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Interval target-based VIKOR method supported on interval distance and preference degree for machine selection

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ABSTRACT

By considering target values for attributes in addition to beneficial and non-beneficial attributes, a traditional MADM technique is converted to a comprehensive form. In many machine selection problems, some attributes have given target values. The target value regarding a machine attribute can be reported as a range of data. Some target-based decision-making methods have recently been developed; however, a research gap exists in the area. For example, fuzzy axiomatic design approach presents a target-based decision-making supported on common area of membership functions of alternative ratings and target values of attributes. However, it has detects on finding a complete ranking because of probable infinite values of assessment index. Two target-based VIKOR models with interval data exist in the literature; however, the target values of attributes or ratings of alternatives on attributes are crisp numbers in the models and their formulations may have some limitations. The present paper tries to fill the gap by developing the VIKOR method with both interval target values of attributes and interval ratings of alternatives on attributes. Moreover, we attempt to utilize the power of interval computations to minimize degeneration of uncertain information. In this regard, we employ interval arithmetic and introduce a new normalization technique based on interval distance of interval numbers. We use a preference matrix to determine extremum and rank interval numbers. Two machine selection problems concerning punching equipment and continuous fluid bed tea dryer are solved employing the proposed method. Preference-degree-based ranking lists are formed by calculating the relative degrees of preference for the arranged assessment values of the candidate machines. The resultant rankings for the problems are compared with the results of fuzzy axiomatic design approach and the interval target-based MULTIMOORA method and its subordinate parts.

1. Introduction

The selection of suitable machine is a crucial decision that leads to a streamlined production environment. Engineers often encounter a pool of candidate machines for selection. A useful alternative may be ignored if machines are only chosen based on experience. A multiple attribute decision making (MADM) method can make a framework for the process of machine selection. Several often conflicting attributes must be considered in the selection process of the best machine. In the traditional MADM methods, only beneficial and non-beneficial attributes exist. For example, cost and machine dimensions often are non-beneficial in most machine selection problems. However, other attributes like safety and user friendliness must be maximized. A number of machine selection problems are more complex. That is, given target values are desired for some attributes. These target values can be crisp numbers or represented as interval, gray, fuzzy, or rough sets. For instance, a given target value may be considered for cost and speed of machines. The target values may be variable as a range of data in some practical cases (Çakır, 2016; Kulak, 2005). The application of target-based MADM techniques is not only restricted to machine selection. In many material selection problems, the chosen materials for a product should be compatible with other materials available in the system. Therefore, given target values are considered for material properties to ensure compatibility between materials (Farag, 2013). For example, a target value for the thermal expansion coefficient is important in the selection process of electrical insulating materials (Jahan et al., 2012). Density and elastic modulus can also be regarded as target-based attributes to have a compatible design. These two material properties are especially important to select suitable biomaterials for implants and prostheses (Bahraminasab et al., 2014; Hafezalkotob and Hafezalkotob, 2015; Jahan and Edwards, 2013b). Generally, the target-based MADM approaches can be regarded as comprehensive

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forms of the traditional MADM methods. Because in target-based decision-making, all kinds of attributes (beneficial, non-beneficial, and target-based attributes) are considered.

Target-based MADM techniques can be generally divided into two categories on the basis of "distance" between alternatives ratings and target values of attributes or "common area" of membership functions of alternatives ratings and target values of attributes. In the first category, a normalization technique is used based on distance between alternatives ratings and target values of attributes. These approaches are named as MADM methods with target-based attributes in the literature. The majority of the studies in this category have focused on the field of material selection process. The fuzzy axiomatic design (FAD) method and its extensions comprise the second category. In this group, information content is obtained based on Suh entropy. In this context, alternatives ratings and target values of attributes are called system and design ranges, respectively. In FAD approach, the common area is the intersection of the areas under membership functions of system and design ranges. Recently, the risk-based fuzzy axiomatic design (RFAD) approach has been developed to solve some real-world decision-making problems (Gören and Kulak, 2014; Kulak et al., 2015; Hafezalkotob and Hafezalkotob, 2016b). The RFAD approach has the ability to model the problems in which the alternatives ratings have some risks regarding their attributes.

The compromise ranking method also named as vlse kriterijumska optimizacija kompromisno resenje (VIKOR) – in Serbian is based on an aggregating function (L_p − metric). The VIKOR method uses L_1 and L_∞ (Opricovic and Tzeng, 2004). Crisp target-based extensions of the method have been previously discussed in several studies (Bahraminasab and Jahan, 2011; Bahraminasab et al., 2014; Cavallini et al., 2013; Jahan, 2012; Jahan and Edwards, 2013b; Jahan et al., 2011; Liu et al., 2014). Only two interval target-based VIKOR models exist in the literature that are not comprehensive (Jahan and Edwards, 2013a; Zeng et al., 2013). In this paper, we develop the VIKOR approach for target-based decision making with interval data to choose appropriate machines. The ratings are normalized based on the concept of interval distance of interval numbers. Moreover, the concept of the preference degree of interval numbers is used for performing comparison as well as finding extremum and ranking. Thus, we try to reduce degenerating interval numbers by employing all capacities of interval computations.

The remainder of the paper has been arranged as follows. A classified literature survey and description of the research gap are presented in Section 2. We introduce the crisp target-based VIKOR method in Section 3. The principles and computations of interval numbers are explained in Section 4. The developed interval target-based VIKOR method and its algorithm are described in Section 5. We discuss two practical machine selection problems in different industrial areas in Section 6. Concluding remarks and some directions for future research are mentioned in Section 7.

2. literature review

2.1. Survey on applications of MADM techniques in machine selection

Various MADM methods have been previously employed for the process of machine selection. Wang et al. (2000) evaluated appropriate machines in a flexible manufacturing cell utilizing a novel fuzzy MADM method. Kulak (2005) employed a decision support system and the FAD approach to choose material handling equipment. Kulak et al. (2005) employed the FAD technique for a punching machine selection problem. Aghdaie et al. (2013) consolidated step-wise weight assessment ratio analysis (SWARA) and complex proportional assessment with gray relations (COPRAS-G) to rank candidate alternatives of machine tools. Chakraborty and Zavadskas (2014) employed the weighted aggregated sum product assessment (WASPAS) to tackle several manufacturing decision-making problems including electroplating machines and industrial robots. Ada et al. (2014) utilized an integrated model based on the technique for order preference by similarity to ideal solution (TOPSIS) and goal programming approach under fuzzy environment in a machine selection problem. Chakraborty et al. (2015) applied the WASPAS technique to select machines in a flexible manufacturing cell. Nguyen et al. (2015) created a hybrid model based on the fuzzy analytic hierarchy process (FAHP) and the fuzzy COPRAS (F-COPRAS) to evaluate a machine tool selection problem. Ozfirat (2015) exploited the FANP method to choose suitable tunneling machine. Kumru and Kumru (2015) also employed the FANP technique to decide on the appropriate 3D coordinate-measuring machine. Khandekar and Chakraborty (2015) utilized the principles of the FAD approach to rank material handling equipment. Ertuɪrul and Öztaş (2015) applied the multi-objective optimization on the basis of ratio analysis (MOORA) technique to choose sewing machine. Özceylan et al. (2016) applied a hybrid model based on the fuzzy analytic network process (FANP) and the preference ranking organization method for enrichment evaluations (PROMETHEE) to select a CNC router machine. Çakır (2016) used an combinatory approach supported on the fuzzy simple multi-attribute rating technique (SMART) and the weighted fuzzy axiomatic design (WFAD) method to find the best continuous fluid bed tea dryer. Wu et al. (2016) developed a multi-criteria group decision-making approach supported on the VIKOR technique to discover a suitable CNC machine tool.

2.2. Survey on the target-based MADM methods

Some researchers have studied target-based MADM techniques on the basis of distance between alternatives ratings and target values of attributes. Zhou et al. (2006) developed a target-based norm to construct a composite environmental index to compare various MADM methods. Jahan et al. (2011) presented a target-based VIKOR approach to choose the best material for a rigid pin related to hip prosthesis. Bahraminasab and Jahan (2011) used the target-based VIKOR to select an appropriate material for the femoral component of knee replacement. Jahan et al. (2012) developed a target-based normalization technique for TOPSIS model. Jahan (2012) compared the results of a goal programming model and the target-based VIKOR for a material selection problem of hip implant. Zeng et al. (2013) proposed a normalization formula based on the distance to target values to extend the VIKOR method for application in healthcare management. Jahan and Edwards (2013a) extended the VIKOR approach with both target values of attributes and interval ratings. Jahan and Edwards (2013b) developed the target-based TOPSIS and VIKOR methods utilizing the integrated weights of attributes. Liu et al. (2014) consolidated the target-based VIKOR and DEMATEL-based ANP methods to choose bush material for the design of a split journal bearing. Hafezalkotob and Hafezalkotob (2015) employed an exponential norm and the integrated weights of attributes to derive a target-based modified MOORA (MULTIMOORA) model for biomaterial selection. Jahan and Edwards (2015) reviewed the applications of target-based norms in decision making models. Aghajani Mir et al. (2016) employed the target-based TOPSIS and VIKOR techniques to evaluate municipal solid waste management methods. Hafezalkotob and Hafezalkotob (2016a) tackled two biomaterial selection problems employing the interval target-based MULTIMOORA technique.

Many researchers have developed target-based MADM techniques on the basis of common area of membership functions of alternatives ratings and target values of attributes. The FAD approach and its extensions constitute this group. Kulak and Kahraman (2005) developed the FAD method. Kulak et al. (2005) added weights of attributes to the FAD model. Kahraman and Çebı (2009) improved the FAD method to solve decision-making problems with hierarchical structures. Their developed method is called hierarchical fuzzy axiomatic design (HFAD) approach. Kulak et al. (2015) employed the FAD method considering risk factors, i.e., the RFAD, to tackle a decision-making problem regarding the selection of medical imaging devices. Some applications of the FAD approach are surveyed in the study of Kulak et al. (2010).

2.3. Survey on the interval MADM methods

A number of MADM approaches have been extended using interval numbers. Pan et al. (2000) employed the linear additive utility function and composite utility variance to generate an interval MADM method. Jahanshahloo et al. (2006) suggested an extension of TOPSIS to tackle decision making problems with interval numbers. Jahanshahloo et al. (2009) formulated an interval TOPSIS model based on interval efficiency. Sayadi et al. (2009) extended the VIKOR technique considering interval data. Fa-Dong et al. (2010) proposed a new interval MADM method by considering loss aversion. Yue (2011) derived an interval TOPSIS model for group decision making. Sayadi and Makui (2012) presented a novel interval-based model of elimination and choice expressing the reality (ELECTRE). Dymova et al. (2013) introduced a direct interval extension of TOPSIS method based on the concept of the distance between midpoints of intervals. Kracka and Zavadskas (2013) derived an interval MULTIMOORA method for an effective selection of structural panels. Dou et al. (2014) developed the VIKOR and TOPSIS algorithms with interval numbers based on reciprocal judgment matrix. Stanujkic et al. (2014) developed the ratio system part of the MOORA method with interval information to choose a grinding circuit. Hafezalkotob et al. (2016) introduced an extended MULTIMOORA approach with interval data by using interval arithmetic and defining a preference matrix.

2.4. Survey on developments and applications of the VIKOR method

The VIKOR method was suggested as a tool to implement within MADM by Opricovic (1998). This approach has been utilized in a wide range of applications such as contractor selection (Vahdani et al., 2013), decision making in healthcare management (Zeng et al., 2013), decision-making on bank investment plans (Hajiagha et al., 2014), project selection (Ghorabaee et al., 2015; Shouzhen and Su, 2015), personnel selection (Liu et al., 2015), evaluating flood vulnerability (Lee et al., 2015), material selection (Anojkumar et al., 2015; Yazdani and Payam, 2015), evaluating the operating performance of semiconductor companies (Hsu, 2015), choosing a tunnel security door (Vučijak et al., 2015), location selection (Bausys and Zavadskas, 2015), evaluating eco-industrial thermal power plants (Li and Zhao, 2016), and evaluating the efficiency of bank branches (Tavana et al., 2016). Based on the concept of neutrosophic (Bausys and Zavadskas, 2015), fuzzy (Vahdani et al., 2013), interval (Sayadi et al., 2009), linguistic (Liu et al., 2013), and stochastic (Tavana et al., 2016) data, a number of extensions have been generated for the VIKOR method. Some researchers have discussed the extensions and applications of the VIKOR approach (Yazdani and Graeml, 2014; Gul et al., 2016; Mardani et al., 2016).

2.5. Research gap

Two group of researchers have analyzed the target-based VIKOR models with interval data; however, the target values of attributes or ratings of alternatives on attributes are crisp numbers in the models and their methodology may have some defects. The first study was conducted by Jahan and Edwards (2013a). In their work, a target-based VIKOR method with interval ratings of alternatives on attributes and crisp target values of attributes was suggested. The target-based norm in their method is supported on Euclidian distance. The second study was undertaken by Zeng et al. (2013). In their study, a target-based VIKOR method with crisp ratings of alternatives on attributes and interval target values of attributes was developed. The interval target values in their model have a normalized distribution function. In the present paper, we introduce a comprehensive interval target-based VIKOR method. Our novelties comparing these two related studies are as follows:

- We consider the interval target values of attributes along with interval ratings of alternatives on attributes.
- We propose a novel interval target-based norm based on the idea of "interval distance" of interval numbers. For this purpose, we define a new formula for interval distance.
- We employ a preference matrix to find extremum and ranking of interval numbers.
- We generate preference-degree-based ranking lists by computing the relative preference degrees for the arranged assessment values of the alternatives.

3. The crisp target-based VIKOR method

An MADM problem can be expressed by a decision matrix **X**. The rating of the decision matrix, i.e., x_{ij}, denotes the response of alternative A_i to attribute a_j, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. Relative weight, i.e., w_j, can be considered for each attribute. Weights of attributes satisfy $\sum_{j=1}^{n} w_j = 1$. The beneficial and non-beneficial attributes are only considered in the traditional MADM methods. However, the necessity of reaching a given target value of an attribute in some practical cases demands modeling the target-based MADM approaches (Jahan et al., 2011). A typical crisp target-based MADM problem can be represented as follows:

$$\begin{array}{c} \quad\; a_1 \quad \cdots \quad a_j \quad \cdots \quad a_n \\ \mathbf{T} = \begin{bmatrix} t_1 & \cdots & t_j & \cdots & t_n \end{bmatrix} \\ \mathbf{X} = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ \vdots & & \vdots & & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & & \vdots & & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix} \begin{matrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} \\ \mathbf{W} = \begin{bmatrix} w_1 & \cdots & w_j & \cdots & w_n \end{bmatrix} \end{array}$$

The target (also named as goal or the most favorable) value, i.e., t_j, for an attribute generally is the maximum or the minimum of the ratings of alternatives on that attribute or may be defined as a given value. The target values of attributes are formulated as:

$$t_j = \begin{cases} \max_i x_{ij}, & \text{if } j \in I, \\ \min_i x_{ij}, & \text{if } j \in J, \\ g_j, & \text{if } j \in K, \end{cases} \tag{2}$$

in which I, J, and K are associated with beneficial, non-beneficial, and target-based attributes, respectively. g_j represents the goal value for each target-based attribute considered by decision-makers. The optimal alternative and rankings can be found by taking the following steps:

Step 1. The crisp target-based normalization

The decision matrix has to be normalized to obtain comparable and dimensionless values. Various norms have been developed for crisp target-based MADM. Jahan and Edwards (2015) have reviewed and compared the norms. A crisp target-based ascending norm can be defined as (Liu et al., 2014):

$$f_{ij} = \frac{|x_{ij} - t_j|}{\max\left\{\max_i x_{ij},\ t_j\right\} - \min\left\{\min_i x_{ij},\ t_j\right\}}. \tag{3}$$

The normalization technique is supported on the concept of "Euclidean distance" of a rating from its associated target value. Eq.

(3) is an ascending norm because the value of the norm raises with the increase of distance from the target value. Based on Eq. (3), smaller normalized rating conveys less distance to target values. Norm f_{ij} also applies to the traditional MADM methods in which only beneficial and non-beneficial attributes are considered.

Step 2. Determine the crisp average group utility and the crisp maximal regret

The crisp average group utility S_i and the crisp maximal regret R_i are calculated as follows (Liu et al., 2014):

$$S_i = \sum_{j=1}^{n} (w_j f_{ij}), \tag{4}$$

$$R_i = \max_j (w_j f_{ij}). \tag{5}$$

Step 3. Calculate the assessment index of the crisp target-based VIKOR method

The assessment index of the crisp target-based VIKOR method, i.e., Q_i, can be specified for each alternative as (Jahan et al., 2011):

$$Q_i = \begin{cases} \frac{R_i - R^+}{R^- - R^+}, & \text{if } S^- = S^+, \\ \frac{S_i - S^+}{S^- - S^+}, & \text{if } R^- = R^+, \\ \frac{S_i - S^+}{S^- - S^+} v + \frac{R_i - R^+}{R^- - R^+}(1-v), & \text{otherwise}, \end{cases} \tag{6}$$

in which $S^+ = \min_i S_i$, $S^- = \max_i S_i$, $R^+ = \min_i R_i$, and $R^- = \max_i R_i$. v is defined as the importance coefficient for the strategy of "the majority of attributes" (or "the maximum group utility"), while $1 - v$ is the importance coefficient of "the individual regret". The value of v is in the range of 0–1 and these strategies can be compromised by $v = 0.5$.

Step 4. Find the optimal alternative and generate the ranking list

The optimal alternative based on the crisp target-based VIKOR method is determined by minimizing the assessment values, i.e., Q_i:

$$A^+_{\text{T-VIKOR}} = \left\{ A_i \middle| \min_i Q_i \right\}, \tag{7}$$

in which "T-VIKOR" stands for "target-based VIKOR". The assessment values are organized in ascending order to generate the ranking list of the crisp target-based VIKOR method.

4. Interval numbers

Uncertainty of data can be mathematically represented by various forms like interval, fuzzy, gray, linguistic, or stochastic numbers. Interval numbers are suitable for utilization in MADM models in uncertain environment because many quantities in real-world applications are reported as a range of information. Thus, they can inherently be regarded as interval data. Sections 4.1–4.4 present the required mathematics for deriving the proposed method.

4.1. Mathematical preliminaries of intervals

Basic definitions regarding interval mathematics are as follows (Trindade et al., 2010):

D1 (Interval). Let y^L and $y^U \in \mathbb{R}$ be such that $y^L \le y^U$. The set $\bar{y} = \{y \in \mathbb{R} | \ y^L \le y \le y^U\}$ is named "a real interval" and also represented as $\bar{y} = [y^L, y^U]$. The set of all real intervals is shown by $\mathbb{IR}$.

D2 (Inclusion order). Let $\bar{y}$ and $\bar{z} \in \mathbb{IR}$. $\bar{y} \subseteq \bar{z}$ if only if $y^L \ge z^L$ and $y^U \le z^U$.

D3 (Kulisch–Miranker order). Let $\bar{y}$ and $\bar{z} \in \mathbb{IR}$. $\bar{y} \le \bar{z}$, if $y^L \le z^L$ and $y^U \le z^U$. Thus, $\bar{y} = \bar{z}$, if $y^L = z^L$ and $y^U = z^U$.

D4 (Positive, negative, and non–negative). An interval, $\bar{y}$, is positive if $y^L > 0$, negative if $y^U < 0$, and non-negative if $y^L \ge 0$.

D5 (Midpoint of an interval). Let $\bar{y} \in \mathbb{IR}$. The midpoint of $\bar{y}$ is defined as:

$$pm(\bar{y}) = y^M = \frac{y^L + y^U}{2}. \tag{8}$$

4.2. Interval arithmetic algebra

A stream of studies have discussed interval arithmetic and its applications (Alefeld and Herzberger, 1983; Hickey et al., 2001; Kearfott and Kreinovich, 1996). Based on Moore interval arithmetic, if $\bar{y} = [y^L, y^U]$ and $\bar{z} = [z^L, z^U]$ are two non-negative real intervals and k is a non-negative real number, then (Moore, 1979):

$$\bar{y} + \bar{z} = [y^L + z^L, \ \ y^U + z^U], \tag{9}$$

$$\bar{y} - \bar{z} = [y^L - z^U, \ \ y^U - z^L], \tag{10}$$

$$\bar{y} \cdot \bar{z} = [y^L \cdot z^L, \ \ y^U \cdot z^U], \tag{11}$$

$$\bar{y}/\bar{z} = [y^L/z^U, \ \ y^U/z^L] \text{with } z^L \text{and} z^U \ne 0, \tag{12}$$

$$k \cdot \bar{y} = [k \cdot y^L, \ \ k \cdot y^U]. \tag{13}$$

4.3. Distance between intervals

Trindade et al. (2010) introduced the idea of "interval distance" between interval numbers $\bar{y} = \{y \in \mathbb{R} | \ y^L \le y \le y^U\}$ and $\bar{z} = \{z \in \mathbb{R} | \ z^L \le z \le z^U\}$ as:

$$\bar{d}(\bar{y}, \bar{z}) = [\inf(|y - z|: y \in \bar{y} \ \text{and} z \in \bar{z}), \ \ \sup(|y - z|: y \in \bar{y} \ \text{and} z \in \bar{z})] \tag{14}$$

The interval distance, i.e., Eq. (14), can have multiple modes (Trindade et al., 2010):

– If $\bar{y} \le \bar{z}$ and $\bar{y} \cap \bar{z} = \varnothing$, then:

$$\bar{d}(\bar{y}, \bar{z}) = [(z^L - y^U), \ \ (z^U - y^L)]. \tag{15}$$

– If $\bar{y} \le \bar{z}$ and $\bar{y} \cap \bar{z} \ne \varnothing$, then:

$$\bar{d}(\bar{y}, \bar{z}) = [0, \ \ (z^U - y^L)]. \tag{16}$$

– If $\bar{y} \subseteq \bar{z}$, then:

$$\bar{d}(\bar{y}, \bar{z}) = [0, \ \ \max\{(y^U - z^L), \ \ (z^U - y^L)\}]. \tag{17}$$

All modes of the interval distance can be integrated into the following equation:

$$\bar{d}(\bar{y}, \bar{z}) = \begin{cases} [\min\{|y^L - z^U|, |y^U - z^L|\}, \ \ \max\{|y^L - z^U|, |y^U - z^L|\}], \\ \quad \text{if } \bar{y} \cap \bar{z} = \varnothing, \\ [0, \ \ \max\{|y^L - z^U|, |y^U - z^L|\}], \\ \quad \text{if } \bar{y} \cap \bar{z} \ne \varnothing. \end{cases} \tag{18}$$

However, Eq. (18) is a general formula for interval distance and may contain some defects in practice. For example, the formula is not sensitive to the degree of intersection and also inclusion of two interval numbers. To correct the defects, we improve Eq. (18) as follows:

$$\bar{d}^{*}(\bar{y},\bar{z}) = \begin{Bmatrix} [\min\{|y^L - z^U|, |y^U - z^L|\}, & |y^M - z^M|], & \text{if } \bar{y}\cap\bar{z} = \varnothing, \\ [0\ , & |y^M - z^M|], & \text{if } \bar{y}\cap\bar{z} \neq \varnothing. \end{Bmatrix} \quad (19)$$

In Section 5, we use $\bar{d}^{*}$ for derivation of the proposed method. We clarify the difference between $\bar{d}$ and $\bar{d}^{*}$ through discussing some examples in the section.

Different formulas have been developed for measuring the distance between real interval numbers as "a crisp value" (Dymova et al., 2013; Khezerloo et al., 2011; Moore et al., 2009). However, we believe that a more inclusive form for "the crisp distance" of two interval numbers can be defined by finding the midpoint of $\bar{d}^{*}(\bar{y},\bar{z})$, i.e., Eq. (19), as:

$$d^{*}(\bar{y},\bar{z}) = pm(\bar{d}^{*}(\bar{y},\bar{z})) = \begin{cases} \frac{\min\{|y^L - z^U|, |y^U - z^L|\} + |y^M - z^M|}{2}, & \text{if } \bar{y}\cap\bar{z} = \varnothing, \\ \frac{|y^M - z^M|}{2}, & \text{if } \bar{y}\cap\bar{z} \neq \varnothing. \end{cases} \quad (20)$$

Eq. (20) can be more robust than the traditional metrics for crisp distance of interval numbers. The reason lies in the fact that $d^{*}(\bar{y},\bar{z})$, i.e., Eq. (20), enjoys two conditional parts whereas all traditional metrics generate the crisp distance of two intervals as one formula without paying attention to the relation (being intersected or not being intersected) of the interval numbers.

4.4. Interval comparison, extremum, and ranking

Comparison of intervals has been analyzed in a number of studies (Levin, 2004; Sevastianov, 2007; Wang et al., 2005a, 2005b). For intervals $\bar{y} = [y^L, y^U]$ and $\bar{z} = [z^L, z^U]$, the preference degree of $\bar{y}$ over $\bar{z}$, denoted by$P(\bar{y} > \bar{z})$, can be defined as follows (Wang et al., 2005a):

$$P(\bar{y} > \bar{z}) = \frac{\max\{0, y^U - z^L\} - \max\{0, y^L - z^U\}}{y^U - y^L + z^U - z^L}. \quad (21)$$

The following principles exist concerning the preference degree (Wang et al., 2005a):

- $P(\bar{z} > \bar{y}) = 1 - P(\bar{y} > \bar{z})$. If $\bar{y} = \bar{z}$ then $P(\bar{y} > \bar{z}) = P(\bar{z} > \bar{y}) = 0.5$.
- If $P(\bar{y} > \bar{z}) > P(\bar{z} > \bar{y})$, then $\bar{y}$ is said to be superior to $\bar{z}$ to the degree of $P(\bar{y} > \bar{z})$, represented by $\bar{y} \overset{P(\bar{y}>\bar{z})}{>} \bar{z}$; If $P(\bar{y} > \bar{z}) = P(\bar{z} > \bar{y}) = 0.5$, then $\bar{y}$ is said to be indifferent to $\bar{z}$, indicated by $\bar{y} \sim \bar{z}$; If $P(\bar{z} > \bar{y}) > P(\bar{y} > \bar{z})$, then $\bar{y}$ is said to be inferior to $\bar{z}$ to the degree of $P(\bar{z} > \bar{y})$, denoted by $\bar{y} \overset{P(\bar{z}>\bar{y})}{<} \bar{z}$.
- If $\bar{y} \leq \bar{z}$ and $\bar{y} \cap \bar{z} = \varnothing$, then $P(\bar{y} \geq \bar{z}) = 0$. If $\bar{y} \leq \bar{z}$ and $\bar{y} \cap \bar{z} \neq \varnothing$, then $0 \leq P(\bar{y} \geq \bar{z}) \leq 0.5$.
- If $\bar{y} \geq \bar{z}$ and $\bar{y} \cap \bar{z} = \varnothing$, then $P(\bar{y} \geq \bar{z}) = 1$. If $\bar{y} \geq \bar{z}$ and $\bar{y} \cap \bar{z} \neq \varnothing$, then $0.5 \leq P(\bar{y} \geq \bar{z}) \leq 1$.

To find extremum (i.e., maximization or minimization) and rank a set of intervals $\{\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_\delta\}$ (δ = the number of intervals), preference degree matrix and preference matrix are defined as follows (Rezaei, 2016):

In Table 1, $P(\bar{y}_\alpha > \bar{y}_\beta)$is the relative preference degree calculated based on Eq. (21):

$$P(\bar{y}_\alpha > \bar{y}_\beta) = \frac{\max\{0, y_\alpha^U - y_\beta^L\} - \max\{0, y_\alpha^L - y_\beta^U\}}{y_\alpha^U - y_\alpha^L + y_\beta^U - y_\beta^L}, \quad (22)$$

in which α and β are the indices denoting the row and column of the preference degree matrix, respectively. If two degenerate interval numbers (crisp numbers) exist successively in the set of intervals, i.e.,$y_\alpha^L = y_\alpha^U$ or $y_\beta^L = y_\beta^U$, a tiny increment (e.g., 1×10^{-6}) is added to the upper boundaries to avoid the zero denominator of Eq. (22).

$P_{\alpha\beta}$ in Table 2 is the relative preference for the set of interval numbers expressed as:

Table 1
Relative preference degrees.

	Relative preference degree					
	$\bar{y}_1$	$\bar{y}_2$	⋯	$\bar{y}_\beta$	⋯	$\bar{y}_\delta$
$\bar{y}_1$	$P(\bar{y}_1 > \bar{y}_1)$	$P(\bar{y}_1 > \bar{y}_2)$	⋯	$P(\bar{y}_1 > \bar{y}_\beta)$	⋯	$P(\bar{y}_1 > \bar{y}_\delta)$
$\bar{y}_2$	$P(\bar{y}_2 > \bar{y}_1)$	$P(\bar{y}_2 > \bar{y}_2)$	⋯	$P(\bar{y}_2 > \bar{y}_\beta)$	⋯	$P(\bar{y}_2 > \bar{y}_\delta)$
⋮	⋮	⋮		⋮		⋮
$\bar{y}_\alpha$	$P(\bar{y}_\alpha > \bar{y}_1)$	$P(\bar{y}_\alpha > \bar{y}_2)$	⋯	$P(\bar{y}_\alpha > \bar{y}_\beta)$	⋯	$P(\bar{y}_\alpha > \bar{y}_\delta)$
⋮	⋮	⋮		⋮		⋮
$\bar{y}_\delta$	$P(\bar{y}_\delta > \bar{y}_1)$	$P(\bar{y}_\delta > \bar{y}_2)$	⋯	$P(\bar{y}_\delta > \bar{y}_\beta)$	⋯	$P(\bar{y}_\delta > \bar{y}_\delta)$

Table 2
Preference matrix for finding the extremum and ranking list of a set of intervals.

	Relative preference						Aggregate preference	Extremum	Ranking
	$\bar{y}_1$	$\bar{y}_2$	⋯	$\bar{y}_\beta$	⋯	$\bar{y}_\delta$			
$\bar{y}_1$	P_{11}	P_{12}	⋯	$P_{1\beta}$	⋯	$P_{1\delta}$	$AP(\bar{y}_1)$	–	rank $(\bar{y}_1)$
$\bar{y}_2$	P_{21}	P_{22}	⋯	$P_{2\beta}$	⋯	$P_{2\delta}$	$AP(\bar{y}_2)$	–	rank $(\bar{y}_2)$
⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮
$\bar{y}_\alpha$	$P_{\alpha1}$	$P_{\alpha2}$	⋯	$P_{\alpha\beta}$	⋯	$P_{\alpha\delta}$	$AP(\bar{y}_\alpha)$	Extremum	rank $(\bar{y}_\alpha) = 1$
⋮	⋮	⋮		⋮		⋮	⋮	⋮	⋮
$\bar{y}_\delta$	$P_{\delta1}$	$P_{\delta2}$	⋯	$P_{\delta\beta}$	⋯	$P_{\delta\delta}$	$AP(\bar{y}_\delta)$	–	rank $(\bar{y}_\delta)$

$$P_{\alpha\beta} = \begin{cases} 1, & \text{if } P(\bar{y}_\alpha > \bar{y}_\beta) > 0.5, \\ 0, & \text{if } P(\bar{y}_\alpha > \bar{y}_\beta) \leq 0.5. \end{cases} \quad (23)$$

To obtain extremum and ranking of the set of interval numbers $\{\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_\delta\}$, the relative preferences of each row of the preference matrix are added to produce the aggregate preference:

$$AP(\bar{y}_\alpha) = \sum_{\beta=1}^{\delta} P_{\alpha\beta}, \quad (24)$$

in which $0 \leq AP(\bar{y}_\alpha) \leq (\delta - 1)$.

The extremum, i.e., the maximum or minimum, of the set of intervals $\{\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_\alpha, \ldots, \bar{y}_\delta\}$ as well as the rank of $\bar{y}_\alpha$in the set can be obtained using the values of aggregate preferences as follows:

$$\max_\alpha \bar{y}_\alpha = \max_\alpha AP(\bar{y}_\alpha), \quad (25)$$

$$\min_\alpha \bar{y}_\alpha = \min_\alpha AP(\bar{y}_\alpha), \quad (26)$$

$$\text{rank}(\bar{y}_\alpha) = \text{rank}(AP(\bar{y}_\alpha)). \quad (27)$$

5. The proposed interval target-based VIKOR method

In this section, the interval target-based VIKOR method is derived based on the crisp model described in Section 3 and the interval mathematics discussed in Section 4. A typical MADM problem with interval target values of attributes and interval ratings can be represented as follows:

$$\begin{matrix} & a_1 & \cdots & a_j & \cdots & a_n \end{matrix}$$

$$\bar{\mathbf{T}} = \begin{bmatrix} \bar{t}_1 & \cdots & \bar{t}_j & \cdots & \bar{t}_n \end{bmatrix}$$

$$\bar{\mathbf{X}} = \begin{bmatrix} \bar{x}_{11} & \cdots & \bar{x}_{1j} & \cdots & \bar{x}_{1n} \\ \vdots & & \vdots & & \vdots \\ \bar{x}_{i1} & \cdots & \bar{x}_{ij} & \cdots & \bar{x}_{in} \\ \vdots & & \vdots & & \vdots \\ \bar{x}_{m1} & \cdots & \bar{x}_{mj} & \cdots & \bar{x}_{mn} \end{bmatrix} \begin{matrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix}$$

The interval rating, i.e., $\bar{x}_{ij} = [x_{ij}^L, x_{ij}^U]$, indicates the response of alternativeA_i to attribute a_j, with interval target value $\bar{t}_j$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. A crisp weight w_j is assigned for each attribute by the decision makers such that $\sum_{j=1}^{n} w_j = 1$. The interval target values of attributes $\bar{t}_j = [t_j^L, t_j^U]$ can be defined as:

$$\bar{t}_j = \begin{cases} \max\limits_i \bar{x}_{ij}, & \text{if } j \in I, \\ \min\limits_i \bar{x}_{ij}, & \text{if } j \in J, \\ \bar{g}_j, & \text{if } j \in K, \end{cases} \tag{29}$$

in which I, J, and K denote the sets of beneficial, non-beneficial, and target-based attributes, respectively. $\bar{g}_j$ shows the interval goal value considered for each target-based attribute assigned by decision-makers. The maximum and minimum of ratings on each attribute are calculated utilizing Eqs. (25) and (26), respectively. By following the subsequent steps, the best alternative and the ranking list are obtained:

Step 1. The interval target-based normalization

We improve the target-based norm, i.e., Eq. (3), as follows:

$$f_{ij}^* = \frac{|x_{ij} - t_j|}{\max\limits_i |x_{ij} - t_j|}. \tag{30}$$

Eq. (30) that is the normalized distance of a rating from the target value can be used as modified norm for target-based MADM problems.

The modified norm, i.e., Eq. (30), can be developed for application in the interval target-based MADM problems using the concept of interval distances of interval numbers. Based on the interval distances $\bar{d}$ and $\bar{d}^*$, i.e., Eqs. (18) and (19), two interval target-based norms are generated as follows:

$$\hat{f}_{ij}^* = \frac{\bar{d}\,(\bar{x}_{ij}, \bar{t}_j)}{\max\limits_i d\,(\bar{x}_{ij}, \bar{t}_j)}, \tag{31}$$

$$\bar{f}_{ij}^* = \frac{\bar{d}^*(\bar{x}_{ij}, \bar{t}_j)}{\max\limits_i d^*(\bar{x}_{ij}, \bar{t}_j)}, \tag{32}$$

in which $d\,(\bar{x}_{ij}, \bar{t}_j) = pm(\bar{d}\,(\bar{x}_{ij}, \bar{t}_j))$ and $d^*(\bar{y}, \bar{z}) = pm(\bar{d}^*(\bar{y}, \bar{z}))$. The crisp distance d and d^* are utilized in the denominator of Eqs. (31) and (32) to avoid zero. However, the values of $\bar{f}_{ij}^*$ are more robust than $\hat{f}_{ij}^*$. To show the difference between $\bar{d}$ and $\bar{d}^*$, we consider some examples as illustrated in Table 3.

The first two examples of Table 3 are the cases of intersection and the last three examples are the cases of inclusion. The first two examples shows that $\bar{d}^*(\bar{x}, \bar{t})$ unlike $\bar{d}\,(\bar{x}, \bar{t})$ is sensitive to the degree of intersection. In these two cases, $\bar{d}\,(\bar{x}, \bar{t})$ equals to $[0, 7]$ while $\bar{d}^*(\bar{x}, \bar{t})$ equals to $[0, 3]$ and $[0, 1]$, respectively. The last three examples indicate three different interval ratings all included in an identical interval target value. In the all three cases, $\bar{d}\,(\bar{x}, \bar{t})$ equals to $[0, 5]$ whereas $\bar{d}^*(\bar{x}, \bar{t})$ is variable.

In target-based decision-making under uncertain environment, the degree of intersection of interval rating $\bar{x}$ and interval target value $\bar{t}$ is important that shows the degree of satisfying the favorable values. Thus, the insensitivity of $\bar{d}\,(\bar{x}, \bar{t})$ to the degree of intersection causes some limitations for its real-world applications. In this regard, we employ $\bar{f}_{ij}^*$, i.e., Eq. (32), for computing measures of Step 2. Also, we utilize $\bar{d}^*$, i.e., Eq. (19), to drive the assessment index of Step 3. We discuss the differences between the values of $\bar{f}_{ij}^*$and $\hat{f}_{ij}^*$ for the first practical case in Subsection 6.1.

Step 2. Determine the interval average group utility and the interval maximal regret

The interval average group utility $\bar{S}_i$ is the sum of the products of normalized interval ratings of alternative A_i and the associated weights on all attributes:

$$\bar{S}_i = \sum_{j=1}^{n} \left(w_j \bar{f}_{ij}^* \right). \tag{33}$$

The interval maximal regret $\bar{R}_i$ is calculated using Eq. (25):

$$\bar{R}_i = \max_j \left(w_j \bar{f}_{ij}^* \right). \tag{34}$$

Step 3. Calculate the assessment index of the interval target-based VIKOR method

The assessment index of the interval target-based VIKOR method, i.e., $\bar{Q}_i$, can be determined for each alternative utilizing Eqs. (18) and (20):

$$\bar{Q}_i = \begin{cases} \dfrac{\bar{d}^*(\bar{R}_i, \bar{R}^+)}{d^*(\bar{R}^-, \bar{R}^+)}, & \text{if } \bar{S}^- = \bar{S}^+, \\ \dfrac{\bar{d}^*(\bar{S}_i, \bar{S}^+)}{d^*(\bar{S}^-, \bar{S}^+)}, & \text{if } \bar{R}^- = \bar{R}^+, \\ \dfrac{\bar{d}^*(\bar{S}_i, \bar{S}^+)}{d^*(\bar{S}^-, \bar{S}^+)} v + \dfrac{\bar{d}^*(\bar{R}_i, \bar{R}^+)}{d^*(\bar{R}^-, \bar{R}^+)} (1 - v), & \text{otherwise}, \end{cases} \tag{35}$$

in which $\bar{S}^+ = \min\limits_i \bar{S}_i$, $\bar{S}^- = \max\limits_i \bar{S}_i$, $\bar{R}^+ = \min\limits_i \bar{R}_i$, and $\bar{R}^- = \max\limits_i \bar{R}_i$. These maximum and minimum values can be calculated applying Eqs. (25) and (26), respectively.

Step 4. Find the optimal alternative and generate the ranking list

The optimal alternative based on the interval target-based VIKOR method is determined by minimizing the assessment values, i.e., $\bar{Q}_i$, through Eq. (26):

$$A^*_{\text{IT-VIKOR}} = \{A_i | \min_i \bar{Q}_i\}, \tag{36}$$

in which "IT-VIKOR" is an abbreviation for "interval target-based VIKOR". The assessment values are organized in ascending order utilizing Eq. (27) to generate the ranking list of the proposed method:

$$\text{rank}\,(\bar{Q}_i) = \text{rank}\,(AP\,(\bar{Q}_i)). \tag{37}$$

Afterwards, the rankings can be developed to a preference-degree-based ranking list by computing the relative preference degrees for the arranged assessment values using Eq. (21). Derivation of the proposed method is summarized as a solution algorithm (Fig. 1).

6. Two case studies on machine selection

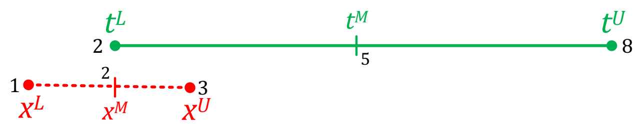
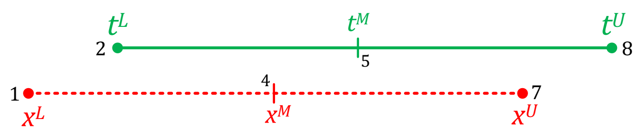
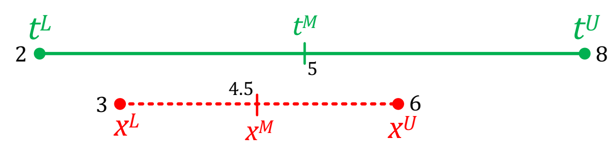
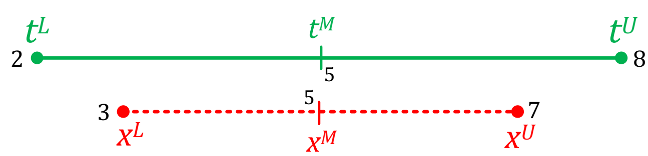
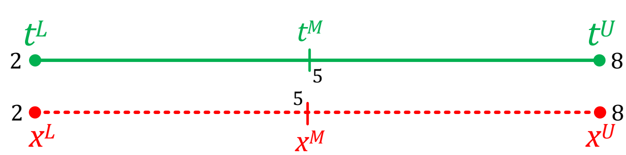
In addition to beneficial and non-beneficial attributes, target values for attributes are commonly considered in machine selection problem. To show the importance of target-based decision making in machine selection process and application of the proposed approach, we examine two real-world examples.

6.1. Example 1: punching machine

The modern punching machines have been computerized and operate with high-speed. Recent advances in technologies regarding punching machines indicate the importance of selection of appropriate alternative. In this example, we discuss a problem regarding choosing the appropriate punching machine to produce electronic parts. This practical case has previously been addressed employing the FAD approach (Kulak et al., 2005). Six candidate punching machines (m=6) along with nine attributes (n=9) have been considered for the practical case as listed in Table 4 (Kulak et al., 2005). The units, weights, and target values of the attributes are also shown in Table 4. Table 5 shows the interval decision matrix for the problem. The arrays of Table 5 are the interval ratings of the candidate machines on their attributes.

The interval ratings are normalized using Eqs. (31) and (32) to generate dimensionless values of Tables 6 and 7. Table 8 shows Suh information contents calculated in the FAD approach for this punching machine selection problem by Kulak et al. (2005). The values of Tables 7 and 8 show a same pattern whereas the values of Table 6 are not

Table 3
Examples for showing the differences between the interval distance suggested by Trindade et al. (2010) and the proposed interval distance, i.e., $\bar{d}$ and $\bar{d}^*$.

$\bar{x}$	$\bar{t}$	$[x^L, x^U]$ versus $[t^L, t^U]$	$\bar{d}\,(\bar{x}, \bar{t})$	$\bar{d}^*(\bar{x}, \bar{t})$
[1, 3]	[2, 8]		[0, 7]	[0, 3]
[1, 7]	[2, 8]		[0, 7]	[0, 1]
[3, 6]	[2, 8]		[0, 5]	[0, 0.5]
[3, 7]	[2,8]		[0, 5]	[0, 0]
[2, 8]	[2,8]		[0, 5]	[0, 0]

corresponding to those of Tables 7 and 8. The issue can be better realized considering the differences between $\bar{d}$ and $\bar{d}^*$ as described in Section 5 as well as the concept of Suh information content. To clarify the similarities and differences between the values of Tables 6–8, we examine some examples (I_{ij} represents the information content of $\bar{x}_{ij}$regarding $\bar{t}_j$):

- Computation of $\hat{f}_{19}^*$ and $\overline{f}_{19}^*$, as well as comparison with I_{19} (the inclusion case)

$$\hat{f}_{19}^* = \frac{\bar{d}\,(\bar{x}_{19}, \bar{t}_9)}{\max_i d\,(\bar{x}_{i9}, \bar{t}_9)} = \frac{\bar{d}\,([16, 20], [16, 20])}{\max_i d\,(\bar{x}_{i9}, [16, 20])} = \frac{[0, 4]}{4} = [0, 1],$$

$$\overline{f}_{19}^* = \frac{\bar{d}^*(\bar{x}_{19}, \bar{t}_9)}{\max_i d^*(\bar{x}_{i9}, \bar{t}_9)} = \frac{\bar{d}^*([16, 20], [16, 20])}{\max_i d^*(\bar{x}_{i9}, [16, 20])} = \frac{[0, 0]}{1.5} = [0, 0],$$

$I_{19} = 0$, (Kulak et al., 2005).

- Computation of $\hat{f}_{17}^*$, $\hat{f}_{27}^*$, $\overline{f}_{17}^*$, and $\overline{f}_{27}^*$, as well as comparison with I_{17}, and I_{27} (the intersection case)

$$\hat{f}_{17}^* = \frac{\bar{d}\,([0, 108], [70, 110])}{\max_i d\,(\bar{x}_{i7}, [70, 110])} = \frac{[0, 110]}{55} = [0, 2],$$

$$\hat{f}_{27}^* = \frac{\bar{d}\,([0, 97], [70, 110])}{\max_i d\,(\bar{x}_{i7}, [70, 110])} = \frac{[0, 110]}{55} = [0, 2],$$

$$\overline{f}_{17}^* = \frac{\bar{d}^*([0, 108], [70, 110])}{\max_i d^*(\bar{x}_{i7}, [70, 110])} = \frac{[0, 36]}{24.5} = [0, 1.47],$$

$$\overline{f}_{27}^* = \frac{\bar{d}^*([0, 97], [70, 110])}{\max_i d^*(\bar{x}_{i7}, [70, 110])} = \frac{[0, 41.5]}{24.5} = [0, 1.69],$$

$I_{17} = 1.507$, (Kulak et al., 2005), $I_{27} = 1.845$, (Kulak et al., 2005).

These comparisons show that the normalized ratings using $\overline{f}_{ij}^*$are more robust than those obtained employing $\hat{f}_{ij}^*$. Besides, the normalized ratings using $\overline{f}_{ij}^*$indicate a similar routine comparing Suh information contents; however, infinity does not appears in the normalized decision matrix obtained using $\overline{f}_{ij}^*$, i.e., Table 7, in contrast with the information contents matrix, i.e., Table 8. That is, when $\bar{x}_{ij}$ and $\bar{t}_j$ are not intersected, Suh information content equals to infinity whereas $\overline{f}_{ij}^*$

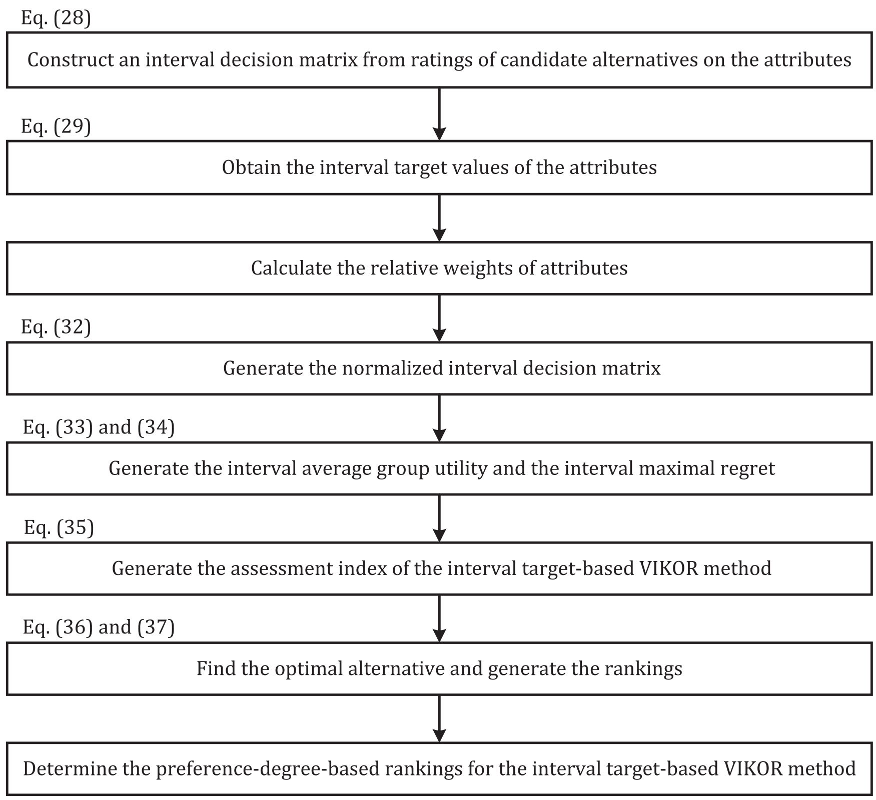


Fig. 1. The algorithm of the proposed IT-VIKOR method.

Table 4
Alternatives and attributes for selection of appropriate punching machine (Example 1).

Candidate machines (alternatives)		Attributes			
Machine name	Machine ID[a]	Attribute name	Unit	Weight	Target value
Punch-A	M1	Fixed costs per hour (FC)	€/h	0.8	[10, 14]
Punch-B	M2	Variable costs per hour (VC)	€/h	0.8	[2, 4]
Punch-C	M3	Equivalent costs of standard tools per hour (ST)	€/h	0.8	[2, 4]
Punch-D	M4	Length of sheet size (L)	mm	0.2	[1200, 2540]
Punch-E	M5	Thickness of sheet metal (T)	mm	0.2	[3, 8]
Punch-F	M6	Number of strokes for 25 mm pitch size sheet metal (NS)	–	0.2	[190, 445]
		Simultaneous axis speed (XY)	m/min	0.2	[70, 110]
		Tool rotation speed (SR)	rpm	0.2	[50, 180]
		Sufficiency of service (SS)	–	0.2	[16, 20]

[a] Machine ID: Machine identification number.

equals to its peak number [0, 2].

Tables 9 and 10 exhibit the measures, assessment indices, and ranking lists of the unweighted and weighted IT-VIKOR solutions, respectively. The interval average group utility $\overline{S}_i$, the interval maximal regret $\overline{R}_i$, and the assessment index $\overline{Q}_i$ are respectively determined using Eqs. (33)–(35). v coefficient in Eq. (35) is assumed to be 0.5. The best machine based on of the unweighted and weighted IT-VIKOR models employing Eq. (36) are $A^{*\,(\text{unweighted})}_{\text{IT-VIKOR}} =$ M3, i.e., Punch-C, and $A^{*\,(\text{weighted})}_{\text{IT-VIKOR}} =$ M2, i.e., Punch-B, respectively. The ranks can be calculated employing Eq. (37). The associated preference-degree-based (PD) ranking lists of the candidate punching machines are obtained for the proposed unweighted and weighted models based on Eq. (21) as follows:

– PD rankings based on the unweighted IT-VIKOR method: $M3 \overset{100\%}{\succ} M1 \overset{73\%}{\succ} M2 \overset{58\%}{\succ} M4 \overset{64\%}{\succ} M5 \overset{59\%}{\succ} M6$.
– PD rankings based on the weighted IT-VIKOR method: $M2 \overset{100\%}{\succ} M1 \overset{54\%}{\succ} M3 \overset{68\%}{\succ} M6 \overset{61\%}{\succ} M5 \overset{51\%}{\succ} M4$.

Table 11 shows the results of the present paper (i.e., the assessment values and rankings based on the unweighted and weighted IT-VIKOR methods) and the study of Kulak et al. (2005) (i.e., the assessment values and rankings based on the unweighted and weighted FAD methods) for this machine selection problem. Based on Table 11, except the proposed weighted IT-VIKOR, the rest of methods show an identical option as optimal machine that is M3, i.e., Punch-C. In the assessment values of the FAD approach, infinite values exist due to the formulation of the method. However, the proposed IT-VIKOR method assigns a finite assessment value for every alternative.

To verify the outcomes of Example 1, we calculated machine rankings using the interval target-based extensions of the MULTIMOORA method and its subordinates, i.e., the ratio system, reference point approach, and full multiplicative form. The algorithm of these extensions is similar to the proposed IT-VIKOR method and has been developed by Hafezalkotob and Hafezalkotob (2016a). They employed the IT-MULTIMOORA approach for selection of biomaterials. Table 12 lists the rankings of the proposed IT-VIKOR and the other MADM methods in two categories named as unweighted and weighted models. M3, i.e., Punch-C, is the best option based on all methods of the unweighted category; whereas, M3, i.e., Punch-C, is also the best alternative in the weighted category except for the FAD approach (Kulak et al., 2005). We employed Spearman rank correlation

Table 5
Interval decision matrix for Example 1.

Machine ID	FC	VC	ST	L	T	NS	XY	SR	SS
M1	[8, 12]	[2, 3]	[2, 4]	[0, 1270]	[0, 6.4]	[0, 420]	[0, 108]	[0, 180]	[16, 20]
M2	[10, 14]	[3, 5]	[2, 4]	[0, 2070]	[0, 6.4]	[0, 220]	[0, 97]	[0, 60]	[16, 20]
M3	[12, 16]	[3, 5]	[2, 4]	[0, 2540]	[0, 6.4]	[0, 445]	[0, 108]	[0, 180]	[16, 20]
M4	[14, 16]	[4, 6]	[2, 4]	[0, 2535]	[0, 8.0]	[0, 445]	[0, 108]	[0, 60]	[16, 20]
M5	[10, 14]	[3, 5]	[4, 6]	[0, 2500]	[0, 6.4]	[0, 400]	[0, 110]	[0, 60]	[12, 18]
M6	[8, 12]	[2, 4]	[3, 5]	[0, 1270]	[0, 6.4]	[0, 200]	[0, 82]	[0, 60]	[12, 18]

Table 6
Normalized interval decision matrix using $\hat{f}_{ij}^{*}$ for Example 1.

Machine ID	FC	VC	ST	L	T	NS	XY	SR	SS
M1	[0, 2]	[0, 1]	[0, 1]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 1]
M2	[0, 1.33]	[0, 1.50]	[0, 1]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 1]
M3	[0, 2]	[0, 1.50]	[0, 1]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 1]
M4	[0, 2]	[0, 2]	[0, 1]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 1]
M5	[0, 1.33]	[0, 1.50]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]
M6	[0, 2]	[0, 1]	[0, 1.50]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]

Table 7
Normalized interval decision matrix using $\overline{f}_{ij}^{*}$ for Example 1.

Machine ID	FC	VC	ST	L	T	NS	XY	SR	SS
M1	[0, 1.33]	[0, 0.50]	[0, 0]	[0, 2]	[0, 2]	[0, 0.99]	[0, 1.47]	[0, 0.59]	[0, 0]
M2	[0, 0]	[0, 1]	[0, 0]	[0, 1.35]	[0, 2]	[0, 1.91]	[0, 1.69]	[0, 2]	[0, 0]
M3	[0, 1.33]	[0, 1]	[0, 0]	[0, 0.97]	[0, 2]	[0, 0.87]	[0, 1.47]	[0, 0.59]	[0, 0]
M4	[0, 2]	[0, 2]	[0, 0]	[0, 0.98]	[0, 1.30]	[0, 0.87]	[0, 1.47]	[0, 2]	[0, 0]
M5	[0, 0]	[0, 1]	[0, 2]	[0, 1.00]	[0, 2]	[0, 1.08]	[0, 1.43]	[0, 2]	[0, 2]
M6	[0, 1.33]	[0, 0]	[0, 1]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]	[0, 2]

Table 8
Suh information contents obtained in the FAD approach for Example 1 (Kulak et al., 2005).

Machine ID	FC	VC	ST	L	T	NS	XY	SR	SS
M1	2.000	1.000	0.000	4.181	0.912	0.869	1.507	0.470	0.000
M2	0.000	2.000	0.000	1.250	0.912	2.874	1.845	2.585	0.000
M3	2.000	2.000	0.000	0.923	0.912	0.803	1.507	0.470	0.000
M4	∞	∞	0.000	0.925	0.678	0.803	1.507	2.585	0.000
M5	0.000	2.000	∞	0.943	0.912	0.930	1.459	2.585	3.391
M6	2.000	0.000	2.000	4.181	0.912	4.322	2.773	2.585	3.391

Table 9
Measures, assessment indices, and rankings of the unweighted IT-VIKOR model for Example 1.

Machine ID	$\overline{S}_i$	$\overline{R}_i$	$\overline{Q}_i$	rank ($\overline{Q}_i$)
M1	[0, 8.879]	[0, 2]	[0, 0.211]	2
M2	[0, 9.954]	[0, 2]	[0, 0.564]	3
M3	[0, 8.236]	[0, 2]	[0, 0]	1
M4	[0, 10.623]	[0, 2]	[0, 0.783]	4
M5	[0, 12.513]	[0, 2]	[0, 1.403]	5
M6	[0, 14.333]	[0, 2]	[0, 2]	6

Table 10
Measures, assessment indices, and rankings of the weighted IT-VIKOR model for Example 1.

Machine ID	$\overline{S}_i$	$\overline{R}_i$	$\overline{Q}_i$	rank ($\overline{Q}_i$)
M1	[0, 2.876]	[0, 1.067]	[0, 0.481]	2
M2	[0, 2.591]	[0, 0.800]	[0,0]	1
M3	[0, 3.047]	[0, 1.067]	[0, 0.569]	3
M4	[0, 4.525]	[0, 1.600]	[0,2]	6
M5	[0, 4.303]	[0, 1.600]	[0, 1.885]	5
M6	[0, 4.267]	[0, 1.067]	[0, 1.200]	4

coefficient to show connection between the rankings. This coefficient introduced by Spearman (1904) is a real number between −1 and 1. The value 1 denotes exact correspondence of compared ranks whereas the value −1 represents complete opposition. The Spearman coefficient between the rankings obtained using the unweighted and weighted IT-VIKOR models is 0.54. Fig. 2 shows the correlation between the rankings of the proposed methods and the other techniques given in Table 12. Generally, because of similar algorithm, the IT-MULTIMOORA method and its subordinate parts are more correlated with IT-VIKOR model comparing the FAD approach (Kulak et al., 2005).

6.2. *Example 2: Continuous fluid bed tea dryer*

The drying process in tea industry is an important step. Besides dehydrating tea leaves, drying process prevents enzymic reaction and oxidation. Selection of the suitable option from the set of available tea dryers leads to high quality products (Çakır, 2016). In this practical case, we evaluate a problem concerning selection of appropriate continuous fluid bed tea dryer. This example has already been tackled based on the FAD approach (Çakır, 2016). Five candidate tea dryers (m=5) and nine attributes (n=9) exist in the problem as shown in

Table 11
Comparison between assessment values and rankings of the proposed IT-VIKOR methods and the FAD approaches for Example 1.

Machine ID	Results of the proposed methods				Results of the FAD methods			
	Unweighted IT-VIKOR		Weighted IT-VIKOR		Unweighted FAD (Kulak et al., 2005)		Weighted FAD (Kulak et al., 2005)	
	Ass.[a] value	Ranking	Ass. value	Ranking	Ass. value	Ranking	Ass. value	Ranking
M1	[0, 0.211]	2	[0, 0.481]	2	10.939	2	6.307	2
M2	[0, 0.564]	3	[0, 0]	1	11.467	3	6.993	3
M3	[0, 0]	1	[0, 0.569]	3	8.615	1	6.226	1
M4	[0, 0.783]	4	[0, 2]	6	∞	5	∞	5
M5	[0, 1.403]	5	[0, 1.885]	5	∞	5	∞	5
M6	[0, 2]	6	[0, 1.200]	4	22.164	4	10.497	4

[a] Ass.: Assessment.

Table 12
Rankings of the proposed methodology and the other approaches for Example 1.

	Unweighted models						Weighted models					
Machine ID	Proposed IT-VIKOR	FAD (Kulak et al., 2005)	IT-RS[a]	IT-RP[a]	IT-MF[a]	IT-MULTIMOORA	Proposed IT-VIKOR	FAD (Kulak et al., 2005)	IT-RS	IT-RP	IT-MF	IT-MULTIMOORA
M1	2	2	2	1	2	2	2	2	2	2	2	2
M2	3	3	3	1	3	3	1	3	1	1	1	1
M3	1	1	1	1	1	1	3	1	3	2	3	3
M4	4	5	4	1	4	4	6	5	4	5	6	6
M5	5	5	3	1	5	4	5	5	3	5	5	5
M6	6	4	6	1	6	6	4	4	6	2	4	4

[a] IT-RS: Interval target-based ratio system, IT-RP: Interval target-based reference point approach, IT-MF: Interval target-based full multiplicative form.

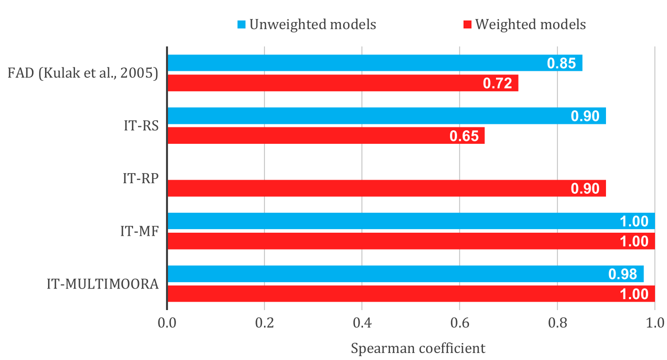


Fig. 2. Correlation between the rankings of the proposed methods and the other approaches for Example 1.

Table 13
Alternatives and attributes for selection of appropriate continuous fluid bed tea dryer (Example 2).

Candidate machines (alternatives)	Attributes		
Machine ID	Attribute name	Weight	Target value
M1	Capacity (CP)	0.162	[0.86, 1]
M2	Water evaporation capacity (WE)	0.152	[0.80, 1]
M3	Fuel consumption (FC)	0.150	[0.80, 1]
M4	Reliability (RL)	0.114	[0.82, 1]
M5	Safety (SF)	0.055	[0.59, 1]
	User Friendliness (UF)	0.109	[0.64, 1]
	Maintenance and service (MS)	0.074	[0.64, 1]
	Price (PC)	0.153	[0, 0.21]
	Space occupied (SO)	0.032	[0, 0.23]

Table 13. The units, weights, and target values of the attributes are also provided in Table 13. Table 14 displays the interval ratings of the candidate equipment on their attributes for the problem. The interval data regarding the target values of Table 13 and the interval ratings of Table 14 are obtained based on $\alpha -$ cut of the original triangular fuzzy numbers given in the study of Çakır (2016). Table 15 represents the normalized decision matrix obtained based on $\overline{f}_{ij}^{*}$.

Tables 16 and 17 demonstrate the measures, assessment indices, and rankings of the unweighted and weighted IT-VIKOR methods for the practical case, respectively. v coefficient of $\overline{Q}_i$ is assumed to be 0.5. The optimal machine based on the unweighted and weighted IT-VIKOR methods are $A^{*\,(\text{unweighted})}_{\text{IT-VIKOR}} = A^{*\,(\text{weighted})}_{\text{IT-VIKOR}} = \text{M4}$. The associated PD ranking lists of the candidate tea dryers based on the proposed unweighted and weighted methods are calculated as follows:

- PD rankings based on the unweighted IT-VIKOR method: $\text{M4} \overset{100\%}{\succ} \text{M3} \overset{52\%}{\succ} \text{M1} \overset{55\%}{\succ} \text{M2} \overset{51\%}{\succ} \text{M5}$.
- PD rankings based on the weighted IT-VIKOR method: $\text{M4} \overset{53\%}{\succ} \text{M3} \overset{67\%}{\succ} \text{M1} \overset{70\%}{\succ} \text{M2} \overset{55\%}{\succ} \text{M5}$.

Table 18 demonstrates the results of the present paper (i.e., the assessment values and rankings based on the unweighted and weighted IT-VIKOR methods) and the study of Çakır (2016) (i.e., the assessment values and rankings based on the unweighted and weighted FAD methods) for this machine selection problem. Table 19 shows the rankings of the proposed IT-VIKOR and the other MADM methods in two categories named as unweighted and weighted models. The Spearman coefficient between the rankings obtained using the unweighted and weighted IT-VIKOR method is 1 that means identical rankings. Fig. 3 shows correlation between our results and those of Çakır (2016) as well as the rankings of the IT-MULTIMOORA method and its subordinate parts through calculating Spearman rank correlation coefficients. Based on Fig. 3, the IT-MULTIMOORA method has a one–to-one correspondence with the IT-VIKOR method in the two unweighted and weighted categories.

7. Conclusion

Target-based decision making is important in many real-world

Table 14
Interval decision matrix for Example 2.

Machine ID	CP	WE	FC	RL	SF	UF	MS	PC	SO
M1	[0.79, 0.92]	[0.72, 0.86]	[0.66, 0.80]	[0.42, 0.57]	[0.06, 0.21]	[0.20, 0.35]	[0.38, 0.53]	[0.40, 0.60]	[0.57, 0.81]
M2	[0.84, 0.96]	[0.34, 0.49]	[0.72, 0.86]	[0.38, 0.53]	[0.06, 0.21]	[0.38, 0.53]	[0.34, 0.48]	[0.57, 0.81]	[0.61, 0.83]
M3	[0.79, 0.92]	[0.58, 0.73]	[0.66, 0.80]	[0.58, 0.73]	[0.12, 0.27]	[0.19, 0.34]	[0.19, 0.34]	[0.36, 0.57]	[0.35, 0.56]
M4	[0.75, 0.89]	[0.62, 0.77]	[0.72, 0.86]	[0.42, 0.57]	[0.21, 0.36]	[0.36, 0.51]	[0.34, 0.48]	[0.00, 0.19]	[0.03, 0.21]
M5	[0.44, 0.59]	[0.72, 0.86]	[0.28, 0.43]	[0.54, 0.69]	[0.21, 0.36]	[0.20, 0.35]	[0.36, 0.51]	[0.40, 0.60]	[0.61, 0.83]

Table 15
Normalized interval decision matrix using $\overline{f}_{ij}^{*}$ for Example 2.

Machine ID	CP	WE	FC	RL	SF	UF	MS	PC	SO
M1	[0, 0.22]	[0, 0.28]	[0, 0.38]	[0.65, 1.09]	[0.73, 1.27]	[0.66, 1.26]	[0.27, 0.86]	[0.41, 0.84]	[0.69, 1.17]
M2	[0, 0.08]	[0.78, 1.22]	[0, 0.25]	[0.78, 1.22]	[0.73, 1.27]	[0.27, 0.86]	[0.38, 0.96]	[0.76, 1.24]	[0.77, 1.23]
M3	[0, 0.22]	[0.17, 0.60]	[0, 0.38]	[0.23, 0.67]	[0.62, 1.17]	[0.70, 1.30]	[0.70, 1.30]	[0.32, 0.76]	[0.23, 0.69]
M4	[0, 0.33]	[0.07, 0.50]	[0, 0.25]	[0.65, 1.09]	[0.45, 0.99]	[0.31, 0.90]	[0.38, 0.96]	[0, 0.02]	[0, 0.01]
M5	[0.78, 1.22]	[0, 0.28]	[0.81, 1.19]	[0.34, 0.78]	[0.45, 0.99]	[0.66, 1.26]	[0.31, 0.90]	[0.41, 0.84]	[0.77, 1.23]

Table 16
Measures, assessment indices, and rankings of the unweighted IT-VIKOR model for Example 2.

Machine ID	$\overline{S}_i$	$\overline{R}_i$	$\overline{Q}_i$	rank ($\overline{Q}_i$)
M1	[3.408, 7.370]	[0.727, 1.273]	[0, 1.612]	3
M2	[4.467, 8.328]	[0.783, 1.217]	[0, 1.931]	4
M3	[2.979, 7.085]	[0.703, 1.297]	[0, 1.499]	2
M4	[1.858, 5.052]	[0.654, 1.091]	[0, 0]	1
M5	[4.527, 8.702]	[0.806, 1.194]	[0, 2]	5

Table 17
Measures, assessment indices, and rankings of the weighted IT-VIKOR model for Example 2.

Machine ID	$\overline{S}_i$	$\overline{R}_i$	$\overline{Q}_i$	rank ($\overline{Q}_i$)
M1	[0.311, 0.729]	[0.094, 0.157]	[0, 0.728]	3
M2	[0.470, 0.875]	[0.117, 0.189]	[0, 1.667]	4
M3	[0.278, 0.720]	[0.077, 0.141]	[0, 0.356]	2
M4	[0.191, 0.551]	[0.094, 0.157]	[0, 0.315]	1
M5	[0.504, 0.958]	[0.127, 0.197]	[0, 2]	5

applications. The models based on such decision making process greatly matter to engineers who deal with machine selection. Decision-makers may consider given target values for some attributes like speed and safety of a machine. The ratings of a machine on such attributes, naturally may have some degrees of uncertainty. Thus, systematic methodologies are required to simultaneously consider target-based attributes and interval data for selection of the best machines. In this paper, we developed the VIKOR method based on interval target values of attributes and interval decision matrix. We presented a novel normalization technique employing interval distance of interval numbers. The employed interval distance is an improved formula comparing the interval distance equation available in the literature. A preference matrix was employed for finding extremum and ranking interval numbers. We evaluated two problems concerning the selection of appropriate punching equipment and continuous fluid bed tea dryer. Preference-degree-based ranking lists were produced by determining the relative degrees of preference for the arranged assessment values of the candidate machines. The rankings of the proposed IT-VIKOR method for the two practical cases were compared with the outcomes of the FAD technique and the interval target-based MULTIMOORA approach and its subordinate parts.

All previous interval MADM studies have degenerated interval numbers in some sections of their models from low to high extents. However, in the proposed method, we attempted to lessen degeneration of interval numbers by utilizing the power of the interval mathematics. We had to degenerate interval numbers in only one application that is the denominator of ratios. Accordingly, we inevitably used crisp distance instead of interval distance in the denominator of the proposed normalization technique and the assessment index.

This paper is related to the studies of Jahan and Edwards (2013a) and Zeng et al. (2013). In their target-based VIKOR methods, the target values of attributes or ratings of alternatives on attributes are crisp numbers. The target-based norm in the study of Jahan and Edwards (2013a) is supported on Euclidian distance, but we used interval distance. The interval target values in in the study of Zeng et al. (2013) have normalized distribution function. However, we developed the VIKOR method with interval target values of attributes and interval ratings of alternatives on attributes. The other novelties of the proposed method comparing the related studies were presented in Research Gap, i.e., Section 2.5.

The FAD approach also models target-based decision-making. It is based on common area of membership functions of alternative ratings

Table 18
Comparison between assessment values and rankings of the proposed IT-VIKOR methods and the FAD approaches for Example 2.

Machine ID	Results of the proposed methods				Results of the FAD methods			
	Unweighted IT-VIKOR		Weighted IT-VIKOR		Unweighted FAD (Çakır, 2016)		Weighted FAD (Çakır, 2016)	
	Ass. value	Ranking	Ass. value	Ranking	Ass. value	Ranking	Ass. value	Ranking
M1	[0, 1.612]	3	[0, 0.728]	3	32.334	3	7.886	2
M2	[0, 1.931]	4	[0, 1.667]	4	∞	4	∞	4
M3	[0, 1.499]	2	[0, 0.356]	2	16.314	1	7.527	1
M4	[0, 0]	1	[0, 0.315]	1	20.805	2	8.939	3
M5	[0, 2]	5	[0, 2]	5	∞	4	∞	4

Table 19
Rankings of the proposed methodology and the other approaches for Example 2.

	Unweighted models						Weighted models					
Machine ID	Proposed IT-VIKOR	FAD (Çakır, 2016)	IT-RS	IT-RP	IT-MF	IT-MULTIMOORA	Proposed IT-VIKOR	FAD (Çakır, 2016)	IT-RS	IT-RP	IT-MF	IT-MULTIMOORA
M1	3	3	3	3	3	3	3	2	3	2	3	3
M2	4	4	4	4	4	4	4	4	4	4	4	4
M3	2	1	2	2	2	2	2	1	2	1	2	2
M4	1	2	1	1	1	1	1	3	1	2	1	1
M5	5	4	3	5	5	5	5	4	3	5	5	5

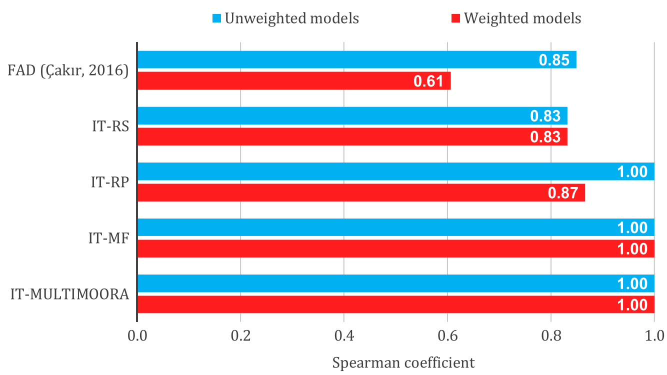


Fig. 3. Correlation between the rankings of the proposed methods and the other approaches for Example 2.

and target values of attributes. It has limitations to find a complete ranking because of probable infinite values of its assessment index. In other words, when the interval rating of an alternative on an attribute and the interval target value of the attribute are not intersected, Suh information content tends to infinity while the proposed distance-based normalized interval rating equals to a finite value. Thus, the proposed IT-VIKOR method generates finite normalized interval ratings and assessment value for every alternative. Moreover, the FAD approach cannot be used in applications in which the target values of attributes or the ratings of alternatives on attributes are crisp. Whereas, the suggested methodology can be utilized in cases in which the target values of attributes or the ratings of alternatives on attributes are crisp or interval. The FAD approach produces crisp assessment values; however, the proposed IT-VIKOR method generates interval assessment values.

As further studies on the field, the proposed method can be developed with fuzzy sets. For the extension, fuzzy distance and uncertain preference degree can be utilized. The development of the proposed method based on stochastic data is also interesting. Besides, different kinds of weighting systems can be considered for the attributes of an uncertain target-based decision-making problem including subjective, objective, and integrated weights.

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