^{\prime}\right) &= |3 \times 0.6 - 5 \times 0.2| + |3 \times 0.3 - 5 \times 0.9| + |3 \times 0.1 - 5 \times 0| = 4.7, \\ d_{\text{SVNL}}\left(x_{2}, x_{2}^{\prime}\right) &= |5 \times 0.2 - 4 \times 0.5| + |5 \times 0.5 - 4 \times 0.7| + |5 \times 0.5 - 4 \times 0.2| = 3, \\ d_{\text{SVNL}}\left(x_{3}, x_{3}^{\prime}\right) &= |6 \times 0.7 - 5 \times 0.4| + |6 \times 0.1 - 5 \times 0.4| + |6 \times 0.1 - 5 \times 0.5| = 5.5, \\ d_{\text{SVNL}}\left(x_{4}, x_{4}^{\prime}\right) &= |1 \times 0.6 - 3 \times 0.5| + |1 \times 0.1 - 3 \times 0.7| + |1 \times 0.6 - 3 \times 0.2| = 2.9, \\ d_{\text{SVNL}}\left(x_{5}, x_{5}^{\prime}\right) &= |4 \times 0.3 - 3 \times 0.4| + |4 \times 0.1 - 3 \times 0.2| + |4 \times 0.9 - 3 \times 0.6| = 2. \end{split}$$

(2) Sort the $d(x_i, x_i')$ (i = 1, 2, ..., 5) in nonincreasing order:

$$d_{\text{SVNL}}(x_{\sigma(1)}, \dot{x'_{\sigma(1)}}) = d_{\text{SVNL}}(x_3, x'_3) = 5.5,$$

$$d_{\text{SVNL}}(x_{\sigma(2)}, \dot{x'_{\sigma(2)}}) = d_{\text{SVNL}}(x_1, x'_1) = 4.7,$$

$$d_{\text{SVNL}}(x_{\sigma(3)}, \dot{x'_{\sigma(3)}}) = d_{\text{SVNL}}(x_2, x'_2) = 3,$$

$$d_{\text{SVNL}}(x_{\sigma(4)}, \dot{x'_{\sigma(4)}}) = d_{\text{SVNL}}(x_4, x'_4) = 2.9,$$

$$d_{\text{SVNL}}(x_{\sigma(5)}, \dot{x'_{\sigma(5)}}) = d_{\text{SVNL}}(x_5, x'_5) = 2.$$
(12)

(3) Let $\omega = (0.15, 0.2, 0.1, 0.35, 0.2)^T$ and $\theta = 0.5$; compute the integrated weights \overline{w}_j according to equation (10):

$$\overline{w}_{1} = \frac{w_{1}^{0.5} \omega_{\sigma(1)}^{1-0.5}}{\sum_{i=1}^{n} \left(w_{j}^{0.5} \omega_{\sigma(j)}^{1-0.5}\right)} = \frac{0.3^{0.5} \times 0.1^{0.5}}{\left(0.3^{0.5} \times 0.1^{0.5} + 0.15^{0.5} \times 0.15^{0.5} + 0.25^{0.5} \times 0.2^{0.5} + 0.2^{0.5} \times 0.35^{0.5} + 0.1^{0.5} \times 0.2^{0.5}\right)} = 0.1791.$$
(13)

Similarly, we can obtain

$$\overline{w}_2 = 0.1901,$$
 $\overline{w}_3 = 0.2183,$
 $\widehat{w}_4 = 0.2366,$
 $\overline{w}_5 = 0.1757.$
(14)

(4) Utilize the SVNLIWD given in equation (9) to compute the distance measure between X and X':

SVNLIWD
$$(X, X') = 0.1791 \times 5.5 + 0.1901 \times 4.7 + 0.2183 \times 3 + 0.2366 \times 2.9 + 0.1757 \times 2 = 3.5719.$$
 (15)

If we use the SVNLOWAD and the SVNLWD to perform the aggregation process, we have

SVNLOWAD
$$(X, X') = 0.3 \times 5.5 + 0.15 \times 4.7 + 0.25 \times 3$$

 $+ 0.2 \times 2.9 + 0.1 \times 2 = 3.885,$
SVNLWD $(X, X') = 0.15 \times 4.7 + 0.2 \times 3 + 0.1 \times 5.5$
 $+ 0.35 \times 2.9 + 0.2 \times 2 = 3.27.$ (16)

Apparently, we obtain different results from three methods. In fact, the SVNLWD model only considers the importance of the individual deviations, while the SVNLOWAD focuses on the weights of the ordered deviations. The SVNLIWD measure unifies the features of both the SVNLOWAD and the SVNLWD measures, and thus, it can not only highlight the ordered weights of positions but also incorporate deviations' importance.

Moreover, some particular SVNL weighted distance measures can be obtained if we sign different weighted schemes for the SVNLIWD measure:

- (i) If $w_1 = 1$ and $w_j = 0$ for $j \in [2, n]$, then we obtain the max-SVNLIWD measure
- (ii) If $w_n = 1$ and $w_j = 0$ for $j \in [1, n-1]$, then the min-SVNLIWD measure is constructed

- (iii) The step-SVNLIWD measure is formed by signing $w_1 = \ldots = w_{k-1} = 0$, $w_k = 1$, and $w_{k+1} = \ldots = w_n = 0$
- (iv) Other special cases of the SVNLIWD can be created by using the similar methods provided in references [15, 33–36]

The SVNLIWD measure is monotonic, bounded, idempotent, and commutative, which can be demonstrated by following theorems.

Theorem 1 (monotonicity). If $d_{SVNL}(y_i, y_i') \ge d_{SVNL}(x_i, x_i')$ for all i, then the following feature holds:

SVNLIWD
$$((y_1, y_1'), ..., (y_n, y_n')) \ge$$
 SVNLIWD $((x_1, x_1'), ..., (x_n, x_n')).$
(17)

Theorem 2 (boundedness). Let $d_{\min} = \min_{i} \{d_{SVNL}(x_i, x_i')\}$ and $d_{\max} = \max_{i} \{d_{SVNL}(x_i, x_i')\}$; then,

$$d_{\min} \le \text{SVNLIWD}\left(\left(x_1, x_1'\right), \dots, \left(x_n, x_n'\right)\right) \le d_{\max}. \tag{18}$$

Theorem 3 (idempotency). If $d_{SVNL}(x_i, x_i') = D$ for all i, then

SVNLIWD
$$((x_1, x_1'), \dots, (x_n, x_n')) = D.$$
 (19)

Theorem 4 (commutativity). This property can also be rendered from the following equation:

SVNLIWD
$$((x_1, x_1'), ..., (x_n, x_n')) = \text{SVNLIWD}((x_1', x_1), ..., (x_n', x_n)).$$
(20)

It is noted that the proof of these theorems are omitted as they are straightforward.

In addition, we can utilize the generalized mean method [37] to achieve a more generalization for SVNL distance measure, obtaining the SVNL generalized integrated weighted distance (SVNLGIWD) measure:

SVNLGIWD
$$((x_1, x_1'), ..., (x_n, x_n')) = \left\{ \sum_{j=1}^n \overline{w}_j (d_{SVNL}(x_{\sigma(j)}, x_{(j)}))^p \right\}^{(1/p)},$$
 (21)

where p is a parameter that meets $p \in (-\infty, +\infty) - \{0\}$. Several representative cases of the SVNLGIWD measure can be determined based on the variation of parameter p; for example, the SVNLIWD is formed when p = 1, the SVNL integrated weighted quadratic distance (SVNLIWQD) is obtained if p = 2, and the SVNL integrated weighted harmonic distance (SVNLIWHD) is rendered if p = -1. Many other cases of the SVNLGIWD measure can be analyzed by using the similar method provided in references [37–43].

4. Application of SVNLIWD in MAGDM Problems

As a more representative distance measurement method, the SVNLIWD can be broadly used in different areas, such as social management, pattern recognition, decision making, data analysis, medical diagnosis, and financial investment. Subsequently, an application of the SVNLIWD measure in MAGDM is presented within SVNL environments. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of finite attributes and $B = \{B_1, B_2, \ldots, B_m\}$ be the set of schemes; then, the decision procedure is summarized as follows.

Step 1. Each expert e_t (t = 1, 2, ..., k) (the weight is δ_t with $\delta_t \ge 0$ and $\sum_{t=1}^k \delta_t = 1$) expresses his or her evaluation on each attribute of the assessed objects in the form of SVNLNs, thus forming the individual SVNL decision matrix $X^t = (x_{ij}^{(t)})_{m \times n}$.

Step 2. Apply the SVNL weighted average (SVNLWA) operator [8] to aggregate all individual evaluations into a group decision matrix:

$$X = \left(x_{ij}\right)_{m \times n} = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix}, \tag{22}$$

where the SVNLN $x_{ij} = \sum_{t=1}^{k} \delta_t x_{ij}^{(t)}$.

Step 3. Determine the ideal gradation of each attribute to construct the ideal solution shown in Table 1.

Step 4. Calculate the deviations between the alternative B_i (i = 1, 2, ..., m) and the ideal alternative I by utilizing the SVNLIWD measure.

Step 5. Rank all alternatives and select the best one(s) according to the distances rendered from the previous step.

Step 6. End.

5. Application in Low-Carbon Supplier Selection

The green and low-carbon economic development mode has received more and more attention from the governments and enterprises all over the world. Choosing a suitable low-carbon supplier has become an important issue for the development of enterprises. As a result, many supplier selection methods have been proposed in the existing literature [44, 45]. In this section, a mathematical case of selecting a low-carbon supplier introduced by Chen et al. [15] is used to verify the usefulness of the proposed method. A company invites three experts to evaluate four potential low-carbon suppliers B_i (i = 1, 2, 3, 4) from the following aspects: low-carbon technology (A_1), cost (A_2), risk factor (A_3), and capacity (A_4). The SVNL decision matrices expressed by the experts regarding these four attributes within set $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ are given in Tables 2–4.

The weights of the experts are supposed to be $\delta_1 = 0.30$, $\delta_2 = 0.37$, and $\delta_3 = 0.33$, respectively. The group SVNL decision matrix is then formed by aggregating the three individual opinions, which are listed in Table 5.

According to the actual performance level of these alternative companies, the experts determine the ideal scheme listed in Table 6.

Let the weighting vectors of the SVNLIWD measure and the attributes be $w = (0.15, 0.3, 0.3, 0.25)^T$ and $\omega = (0.2, 0.3, 0.3, 0.2)^T$, respectively. According to the available information, let the parameter $\theta = 0.5$; then, the SVNLIWD can be used to compute the deviations between the alternative B_i (i = 1, 2, 3, 4) and the ideal supplier I:

SVNLIWD
$$(I, B_1) = 5.0563$$
,
SVNLIWD $(I, B_2) = 5.7334$,
SVNLIWD $(I, B_3) = 6.5700$,
SVNLIWD $(I, B_4) = 6.5798$. (23)

The smaller the value of SVNLIWD (I, B_i) is, the closer the alternative B_i is to the ideal scheme and the better scheme B_i is. Thus, the ranking of all alternatives yields

$$B_1 > B_2 > B_3 > B_4. \tag{24}$$

The results show that B_1 is the most desirable alternative as it possesses the smallest distance from the ideal scheme.

To more effectively show the superiority of the SVNLIWD measure, we also utilize the SVNLOWAD and the SVNLWD measures to calculate the subsequent distances of each alternative to the ideal supplier. For the SVNLOWAD measure, we have

	A_1	A_2	•••	A_n
I	${ ilde y}_1$	\widetilde{y}_2	•••	$\widetilde{\mathcal{Y}}_n$

Table 2: SVNL decision matrix X^1 .

	A_1	A_2	A_3	A_4
B_1	$\langle s_5^{(3)}, (0.7, 0.2, 0.1) \rangle$	$\langle s_4^{(3)}, (0.5, 0.2, 0.2) \rangle$	$\langle s_3^{(3)}, (0.4, 0.1, 0.1) \rangle$	$\langle s_4^{(3)}, (0.6, 0.1, 0.2) \rangle$
B_2	$\langle s_6^{(3)}, (0.4, 0.6, 0.2) \rangle$	$\langle s_4^{(3)}, (0.7, 0.2, 0.2) \rangle$	$\langle s_5^{(3)}, (0.7, 0.2, 0.1) \rangle$	$\langle s_5^{(3)}, (0.5, 0.2, 0.3) \rangle$
B_3	$\langle s_4^{(3)}, (0.3, 0.6, 0.2) \rangle$	$\langle s_5^{(3)}, (0.6, 0.1, 0.3) \rangle$	$\langle s_4^{(3)}, (0.6, 0.2, 0.1) \rangle$	$\langle s_6^{(3)}, (0.5, 0.1, 0.3) \rangle$
B_4	$\langle s_4^{(3)}, (0.5, 0.2, 0.3) \rangle$	$\langle s_6^{(3)}, (0.6, 0.2, 0.4) \rangle$	$\langle s_5^{(3)}, (0.2, 0.1, 0.6) \rangle$	$\langle s_6^{(3)}, (0.5, 0.2, 0.3) \rangle$

TABLE 3: SVNL decision matrix X^2 .

	A_1	A_2	A_3	A_4
B_1	$\langle s_6^{(1)}, (0.6, 0.1, 0.2) \rangle$	$\langle s_4^{(1)}, (0.6, 0.1, 0.2) \rangle$	$\langle s_3^{(1)}, (0.3, 0.1, 0.2) \rangle$	$\langle s_5^{(1)}, (0.7, 0.0, 0.1) \rangle$
B_2	$\langle s_3^{(1)}, (0.6, 0.2, 0.4) \rangle$	$\langle s_5^{(1)}, (0.6, 0.1, 0.2) \rangle$	$\langle s_4^{(1)}, (0.5, 0.2, 0.2) \rangle$	$\langle s_6^{(1)}, (0.6, 0.1, 0.2) \rangle$
B_3	$\langle s_5^{(1)}, (0.3, 0.5, 0.2) \rangle$	$\langle s_4^{(1)}, (0.5, 0.2, 0.3) \rangle$	$\langle s_3^{(1)}, (0.5, 0.3, 0.1) \rangle$	$\langle s_4^{(1)}, (0.3, 0.2, 0.3) \rangle$
B_4	$\langle s_4^{(1)}, (0.5, 0.3, 0.3) \rangle$	$\langle s_5^{(1)}, (0.4, 0.2, 0.3) \rangle$	$\langle s_3^{(1)}, (0.3, 0.2, 0.5) \rangle$	$\langle s_5^{(1)}, (0.4, 0.2, 0.3) \rangle$

TABLE 4: SVNL decision matrix X^3 .

	A_1	A_2	A_3	${ m A}_4$
B_1	$\langle s_6^{(2)}, (0.6, 0.3, 0.3) \rangle$	$\langle s_5^{(2)}, (0.7, 0.2, 0.3) \rangle$	$\langle s_4^{(2)}, (0.4, 0.2, 0.2) \rangle$	$\langle s_4^{(2)}, (0.8, 0.1, 0.2) \rangle$
B_2	$\langle s_4^{(2)}, (0.5, 0.4, 0.2) \rangle$	$\langle s_6^{(2)}, (0.7, 0.2, 0.3) \rangle$	$\langle s_5^{(2)}, (0.6, 0.2, 0.2) \rangle$	$\langle s_6^{(2)}, (0.7, 0.2, 0.3) \rangle$
B_3	$\langle s_5^{(2)}, (0.4, 0.4, 0.1) \rangle$	$\langle s_6^{(2)}, (0.6, 0.3, 0.4) \rangle$	$\langle s_4^{(2)}, (0.6, 0.1, 0.3) \rangle$	$\langle s_6^{(2)}, (0.4, 0.2, 0.4) \rangle$
B_4	$\langle s_3^{(2)}, (0.7, 0.1, 0.1) \rangle$	$\langle s_6^{(2)}, (0.5, 0.1, 0.2) \rangle$	$\langle s_5^{(2)}, (0.3, 0.1, 0.6) \rangle$	$\langle s_5^{(2)}, (0.4, 0.3, 0.4) \rangle$

Table 5: Group SVNL decision matrix R.

	A_1	A_2	A_3	A_4
$\overline{C_1}$	$\langle s_{5.70}, (0.633, 0.180, 0.186) \rangle$	$\langle s_{4.33}, (0.611, 0.155, 0.229) \rangle$	$\langle s_{3.67}, (0.365, 0.128, 0.163) \rangle$	$\langle s_{4.37}, (0.714, 0.000, 0.155) \rangle$
C_2	$\langle s_{4.23}, (0.514, 0.350, 0.258) \rangle$	$\langle s_{4.70}, (0.666, 0.155, 0.229) \rangle$	$\langle s_{2.37}, (0.602, 0.200, 0.162) \rangle$	$\langle s_{5.70}, (0.611, 0.155, 0.258) \rangle$
C_3	$\langle s_{4.70}, (0.335, 0.491, 0.159) \rangle$	$\langle s_{4.96}, (0.566, 0.186, 0.330) \rangle$	$\langle s_{3.37}, (0.566, 0.185, 0.144) \rangle$	$\langle s_{5.26}, (0.399, 0.163, 0.330) \rangle$
C_4	$\langle s_{3.67}, (0.578, 0.185, 0.209) \rangle$	$\langle s_{5.63}, (0.450, 0.159, 0.286) \rangle$	$\langle s_{2.37}, (0.271, 0.129, 0.561) \rangle$	$\langle s_{5.30}, (0.432, 0.229, 0.330) \rangle$

TABLE 6: Ideal scheme.

	A_1	A_2	A_3	A_4
I	$\langle s_7, (0.9, 0.1, 0) \rangle$	$\langle s_7, (1,0,0.1) \rangle$	$\langle s_7, (0.9, 0, 0.1) \rangle$	$\langle s_6, (0.9, 0, 0) \rangle$

$$SVNLOWAD(I, B_1) = 5.0171, \qquad And for the SVNLWD measure, we have \\ SVNLOWAD(I, B_2) = 5.6742, \\ SVNLOWAD(I, B_3) = 6.5613, \\ SVNLWD(I, B_2) = 5.8078, \\ SVNLWD(I, B_3) = 6.6038, \\ SVNLWD(I, B_4) = 6.5743. \end{aligned}$$
 (26)

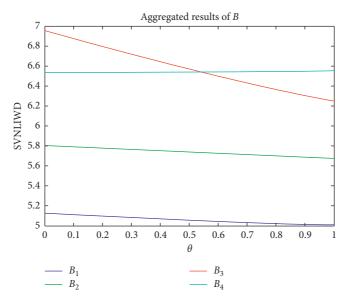


FIGURE 1: The performance of alternatives rendered by the SVNLIWD measure with θ .

From the results, one can find that B_1 is the best choice for both the SVNLOWAD and SVNLWD measures, which is consistent with the result obtained by the SVNLIWD measure. However, from the comparisons with the SVNLWD and SVNLOWAD measures discussed in the previous example, one can see that the SVNLIWD measure can not only overcome the SVNLWD's disadvantage of just considering the importance of attributes but also make up for the SVNLOWAD's defects of only reflecting expert's risk preference but fails to integrate attributes' weights; therefore, it can yield a more reasonable result. Furthermore, the SVNLIWD-based MAGDM method will not be affected by the parameter θ change, which can be verified by Figure 1.

Following the results from Figure 1, the best alternative is A_1 for all $\theta \in [0,1]$. It shows that the variation of parameter θ will not affect the final integration results; that is, the MAGDM approach based on the SVNLIWD will not be affected by the parameter variation. Thus, the proposed method has certain stability and robustness.

6. Conclusions

This paper introduces a new integrated aggregation distance method for handling single-valued neutrosophic linguistic MAGDM problems. Thus, we obtain the SVNL integrated weighted (SVNLIWD) measure. Given that the presented distance measure generalizes both advantages of the arithmetic weight and ordered weight approaches during aggregating process, the importance for separate attributes and attitudes towards ordered deviations is taken into account. Moreover, the SVNLIWD measure generalizes a wide type of SVNL distance measures, such as the SVNLWD and the SVNLOWAD measures. Therefore, it provides a much wider model to solve complex situations in a more efficient and flexible way, which further illustrates the promotion of the previous methods. The application of the proposed model is taken to deal with the supplier selection problem, which

demonstrates that the presented methodology can consider capricious decision makers' preferences as well as the different importance of attributes during the decision process. Finally, we verify that the presented SVNLIWD-based MAGDM method will not be affected by the parameter variation. Therefore, this method has certain stability and robustness and can achieve more accurate results.

In future work, both extensions of mathematical formula and application in different areas will be considered. Various variables can be considered in the SVNLIWD for future analysis, such as the induced variables, heavy aggregation, and q-rung orthopair fuzzy set [46]. Also, the method of entropy will be considered to account for the weighting schemes.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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