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Interval Neutrosophic Reducible Weighted Maclaurin Symmetric Means With Internet of Medical Things (IoMt) Industry Evaluation

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ABSTRACT The Internet of Medical Things (IoMT) is a global infrastructure composing of plentiful applications and medical devices that are interconnected by ICT. In considering the problem of the IoMT industry evaluation, the requisite issue that concerns strong interaction and incertitude. The Maclaurin symmetric mean (MSM), as a resultful information concordant instrument, can capture the interrelation among multiple arguments more efficiently. The abundance of the weighted MSMs has been presented to manage the different uncertain information aggregation issues by reason that the attribute variables are frequently diverse. However, these existing weighted form of MSM operators fail to possess the fundamental properties of idempotency and reducibility. To solve the above issues, we explore the interval neutrosophic reducible weighted MSM (INRWMSM) operator and the interval neutrosophic reducible weighted dual MSM (INRWDMSM) operator. Moreover, momentous properties and some special cases of the INRWMSM and INRWDMSM operators are discussed in detail. Whereafter, we propose some multiple attribute decision making (MADM) algorithms based on INRWMSM and INRWDMSM. The availability of proposed algorithms is stated by an IoMT evaluation issue. Finally, a comparison of the developed with the existing interval neutrosophic decision making algorithms has been formed for showing their validity.

INDEX TERMS IoMT, interval neutrosophic set, aggregation operator, idempotency, reducible weighted MSM.

I. INTRODUCTION

Internet of Medical Things (IoMT), also named the Internet of Health Things (IoHT) [1], is an application scenario of the Internet of Things (IoT) for medical or health interrelated objectives, data collection and analysis for researching, and monitoring. This "Smart Healthcare" [2], as it can also be called, give rise to the creation of a digitized healthcare system, connecting doable medical resources and healthcare services.

Up to now, IoMT has already been not only applied in the healthcare and health insurance industries [3]–[9], but also in the clinical laboratory industry [10]. The healthcare industry of IoMT is now allowing patients, doctors and others involved (i.e. guardians of patients, nurses, families, etc.) to be part of a system, where patient records are preserved in a medical database, permitting doctors and the rest of the medical staff to have access to the patient's information. Furthermore, IoT-based systems are patient-centered, which involves being flexible to the patient's medical conditions. IoMT in the insurance industry provides access to better and new types of dynamic information [2]. The stock god Warren Buffett once said, IoMT is the future of technology which will yield unusually brilliant results. The first rate or below hospitals want to seek their own quick development in the China's hospital ranking. However, if they want to possess a higher hospital ranking, it is insufficient to rely on themselves alone. Therefore, they should better seek some excellent IoMT companies for collaborating and remoulding which can accelerate their digitization, intelligentization and integration.

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Consequently, the novel thought that regards the process to pick the some ideal IoMT companies to collaborate is an interesting topic which can classify into multiple attribute decision making (MADM) issues. However, the increasingly complicated decision making atmosphere and vacillating decision makers (DMs) have difficulty in signifying decision information with indeterminate numbers.

Interval neutrosophic set (INS), initiatively conceived by Wang et al. [11], has perceived as a more underlying measure for describing indeterminate information, which is a generalization of materialization neutrosophic set (NS) [12] (called single valued neutrosophic set (SVNS) [13]). Up to now, INS has achieved an incredible success [14]. Peng and Dai [15] initiatively investigated some decision making algorithms based on MABAC, similarity measure and EDAS and applied them in selection of investment companies and C Programming Language teachers. Broumi and Smarandache [16] developed the correlation coefficient for INS with their breakthrough proof. Broumi et al. [17] employed the shortest path problem in INS. Yang *et al.* [18] conceived the linear assignment method for solving the interval neutrosophic MADM problem. Karaş an and Kahraman [19] has triumphantly solved the prioritization of the UN national sustainable development goals. based on EDAS method using the interval neutrosophic data. Bolturk and Kahraman [20] explored the interval neutrosophic AHP method with cosine similarity measure for energy alternative selection. Wang *et al.* [21] introduced the notion of probability into stochastic interval neutrosophic MADM problem in virtue of regret theory.

The most popular tool to deal MADM problems is aggregation operators, which aggregate all the presumptive individual arguments into a holistic argument. Nevertheless, the existing interval neutrosophic aggregation operators [22]–[33] only seize the pertinence between a fixed number of parameters. In order to facilitate the flexibility of information integration, Maclaurin [34] developed the Maclaurin symmetric mean (MSM), which can trap the relatedness among any number of arguments. Qin and Liu [35] originally united the MSM with the intuitionistic fuzzy circumstance in vague domain and explored the weighted intuitionistic fuzzy MSM (WIFMSM) for integrating the experts' assessment information. At present, there are plentiful generalizations of MSM operators employing in different indeterminate circumstance [36]–[42].

By exploring the existing WMSM operators in different indeterminate environment, we can find certain unconscionable problems as follows: (i) If the attribute values of all arguments are equal, the diverse uncertain environment of WMSM operators [36]–[42] cannot reduce to the corresponding MSM operators, which is a fundamental feature of the traditional weighted operators. (ii) The existing WMSM operators [36]–[42] do not have the properties of idempotency. That is to say, it is illogical that the weighted average value of some uniform aggregated arguments rely on the weight values. Inspired by reducible weighted MSM operator and reducible weighted dual MSM (RWDMSM) operator [47], we unite them with INS to aggregate interval neutrosophic values and deal interval neutrosophic MADM problems by thinking about the virtues of both.

The above discussion elicitates the major contributions in the following.

(1) Develop two fire-new aggregation operators (interval neutrosophic RWMSM (INRWMSM) and interval neutrosophic RWDMSM (INRWDMSM)) for coalescing the decision information;

(2) Propose certain algorithms based on presented aggregate operators for dealing the division by zero issue [48]–[50] and unauthentic problem [51];

(3) Explore the sensitivity analysis of different parameter values on the conclusive ranking;

(4) Give an example for showing the availability of presented methods.

The rest of the paper is listed as follows: In Section 2, we concisely review basic notions of INS, and the definitions of MSM, DMSM, RWMSM and RWDMSM operator. In Section 3, we develop the interval neutrosophic Maclaurin symmetric means operators such as INRWMSM and INRWDMSM operator. In Section 4, we present two MADM methods based on the proposed INRWMSM operator and INRWDMSM operator with INNs. In addition, an example to state the effectiveness is presented with discussing the effect of the different parameter values on final ordering. In Section 5, a comparison with some existing methods of different interval neutrosophic aggregation operators and the characteristic comparisons of different indeterminate circumstance are discussed in detailed. Finally, Section 6 achieves the whole-length concluding.

II. PRELIMINARIES

A. INTERVAL NEUTROSOPHIC SET

Definition 1 [11]: Let X be domain of discourse, with a series of elements in X denoted by x. An INS A in X is summarized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then an INS A can be denoted as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) > \mid x \in X\}. \tag{1}$$

For each point x in X, $T_A(x) = [T_A^L(x), T_A^U(x)], I_A(x) = [I_A^L(x), I_A^U(x)], F_A(x) = [F_A^L(x), F_A^U(x)] \subseteq [0, 1]$, and $0 \leq T_A^U(x) + I_A^U(x) + F_A^U(x) \leq 3$. For convenience, Peng and Dai [15] can simply use $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ to represent an INN as an element in the INS A.

Definition 2 [11]: An INS N is contained in other INS M, $N \subseteq M$ if and only if $T_N^L(x) \leq T_M^L(x), T_N^U(x) \leq T_M^U(x), I_N^L(x) \geq I_M^L(x), I_N^U(x) \geq I_M^U(x), F_N^L(x) \geq F_M^L(x), F_N^U(x) \geq F_M^U(x)$ for $\forall x$.

Definition 3 [22]: Let $x = ([T_x^L, T_x^U], [I_x^L, I_x^U], [F_x^L, F_x^U])$ and $y = ([T_y^L, T_y^U], [I_y^L, I_y^U], [F_y^L, F_y^U])$ be two INNs, and $\lambda > 0$, then the operations for the INNs are defined as follows:

(1) $\lambda x = ([1-(1-T_x^L)^\lambda, 1-(1-T_x^U)^\lambda], [(I_x^L)^\lambda, (I_x^U)^\lambda], [(F_x^L)^\lambda, (F_x^U)^\lambda]);$

(2) $x^\lambda = ([(T_x^L)^\lambda, (T_x^U)^\lambda], [1-(1-I_x^L)^\lambda, 1-(1-I_x^U)^\lambda], [1-(1-F_x^L)^\lambda, 1-(1-F_x^U)^\lambda]);$

(3) $x \bigoplus y = ([T_x^L + T_y^L - T_x^L T_y^L, T_x^U + T_y^U - T_x^U T_y^U], [I_x^L * I_y^L, I_x^U * I_y^U], [F_x^L * F_y^L, F_x^U * F_y^U]);$

(4) $x \bigotimes y = ([T_x^L * T_y^L, T_x^U * T_y^U], [I_x^L + I_y^L - I_x^L I_y^L, I_x^U + I_y^U - I_x^U I_y^U], [F_x^L + F_y^L - F_x^L F_y^L, F_x^U + F_y^U - F_x^U F_y^U]);$

(5) $x^c = ([F_x^L, F_x^U], [1-I_x^U, 1-I_x^L], [T_x^L, T_x^U]).$

Definition 4 [15]: Let $x = ([T_x^L, T_x^U], [I_x^L, I_x^U], [F_x^L, F_x^U])$ be an INN, then the proposed score function $s(x)$ is defined as follows:

$$s(x) = \frac{2}{3} + \frac{T_x^L + T_x^U}{6} - \frac{I_x^L + I_x^U}{6} - \frac{F_x^L + F_x^U}{6}. \quad (2)$$

B. REDUCIBLE WEIGHTED MACLAURIN SYMMETRIC MEANS

The Maclaurin symmetric mean (MSM), initially developed by Maclaurin [34], can trap the relevancy among multiple arguments more efficaciously. Up to now, the MSM is used in integrating indeterminate information during the process of decision making.

Definition 5 [34]: Let $x_i (i = 1, 2, \cdots, n)$ be an amount of nonnegative real numbers, and $k = 1, 2, \cdots, n$, then the Maclaurin symmetric mean (MSM) operator is denoted in the following.

$$\mathrm{MSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} x_{i_j}}{C_n^k} \right)^{1/k}, \quad (3)$$

where $(i_1, i_2, \cdots, i_k)$ traverses all the $k-$ permutations of $(1, 2, \cdots, n)$, and the C_n^k is the binomial coefficient meeting the formula: $C_n^k = \frac{n!}{k!(n-k)!}$.

Definition 6 [36]: Let $x_i (i = 1, 2, \cdots, n)$ be an amount of nonnegative real numbers, and $k = 1, 2, \cdots, n$, then the dual Maclaurin symmetric mean (DMSM) operator is denoted as follows:

$$\mathrm{DMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} x_{i_j}}{C_n^k} \right)^{1/k}, \quad (4)$$

where $(i_1, i_2, \cdots, i_k)$ traverses all the $k-$ permutations of $(1, 2, \cdots, n)$, and the C_n^k is the binomial coefficient satisfying following formula: $C_n^k = \frac{n!}{k!(n-k)!}$.

For the sake of solving the problems of idempotency and reducibility in some existing MSM operators, Shi and Xiao [47] presented the reducible weighted MSM (RWMSM) and the reducible weighted dual MSM (RWDMSM) as follows:

Definition 7 [47]: Let $x_i (i = 1, 2, \cdots, n)$ be an amount of nonnegative real numbers, $k = 1, 2, \cdots, n$, and $W = (w_1, w_2, \cdots, w_n)^T$ with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$, then the reducible weighted MSM (RWMSM) operator is denoted as follows:

$$\mathrm{RWMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}} \right)^{1/k}. \quad (5)$$

Definition 8 [47]: Let $x_i (i = 1, 2, \cdots, n)$ be an amount of nonnegative real numbers, $k = 1, 2, \cdots, n$, and $W = (w_1, w_2, \cdots, w_n)^T$ with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$, then the reducible weighted DMSM (RWDMSM) operator is denoted as follows:

$$\mathrm{RWDMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \frac{\prod_{1 \le i_1 < \cdots < i_k \le n} \left(\sum_{j=1}^{k} x_{i_j} \right)^{\frac{\sum_{j=1}^{k} w_{i_j}}{\sum_{1 \le i_1 < \cdots < i_k \le n} \sum_{j=1}^{k} w_{i_j}}}}{k}. \quad (6)$$

III. INTERVAL NEUTROSOPHIC REDUCIBLE WEIGHTED MACLAURIN SYMMETRIC MEANS

In this section, according to the operational rules of INNs with the RWMSM and RWDMSM operators, we propose interval neutrosophic RWMSM (INRWMSM) operator and interval neutrosophic RWDMSM (INRWDMSM) operator. In addition, some interesting properties and certain special cases of proposed aggregation operators are discussed in detailed.

A. INTERVAL NEUTROSOPHIC REDUCIBLE WEIGHTED MSM OPERATOR

Definition 9: Let $x_i = ([T_{x_i}^L, T_{x_i}^U], [I_{x_i}^L, I_{x_i}^U], [F_{x_i}^L, F_{x_i}^U]) (i = 1, 2, \cdots, n)$ be a sets of INNs, and let $W = (w_1, w_2, \cdots, w_n)^T$ be a weight vector with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. The INRWMSM: $\Omega^m \to \Omega$, an INRWMSM operator is given as follows:

$$\mathrm{INRWMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}} \right)^{1/k} \quad (7)$$

where Ω is the set of all INNs, then INRWMSM is called the interval neutrosophic reducible weighted MSM operator.

Based on the operational laws of the INNs described in Definition 3, from Eq. (7), we can have the aggregated result shown in Theorem 1.

$$
\begin{aligned}
&\mathrm{INRWMSM}^{(k)}(x_1, x_2, \cdots, x_n)\\
&= \left(\left[\left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T^L_{x_{i_j}} \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k}, \right.\right.\\
&\quad \left. \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T^U_{x_{i_j}} \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k} \right],\\
&\quad \left[1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} (1 - I^L_{x_{i_j}}) \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k}, \right.\\
&\quad \left. 1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} (1 - I^U_{x_{i_j}}) \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k} \right],\\
&\quad \left[1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} (1 - F^L_{x_{i_j}}) \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k}, \right.\\
&\quad \left.\left. 1 - \left(1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} (1 - F^U_{x_{i_j}}) \right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right)^{1/k} \right] \right) \qquad (8)
\end{aligned}
$$

Theorem 1: Let $x_i = ([T^L_{x_i}, T^U_{x_i}], [I^L_{x_i}, I^U_{x_i}], [F^L_{x_i}, F^U_{x_i}])(i = 1, 2, \cdots, n)$ be a sets of INNs, and let $W = (w_1, w_2, \cdots, w_n)^T$ be a weight vector with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, then the final result of INRWMSM operator is an INN.

Theorem 2 (Monotonicity): Let $x_i (i = 1, 2, \cdots, n)$ and $x'_i (i = 1, 2, \cdots, n)$ be two series of INNs, if $x'_i = ([T^L_{x'_i}, T^U_{x'_i}], [I^L_{x'_i}, I^U_{x'_i}], [F^L_{x'_i}, F^U_{x'_i}])$, $x_i = ([T^L_{x_i}, T^U_{x_i}], [I^L_{x_i}, I^U_{x_i}], [F^L_{x_i}, F^U_{x_i}])$, $T^L_{x_i} \ge T^L_{x'_i}, T^U_{x_i} \ge T^U_{x'_i}, I^L_{x_i} \le I^L_{x'_i}, I^U_{x_i} \le I^U_{x'_i}, F^L_{x_i} \le F^L_{x'_i}, I^U_{x_i} \le F^U_{x'_i}$ for all $i = 1, 2, \cdots, n$, then

$$
\begin{aligned}
&INRWMSM^{(k)}(x_1, x_2, \cdots, x_n)\\
&\quad \ge INRWMSM^{(k)}(x'_1, x'_2, \cdots, x'_n). \qquad (9)
\end{aligned}
$$

Theorem 3 (Commutativity): Let $(x'_1, x'_2, \cdots, x'_n)$ be any permutation of $(x_1, x_2, \cdots, x_n)$, then

$$
\begin{aligned}
&INRWMSM^{(k)}(x_1, x_2, \cdots, x_n)\\
&\quad = INRWMSM^{(k)}(x'_1, x'_2, \cdots, x'_n). \qquad (10)
\end{aligned}
$$

Proof:

$$
\begin{aligned}
&\mathrm{INRWMSM}^{(k)}(x_1, x_2, \cdots, x_n)\\
&= \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}} \right)^{1/k}\\
&= \left(\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w'_{i_j})(\prod_{j=1}^{k} x'_{i_j})}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w'_{i_j}} \right)^{1/k}\\
&= \mathrm{INRWMSM}^{(k)}(x'_1, x'_2, \cdots, x'_n).
\end{aligned}
$$

Theorem 4 (Idempotency): Let $x_i (i = 1, 2, \cdots, n)$ be a series of INNs, if $x_i = x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$

The Proof of Theorem 1:

Based on the Definition 3, we can obtain $\prod_{j=1}^{k} x_{i_j} = \left(\left[\prod_{j=1}^{k} T_{x_{i_j}}^{L}, \prod_{j=1}^{k} T_{x_{i_j}}^{U} \right], \left[1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}^{L}), 1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}^{U}) \right] \right.$ $\left. \left[1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}^{L}), 1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}^{U}) \right] \right)$ and $\left(\prod_{j=1}^{k} w_{i_j}\right)\left(\prod_{j=1}^{k} x_{i_j}\right) = \left(\left[1 - \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}^{L}\right)^{\prod_{j=1}^{k} w_{i_j}}, 1 - \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}^{U}\right)^{\prod_{j=1}^{k} w_{i_j}} \right], \right.$ $\left[\left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}^{L})\right)^{\prod_{j=1}^{k} w_{i_j}}, \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}^{U})\right)^{\prod_{j=1}^{k} w_{i_j}} \right], \left. \left[\left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}^{L})\right)^{\prod_{j=1}^{k} w_{i_j}}, \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}^{U})\right)^{\prod_{j=1}^{k} w_{i_j}} \right] \right).$

Further,

$$
\begin{aligned}
\sum_{1 \le i_1 < \cdots < i_k \le n} \left(\prod_{j=1}^{k} w_{i_j}\right)\left(\prod_{j=1}^{k} x_{i_j}\right) = & \left(\left[1 - \prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}^{L}\right)^{\prod_{j=1}^{k} w_{i_j}}, 1 - \prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}^{U}\right)^{\prod_{j=1}^{k} w_{i_j}} \right], \right. \\
[3pt] \quad & \left[\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}^{L})\right)^{\prod_{j=1}^{k} w_{i_j}}, \prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}^{U})\right)^{\prod_{j=1}^{k} w_{i_j}} \right], \\
[3pt] \quad & \left. \left[\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}^{L})\right)^{\prod_{j=1}^{k} w_{i_j}}, \prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}^{U})\right)^{\prod_{j=1}^{k} w_{i_j}} \right] \right).
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
\frac{\sum_{1 \le i_1 < \cdots < i_k \le n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}} = & \left(\left[1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}^{L}\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}}, \right. \right. \\
& 1 - \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k} T_{x_{i_j}}^{U}\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \Bigg], \left[\left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}^{L})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}}, \right. \\
& \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - I_{x_{i_j}}^{U})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \Bigg], \left[\left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}^{L})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}}, \right. \\
& \left. \left. \left(\prod_{1 \le i_1 < \cdots < i_k \le n} \left(1 - \prod_{j=1}^{k}(1 - F_{x_{i_j}}^{U})\right)^{\prod_{j=1}^{k} w_{i_j}} \right)^{\frac{1}{\sum_{1 \le i_1 < \cdots < i_k \le n} \prod_{j=1}^{k} w_{i_j}}} \right] \right).
\end{aligned}
$$

Finally, we can have

$$\mathrm{INRWMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \Bigg(\Bigg[\Bigg(1 - \Bigg(\prod_{1\le i_1<\cdots<i_k\le n}\Big(1-\prod_{j=1}^{k} T^L_{x_{i_j}}\Big)^{\prod_{j=1}^{k} w_{i_j}}\Bigg)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\prod_{j=1}^{k} w_{i_j}}}\Bigg)^{1/k},$$

$$\Bigg(1 - \Bigg(\prod_{1\le i_1<\cdots<i_k\le n}\Big(1-\prod_{j=1}^{k} T^U_{x_{i_j}}\Big)^{\prod_{j=1}^{k} w_{i_j}}\Bigg)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\prod_{j=1}^{k} w_{i_j}}}\Bigg)^{1/k}\Bigg],$$

$$\Bigg[1 - \Bigg(1 - \Bigg(\prod_{\substack{1\le i_1<\\ \cdots<i_k\le n}}\Big(1-\prod_{j=1}^{k}(1-I^L_{x_{i_j}})\Big)^{\prod_{j=1}^{k} w_{i_j}}\Bigg)^{\frac{1}{\sum_{\substack{1\le i_1<\\ \cdots<i_k\le n}}\prod_{j=1}^{k} w_{i_j}}}\Bigg)^{1/k},$$

$$1 - \Bigg(1 - \Bigg(\prod_{\substack{1\le i_1<\\ \cdots<i_k\le n}}\Big(1-\prod_{j=1}^{k}(1-I^U_{x_{i_j}})\Big)^{\prod_{j=1}^{k} w_{i_j}}\Bigg)^{\frac{1}{\sum_{\substack{1\le i_1<\\ \cdots<i_k\le n}}\prod_{j=1}^{k} w_{i_j}}}\Bigg)^{1/k}\Bigg],$$

$$\Bigg[1 - \Bigg(1 - \Bigg(\prod_{\substack{1\le i_1<\\ \cdots<i_k\le n}}\Big(1-\prod_{j=1}^{k}(1-F^L_{x_{i_j}})\Big)^{\prod_{j=1}^{k} w_{i_j}}\Bigg)^{\frac{1}{\sum_{\substack{1\le i_1<\\ \cdots<i_k\le n}}\prod_{j=1}^{k} w_{i_j}}}\Bigg)^{1/k},$$

$$1 - \Bigg(1 - \Bigg(\prod_{\substack{1\le i_1<\\ \cdots<i_k\le n}}\Big(1-\prod_{j=1}^{k}(1-F^U_{x_{i_j}})\Big)^{\prod_{j=1}^{k} w_{i_j}}\Bigg)^{\frac{1}{\sum_{\substack{1\le i_1<\\ \cdots<i_k\le n}}\prod_{j=1}^{k} w_{i_j}}}\Bigg)^{1/k}\Bigg]\Bigg).$$

Then we can know that Eq. (8), as shown at the top of page 4 is correct. It is readily-easily known that

$$0 \le \Bigg(1 - \Bigg(\prod_{1\le i_1<\cdots<i_k\le n}\Big(1-\prod_{j=1}^{k} T^L_{x_{i_j}}\Big)^{\prod_{j=1}^{k} w_{i_j}}\Bigg)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\prod_{j=1}^{k} w_{i_j}}}\Bigg)^{1/k}$$

$$\le \Bigg(1 - \Bigg(\prod_{1\le i_1<\cdots<i_k\le n}\Big(1-\prod_{j=1}^{k} T^U_{x_{i_j}}\Big)^{\prod_{j=1}^{k} w_{i_j}}\Bigg)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\prod_{j=1}^{k} w_{i_j}}}\Bigg)^{1/k} \le 1.$$

Similarly, the later two formulae are all tenable.
Consequently, we can know that the aggregated result from Eq. (8) is still an INN.

The Proof of Theorem 2:
Suppose that

$$INRWMSM^{(k)}(x_1, x_2, \cdots, x_n) = ([T^L, T^U], [I^L, I^U], [F^L, F^U]),$$
$$INRWMSM^{(k)}(x_1', x_2', \cdots, x_n') = ([(T^L)', (T^U)'], [(I^L)', (I^U)'], [(F^L)', (F^U)']).$$

Since $T^L_{x_i} \geq T^L_{x_i'}$, then we can easily obtain $1 - \prod_{j=1}^{k} T^L_{x_{i_j}} \leq 1 - \prod_{j=1}^{k} T^L_{x'_{i_j}}$.
Further, we can have

$$\left(1 - \left(\prod_{1\leq i_1<\cdots<i_k\leq n} \left(1 - \prod_{j=1}^{k} T^L_{x_{i_j}}\right)^{\prod_{j=1}^{k} w_{i_j}}\right)^{\frac{1}{\sum_{1\leq i_1<\cdots<i_k\leq n}\prod_{j=1}^{k} w_{i_j}}}\right)^{1/k}$$
$$\geq \left(1 - \left(\prod_{1\leq i_1<\cdots<i_k\leq n} \left(1 - \prod_{j=1}^{k} T^L_{x'_{i_j}}\right)^{\prod_{j=1}^{k} w_{i_j}}\right)^{\frac{1}{\sum_{1\leq i_1<\cdots<i_k\leq n}\prod_{j=1}^{k} w_{i_j}}}\right)^{1/k}.$$

Therefore, $T^L \geq (T^L)'$.
Similar to above, we can also prove that $T^U \geq (T^U)'$, $I^L \leq (I^L)'$, $I^U \leq (I^U)'$, $F^L \leq (F^L)'$, $F^U \leq (F^U)'$.
Finally, we can achieve

$$([T^L, T^U], [I^L, I^U], [F^L, F^U]) \geq ([(T^L)', (T^U)'], [(I^L)', (I^U)'], [(F^L)', (F^U)']).$$

That is to say, $INRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \geq INRWMSM^{(k)}(x_1', x_2', \cdots, x_n')$.

---

for $\forall i$, then

$$INRWMSM^{(k)}(x_1, x_2, \cdots, x_n) = x. \tag{11}$$

Proof:

$$\begin{aligned}
&\mathrm{INRWMSM}^{(k)}(x_1, x_2, \cdots, x_n)\\
&= \left(\frac{\sum_{1\leq i_1<\cdots<i_k\leq n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x_{i_j})}{\sum_{1\leq i_1<\cdots<i_k\leq n} \prod_{j=1}^{k} w_{i_j}}\right)^{1/k}\\
&= \left(\frac{\sum_{1\leq i_1<\cdots<i_k\leq n} (\prod_{j=1}^{k} w_{i_j})(\prod_{j=1}^{k} x)}{\sum_{1\leq i_1<\cdots<i_k\leq n} \prod_{j=1}^{k} w_{i_j}}\right)^{1/k}\\
&= \left(\frac{x^k \sum_{1\leq i_1<\cdots<i_k\leq n} \prod_{j=1}^{k} w_{i_j}}{\sum_{1\leq i_1<\cdots<i_k\leq n} \prod_{j=1}^{k} w_{i_j}}\right)^{1/k}\\
&= x.
\end{aligned}$$

Theorem 5 (Boundedness): Let $x_i (i = 1, 2, \cdots, n)$ be a series of INNs, and

$$\begin{aligned}
x^+ &= \left(\left[\max_{i=1}^{n} T_i^L, \max_{i=1}^{n} T_i^U\right], \left[\min_{i=1}^{n} I_i^L, \min_{i=1}^{n} I_i^U\right], \left[\min_{i=1}^{n} F_i^L, \min_{i=1}^{n} F_i^U\right]\right),\\
x^- &= \left(\left[\min_{i=1}^{n} T_i^L, \min_{i=1}^{n} T_i^U\right], \left[\max_{i=1}^{n} I_i^L, \max_{i=1}^{n} I_i^U\right], \left[\max_{i=1}^{n} F_i^L, \max_{i=1}^{n} F_i^U\right]\right),
\end{aligned}$$

then

$$x^- \leq INRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \leq x^+. \tag{12}$$

Proof: Based on the above monotonicity and idempotency, we can have

$$INRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \leq INRWMSM^{(k)}(x^+, x^+, \cdots, x^+)$$

and

$$INRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \geq INRWMSM^{(k)}(x^-, x^-, \cdots, x^-).$$

Consequently, we can obtain

$$x^- \leq INRWMSM^{(k)}(x_1, x_2, \cdots, x_n) \leq x^+.$$

Next, we present certain special cases of the INRWMSM by adjusting the value of the argument k:

Case 1: If $k = 1$, the INRWMSM operator degenerates into an interval neutrosophic weighted averaging (INWA) operator (Zhang *et al.* [22]):

$$
\begin{aligned}
&\mathrm{INRWMSM}^{(1)}(x_1, x_2, \cdots, x_n)\\
&= \left(\left[\left(1 - \left(\prod_{1 \le i_1 \le n} \left(1 - T^L_{x_{i_1}}\right)^{w_{i_1}} \right)^{\frac{1}{\sum_{1\le i_1 \le n} w_{i_1}}} \right)^{1/1}, \right.\right.\\
&\quad \left. \left(1 - \left(\prod_{1 \le i_1 \le n} \left(1 - T^U_{x_{i_1}}\right)^{w_{i_1}} \right)^{\frac{1}{\sum_{1\le i_1 \le n} w_{i_1}}} \right)^{1/1} \right],\\
&\quad \left[1 - \left(1 - \left(\prod_{1 \le i_1 \le n} (I^L_{x_{i_1}})^{w_{i_1}} \right)^{\frac{1}{\sum_{1\le i_1 \le n} w_{i_1}}} \right)^{1/1}, \right.\\
&\quad \left. 1 - \left(1 - \left(\prod_{1 \le i_1 \le n} (I^U_{x_{i_1}})^{w_{i_1}} \right)^{\frac{1}{\sum_{1\le i_1 \le n} w_{i_1}}} \right)^{1/1} \right],\\
&\quad \left[1 - \left(1 - \left(\prod_{1 \le i_1 \le n} (F^L_{x_{i_1}})^{w_{i_1}} \right)^{\frac{1}{\sum_{1\le i_1 \le n} w_{i_1}}} \right)^{1/1}, \right.\\
&\quad \left.\left. 1 - \left(1 - \left(\prod_{1 \le i_1 \le n} (F^U_{x_{i_1}})^{w_{i_1}} \right)^{\frac{1}{\sum_{1\le i_1 \le n} w_{i_1}}} \right)^{1/1} \right] \right)\\
&= \left(1 - \prod_{1 \le i_1 \le n} \left(1 - T_{x_{i_1}}\right)^{w_{i_1}}, \prod_{1 \le i_1 \le n} I_{x_{i_1}}^{w_{i_1}}, \prod_{1 \le i_1 \le n} F_{x_{i_1}}^{w_{i_1}} \right)\\
&= \mathrm{INWA}(x_1, x_2, \cdots, x_n)
\end{aligned}
$$

Case 2: If $k = n$, the INRWMSM operator degenerates into an interval neutrosophic geometric (ING) operator (Zhang *et al.* [22]):

$$
\begin{aligned}
&\mathrm{INRWMSM}^{(n)}(x_1, x_2, \cdots, x_n)\\
&= \left(\left[\left(1 - \left(\left(1 - \prod_{j=1}^{n} T^L_{x_{i_j}} \right)^{\prod_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\prod_{j=1}^{n} w_{i_j}}} \right)^{1/n}, \right.\right.\\
&\quad \left. \left(1 - \left(\left(1 - \prod_{j=1}^{n} T^U_{x_{i_j}} \right)^{\prod_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\prod_{j=1}^{n} w_{i_j}}} \right)^{1/n} \right],\\
&\quad \left[1 - \left(1 - \left(\left(1 - \prod_{j=1}^{n} (1 - I^L_{x_{i_j}}) \right)^{\prod_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\prod_{j=1}^{n} w_{i_j}}} \right)^{1/n}, \right.\\
&\quad \left. 1 - \left(1 - \left(\left(1 - \prod_{j=1}^{n} (1 - I^U_{x_{i_j}}) \right)^{\prod_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\prod_{j=1}^{n} w_{i_j}}} \right)^{1/n} \right],\\
&\quad \left[1 - \left(1 - \left(\left(1 - \prod_{j=1}^{n} (1 - F^L_{x_{i_j}}) \right)^{\prod_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\prod_{j=1}^{n} w_{i_j}}} \right)^{1/n}, \right.\\
&\quad \left.\left. 1 - \left(1 - \left(\left(1 - \prod_{j=1}^{n} (1 - F^U_{x_{i_j}}) \right)^{\prod_{j=1}^{n} w_{i_j}} \right)^{\frac{1}{\prod_{j=1}^{n} w_{i_j}}} \right)^{1/n} \right] \right)\\
&= \left(\left[1 - \prod_{1 \le i_1 \le n} \left(1 - T^L_{x_{i_1}}\right)^{\frac{1}{n}}, 1 - \prod_{1 \le i_1 \le n} \left(1 - T^U_{x_{i_1}}\right)^{\frac{1}{n}} \right], \right.\\
&\quad \left[\prod_{1 \le i_1 \le n} (I^L_{x_{i_1}})^{\frac{1}{n}} \prod_{1 \le i_1 \le n} (I^U_{x_{i_1}})^{\frac{1}{n}} \right],\\
&\quad \left. \times \left[\prod_{1 \le i_1 \le n} (F^L_{x_{i_1}})^{\frac{1}{n}}, \prod_{1 \le i_1 \le n} (F^U_{x_{i_1}})^{\frac{1}{n}} \right] \right)\\
&= \mathrm{ING}(x_1, x_2, \cdots, x_n)
\end{aligned}
$$

B. INTERVAL NEUTROSOPHIC REDUCIBLE WEIGHTED DUAL MSM OPERATOR

Definition 10: Let $x_i = ([T^L_{x_i}, T^U_{x_i}], [I^L_{x_i}, I^U_{x_i}], [F^L_{x_i}, F^U_{x_i}])$ $(i = 1, 2, \cdots, n)$ be a series of INNs, and let $W = (w_1, w_2, \cdots, w_n)^T$ be a weight vector with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. The INRWDMSM: $\Omega^m \rightarrow \Omega$, an INRWDMSM operator is defined as follows:

$$
\mathrm{INRWDMSM}^{(k)}(x_1, x_2, \cdots, x_n) = \frac{\prod_{1 \le i_1 < \cdots < i_k \le n} \left(\sum_{j=1}^{k} x_{i_j} \right)^{\frac{\sum_{j=1}^{k} w_{i_j}}{\sum_{1 \le i_1 < \cdots < i_k \le n} \sum_{j=1}^{k} w_{i_j}}}}{k} \tag{13}
$$

where Ω is a set of all INNs, then INRWDMSM is called the interval neutrosophic reducible weighted dual MSM operator.

Based on the operational laws of the INNs presented in Definition 3, from Eq. (13), we can have the aggregated result shown in Theorem 6.

Theorem 6: Let $x_i = ([T^L_{x_i}, T^U_{x_i}], [I^L_{x_i}, I^U_{x_i}], [F^L_{x_i}, F^U_{x_i}])(i = 1, 2, \cdots, n)$ be a set of INNs, and let $W = (w_1, w_2, \cdots, w_n)^T$

$$
\begin{aligned}
&\mathrm{INRWDMSM}^{(k)}(x_1, x_2, \cdots, x_n)\\
&= \left(\left[1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}(1-T^L_{x_{i_j}})\right)^{\sum_{j=1}^{k} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k} w_{i_j}}}\right)^{1/k},\right.\right.\\
&\qquad \left.1-\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}(1-T^U_{x_{i_j}})\right)^{\sum_{j=1}^{k} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k} w_{i_j}}}\right)^{1/k}\right],\\
&\qquad \left[\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}I^L_{x_{i_j}}\right)^{\sum_{j=1}^{k} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k} w_{i_j}}}\right)^{1/k},\right.\\
&\qquad \left.\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}I^U_{x_{i_j}}\right)^{\sum_{j=1}^{k} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k} w_{i_j}}}\right)^{1/k}\right],\\
&\qquad \left[\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}F^L_{x_{i_j}}\right)^{\sum_{j=1}^{k} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k} w_{i_j}}}\right)^{1/k},\right.\\
&\qquad \left.\left.\left(1-\left(\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}F^U_{x_{i_j}}\right)^{\sum_{j=1}^{k} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1<\cdots<i_k\le n}\sum_{j=1}^{k} w_{i_j}}}\right)^{1/k}\right]\right) \qquad (14)
\end{aligned}
$$

be a weight vector with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, then the final result of INRWDMSM operator is still an INN.

Remark 1: The INRWDMSM operator also possesses the characters of idempotency, commutativity, monotonicity and boundedness.

Next, some special cases of the INRWDMSM by adjusting the value of the argument k are shown as follows:

Case 1: If $k = 1$, the INRWDMSM operator degenerates into an interval neutrosophic weighted geometric (INWG) operator (Zhang *et al.* [22]):

$$
\begin{aligned}
&\mathrm{INRWDMSM}^{(1)}(x_1, x_2, \cdots, x_n)\\
&= \left(\left[1-\left(1-\left(\prod_{1\le i_1\le n}\left(1-\prod_{j=1}^{1}(1-T^L_{x_{i_j}})\right)^{\sum_{j=1}^{1} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1\le n}\sum_{j=1}^{1} w_{i_j}}}\right)^{1/1},\right.\right.\\
&\qquad \left.1-\left(1-\left(\prod_{1\le i_1\le n}\left(1-\prod_{j=1}^{1}(1-T^U_{x_{i_j}})\right)^{\sum_{j=1}^{1} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1\le n}\sum_{j=1}^{1} w_{i_j}}}\right)^{1/1}\right],\\
&\qquad \left[\left(1-\left(\prod_{1\le i_1\le n}\left(1-\prod_{j=1}^{1}I^L_{x_{i_j}}\right)^{\sum_{j=1}^{1} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1\le n}\sum_{j=1}^{1} w_{i_j}}}\right)^{1/1},\right.\\
&\qquad \left.\left(1-\left(\prod_{1\le i_1\le n}\left(1-\prod_{j=1}^{1}I^U_{x_{i_j}}\right)^{\sum_{j=1}^{1} w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1\le n}\sum_{j=1}^{1} w_{i_j}}}\right)^{1/1}\right],
\end{aligned}
$$

$$
\left[\left(1-\left(\prod_{1\le i_1<\le n}\left(1-\prod_{j=1}^{1}F^L_{x_{i_j}}\right)^{\sum_{j=1}^{1}w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1\le n}\sum_{j=1}^{1}w_{i_j}}}\right)^{1/1},\right.
$$

$$
\left.\left.\left(1-\left(\prod_{1\le i_1<\le n}\left(1-\prod_{j=1}^{1}F^U_{x_{i_j}}\right)^{\sum_{j=1}^{1}w_{i_j}}\right)^{\frac{1}{\sum_{1\le i_1\le n}\sum_{j=1}^{1}w_{i_j}}}\right)^{1/1}\right]\right)
$$

$$
=\left(\left[\prod_{1\le i_1\le n}(T^L_{x_{i_j}})^{w_{i_j}},\prod_{1\le i_1\le n}(T^U_{x_{i_j}})^{w_{i_j}}\right],\right.
$$

$$
\left[\prod_{1\le i_1\le n}1-(1-I^L_{x_{i_j}})^{w_{i_j}},\prod_{1\le i_1\le n}1-(1-I^U_{x_{i_j}})^{w_{i_j}}\right],
$$

$$
\left.\left[\prod_{1\le i_1\le n}1-(1-F^L_{x_{i_j}})^{w_{i_j}},\prod_{1\le i_1\le n}1-(1-F^U_{x_{i_j}})^{w_{i_j}}\right]\right)
$$

$$
=\mathrm{INWG}(x_1,x_2,\cdots,x_n)
$$

Case 2: If $k = n$, the INRWDMSM operator degenerates into an interval neutrosophic averaging (INA) operator (Zhang *et al.* [22]):

$$
\mathrm{INRWMSM}^{(n)}(x_1,x_2,\cdots,x_n)
$$

$$
=\left(\left[1-\left(1-\left(\left(1-\prod_{j=1}^{n}(1-T^L_{x_{i_j}})\right)^{\sum_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n}w_{i_j}}}\right)^{1/n},\right.\right.
$$

$$
\left.1-\left(1-\left(\left(1-\prod_{j=1}^{n}(1-T^U_{x_{i_j}})\right)^{\sum_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n}w_{i_j}}}\right)^{1/n}\right],
$$

$$
\left[\left(1-\left(\left(1-\prod_{j=1}^{n}I^L_{x_{i_j}}\right)^{\sum_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n}w_{i_j}}}\right)^{1/n},\right.
$$

$$
\left.\left(1-\left(\left(1-\prod_{j=1}^{n}I^U_{x_{i_j}}\right)^{\sum_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n}w_{i_j}}}\right)^{1/n}\right],
$$

$$
\left[\left(1-\left(\left(1-\prod_{j=1}^{n}F^L_{x_{i_j}}\right)^{\sum_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n}w_{i_j}}}\right)^{1/n},\right.
$$

$$
\left.\left.\left(1-\left(\left(1-\prod_{j=1}^{n}F^U_{x_{i_j}}\right)^{\sum_{j=1}^{n}w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n}w_{i_j}}}\right)^{1/n}\right]\right)
$$

$$
=\left(\left[1-\prod_{1\le i_1\le n}\left(1-T^L_{x_{i_1}}\right)^{\frac{1}{n}},1-\prod_{1\le i_1\le n}\left(1-T^U_{x_{i_1}}\right)^{\frac{1}{n}}\right],\right.
$$

$$
\left[\prod_{1\le i_1\le n}(I^L_{x_{i_1}})^{\frac{1}{n}},\prod_{1\le i_1\le n}(I^U_{x_{i_1}})^{\frac{1}{n}}\right],
$$

$$
\left.\left[\prod_{1\le i_1\le n}(F^L_{x_{i_1}})^{\frac{1}{n}},\prod_{1\le i_1\le n}(F^U_{x_{i_1}})^{\frac{1}{n}}\right]\right)
$$

$$
=\mathrm{INA}(x_1,x_2,\cdots,x_n)
$$

Remark 2: Markedly, it is significant that INRWMSM or INRWDMSM operator cannot obtain the correlation among the given arguments when $k = 1$ or $k = n$. That is to say, both of them degenerate into the independent operators such as INA, ING, INWA and INWG (Zhang *et al.* [22]).

IV. THE MADM ALGORITHMS BASED ON INRWMSM AND INRWDMSM OPERATORS

A. THE DESCRIPTION OF THE MADM ISSUE

Let $A = \{A_1, A_2, \cdots, A_m\}$ be a series of alternatives, $C = \{C_1, C_2, \cdots, C_n\}$ be a set of n attributes, and $W = \{w_1, w_2, \cdots, w_n\}$ be a weight vector assigned to the attributes by the experts with the standard restriction $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$. Suppose that the global evaluation of the alternatives with respect to attributes is denoted by an interval neutrosophic matrix $P = (p_{ij})_{m\times n} = ([T^L_{ij}, T^U_{ij}], [I^L_{ij}, I^U_{ij}], [F^L_{ij}, F^U_{ij}])_{m\times n}$. By this we mean that the values associated with the alternatives for the formalization of MADM issue can be presented in Table 1.

B. THE MADM METHOD BASED ON INRWMSM OR INRWDMSM OPERATOR

For the case of making decisions in our setting, the framework for employing the developed algorithm is shown in Fig. 1.

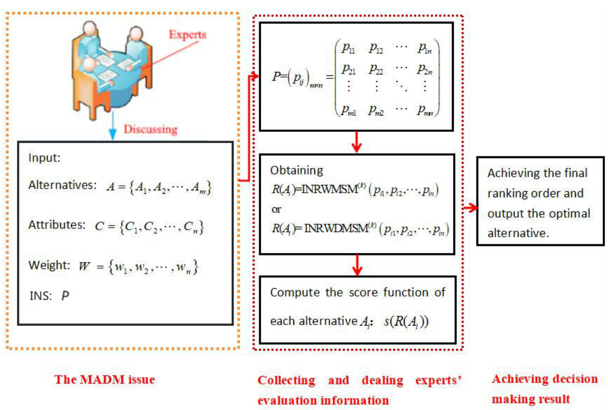

FIGURE 1. The framework for using the proposed method.

TABLE 1. The interval neutrosophic MADM matrix.

	C_1	C_2	$\cdots$	C_n
A_1	$([T_{11}^L, T_{11}^U], [I_{11}^L, I_{11}^U], [F_{11}^L, F_{11}^U])$	$([T_{12}^L, T_{12}^U], [I_{12}^L, I_{12}^U], [F_{12}^L, F_{12}^U])$	$\cdots$	$([T_{1n}^L, T_{1n}^U], [I_{1n}^L, I_{1n}^U], [F_{1n}^L, F_{1n}^U])$
A_2	$([T_{21}^L, T_{21}^U], [I_{21}^L, I_{21}^U], [F_{21}^L, F_{21}^U])$	$([T_{22}^L, T_{22}^U], [I_{22}^L, I_{22}^U], [F_{22}^L, F_{22}^U])$	$\cdots$	$([T_{2n}^L, T_{2n}^U], [I_{2n}^L, I_{2n}^U], [F_{2n}^L, F_{2n}^U])$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$([T_{m1}^L, T_{m1}^U], [I_{m1}^L, I_{m1}^U], [F_{m1}^L, F_{m1}^U])$	$([T_{m2}^L, T_{m2}^U], [I_{m2}^L, I_{m2}^U], [F_{m2}^L, F_{m2}^U])$	$\cdots$	$([T_{mn}^L, T_{mn}^U], [I_{mn}^L, I_{mn}^U], [F_{mn}^L, F_{mn}^U])$

At the same time, the following algorithm is self-explanatory:

Algorithm 1 INRWMSM or INRWDMSM Operator.

1: Determine the input alternatives and attributes, and obtain the interval neutrosophic matrix $P = (p_{ij})_{m\times n}$ which is given in the format of Table 1.

2: Transform the matrix $P = (p_{ij})_{m\times n}$ into a normalized interval neutrosophic matrix $P' = (p'_{ij})_{n\times m}$ by Eq. (15).

$$p'_{ij} = \begin{cases} ([T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U]), C_j \in B, \\ ([F_{ij}^L, F_{ij}^U], [1 - I_{ij}^U, 1 - I_{ij}^L], [T_{ij}^L, T_{ij}^U]),\ C_j \in C, \end{cases} \tag{15}$$

where B is benefit attributes set and C is cost attributes set.

3: Employ the INRWMSM operator

$$R(A_i) = (T_i, I_i, F_i) = INRWMSM^{(k)}(p'_{i1}, p'_{i2}, \cdots, p'_{in}) \tag{16}$$

or

Employ the INRWDMSM operator

$$R(A_i) = (T_i, I_i, F_i) = INRWDMSM^{(k)}(p'_{i1}, p'_{i2}, \cdots, p'_{in}) \tag{17}$$

to determine the aggregated decision value.

4: Compute the score function $s(R(A_i))$ of the whole values $R(A_i)(i = 1, 2, \cdots, m)$.

5: Select the optimal alternative(s) by maximization of their scores.

C. A CASE OF INTERNET OF MEDICAL THINGS (IOMT)

According to the authoritative medical institute (NIH), the global population is aging rapidly. About 8.5 percent of the world's population will be over 65 by 2025, and 17 percent by 2050. And the average life expectancy of the world's population will go from 68.6 years in 2015 to 76.2 years in 2050, an increase of eight years. With the growth of various chronic diseases, the quantity and quality of medical resources including hospitals, medical equipment and medical staff is a huge challenge. Internet of medical things (IoMT) technology is undoubtedly one of the effective tools to solve the contradiction between the shortage of medical resources and the rapid growth of demand. IoMT technology has four effects on healthcare as follows:

(1) Generating massive amounts of medical data by IoT terminals

With the increase of various kinds of wearable medical devices, they are not only responsible for monitoring patients' condition and timely reporting to doctors; They also generate vast amounts of data, and patients' personal data, such as medical history, allergens, medication history, pathological or physiological tests and test reports, are stored in hospital equipment or cloud systems. These data are not only extremely useful for individual patients, but also provide useful reference for other people's diagnosis.

(2) Accelerating the application of artificial intelligence in the medical field

Based on the massive data generated by IoMT terminals, through machine learning and analysis of vast amounts of medical data, medical institutions and medical device manufacturers can combine artificial intelligence and the IoT to develop or develop more advanced medical applications or equipments that provide a more personalized service from initial diagnosis to further treatment. For example, the robot "nurse" can integrate the face recognition technology and the patient's massive data, he can identify the patient's emotions and have different coping styles, such as reminding the patient to take medicine or reminding the patient to go to the doctor on time. Imagine the collaborative work between the robot "nurse" and the real nurse in the hospital, it will be a thing that is not going to happen in the distant future! Artificial intelligence is growing rapidly in the medical field. According to Accenture, by 2026, AI technology will save the US medical system 150 billion.

(3) Producing new kinds of medical first aid equipment

IoT applications have spawned new medical emergency or rescue devices, and medical drones (some called "flight Internet of Things") are one of these rescue devices. Due to its small size and flexibility, drones can send medical first-aid supplies to places where other tools are not easily delivered, and at low cost. In a recent emergency operation in Rwanda, drones were reportedly used to deliver bags of plasma, drugs and other drugs to designated locations for emergency treatment. A number of companies are already developing drone systems for the delivery of medical

TABLE 2. **The evaluation attributes of hospitals using IoMT.**

Attributes	Brief description
Insurance system (C_1)	It involves organizational guarantee, system guarantee, financial guarantee and human resource guarantee.
Infrastructure configuration (C_2)	It consists of equipment configuration, network configuration, machine room construction.
Application scenarios (C_3)	It denotes the personnel management, goods management, medical care, environmental monitoring, hospital information management. The better the hospital, the richer application scenarios.
Information security (C_4)	It is mainly examined from four aspects: network security, data security, system security and security management system.
Information sharing (C_5)	It presents hospital internal information sharing and regional information sharing (such as information sharing between medical and health institutions, information sharing between hospitals and residents' communities).
Technical standard (C_6)	It presents IoT technology perception layer standard, network transmission layer standard, and medical application layer standard.

TABLE 3. **The interval neutrosophic matrix in example 1.**

	C_1	C_2	C_3
A_1	$([0.8,0.9],[0.1,0.2],[0.1,0.3])$	$([0.8,0.9],[0.1,0.2],[0.2,0.3])$	$([0.8,0.9],[0.2,0.3],[0.1,0.2])$
A_2	$([0.8,0.9],[0.1,0.2],[0.2,0.3])$	$([0.8,0.9],[0.2,0.3],[0.2,0.3])$	$([0.7,0.8],[0.2,0.3],[0.2,0.3])$
A_3	$([0.7,0.8],[0.1,0.3],[0.2,0.3])$	$([0.7,0.8],[0.2,0.3],[0.3,0.4])$	$([0.6,0.7],[0.2,0.3],[0.1,0.3])$
A_4	$([0.5,0.6],[0.1,0.3],[0.2,0.3])$	$([0.6,0.8],[0.2,0.3],[0.3,0.4])$	$([0.6,0.7],[0.2,0.3],[0.2,0.3])$
A_5	$([0.4,0.5],[0.1,0.3],[0.2,0.3])$	$([0.5,0.6],[0.2,0.3],[0.3,0.4])$	$([0.5,0.6],[0.2,0.3],[0.2,0.3])$
	C_4	C_5	C_6
A_1	$([0.8,0.9],[0.1,0.2],[0.1,0.3])$	$([0.8,0.9],[0.1,0.2],[0.2,0.3])$	$([0.8,0.9],[0.2,0.3],[0.1,0.2])$
A_2	$([0.8,0.9],[0.1,0.2],[0.2,0.3])$	$([0.8,0.9],[0.1,0.3],[0.2,0.3])$	$([0.7,0.8],[0.2,0.4],[0.1,0.3])$
A_3	$([0.6,0.7],[0.1,0.2],[0.2,0.3])$	$([0.6,0.8],[0.1,0.3],[0.3,0.4])$	$([0.6,0.8],[0.2,0.4],[0.1,0.3])$
A_4	$([0.5,0.7],[0.1,0.2],[0.2,0.3])$	$([0.6,0.7],[0.1,0.3],[0.3,0.5])$	$([0.6,0.7],[0.2,0.4],[0.2,0.3])$
A_5	$([0.4,0.6],[0.1,0.2],[0.2,0.3])$	$([0.5,0.7],[0.1,0.3],[0.3,0.5])$	$([0.4,0.6],[0.2,0.5],[0.2,0.4])$

supplies or personnel. In the United States, Australia and Switzerland, the use of drones for medical purposes is gaining traction across the country.

(4) Helping telemedicine gradually rise

Most people would rather have a medical diagnosis or treatment at home than the noise in a hospital or medical facility. A 2016 survey showed that 94%-96% of the 3,000 respondents were very satisfied with telemedicine. One-third of respondents prefer telephone medical care rather than going to a doctor's clinic. I think this is even more true in China, as long as you go to the Children's Hospital or the Institute to experience it. The increasing popularity of medical Internet of things applications, such as remote health monitoring, remote transmission of medical images, and convenient use of remote medical equipment, makes telemedicine or telemedicine possible. This is very effective for the treatment of some chronic diseases and for some residents in more remote places.

The prime minister of this paper first searches the authoritative database at home and abroad, and finds that there are few studies on the evaluation of IoMT level. However, the evaluation research on medical informatization, enterprise informatization, regional informatization, and IoT is relatively mature. In a broad sense, the IoT is an extension and extension of the Internet. The IoT is a stage of information development. Therefore, the development of IoMT is based on medical informationization, which provides a new idea for the construction of medical IoT level evaluation index system, which is to draw on the more mature information and IoT related evaluation models at home and abroad. And focus on the application of IoT technology in the medical field, combined with important policy documents promulgated by the state in recent years.

A preliminary model of the evaluation index system of IoMT level is established. Six preliminary attributes $(C_1, C_2, C_3, C_4, C_5, C_6)$ are established in the preliminary model related to hospitals. We give the detailed description of each attribute in Table 2.

Example 1: Suppose that there are five influential hospitals $A = \{A_1, A_2, A_3, A_4, A_5\}$ to be considered for the assessment. The experts choose the highly representative attribute set $C = \{C_1(\textit{Insurance system}), C_2(\textit{Infrastructure configuration}), C_3(\textit{Application scenarios}), C_4(\textit{Information security}), C_5(\textit{Information sharing}), C_6(\textit{Technical standard})\}$. According to the general evolving principle and the features of the IoMT, we can ascertain that whole attributes are benefit attributes. Assume that the expert has given the weight information as

TABLE 4. The interval neutrosophic matrix in example 2.

	C_1	C_2	C_3
A_1	$([0.8, 0.9], [0.1, 0.2], [0.1, 0.3])$	$([0.8, 0.9], [0.1, 0.2], [0.2, 0.3])$	$([0.8, 0.9], [0.2, 0.3], [0.1, 0.2])$
A_2	$([0.8, 0.9], [0.1, 0.2], [0.2, 0.3])$	$([0.8, 0.9], [0.2, 0.3], [0.2, 0.3])$	$([0.7, 0.8], [0.2, 0.3], [0.2, 0.3])$
A_3	$([0.7, 0.8], [0.1, 0.3], [0.2, 0.3])$	$([0.7, 0.8], [0.2, 0.3], [0.3, 0.4])$	$([0.6, 0.7], [0.2, 0.3], [0.1, 0.3])$
A_4	$([0.5, 0.6], [0.1, 0.3], [0.2, 0.3])$	$([0.6, 0.8], [0.2, 0.3], [0.3, 0.4])$	$([0.6, 0.7], [0.2, 0.3], [0.2, 0.3])$
A_5	$([0.4, 0.5], [0.1, 0.3], [0.2, 0.3])$	$([0.5, 0.6], [0.2, 0.3], [0.3, 0.4])$	$([0.5, 0.6], [0.2, 0.3], [0.2, 0.3])$
	C_4	C_5	C_6
A_1	$([0.8, 0.9], [0.1, 0.2], [0.1, 0.3])$	$([0.8, 0.9], [0.1, 0.2], [0.2, 0.3])$	$([0.8, 0.9], [0.2, 0.3], [0.1, 0.2])$
A_2	$([0.8, 0.9], [0.1, 0.2], [0.2, 0.3])$	$([0.8, 0.9], [0.1, 0.3], [0.2, 0.3])$	$([0.7, 0.8], [0.2, 0.4], [0.1, 0.3])$
A_3	$([0.6, 0.7], [0.1, 0.2], [0.2, 0.3])$	$([0.6, 0.8], [0.1, 0.3], [0.3, 0.4])$	$([0.6, 0.8], [0.2, 0.4], [0.1, 0.3])$
A_4	$([0.5, 0.7], [0.1, 0.2], [0.2, 0.3])$	$([0.6, 0.7], [0.1, 0.3], [0.3, 0.5])$	$([0.6, 0.7], [0.2, 0.4], [0.2, 0.3])$
A_5	$([0.4, 0.6], [0.1, 0.2], [0.2, 0.3])$	$([0.5, 0.7], [0.1, 0.3], [0.3, 0.5])$	$([0.0, 0.0], [0.0, 0.0], [0.0, 0.0])$

$w = (w_1, w_2, w_3, w_4, w_5, w_6) = (0.2, 0.3, 0.1, 0.1, 0.2, 0.1)$. The evaluations for hospitals employing IoMT arising from questionnaire investigation by the veteran expert group and generating the final interval neutrosophic matrix with its tabular form given in Table 4.

In the following, we use the algorithm proposed above in choosing optimal hospital to collaborate under interval neutrosophic text.

Step 1: Obtain the alternatives and attributes, and obtain the interval neutrosophic matrix $P = (p_{ij})_{5\times 6}$ which is shown in the format of Table 4.

Step 2: No conversion is needed because all attributes are beneficial attributes.

Step 3: Employ the $INRWMSM^{(1)}$ operator to integrate the decision value as follows:

$R(A_1) = ([0.800000, 0.900000], [0.114870, 0.216894], [0.141421, 0.276632])$,

$R(A_2) = ([0.783106, 0.885130], [0.141421, 0.273393], [0.186607, 0.300000])$,

$R(A_3) = ([0.653590, 0.783106], [0.141421, 0.296487], [0.213240, 0.346410])$,

$R(A_4) = ([0.572306, 0.718628], [0.141421, 0.296487], [0.244949, 0.362220])$,

$R(A_5) = ([0.462173, 0.605130], [0.141421, 0.303178], [0.244949, 0.372792])$.

or

Employ the $INRWDMSM^{(1)}$ operator to integrate the preference value as follows:

$R(A_1) = ([0.800000, 0.900000], [0.120953, 0.221082], [0.151472, 0.281054])$,

$R(A_2) = ([0.778918, 0.879047], [0.151472, 0.282535], [0.190522, 0.300000])$,

$R(A_3) = ([0.648074, 0.778918], [0.151472, 0.301442], [0.233831, 0.351926])$,

$R(A_4) = ([0.568063, 0.706490], [0.151472, 0.301442], [0.251669, 0.375132])$,

$R(A_5) = ([0.457305, 0.596629], [0.151472, 0.314063], [0.251669, 0.384690])$.

Step 4: Compute the score function $s(R(A_i))$ of the whole values $R(A_i)(i = 1, 2, 3, 4, 5)$ as follows:

INRWMSM:

$s(R(A_1)) = 0.825030$, $s(R(A_2)) = 0.794469$, $s(R(A_3)) = 0.739856$, $s(R(A_4)) = 0.707643$, $s(R(A_5)) = 0.667494$.

INRWDMSM:

$s(R(A_1)) = 0.820906$, $s(R(A_2)) = 0.788906$, $s(R(A_3)) = 0.731387$, $s(R(A_4)) = 0.699140$, $s(R(A_5)) = 0.658673$.

Step 5: According to the above score function $s(R(A_i))(i = 1, 2, 3, 4, 5)$, we can achieve the ordering of the given hospitals $\{A_1, A_2, A_3, A_4, A_5\}$ as follows: $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

D. SENSITIVITY ANALYSIS OF THE PARAMETER K ON THE ORDERING IN PROPOSED ALGORITHMS

For analyzing the sensitivity of the parameters k on the decision values, an experiment (Example 1) was given by employing diverse values of argument $k(k = 1, 2, 3, 4, 5, 6)$.

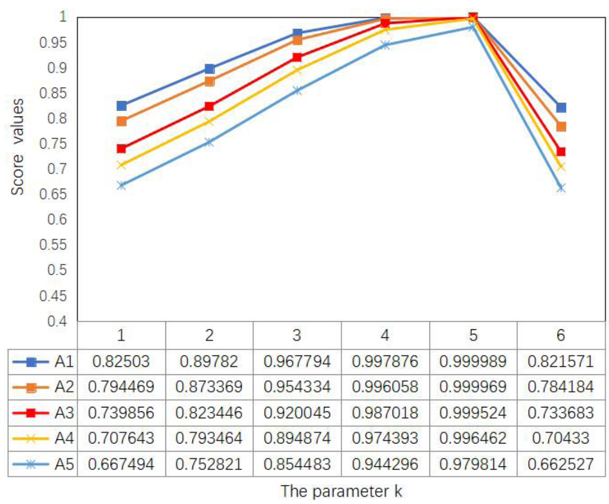

FIGURE 2. The total changing trend of parameter k in INRWMSM algorithm.

According to the INRWMSM or INRWDMSM operator, the eventual decision making results are given in Fig. 2 or Fig. 3. From the figures, some important points have been concluded in the following.

TABLE 5. A comparison study with some existing methods in example 2.

Algorithms	Ranking	Optimal alternative
INRWMSM$^{(k)}$	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
INRWDMSM$^{(k)}$	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhang et al. [22]: INNWA	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhang et al. [22]: INNWG	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhang et al. [22]: INNEWG	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhao et al. [24]: IVNSGWA ($\lambda = 1$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhao et al. [24]: IVNSGWA ($\lambda \to 0$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [27]: CIINWAA	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [27]: CIINWGA	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Liu and Tang [29]: INPGWA ($\lambda = 1$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Liu and Tang [29]: INPGWA ($\lambda \to 0$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Liu and You [31]: INWMMP$^{(1,0,0,0,0)}$	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Liu and You [31]: INDWMMP$^{(1,0,0,0,0)}$	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [51]:Dice measure G_{WINN3} ($\lambda = 0$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [51]:Dice measure G_{WINN3} ($\lambda = 1$)	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$	A_5
Ye [51]:Dice measure G_{WINN4} ($\lambda = 0$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [51]:Dice measure G_{WINN4} ($\lambda = 1$)	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$	A_5
Broumi and Smarandache [49]: Similarity measure	N/A	$*$
Broumi and Smarandache [50]: Similarity measure	N/A	$*$
Ye [48]: Similarity measure	N/A	$*$
Ye [52]: Similarity measure	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Mondal et al. [53]: Similarity measure	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1

$k = 1, 2, 3, 4, 5, 6$ in INRWMSM and INRWDMSM;
The green background color denotes the unauthentic results.
The red background color denotes that it cannot make a decision due to the division by zero issue;
"*" denotes that it cannot obtain ranking;
"N/A" denotes that it cannot compute the degree of similarity due to "the division by zero issue".

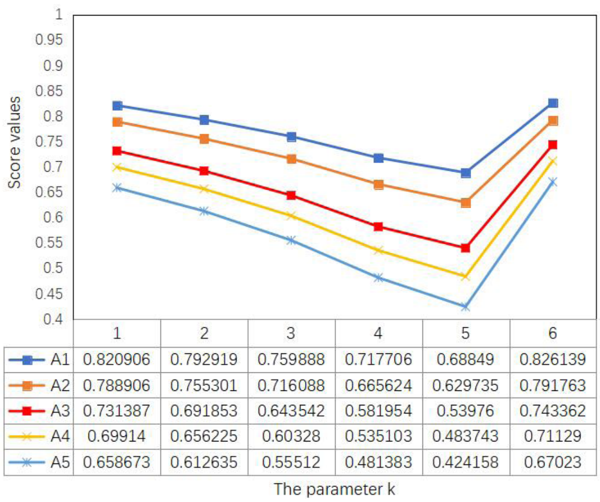

FIGURE 3. The total changing trend of parameter *k* in INRWDMSM algorithm.

(1) With regard to the INRWMSM algorithm, the score values of proprietary five hospitals are firstly monotonically increases when $k \in [1, 5]$, and later monotonically decreases when $k \in [5, 6]$. In addition, it is not very unambiguous to see the ultimate ranking owing to achieving the similar values which vary from 0.979814 to 0.999989 with difference value of 0.02 when $k = 5$. The ultimate results all stay around as $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

(2) With regard to INRWDMSM algorithm, the score values of proprietary five hospitals are firstly monotonically decreases when $k \in [1, 5]$, and later monotonically increases when $k \in [5, 6]$. Moreover, it is very unambiguous to see the ultimate ranking compared with the INRWMSM algorithm. The ultimate results all also stay around as $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

(3) The cause for the inflection point is that $k = 1$ is the form of averaging operator at the beginning and $k = 6$ is the form of geometric operator at the end for INRWMSM algorithm. Similarly, for INRWDMSM algorithm, it experiences the transformation from geometric operator to averaging operator.

(4) The values at both ends ($k = 1$ and $k = 6$) have an interesting case. With regard to INRWMSM algorithm, the score values of proprietary five hospitals are the minimum compared with $k = 1, 2, 3, 4, 5$ when $k = 6$ (geometric form). With regard to INRWDMSM algorithm, the score values of all five hospitals are the maximal compared with $k = 1, 2, 3, 4, 5$ when $k = 6$ (averaging form).

V. COMPARATIVE ANALYSIS AND DISCUSSION

Next, some existing MADM algorithms [22], [24], [27], [29], [31], [48]–[53] with their restrictions and features are analyzed in detail.

Example 2: Continue to Example 1. Suppose that the evaluation for hospitals employing IoMT arising from novel experts are presented which is shown in Table 4.

Remark 3: From the Table 5, we can see that the red background color denote some unreasonable results due to the division by zero issue. For G_{WINN3} and G_{WINN4} in [51], it would cause the unauthentic situation because the selection

TABLE 6. **Characteristic comparisons of diverse interval neutrosophic aggregation operators.**

Aggregation operators	Whether consider interrelationships between aggregating two attributes	Whether make the information aggregation process more flexible by a parameter	Whether consider interrelationships between aggregating multiple attributes
INNWA [22]	No	No	No
INNWG [22]	No	No	No
INNEWG [22]	No	No	No
INNCI [23]	No	No	No
IVNSGWA [24]	No	No	No
INNOWA [25]	No	No	No
INNOWG [25]	No	No	No
INWEA [26]	No	No	No
DINWEA [26]	No	No	No
CIINWAA [27]	No	No	No
CIINWGA [27]	No	No	No
INPOWA [28]	No	No	No
INPGWA [29]	No	Yes	No
IGINSHAA [30]	No	Yes	No
IGINSHGM [30]	No	Yes	No
INWMM [31]	Yes	No	No
INDWMM [31]	Yes	No	No
INWEA [32]	No	No	No
G-INCOA [33]	No	Yes	No
G-INCOG [33]	No	Yes	No
INRWMSM	Yes	Yes	Yes
INRWDMSM	Yes	Yes	Yes

TABLE 7. **Characteristic comparisons of diverse uncertain environment of MSM aggregation operators.**

Sets	Aggregation operators	Whether possess the property of idempotency	Whether possess the property of reducibility
SVNS	SVNRWMSM	Yes	Yes
SVNS	SVNRWDMSM	Yes	Yes
IFS	WIFMSM [35]	No	No
ULS	ULWDMSM [36]	No	No
ILS	WILMSM [37]	No	No
IULS	WIULMSM [37]	No	No
LIFS	WLIFMSM [38]	No	No
LIFS	WLIFDMSM [38]	No	No
PFS	PFIWMSM [39]	No	No
HFLS	HFLWMSM [40]	No	No
PFS	PFWMSM [41]	No	No
HFS	WHFMSM [42]	No	No
SVTNS	SVTNWMSM [43]	No	No
INLS	SVTNWMSM [44]	No	No
SVN-I2TLS	SVN-ITLWMSM [45]	No	No
2TLS	2TLWMSM [46]	No	No

2-tuple linguistic set (2TLS) Interval neutrosophic linguistic set (INLS) Intuitionistic fuzzy set (IFS), Intuitionistic linguistic set (ILS), Intuitionistic uncertain linguistic set (IULS), Linguistic intuitionistic fuzzy set (LIFS), Hesitant fuzzy linguistic set (HFLS), Hesitant fuzzy set (HFS), Pythagorean Fuzzy set (PFS), Single-valued neutrosophic interval 2-tuple linguistic set (SVN-I2TLS), Single-valued neutrosophic set (SVNS), Single-valued trapezoidal neutrosophic set (SVTNS), Uncertain linguistic set (ULS).

of λ. In other words, it will not obtain a convincing result. It is easily known that the optimal alternative and corresponding the ordering are same as the results of Zhang *et al.* [22], Zhao *et al.* [24], Ye [27], [52], Liu and Tang [29], Liu and You [31], Ye [51] (G_{WINN3}, G_{WINN4} ($\lambda = 0$)) and Mondal *et al.* [53].

With regard to aggregation functions, only just the aggregation operators [31] take the relevance of the attributes into consideration. In the sake of better distinguishing the features of existing interval neutrosophic aggregation operators, we make a summary of them presented in Table 6. According to Table 6, we can see the developed aggregation operators are based on RWMSM and RWDMSM operators with an argument k. Therefore, the developed aggregation operators (INRWMSM and INRWDMSM) are more common and more agile than some existing aggregation operators. Meanwhile, they can take the interrelationship of the multiple attributes into consideration for dealing with MADM issues.

With respect to better comparison with certain MSM operators in different indeterminate circumstance [35]–[46], we make an overview of them shown in Table 7.

According to Table 7, we can find some existing WMSM operators do not have the peculiarity of idempotency. Moreover, the WMSM cannot degrade to the MSM when their weights information are equal. That is to say, it lost the peculiarity of reducibility.

VI. CONCLUSION

The main contributions can be stated and summarized as follows:

(1) Two bran-new interval neutrosophic aggregation operators are developed (INRWMSM operator and INRWDMSM operator).

(2) Some interesting characters such as monotonicity, commutativity, idempotency and boundedness are explored in detailed. Some existing MSM operators in different indeterminate circumstance [35]–[46] fail to possess the features of idempotency and reducibility (Table 7).

(3) Two algorithms for dealing interval neutrosophic decision making problems by INRWMSM and INRWDMSM operators are presented. The sensitivity analysis of the argument k on the ordering is investigated in detailed (Figs. 2 and 3). Compared with the existing interval neutrosophic MADM algorithms (Table 5), are (i) they have no division by zero issue [48]–[50]; (ii) they have no unauthentic problem [51].

In the future, we will employ the INRWMSM operator and INRWDMSM operator in other domains such as cancer classification [54]. In addition, we will also take RWMSM and RWDMSM operators into different uncertain environment [55]–[59].

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