Malaysian Journal of Mathematical Sciences 10(3): 309–329 (2016)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

Interval Valued Q-Fuzzy Multiparameterized Soft Set and its Application

Fatma Adam and Nasruddin Hassan *

School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

E-mail: nas@ukm.edu.my
**Corresponding author*

ABSTRACT

Molodtsov's soft set theory is a newly emerging mathematical tool for handling uncertainty. However, classical multiparameterized soft sets are not appropriate for imprecise and Q-fuzzy parameters. This work aims to extend the classical multiparameterized soft sets to interval-valued Q-fuzzy multiparameterized soft (IVQFMP -soft) sets by giving an importance degree for each element in the multiparameterized sets. The equality, subset, complement, union, intersection, $\wedge$ and $\vee$-product operations are defined for the IVQFMP-soft sets. Finally, the proposed IVQFMP-soft set is applied to a decision- making problem, and its effectiveness is demonstrated through a numerical example.

Keywords: Approximate function, decision making, membership, multi Q-fuzzy set.

1. Introduction

Most real-life problems in the medical, engineering, management, environmental, and social sciences involve various uncertainties. Thus, the inherent data for these problems is not crisp, precise, or deterministic. Such uncertainties are usually handled using probabilistic techniques, fuzzy sets Zadeh (1965) , rough sets Pawlak (1982) and other well-known mathematical tools. However, all of these theories have limitations. To overcome these weaknesses, Molodtsov (1999) proposed a novel approach for modelling vagueness and uncertainty. This so-called soft set theory has potential applications in many different fields, such as in genetics by Varnamkhasti and Hassan (2012, 2013). Ali et al. (2009) introduced some new operations and discussed their basic properties.

Babitha and Sunil (2010) introduced the concept of soft set relation and function and discussed many related concepts. Moreover, Babitha and Sunil (2011) further worked on soft set relation and ordering by introducing the concept of anti-symmetric relation and transitive closure of a soft set relation. Cagman and Enginoglu (2010) developed soft matrix theory and successfully applied it to a decision making problem. Sezgin and Atagun (2011) and Ge and Yang (2011) gave some modifications of the work by Maji et al. (2003) and also established some new results. Singh and Onyeozili (2012) proved that the operations defined on soft sets are equivalent to the corresponding operations defined on their soft matrices.

Although Zhu and Wen (2013) proposed some operations in soft sets, their effort was inadequate for dealing with complex problems. Cagman (2014) added contributions to the theory of soft sets and made modifications to soft set operations that serve as the foundations of further research. Earlier, Majumdar and Samanta (2013) proposed the softness of soft sets. Before that, Maji et al. (2003) studied the theory of fuzzy soft sets. Chen et al. (2005) presented a new definition of soft set parametrization reduction so as to improve the soft set based decision making. Cagman et al. (2010, 2011) further developed a form of parameterized fuzzy soft set theory and its applications. Yang et al. (2009) introduced interval-valued fuzzy soft sets which realize a common extension of both Molodtsov's soft sets and interval-valued fuzzy sets.

Alkhazaleh et al. (2011) introduced fuzzy parameterized interval-valued fuzzy soft set theory followed by soft intuitionistic fuzzy sets by Alhazaymeh et al. (2012), while Alhazaymeh and Hassan (2012a, 2012b, 2012c) applied vague soft sets to decision making, and further explored interval-valued vague soft set (2013a, 2013c), generalized vague soft expert set (2013b, 2014a, 2014b,

2014c), vague soft multiset theory (2014d), and vague soft set relations and functions (2015). Al-Quran and Hassan (2016) then extended the work of Hassan and Alhazaymeh (2013) on vague soft expert set to neutrosophic vague soft expert set.
Recently, in order to establish the degree of multi-membership of elements in Q-fuzzy sets, Adam and Hassan (2014a, 2014b, 2014c, 2014d, 2015, 2016) proposed the concept of multi-Q-fuzzy soft set. These concepts will be extended further to a multiparameterized form to use fuzziness advantages to represent descriptions of objects and give an importance degree for each element in the multiparameterized set. Its properties, applications and algorithm will also be studied.

The rest of this paper is organized as follows. The background on multiparameterized soft set and multi-Q-fuzzy sets will initially be reviewed. Then, concepts and operations of interval-valued Q-fuzzy sets are proposed. Next, interval valued Q-fuzzy multiparameterized soft (IVQFMP-soft) set is introduced and the properties of union and intersection, $\wedge$ and $\vee$-product are discussed. The proposed IVQFMP-soft set is later applied to a decision-making problem, and an explicit algorithm is designed, before the remarks of the conclusion of this study.

2. Preliminaries

In this section the basic definitions of multiparameterized soft set theory and multi Q-fuzzy set required as preliminaries are presented.

Let U be a universe set and E_i be a set of parameters for all $i \in I$ such that $\bigcap_{i\in I} E_i = \emptyset$, $E = \bigcup_{i\in I} E_i$. Let $P(U)$ denote the power set of U, $P(E)$ denotes the power set of E and $A \subset P(E)$.

The following three definitions on multiparameterized soft set were proposed by Salleh et al. (2012).

Definition 2.1. (F, A) *is called a multiparameterized soft set over* U *where* F *is a mapping given by*

$$F : A \longrightarrow P(U).$$

In other words, a multiparameterized soft set over U *is a parameterized family of subsets of the universe* U*. For all* $e \in A$, $F(e)$ *may be considered as the set* *of e-approximate sets of the multiparameterized soft set* (F, A).

Definition 2.2. *The intersection of two multiparameterized soft sets* (F, A) *and* (G, B) *over a common universe set* U *is the multiparameterized soft set* (H, C), *where* $C = A \cap B$, *and* $\forall e \in C, H(e) = F(e) \,\mathrm{or}\, G(e)$, *(as both are in the same set). We write* $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Definition 2.3. *The union of two multiparameterized soft sets of* (F, A) *and* (G, B) *over the common universe* U *is the multiparameterized soft set* (H, C), *where* $C = A \cup B$, *and* $\forall e \in C$

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \ ; \\ G(e), & \text{if } e \in B - A; \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

The following definition on multi Q-fuzzy set was proposed by Adam and Hassan (2014d).

Definition 2.4. *Let* I *be a unit interval [0, 1],* k *be a positive integer,* U *be a universal set and* Q *be a non-empty set. A multi Q-fuzzy set* $\tilde{A}_Q$ *in* U *and* Q *is a set of ordered sequences* $\tilde{A}_Q = \{((u, q), (\mu_1(u, q), \mu_2(u, q), ..., \mu_k(u, q))) : u \in U, q \in Q\}$,
where

$$\mu_i(u, q) \in I, \ for \ all \ i = 1, 2, ..., k$$

The function $(\mu_1(u, q), \mu_2(u, q), ..., \mu_k(u, q))$ *is called the membership function of multi Q-fuzzy set* $\tilde{A}_Q$; *and* $\mu_1(u, q) + \mu_2(u, q) + ... + \mu_k(u, q)) \leq 1$, k *is called the dimension of* $\tilde{A}_Q$. *In other words, if the sequences of the membership functions have only k-terms (finite number of terms) the multi Q-fuzzy set is a function from* $U \times Q$ *to* I^k *such that for all* $(u, q) \in U \times Q$, $\mu_{\tilde{A}_Q} = (\mu_1(u, q), \mu_2(u, q), ..., \mu_k(u, q))$. *The set of all multi Q-fuzzy sets of dimension* k *in* U *and* Q *is denoted by* $M^kQF(U)$.

3. Interval Valued Q-Fuzzy Set

In this section the authors define the concept of an interval valued Q-fuzzy set and its basic operations namely subset, equality, complement, union and intersection.

Definition 3.1. *An interval valued Q-fuzzy set on the universe U and non-empty set Q is a mapping such that*

$$F_Q : U \times Q \to Int([0,1])$$

where $Int([0,1])$ stands for all closed subintervals of $[0,1]$, while the set of all interval valued Q-fuzzy sets on U and Q is denoted by $IVQF(U)$.

Suppose that $F_Q \in IVQF(U)$ $\forall x \in U, q \in Q$, then $\mu_{F_Q}(x,q) = [\mu^-_{F_Q}(x,q), \mu^+_{F_Q}(x,q)]$ is called the degree of membership of an element (x,q) to F_Q. $\mu^-_{F_Q}(x,q)$ and $\mu^+_{F_Q}(x,q)$ are referred to as the lower and upper degrees of membership of (x,q) to F_Q where $0 \leq \mu^-_{F_Q}(x,q) \leq \mu^+_{F_Q}(x,q) \leq 1$.

Example 3.1. *Let $U = \{u_1, u_2, u_3\}$ be a universal set, $Q = \{p, q, r\}$ be a non-empty set. The set $A_Q = \{((u_1,p),[0.1,0.3]),((u_1,q),[0.4,0.7]),((u_1,r),[0.6,0.8]),$ $((u_2,p),[0.1,0.3]),((u_2,q),[0.2,0.4]),((u_2,r),[0,0.1]),((u_3,p),[0.1,0.7]),$ $((u_3,q),[0.4,0.6]),((u_3,r),[0,0.8])\}$ is an interval valued Q-fuzzy set.*

Definition 3.2. *Let A_Q and B_Q be two interval valued Q-fuzzy sets,* $A_Q = \{(u,q),[\mu^-_{A_Q}(u,q), \mu^+_{A_Q}(u,q)] : u \in U, q \in Q\}$, *and* $B_Q = \{(u,q),[\nu^-_{B_Q}(u,q), \nu^+_{B_Q}(u,q)] : u \in U, q \in Q\}$. *Then we have the following relations and operations for all* $u \in U$ and $q \in Q$.

1. $A_Q \subseteq B_Q$ *if and only if* $\mu^-_{A_Q}(u,q) \leq \nu^-_{B_Q}(u,q)$, *and* $\mu^+_{A_Q}(u,q) \leq \nu^+_{B_Q}(u,q)$.
2. $A_Q = B_Q$ *if and only if* $\mu^-_{A_Q}(u,q) = \nu^-_{B_Q}(u,q)$, *and* $\mu^+_{A_Q}(u,q) = \nu^+_{B_Q}(u,q)$.
3. $A_Q \cup B_Q = \{(u,q),[\sup(\mu^-_{A_Q}(u,q), \nu^-_{B_Q}(u,q)), \sup(\mu^+_{A_Q}(u,q), \nu^+_{B_Q}(u,q))]\}$.
4. $A_Q \cap B_Q = \{(u,q),[\inf(\mu^-_{A_Q}(u,q), \nu^-_{B_Q}(u,q)), \inf(\mu^+_{A_Q}(u,q), \nu^+_{B_Q}(u,q))]\}$.

Example 3.2. *Let $U = \{u_1, u_2, u_3\}$ be a universal set, $Q = \{p, q, r\}$ be a non-empty set. Suppose $A_Q = \{((u_1,p),[0.2,0.3]),((u_1,q),[0.2,0.5]),((u_1,r),[0.3,0.5]),$ $((u_2,p),[0.1,0.3]),((u_2,q),[0.2,0.3]),((u_2,r),[0,0.1]),((u_3,p),[0.1,0.6]),$ $((u_3,q),[0.1,0.4]),((u_3,r),[0,0.4])\}$ and $B_Q = \{((u_1,p),[0.3,0.5]),((u_1,q),[0.3,0.7]),$ $((u_1,r),[0.4,0.6]),((u_2,p),[0.1,0.4]),((u_2,q),[0.4,0.5]),((u_2,r),[0.1,0.3]),((u_3,p),[0.2,0.7]),$ $((u_3,q),[0.2,0.6]),((u_3,r),[0.1,0.6])\}$ are interval valued Q-fuzzy sets, then*

1. $A_Q \subseteq B_Q$.

2. $A_Q \cup B_Q = \{((u_1, p), [0.3, 0.5]), ((u_1, q), [0.3, 0.7]), ((u_1, r), [0.4, 0.6]), ((u_2, p), [0.1, 0.4]), ((u_2, q), [0.4, 0.5]), ((u_2, r), [0.1, 0.3]), ((u_3, p), [0.2, 0.7]), ((u_3, q), [0.2, 0.6]), ((u_3, r), [0.1, 0.6])\}$.

3. $A_Q \cap B_Q =_Q= \{((u_1, p), [0.2, 0.3]), ((u_1, q), [0.2, 0.5]), ((u_1, r), [0.3, 0.5]), ((u_2, p), [0.1, 0.3]), ((u_2, q), [0.2, 0.3]), ((u_2, r), [0, 0.1]), ((u_3, p), [0.1, 0.6]), ((u_3, q), [0.1, 0.4]), ((u_3, r), [0, 0.4])\}$.

Definition 3.3. *Let A_Q be an interval valued Q-fuzzy set. The complement of interval valued Q-fuzzy set A_Q is an interval-valued Q-fuzzy set denoted by $C(A_Q)$ and defined by*

$$C(A_Q) = \{(u, q), [1 - \mu^{+}_{A_Q}(u, q), 1 - \mu^{-}_{A_Q}(u, q)] : u \in U, q \in Q\}.$$

Example 3.3. *From Example 3.1. the complement of interval valued Q-fuzzy set A_Q is* $C(A_Q) = \{((u_1, p), [0.7, 0.9]), ((u_1, q), [0.3, 0.6]), ((u_1, r), [0.2, 0.4]), ((u_2, p), [0.7, 0.9]), ((u_2, q), [0.6, 0.8]), ((u_2, r), [0.9, 1]), ((u_3, p), [0.3, 0.9]), ((u_3, q), [0.4, 0.6]), ((u_3, r), [0.2, 1])\}$.

4. Interval Valued Q-Fuzzy Multiparameterized Soft Set

In this section the authors introduce the concept of an interval valued Q-fuzzy multiparameterized soft set and define some operations on an interval-valued Q-fuzzy multiparameterized soft set, namely subset, equality, null, absolute, complement, union, intersection, $\wedge$ and $\vee$-product.

Definition 4.1. *The interval-valued Q-fuzzy multiparameterized soft set (IVQFMP-soft set) F_{Q_X} on the universe U is defined by the set of ordered pairs*

$$F_{Q_X} = \{((x, q), [\mu^{-}_{Q_X}(x, q), \mu^{+}_{Q_X}(x, q)], f_{Q_X}(x)) : x \in A, f_{Q_X} \in P(U)\}$$

where the function $f_{Q_X} : A \to P(U)$ is called the approximate function such that $f_{Q_X}(x) = \emptyset$, if $[\mu^{-}_{Q_X}(x, q), \mu^{+}_{Q_X}(x, q)] = \emptyset$, and the function $\mu^{-}_{Q_X}(x, q), \mu^{+}_{Q_X}(x, q) : A \times Q \to int([0, 1])$ is called the membership function of IVQFMP-soft set F_{Q_X}. The value of $[\mu^{-}_{Q_X}(x, q), \mu^{+}_{Q_X}(x, q)]$ is the degree of importance of the parameter x, and depends on the decision maker's requirements.

Note that from now on the sets of all IVQFMP-soft sets over U will be denoted by $IVQFMPS(U)$.

Below is an illustration of an interval valued Q-fuzzy multiparameterized soft set.

Example 4.1. *Let us consider a multiparameterized soft set which describes the attractiveness of houses that a person is considering to purchase. Suppose that there are seven houses in the universe* $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$, $Q = \{q, r\}$ *be a set of cities under consideration and that* $E_i = \{E_1, E_2\}$ *is a set of decision parameters. Let* E_1 *be a set of cost parameters given by* $E_1 = \{e_{1,1} = expensive, e_{1,2} = cheap, e_{1,3} = very\, expensive\}$, E_2 *is a set of location parameters given by* $E_2 = \{e_{2,1} = Piazza, e_{2,2} = Valencia, e_{2,3} = Atrium\}$, *let* $E = \bigcup_i E_i$ *and* $A \subset P(E)$ *such that* $A = \{x_1 = \{e_{1,1}, e_{2,1}\}, x_2 = \{e_{1,2}, e_{2,2}\}, x_3 = \{e_{1,3}, e_{2,3}\}\}$.
Let $X = \{((x_1, q), [0.1, 0.4]), ((x_1, r), [0.3, 0.7]), ((x_2, q), [0.3, 0.6]), ((x_2, r),$
$((x_3, q), [0.2, 0.8]), ((x_3, r), [0.6, 0.7])\}$, *and* $f_{Q_X}(x_1) = \{u_1, u_3, u_4\}, f_{Q_X}(x_2) = \{u_2, u_5\}$, *and* $f_{Q_X}(x_3) = \{u_6, u_7\}$, *then*
$F_{Q_X} = \{(((x_1, q), [0.1, 0.4]), ((x_1, r), [0.3, 0.7]), \{u_1, u_3, u_4\}), (((x_2, q), [0.3, 0.6]), ((x_2, r), [0.1, 0.5]), \{u_2, u_5\}), (((x_3, q), [0.2, 0.8]), ((x_3, r), [0.6, 0.7]), \{u_6, u_7\})\}$.

Thus an interval valued Q-fuzzy multiparameterized soft set which gives us a collection of approximate descriptions of an object is as follows.

The houses u_1, u_3 *and* u_4 *are at Piazza and expensive with membership* $[0.1, 0.4]$ *and* $[0.3, 0.7]$ *in cities* q *and* r *respectively, while the houses* u_2 *and* u_5 *are at Valencia and cheap with membership* $[0.3, 0.6]$ *and* $[0.1, 0.5]$ *in cities* q *and* r *respectively. The houses* u_6 *and* u_7 *are at Atrium and very expensive with membership* $[0.2, 0.8]$ *and* $[0.3, 0.7]$ *in cities* q *and* r *respectively.*

In the following, we introduce the definitions of the empty IVQFMP-soft set and the absolute IVQFMP-soft set.

Definition 4.2. *Let* $F_{Q_X} \in IVQFMPS(U)$. *If* $f_{Q_X}(x) = \emptyset$, *for all* $x \in A$, *then* F_{Q_X} *is called an empty IVQFMP-soft set denoted by* F_{Q_Φ}.

Definition 4.3. *Let* $F_{Q_X} \in IVQFMPS(U)$. *If* $f_{Q_X}(x) = U$ *for all* $x \in A$, *then* F_{Q_X} *is called an absolute IVQFMP-soft set, denoted by* $F_{Q_{\tilde{A}}}$.

The following are examples related to the empty IVQFMP-soft set and the absolute IVQFMP-soft set.

Example 4.2. *Consider* $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ *as a universal set,* $Q = \{q, p\}$ *be a non-empty set,* $E_i = \{E_1, E_2, E_3\}$ *be a set of decision parameters,*

such that $E_1 = \{x_{1,1}, x_{1,2}, x_{1,3}\}$, $E_2 = \{x_{2,1}, x_{2,2}, x_{2,3}\}$, $E_3 = \{x_{3,1}, x_{3,2}, x_{3,3}\}$, $A \subset P(E)$ *such that* $A = \{x_1 = \{x_{1,1}, x_{2,1}, x_{3,1}\}, x_2 = \{x_{1,2}, x_{2,2}, x_{1,3}\}, x_3 = \{x_{2,2}, x_{3,2}, x_{3,3}\}\}$. *Let* $X = \{((x_1, q), [0.1, 0.3]), ((x_1, p), [0.2, 0.8]), ((x_2, q), [0, 0.1]), ((x_2, p), [0.3, 0.6]), ((x_3, q), [0.2, 0.8]), ((x_3, p), [0.4, 0.5])\}$.
If $f_{Q_X}(x_1) = \emptyset$, $f_{Q_X}(x_2) = \emptyset$, $f_{Q_X}(x_3) = \emptyset$, *and* $F_{Q_X} = \{(((x_1, q), [0.1, 0.3]), ((x_1, p), [0.2, 0.8]), \emptyset), (((x_2, q), [0, 0.1]), ((x_2, p), [0.3, 0.6]), \emptyset), (((x_3, q), [0.2, 0.8]), ((x_3, p), [0.4, 0.5]), \emptyset)\}$, *then* $F_{Q_X} = F_{Q_\Phi}$.
If $f_{Q_X}(x_1) = U$, $f_{Q_X}(x_2) = U$, $f_{Q_X}(x_3) = U$, *and* $F_{Q_X} = \{(((x_1, q), [0.1, 0.3]), ((x_1, p), [0.2, 0.8]), U), (((x_2, q), [0, 0.1]), ((x_2, p), [0.3, 0.6]), U), (((x_3, q), [0.2, 0.8]), ((x_3, p), [0.4, 0.5]), U)\}$, *then* $F_{Q_X} = F_{Q_{\tilde{A}}}$.

Definition 4.4. *Let* $F_{Q_X} \in IVQFMPS(U)$. *The complement of* F_{Q_X}, *denoted by* $F^c_{Q_X}$, *is an IVQFMP-soft set defined by the approximate and membership functions as*

$$\mu_{Q^c{}_X}(x, q) = [1 - \mu^+_{Q^c_X}(x, q), 1 - \mu^-_{Q^c_X}(x, q)] \text{ and } f_{Q^c{}_X}(x) = U - f_{Q_X}(x).$$

Proposition 4.1. *Let* $F_{Q_X} \in IVQFMP(U)$. *Thus*

1. $(F^c_{Q_X})^c = F_{Q_X}$.
2. $F^c_{Q_\Phi} = F_{Q_{\tilde{A}}}$.

Proof. By using the approximate and membership functions of the IVQFMP-soft sets, the proof is straightforward. □

In the following, the concept of the subset of two IVQFMP-soft sets is introduced.

Definition 4.5. *Let* $F_{Q_X}, F_{Q_Y} \in IVQFMPS(U)$. F_{Q_X} *is a IVQFMP-soft subset of* F_{Q_Y}, *denoted by* $F_{Q_X} \tilde{\subseteq} F_{Q_Y}$, *if* $\mu^-_{Q_X}(x, q) \leq \mu^-_{Q_Y}(x, q)$, *and* $\mu^+_{Q_X}(x, q) \leq \mu^+_{Q_Y}(x, q)$, *and* $f_{Q_X}(x) \subseteq f_{Q_Y}(x)$ *for all* $x \in A$ *and* $q \in Q$.

In the following, the proposition on the IVQFMP- soft subset is provided.

Proposition 4.2. *Let* $F_{Q_X}, F_{Q_Y} \in IVQFMPS(U)$. *Therefore*

1. $F_{Q_X} \tilde{\subseteq} F_{Q_{\tilde{A}}}$.
2. $F_{Q_\Phi} \tilde{\subseteq} F_{Q_X}$.
3. *If* $F_{Q_X} \tilde{\subseteq} F_{Q_Y}$ *and* $F_{Q_Y} \tilde{\subseteq} F_{Q_H}$, *then* $F_{Q_X} \tilde{\subseteq} F_{Q_H}$.

Proof. These can be proven easily by using the approximate and membership functions of the IVQFMP-soft sets. □

In the following, the equality of two IVQFMP- soft sets is introduced.

Definition 4.6. *Let $F_{Q_X}, F_{Q_Y} \in IVQFPS(U)$. F_{Q_X} and F_{Q_Y} are equal, written as $F_{Q_X} = F_{Q_Y}$ if and only if $f_{Q_X}(x) = f_{Q_Y}(x)$ and $\mu^-_{Q_X}(x,q) = \mu^-_{Q_Y}(x,q)$, and $\mu^+_{Q_X}(x,q) = \mu^+_{Q_Y}(x,q)$, for all $x \in A$, and $q \in Q$.*

5. Operations on interval valued Q-fuzzy multiparameterized soft set

In this section the authors introduce the union, intersection, $\wedge$ and $\vee$-product of interval valued Q-fuzzy multiparameterized soft set.

Definition 5.1. *Let $F_{Q_X}, F_{Q_Y} \in IVQFMPS(U)$. The union of two IVQFMP-soft sets F_{Q_X} and F_{Q_Y} is the IVQFMP-soft set denoted by $F_{Q_X} \tilde{\cup} F_{Q_Y}$ and defined by $\mu_{Q_X \tilde{\cup} Q_Y} = [\sup(\mu^-_{Q_X}(x,q), \mu^-_{Q_Y}(x,q)), \sup(\mu^+_{Q_X}(x,q), \mu^+_{Q_Y}(x,q))]$,* and *$f_{Q_X \tilde{\cup} Q_Y} = f_{Q_X}(x) \cup f_{Q_Y}(x)$ for all $x \in A$, and $q \in Q$.*

An example and a proposition on the union of two IVQFMP-soft sets are as follows.

Example 5.1. *Consider $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ as a universal set, $Q = \{q, p\}$ be a non-empty set, $E_i = \{E_1, E_2, E_3\}$ be a set of decision parameters, $E_1 = \{x_{1,1}, x_{1,2}, x_{1,3}\}$, $E_2 = \{x_{2,1}, x_{2,2}, x_{2,3}\}$, $E_3 = \{x_{3,1}, x_{3,2}, x_{3,3}\}$, $A \subset P(E)$ such that $A = \{x_1 = \{x_{1,1}, x_{2,2}, x_{3,1}\}, x_2 = \{x_{1,2}, x_{2,1}, x_{1,3}\}, x_3 = \{x_{1,3}, x_{2,3}, x_{3,3}\}\}$.*

Let $X = \{((x_1,q),[0.2,0.8]), ((x_1,p),[0.1,0.2]), ((x_2,q),[0.4,0.6]), ((x_2,p),[0.3,0.5]), ((x_3,q),[0.5,0.5]), ((x_3,p),[0,0.4])\}$. $f_{Q_X}(x_1) = \{u_1,u_3,u_5\}$, $f_{Q_X}(x_2) = \{u_1,u_6,u_7\}$, $f_{Q_X}(x_3) = \{u_1,u_2,u_4,u_5\}$, and
$Y = \{((x_1,q),[0.3,0.7]), ((x_1,p),[0.4,0.5]), ((x_2,q),[0.5,0.7]), ((x_2,p),[0.3,0.6]), ((x_3,q),[0,1]), ((x_3,p),[0.1,0.9])\}$, $f_{Q_Y}(x_1) = \{u_1,u_2,u_7\}$, $f_{Q_Y}(x_2) = \{u_3,u_4,u_7\}$, $f_{Q_Y}(x_3) = \{u_4,u_8\}$.

Suppose $F_{Q_X} = \{(((x_1,q),[0.2,0.8]), ((x_1,p),[0.1,0.2]), \{u_1,u_3,u_5\}), (((x_2,q),[0.4,0.6]), ((x_2,p),[0.3,0.5]), \{u_1,u_6,u_7\}), (((x_3,q),[0.5,0.5]), ((x_3,p),[0,0.4]), \{u_1,u_2,u_4,u_5\})\}$, and
$F_{Q_Y} = \{(((x_1,q),[0.3,0.7]), ((x_1,p),[0.4,0.5]), \{u_1,u_2,u_7\}), (((x_2,q),[0.5,0.7]), ((x_2,p),[0.3,0.6]), \{u_3,u_4,u_7\}), (((x_3,q),[0,1]), ((x_3,p),[0.1,0.9]), \{u_4,u_8\})\}$.

Then $F_{Q_X} \tilde{\cup} F_{Q_Y} = \{(((x_1, q), [0.3, 0.8]), ((x_1, p), [0.4, 0.5]), \{u_1, u_2, u_3, u_5, u_7\}), (((x_2, q), [0.5, 0.7]), ((x_2, p), [0.3, 0.6]), \{u_1, u_3, u_4, u_6, u_7\}), (((x_3, q), [0.5, 1]), ((x_3, p), [0.1, 0.9], \{u_1, u_2, u_4, u_5, u_8\})\}$.

Proposition 5.1. *Let* $F_{Q_X}, F_{Q_Y}, F_{Q_H} \in IVQFMPS(U)$. *Therefore*

1. $F_{Q_X} \tilde{\cup} F_{Q_\Phi} = F_{Q_X}$.
2. $F_{Q_X} \tilde{\cup} F_{Q_{\tilde{A}}} = F_{Q_{\tilde{A}}}$.
3. $F_{Q_X} \tilde{\cup} F_{Q_Y} = F_{Q_Y} \tilde{\cup} F_{Q_X}$.
4. $(F_{Q_X} \tilde{\cup} F_{Q_Y}) \tilde{\cup} F_{Q_H} = F_{Q_X} \tilde{\cup} (F_{Q_Y} \tilde{\cup} F_{Q_H})$.

Proof. The proof can be easily obtained from Definition 5.1. □

Definition 5.2. *Let* $F_{Q_X}, F_{Q_Y} \in IVQFMPS(U)$. *The intersection of two IVQFMP-soft sets* F_{Q_X} *and* F_{Q_Y} *is the IVQFMP-soft set denoted by* $F_{Q_X} \tilde{\cap} F_{Q_Y}$ *and defined by* $\mu_{Q_X \tilde{\cap} Q_Y} = [\inf(\mu^-_{Q_X}(x, q), \mu^-_{Q_Y}(x, q)), \inf(\mu^+_{Q_X}(x, q), \mu^+_{Q_Y}(x, q))]$, and $f_{Q_X \tilde{\cap} Q_Y} = f_{Q_X}(x) \cap f_{Q_Y}(x)$ *for all* $x \in A$.

Example 5.2. *From Example 5.1, suppose* $F_{Q_X} = \{(((x_1, q), [0.2, 0.8]), ((x_1, p), [0.1, 0.2]), \{u_1, u_3, u_5\}), (((x_2, q), [0.4, 0.6]), ((x_2, p), [0.3, 0.5]), \{u_1, u_6, u_7\}), (((x_3, q), [0.5, 0.5]), ((x_3, p), [0, 0.4]), \{u_1, u_2, u_4, u_5\})\}$, and $F_{Q_Y} = \{(((x_1, q), [0.3, 0.7]), ((x_1, p), [0.4, 0.5]), \{u_1, u_2, u_7\}), (((x_2, q), [0.5, 0.7]), ((x_2, p), [0.3, 0.6]), \{u_3, u_4, u_7\}), (((x_3, q), [0, 1]), ((x_3, p), [0.1, 0.9]), \{u_4, u_8\})\}$.
Then $F_{Q_X} \tilde{\cap} F_{Q_Y} = \{(((x_1, q), [0.2, 0.7]), ((x_1, p), [0.1, 0.2]), \{u_1\}), (((x_2, q), [0.4, 0.6]), ((x_2, p), [0.3, 0.5]), \{u_7\}), (((x_3, q), [0, 0.5]), ((x_3, p), [0, 0.4]), \{u_4\})\}$.

Now we give some propositions on the union and intersection of IVQFMP-soft sets.

Proposition 5.2. *Let* $F_{Q_X}, F_{Q_Y}, F_{Q_H} \in IVQFMPS(U)$. *Therefore*

1. $F_{Q_X} \tilde{\cap} F_{Q_\Phi} = F_{Q_\Phi}$.
2. $F_{Q_X} \tilde{\cap} F_{Q_{\tilde{A}}} = F_{Q_X}$.
3. $F_{Q_X} \tilde{\cap} F_{Q_Y} = F_{Q_Y} \tilde{\cap} F_{Q_X}$.
4. $(F_{Q_X} \tilde{\cap} F_{Q_Y}) \tilde{\cap} F_{Q_H} = F_{Q_X} \tilde{\cap} (F_{Q_Y} \tilde{\cap} F_{Q_H})$.

Proof. Result follows trivially from Definition 5.2. □

Proposition 5.3. *Let $F_{Q_X}, F_{Q_Y}, F_{Q_H} \in IVQFMPS(U)$. Therefore*

1. $(F_{Q_X} \tilde{\cap} (F_{Q_Y} \tilde{\cup} F_{Q_H})) = (F_{Q_X} \tilde{\cap} F_{Q_Y}) \tilde{\cup} (F_{Q_X} \tilde{\cap} F_{Q_H})$.
2. $(F_{Q_X} \tilde{\cup} (F_{Q_Y} \tilde{\cap} F_{Q_H})) = (F_{Q_X} \tilde{\cup} F_{Q_Y}) \tilde{\cap} (F_{Q_X} \tilde{\cup} F_{Q_H})$.

Proof. Let $[\mu^-_{Q_X}(x,q), \mu^+_{Q_X}(x,q)] = \mu_{Q_X}(x,q)$, $[\mu^-_{Q_Y}(x,q), \mu^+_{Q_Y}(x,q)] = \mu_{Q_Y}(x,q)$, and $[\mu^-_{Q_H}(x,q), \mu^+_{Q_H}(x,q)] = \mu_{Q_H}(x,q)$, for all $x \in A$, and $q \in Q$.

1. Assume that $\mu_{Q_X}(x,q), \mu_{Q_Y}(x,q)$ and $\mu_{Q_H}(x,q)$, are elements of Q_X, Q_Y and Q_H respectively, then we have
$\mu_{Q_X \tilde{\cap} (Q_Y \tilde{\cup} Q_H)}(x,q) = \mu_{Q_X}(x,q) \tilde{\cap} (\mu_{Q_Y}(x,q) \tilde{\cup} \mu_{Q_H}(x,q))$,
$= \inf(\mu_{Q_X}(x,q), \sup(\mu_{Q_Y}(x,q), \mu_{Q_H}(x,y)))$,
$= \sup(\inf(\mu_{Q_X}(x,q), \mu_{Q_Y}(x,q)), \inf(\mu_{Q_X}(x,q), \mu_{Q_H}(x,q)))$,
$= \mu_{(Q_X \tilde{\cap} Q_Y) \tilde{\cup} (Q_X \tilde{\cap} Q_H)}(x,q)$.

This implies $f_{(Q_X \tilde{\cap} (Q_Y \tilde{\cup} Q_H))}(x) = f_{Q_X}(x) \tilde{\cap} (f_{Q_Y}(x) \tilde{\cup} f_{Q_H}(x))$, therefore
$= (f_{Q_X}(x) \tilde{\cap} f_{Q_Y}(x)) \tilde{\cup} (f_{Q_H}(x) \tilde{\cap} f_{Q_H}(x))$.

Thus $(F_{Q_X} \tilde{\cap} (F_{Q_Y} \tilde{\cup} F_{Q_H})) = (F_{Q_X} \tilde{\cap} F_{Q_Y}) \tilde{\cup} (F_{Q_X} \tilde{\cap} F_{Q_H})$.

Assertion 2 can be proven in a similar fashion. □

Proposition 5.4. *(De Morgan's Law). Let $F_{Q_X}, F_{Q_Y}, F_{Q_H} \in IVQFMPS(U)$. Therefore*

1. $(F_{Q_X} \tilde{\cup} F_{Q_Y})^c = F^c_{Q_X} \tilde{\cap} F^c_{Q_Y}$.
2. $(F_{Q_X} \tilde{\cap} F_{Q_Y})^c = F^c_{Q_X} \tilde{\cup} F^c_{Q_Y}$.

Proof.

Let $[\mu^-_{Q_X}(x,q), \mu^+_{Q_X}(x,q)] = \mu_{Q_X}(x,q)$, $[\mu^-_{Q_Y}(x,q), \mu^+_{Q_Y}(x,q)] = \mu_{Q_Y}(x,q)$, and $[\mu^-_{Q_H}(x,q), \mu^+_{Q_H}(x,q)] = \mu_{Q_H}(x,q)$, for all $x \in A$, and $q \in Q$.

1. By Definitions 5.1 and 5.2 we have
$(\mu_{Q_X}(x,q) \tilde{\cup} \mu_{Q_Y}(x,q))^c = (sup(\mu_{Q_X}(x,q), \mu_{Q_Y}(x,q)))^c$,
$= 1 - sup(\mu_{Q_X}(x,q), \mu_{Q_Y}(x,q))$,
$= inf(1 - \mu_{Q_X}(x,q), 1 - \mu_{Q_Y}(x,q))$,

$= (1 - \mu_{Q_X}(x, q) \cap 1 - \mu_{Q_Y}(x, q)),$
$= \mu^c_{Q_X}(x, q) \cap \mu^c_{Q_Y}(x, q)).$

This implies $f_{(Q_X \tilde{\cup} Q_Y)^c} = U - (f_{Q_X}(x) \tilde{\cup} f_{Q_Y}(x))$
$= (U - f_{Q_X}(x) \tilde{\cap} U - f_{Q_Y}(x)),$
$= f^c_{Q_X}(x) \tilde{\cap} f^c_{Q_Y}(x).$

Thus $(F_{Q_X} \tilde{\cup} F_{Q_Y})^c = F^c_{Q_X} \tilde{\cap} F^c_{Q_Y}$.

Assertion 2 can be proven in a similar fashion. □

Definition 5.3. *Let F_{Q_X} and F_{Q_Y} be interval valued Q-fuzzy multiparameterized soft sets over U. The $\wedge$-product (also called AND operation) of interval valued Q-fuzzy multiparameterized soft sets F_{Q_X} and F_{Q_Y} is an interval valued Q-fuzzy multiparameterized soft sets defined by $F_{Q_X} \wedge F_{Q_Y} = F_{Q_{X \times Y}}$ where $X \times Y = min((\mu_{Q_X}(x, q), \mu_{Q_Y}(x, q))$ for all $(x, q) \in A \times Q$, and $f_{Q_{X \wedge Y}}(a, b) = f_{Q_X}(a) \cap f_{Q_Y}(b)$ for all $(a, b) \in A \times A$.*

The following example illustrates the $\wedge$-product of interval valued Q-fuzzy multiparameterized soft sets.

Example 5.3. *Let us consider a staff selection problem to fill a position in a private company. Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be the universe set consisting of five candidates, $Q = \{r\}$ be a qualification set. Let us consider $E_i = \{E_1, E_2, E_3\}$ the set of decision parameters where $E_1 = \{x_{1,1} = experience, x_{1,2} = computer\, knowledge\}, E_2 = \{x_{2,1} = young, x_{2,2} = charming\}$, and $E_3 = \{x_{3,1} = genial, x_{3,2} = fluent\}$, and $A \in P(E)$ such that $A = \{x_1 = \{x_{1,1}, x_{2,1}\}, x_2 = \{x_{1,2}, x_{2,2}\}, x_3 = \{x_{1,1}, x_{1,3}\}, x_4 = \{x_{2,2}, x_{3,3}\}\}$.*

Let $X = \{((x_1, r), [0.1, 0.3]), ((x_2, r), [0.1, 0.2]), ((x_3, r), [0.2, 0.3]), ((x_4, r), [0, 0.1])\}$. Let $f_{Q_X}(x_1) = \{u_1, u_2, u_5\}, f_{Q_X}(x_2) = \{u_1, u_4, u_6\}, f_{Q_X}(x_3) = \{u_1, u_2, u_4, u_5\}$, and $f_{Q_X}(x_4) = \{u_1, u_2\}$. Suppose $Y = \{((x_1, r), [0.3, 0.9]), ((x_2, r), [0.4, 0.5]), ((x_3, r), [0.3, 0.5]), ((x_4, r), [0.2, 0.3])\}$, $f_{Q_Y}(x_1) = \{u_1, u_2, u_4, u_5\}$, $f_{Q_Y}(x_2) = \{u_3, u_4, u_5\}, f_{Q_Y}(x_3) = \{u_4, u_5\}$ and $f_{Q_Y}(x_4) = \{u_1, u_2\}$.

The representation of these sets F_{Q_X} and F_{Q_Y} are as follows:
$F_{Q_X} = \{(((x_1, r), [0.1, 0.3]), \{u_1, u_2, u_5\}), (((x_2, r), [0.1, 0.2]), \{u_1, u_4, u_6\}), ((x_3, r), [0.3, 0.2]), \{u_1, u_2, u_4, u_5\}), (((x_4, r), [0, 0.1]), \{u_1, u_2\})\}$, *and* $F_{Q_Y} = \{(((x_1, r), [0.3, 0.9]), \{u_1, u_2, , u_4, u_5\}), (((x_2, r), [0.4, 0.5]), \{u_3, u_4, u_5\}), ((x_3, r), [0.3, 0.5]), \{u_4, u_5\}), (((x_4, r), [0.2, 0.3]), \{u_1, u_2\})\}$.

Then the $\wedge$-product of interval valued Q-fuzzy multiparameterized soft sets F_{Q_X} and F_{Q_Y} are as follows

$X \times Y = \{(((x_1, r), (x_1, r)), [0.1, 0.3]), (((x_1, r), (x_2, r)), [0.1, 0.3]), (((x_1, r), (x_3, r)), [0.1, 0.3]), (((x_1, r), (x_4, r)), [0.1, 0.3]), (((x_2, r), (x_1, r)), [0.1, 0.2]), (((x_2, r), (x_2, r)), [0.1, 0.2]), (((x_2, r), (x_3, r)), [0.1, 0.2]), (((x_2, r), (x_4, r)), [0.1, 0.2]), (((x_3, r), (x_1, r)), [0.2, 0.3]), (((x_3, r), (x_2, r)), [0.2, 0.3]), (((x_3, r), (x_3, r)), [0.2, 0.3]), (((x_3, r), (x_4, r)), [0.2, 0.3]), (((x_4, r), (x_1, r)), [0, 0.1]), (((x_4, r), (x_2, r)), [0, 0.1]), (((x_4, r), (x_3, r)), [0, 0.1]), (((x_4, r), (x_4, r)), [0, 0.1])\}$, *and*
$f_{Q_{X\times Y}}(x_1, x_1) = \{u_1, u_2, u_5\}, f_{Q_{X\times Y}}(x_1, x_2) = \{u_5\}$,
$f_{Q_{X\times Y}}(x_1, x_3) = \{u_5\}, f_{Q_{X\times Y}}(x_1, x_4) = \{u_1, u_2\}, f_{Q_{X\times Y}}(x_2, x_1) = \{u_1, u_4\}$,
$f_{Q_{X\times Y}}(x_2, x_2) = \{u_4\}, f_{Q_{X\times Y}}(x_2, x_3) = \{u_4\}$, *and* $f_{Q_{X\times Y}}(x_2, x_4) = \{u_1\}$.

Similarly we can get $f_{Q_{X\times Y}}(x_3, x_1), f_{Q_{X\times Y}}(x_3, x_2), f_{Q_{X\times Y}}(x_3, x_3), f_{Q_{X\times Y}}(x_3, x_4)$, $f_{Q_{X\times Y}}(x_4, x_1)$, $f_{Q_{X\times Y}}(x_4, x_2), f_{Q_{X\times Y}}(x_4, x_3)$, *and* $f_{Q_{X\times Y}}(x_4, x_4)$.

Thus $F_{Q_X} \wedge F_{Q_Y} = \{((((x_1, r), (x_1, r)), [0.1, 0.3]), \{u_1, u_2, u_5\}), ((((x_1, r), (x_2, r)), [0.1, 0.3]), \{u_5\}), ((((x_1, r), (x_3, r)), [0.1, 0.3]), \{u_5\}), (((x_1, r), (x_4, r)), [0.1, 0.3]), \{u_1, u_2\}), ((((x_2, r), (x_1, r)), [0.1, 0.2]), \{u_1, u_4\})), ((((x_2, r), (x_2, r)), [0.1, 0.2]), \{u_4\}), ((((x_2, r), (x_3, r)), [0.1, 0.2]), \{u_4\}), ((((x_2, r), (x_4, r)), [0.1, 0.2]), \{u_1\}), ..., ((((x_4, r), (x_4, r)), [0, 0.1]), \{u_1, u_2\})\}$.

Definition 5.4. *Let F_{Q_X} and F_{Q_Y} be interval valued Q-fuzzy multiparameterized soft sets over U. The $\vee$-product (also called AND operation) of interval valued Q-fuzzy multiparameterized soft sets F_{Q_X} and F_{Q_Y} is an interval valued Q-fuzzy multiparameterized soft sets defined by $F_{Q_X} \vee F_{Q_Y} = F_{Q_{X\times Y}}$ where $X \times Y = max((\mu_{Q_X}(x, q), \mu_{Q_Y}(x, q))$ for all $(x, q) \in A \times Q$, and $f_{Q_{X\vee Y}}(a, b) = f_{Q_X}(a) \cup f_{Q_Y}(b)$ for all $(a, b) \in A \times A$.*

Example 5.4. *From Example 5.3., the $\vee$-product of interval valued Q-fuzzy multiparameterized soft sets F_{Q_X} and F_{Q_Y} are as follows:* $X \times Y = \{(((x_1, r), (x_1, r)), [0.3, 0.9]), (((x_1, r), (x_2, r)), [0.4, 0.5]), (((x_1, r), (x_3, r)), [0.3, 0.5]), (((x_1, r), (x_4, r)), [0.2, 0.3]), (((x_2, r), (x_1, r)), [0.3, 0.9]), (((x_2, r), (x_2, r)), [0.4, 0.5]), (((x_2, r), (x_3, r)), [0.3, 0.5]), (((x_2, r), (x_4, r)), [0.2, 0.3]), (((x_3, r), (x_1, r)), [0.3, 0.9]), (((x_3, r), (x_2, r)), [0.4, 0.5]), (((x_3, r), (x_3, r)), [0.3, 0.5]), (((x_3, r), (x_4, r)), [0.2, 0.3]), (((x_4, r), (x_1, r)), [0.3, 0.9]), (((x_4, r), (x_2, r)), [0.4, 0.5]), (((x_4, r), (x_3, r)), [0.3, 0.5]), (((x_4, r), (x_4, r)), [0.2, 0.3])\}$, *and* $f_{Q_{X\times Y}}(x_1, x_1) = \{u_1, u_2, u_4, u_5\}, f_{Q_{X\times Y}}(x_1, x_2) = \{u_1, u_2, u_3, u_4, u_5\}$, $f_{Q_{X\times Y}}(x_1, x_3) = \{u_1, u_2, u_4, u_5\}, f_{Q_{X\times Y}}(x_1, x_4) = \{u_1, u_2, u_5\}, f_{Q_{X\times Y}}(x_2, x_1) = \{u_1, u_2, u_4, u_5, u_6\}, f_{Q_{X\times Y}}(x_2, x_2) = \{u_1, u_3, u_4, u_5, u_6\}, f_{Q_{X\times Y}}(x_2, x_3) = \{u_1, u_4, u_5, u_6\}$, *and* $f_{Q_{X\times Y}}(x_2, x_4) = \{u_1, u_2, u_4, u_6\}$.

Similarly we can get $f_{Q_{X\times Y}}(x_3,x_1), f_{Q_{X\times Y}}(x_3,x_2), f_{Q_{X\times Y}}(x_3,x_3), f_{Q_{X\times Y}}(x_3,x_4),$ $f_{Q_{X\times Y}}(x_4,x_1), f_{Q_{X\times Y}}(x_4,x_2), f_{Q_{X\times Y}}(x_4,x_3),$ *and* $f_{Q_{X\times Y}}(x_4,x_4)$*. Then* $F_{Q_X} \vee F_{Q_Y} = \{((((x_1,r),(x_1,r)),[0.3,0.9]),\{u_1,u_2,u_4,u_5\}),,((((x_1,r),(x_2,r)),$ $[0.4,0.5]),\{u_1,u_2,u_3,u_4,u_5\}),((((x_1,r),(x_3,r)),[0.3,0.5]),\{u_1,u_2,u_4,u_5\}),(((x_1,r),$ $(x_4,r)),[0.2,0.3]),\{u_1,u_2,u_5\}),((((x_2,r),(x_1,r)),[0.3,0.9]),\{u_1,u_2,u_4,u_5,u_6\}),$ $((((x_2,r),(x_2,r)),[0.4,0.5]),\{u_1,u_3,u_4,u_5,u_6\}),((((x_2,r),(x_3,r)),[0.3,0.5]),$ $\{u_1,u_4,u_5,u_6\}),((((x_2,r),(x_4,r)),[0.2,0.3]),\{u_1,u_2,u_4,u_6\}),...,((((x_4,r),$ $(x_4,r)),[0.2,0.3]),\{u_1,u_2\})\}$.

Now we give some propositions on the $\wedge$ and $\vee$-product of interval valued Q-fuzzy multiparameterized soft sets

Proposition 5.5. *Let* F_{Q_X} *and* F_{Q_Y} *be interval valued Q-fuzzy multiparameterized soft sets over* U*. Then we have*

1. $F_{Q_X} \subseteq F_{Q_X} \vee F_{Q_Y}$.
2. $F_{Q_Y} \subseteq F_{Q_X} \vee F_{Q_Y}$.
3. $F_{Q_X} \wedge F_{Q_Y} \subseteq F_{Q_X}$.
4. $F_{Q_X} \wedge F_{Q_Y} \subseteq F_{Q_Y}$.

Proof. (1) Let $[\mu^-_{Q_X}(u,q), \mu^+_{Q_X}(u,q)] = \mu_{Q_X}(u,q), [\mu^-_{Q_Y}(u,q), \mu^+_{Q_Y}(u,q)] = \mu_{Q_Y}(u,q)$. Assume that $\mu_{Q_X}(u,q)$ and $\mu_{Q_Y}(u,q)$ are elements of Q_X and Q_Y respectively.

Then from Definition 5.4 we have $\mu_{Q_X\vee Q_Y}(x,q) = max(\mu_{Q_X}(u,q), \mu_{Q_Y}(u,q))$, $\mu_{Q_X}(u,q) \leq max(\mu_{Q_X}(u,q), \mu_{Q_Y}(u,q))$,

This implies $f_{Q_X\cup Q_Y}(x) = f_{Q_X}(x) \cup f_{Q_Y}(x)$, therefore $f_{Q_X}(x) \subseteq f_{Q_X}(x) \cup f_{Q_Y}(x)$.

Thus $F_{Q_X} \subseteq F_{Q_X} \vee F_{Q_Y}$.
Assertion (2) can be proven in a similar fashion.

(3). Assume $F_{Q_X} \wedge F_{Q_Y} = (\mu_{Q_X}(u,q) \wedge \mu_{Q_Y}(u,q))$, and from Definition 5.3 we have $\mu_{Q_X\wedge Q_Y}(x,q) = min(\mu_{Q_X}(u,q), \mu_{Q_Y}(u,q))$, $min(\mu_{Q_X}(u,q), \mu_{Q_Y}(u,q)) \subseteq \mu_{Q_X}(u,q)$,

This implies $f_{Q_X\cap Q_Y}(x) = f_{Q_X}(x) \cap f_{Q_Y}(x)$, therefore $f_{Q_X}(x) \cap f_{Q_Y}(x) \subseteq f_{Q_X}(x)$.

Thus $F_{Q_X} \wedge F_{Q_Y} \subseteq F_{Q_X}$. The proof of (4) can be obtained in a similar fashion. □

Proposition 5.6. *Let F_{Q_X}, F_{Q_Y} and F_{Q_H} be interval valued Q-fuzzy multiparameterized soft sets over U. If $F_{Q_X} \subseteq F_{Q_Y}$, then we have*

1. $F_{Q_X} \wedge F_{Q_H} \subseteq F_{Q_Y} \wedge F_{Q_H}$.
2. $F_{Q_X} \wedge F_{Q_H} \subseteq F_{Q_Y} \wedge F_{Q_H}$.

Proof. (1) Let $[\mu^-_{Q_X}(u,q), \mu^+_{Q_X}(u,q)] = \mu_{Q_X}(u,q), [\mu^-_{Q_Y}(u,q), \mu^+_{Q_Y}(u,q)] = \mu_{Q_Y}(u,q)$ and $[\mu^-_{Q_H}(u,q), \mu^+_{Q_H}(u,q)] = \mu_{Q_H}(u,q)$, Assume that $\mu_{Q_X}(u,q),\ \mu_{Q_Y}(u,q)$ and $\mu_{Q_H}(u,q)$ are elements of Q_X, Q_Y and Q_H respectively.Thus
(i) $\mu_{Q_X \wedge Q_H}(x,q) = min(\mu_{Q_X}(u,q), \mu_{Q_H}(u,q))$, and
(ii) $\mu_{Q_Y \wedge Q_H}(x,q) = min(\mu_{Q_Y}(u,q), \mu_{Q_H}(u,q))$.

There are three cases to be considered.

1 . If the minimum membership of (i) is $\mu_{Q_X}(u,q)$, and for (ii) is $\mu_{Q_Y}(u,q)$, then $F_{Q_X} \subseteq F_{Q_Y}$. Therefore $F_{Q_X} \wedge F_{Q_H} \subseteq F_{Q_Y} \wedge F_{Q_H}$,

2 . If the minimum membership of (i) is $\mu_{Q_X}(u,q)$, and for (ii) is $\mu_{Q_H}(u,q)$, then $\mu_{Q_X}(u,q) \leq \mu_(Q_H)(u,q)$, implies $f_{Q_X} \subseteq f_{Q_H}$ and consequently $F_{Q_X} \subseteq F_{Q_H}$. Thus $F_{Q_X} \wedge F_{Q_H} \subseteq F_{Q_Y} \wedge F_{Q_H}$.

3 . If the minimum membership of (i) is $\mu_{Q_H}(u,q)$, and for (ii) is $\mu_{Q_Y}(u,q)$, then $\mu_{Q_H}(u,q) \leq Q_X(u,q)$, implies $f_{Q_H} \subseteq f_{Q_X}$, and $F_{Q_H} \subseteq F_{Q_X}$. Consequently if $F_{Q_H} \subseteq F_{Q_X}$ and $F_{Q_X} \subseteq F_{Q_Y}$, then $F_{Q_H} \subseteq F_{Q_Y}$.

Therefore $F_{Q_X} \wedge F_{Q_H} \subseteq F_{Q_Y} \wedge F_{Q_H}$.

Assertion (2) can be proven in a similar fashion □

Proposition 5.7. *Let F_{Q_X}, F_{Q_Y} and F_{Q_H} be interval valued Q-fuzzy multiparameterized soft sets over U. Therefore*

1. $F_{Q_X} \vee (F_{Q_X} \wedge F_{Q_Y}) \subseteq F_{Q_X} \vee F_{Q_X}$.
2. $F_{Q_X} \wedge F_{Q_X} \subseteq F_{Q_X} \wedge (F_{Q_X} \vee F_{Q_Y})$.

Proof. (1) Let us write $F_{Q_{X\wedge Y}} = F_{Q_X} \wedge F_{Q_Y}$, where $F_{Q_{X\wedge Y}}(a,b) = F_{Q_X}(a) \cap F_{Q_Y}(b)$, for all $(a,b) \in A \times A$, Then let $F_{Q_X} \vee F_{Q_{X\wedge Y}} = F_{Q_{X\vee(X\wedge Y)}}$ where

$F_{Q_{X\vee(X\wedge Y)}}(\acute{a},(a,b)) = F_{Q_X}(\acute{a}) \cup (F_{Q_X}(a) \cap F_{Q_Y}(b))$, for all $(\acute{a},(a,b)) \in A \times (A \times A)$. Let $F_{Q_{X\vee X}} = F_{Q_X} \vee F_{Q_X}$ where $F_{Q_{X\vee X}}(a_1,a_2) = F_{Q_X}(a) \cup F_{Q_X}(a_2)$, for all $(a_1,a_2) \in A \times A$. Note also that $F_{Q_X}(\acute{a}) \cup (F_{Q_X}(a) \cap F_{Q_Y}(b)) = (F_{Q_X}(a) \cap F_{Q_X}(a)) \cup (F_{Q_X}(\acute{a}) \cap F_{Q_Y}(b))$. Hence we deduce that $F_{Q_{X\vee(X\wedge Y)}}(\acute{a},(a,b)) \subseteq (F_{Q_X}(\acute{a}) \cup F_{Q_X}(a)) = F_{Q_{X\vee X}}(\acute{a},a)$. This shows that $F_{Q_{X\vee(X\wedge Y)}} \subseteq F_{Q_{X\wedge X}}$ as required.

Assertion (2) can be proven in a similar fashion. □

6. Interval Valued Q-Fuzzy Decision Set of an IVQFMP-Soft Set

In this section, the authors define an interval value Q-fuzzy decision set of an IVQFMP-soft set to construct a decision method by which approximate functions of a soft set are combined to produce a single Q-fuzzy set that can be used to evaluate each alternative.

Definition 6.1. *Let $F_{Q_X} \in IVQFMPS(U)$. An interval valued Q-fuzzy decision set of F_{Q_X}, denoted by $F^d_{Q_X}$ is defined by*

$$F^d_{Q_X} = \{(u,q), \mu_{F^d_{Q_X}}(u,q) \, : u \in U, q \in Q\},$$

which is Q-fuzzy set over U, whereby its membership function $\mu_{F^d_{Q_X}}(u,q)$ is defined by $\mu_{F^d_{Q_X}} : U \times Q \to [0,1]$ and

$$\mu_{F^d_{Q_X}}(u,q) = \frac{1}{|supp(X)|} \sum_{(x,q)\in supp(X)} \{\frac{[\mu^-_{Q_X}(x,q) + \mu^+_{Q_X}(x,q)]}{2}\} \chi f_{Q_X(x)}(u,q),$$

where $supp(X)$ is the support set of X, $f_{Q_X}(x)$ is the crisp subset determined by the parameter x and

$$f_{Q_X(x)} = \left\{ \begin{array}{ll} 1, & \text{if; } u \in f_{Q_X} \text{ and } q \in X \\ 0, & \text{if; } u \notin f_{Q_X} \text{ or } q \notin X \end{array} \right\}.$$

Using the definition of interval valued Q-fuzzy decision set of an IVQFMP-soft set, a decision method by the following algorithm can be constructed.

Step 1: Construct a F_{Q_X} over U.

Step 2: Compute the interval valued Q-fuzzy decision set $F^d_{Q_X}$.

Step 3: Select the largest membership grade max $\mu_{F^d_{Q_X}}(u, q)$.

The following example will illustrate the idea of the algorithm given above.

Example 6.1. *Let us assume that a person goes to buy a car from a set of five cars* $U = \{u_1, u_2, u_3, u_4, u_5\}$, $Q = \{q, p\}$ *be a set years of car made and* $E = \{E_1, E_2\}$ *be a set of parameters where* E_1 *is a set of cost parameters given by* $\{x_{1,1} = cheap, x_{1,2} = expensive, x_{1,3} = very\, expensive, x_{1,4} = very\, cheap\}$, E_2 *is a set of color parameters given by* $\{x_{2,1} = red, x_{2,2} = black, x_{2,3} = white\}$, *with* $A \subseteq P(E)$ *such that* $A = \{x_1 = \{x_{1,1}, x_{2,1}\}, x_2 = \{x_{1,4}, x_{2,2}\}, x_3 = \{x_{1,3}, x_{2,3}\}\}$. *The person considers two multi parameters namely, (cheap, red color) and (very cheap, white color). That is, the subset of parameters is* $X = \{((x_1, q), [0.6, 0.8]), ((x_1, p), [0.6, 0.9]), ((x_2, q), [0.4, 0.7]), ((x_2, p), [0.3, 0.5])\}$.

The steps below can be used to find a suitable car for the person to buy.

Step 1: Construct F_{Q_X} *over* U,

$$F_{Q_X} = \{(((x_1, q), [0.6, 0.8]), \{u_1, u_2\}), (((x_1, p), [0.6, 0.9]), \{u_1, u_5\}),$$
$$(((x_2, q), [0.4, 0.7]), \{u_2, u_4, u_3\}), (((x_2, p), [0.3, 0.5]), \{u_1, u_4, u_5\})\}.$$

Step 2: The Q-fuzzy decision set of F_{Q_X} *can be found as*

$$F^d_{Q_X} = \{((u_1, q), 0.1), ((u_1, p), 0.29), ((u_2, q), 0.31), ((u_2, p), 0), ((u_3, q), 0.14),$$
$$((u_3, p), 0), ((u_4, q), 0.14), ((u_4, p), 0.1), ((u_5, q), 0.19), ((u_5, p), 0.1)\}.$$

Step 3: Finally, the largest membership grade can be chosen by

$$max \mu_{F^d_{Q_X}}(u, q) = 0.31.$$

This implies that the candidate (u_2, q) *has the largest membership grade, hence the car* u_2 *made in year* q *is most suitable to be bought.*

The following example will illustrate that IVQFMP-soft set can better reflect the decision maker's preferences compared to the multi Q-fuzzy set.

Example 6.2. *Suppose that a university wants to select a student for the " Best outgoing student award" and there are nominees who form the universe* $U = \{p_1, p_2, p_3, p_4, p_5\}$, $Q = \{q, p\}$ *be the nationality and* $E_i = \{E_1, E_2, E_3\}$, *be a set of parameters where* E_1 *is a set of fields of study parameters* $E_1 = \{x_{1,1} = Biochemistry, x_{1,2} = Biology, x_{1,3} = Ecology\}$, E_2 *is a set of achievement of the students parameters given by* $E_2 = \{x_{2,1} = academic\ performance, x_{2,2} = cocurricular\ activities, x_{2,3} = good\ conduct\}$, *and* E_3 *is a set of programme of study parameters given by* $E_3 = \{x_{3,1} = graduate\ students, x_{3,2} = undergraduate\ students\}$. *Consider a set of parameters,* $\{x_{1,1}, x_{2,1}\}$, $\{x_{1,3}, x_{2,2}, x_{3,2}\}$, $\{x_{1,2}, x_{2,3}\}$, *and* $\{x_{1,1}, x_{3,1}\}$.

In this example it will be difficult to explain the universal U with two or three membership function using multi Q-fuzzy set, especially when there are different numbers of parameters involved. However, IVQFMP-soft set can reflect the decision maker's preferences. Thus, the expert committee can consider sets of multi parameters namely,
$x_1 = \{Biochemistry, high\ academic\ performance\}$,
$x_2 = \{Ecology, cocurricular\ activities, undergraduate\ students\}$,
$x_3 = \{Biology, good\,conduct\}$, *and*
$x_4 = \{Biochemistry, graduate\ students\}$.
That is, the subset of parameter and IVQFMP-soft sets are as follows.

$X = \{((x_1, q), [0.3, 0.5]), ((x_1, p), [0.2, 0.9]), ((x_2, q), [0.4, 0.8]), ((x_2, p), [0.3, 0.5]), ((x_3, q), [0.3, 0.6]), ((x_3, p), [0.1, 0.7]), ((x_4, q), [0.1, 0.3]), ((x_4, p), [0.5, 0.7])\}$.

$F_{Q_X} = \{(((x_1, q), [0.3, 0.5]), \{u_1, u_5\}), (((x_1, p), [0.2, 0.9]), \{u_3, u_6\}), (((x_2, q), [0.4, 0.8]), \{u_2, u_3, u_4\}), (((x_2, p), [0.3, 0.5]), \{u_1, u_4, u_5\}), (((x_3, p), [0.3, 0.6]), \{u_1, u_4, u_5\}), (((x_3, q), [0.1, 0.7]), \{u_3, u_4, u_5\}), (((x_4, q), [0.3, 0.6]), \{u_4, u_5\}), ((x_4, p), [0.1, 0.7]), \{u_1\})\}$.

7. Conclusion

The authors introduced the concepts of an interval-valued Q-fuzzy set and IVQFMP-soft set and described their equality, subset, union, intersection, $\wedge$ and $\vee$-product. An algorithm for a decision method using an IVQFMP-soft set was constructed. The proposed IVQFMP-soft set is then successfully applied to a decision making problem which shows that it can more accurately reflect the decision maker's hesitancy in stating his preferences over choice alternatives, as compared to the multi Q- fuzzy set.

References

Adam, F., and Hassan, N. (2014a). Q-fuzzy soft matrix and its application. AIP Conference Proceedings, 1602, 772-778.

Adam, F., and Hassan, N. (2014b). Q-fuzzy soft set. *Applied Mathematical Sciences,* 8 (174): 8689-8695.

Adam, F., and Hassan, N. (2014c). Operations on Q-fuzzy soft set. *Applied Mathematical Sciences,* 8 (175): 8697 - 8701.

Adam, F., and Hassan, N. (2014d). Multi Q-fuzzy parameterized soft set and its application. *Journal of Intelligent and Fuzzy Systems,* 27 (1):419-424.

Adam, F., and Hassan, N. (2015). Multi Q-fuzzy soft set and its application. *Far East Journal of Mathematical Sciences,* 97(7): 871-881.

Adam, F., and Hassan, N. (2016). Multi Q-fuzzy soft expert set and its application. *Journal Intelligent and Fuzzy Systems,* 30(2): 943-950.

Alhazaymeh, K., Halim, S.A., Salleh, A.R. and Hassan, N. (2012). Soft intuitionistic fuzzy sets. *Applied Mathematical Sciences,* 6(54): 2669-2680.

Alhazaymeh, K. and Hassan, N. (2012a). Generalized vague soft set and its application. *International Journal of Pure and Applied Mathematics,* 77(3): 391-401.

Alhazaymeh, K. and Hassan, N. (2012b). Interval-valued vague soft sets and its application. *Advances in Fuzzy Systems,* volume 2012, Article ID 208489.

Alhazaymeh, K. and Hassan, N. (2012c). Possibility vague soft set and its application in decision making. *International Journal of Pure and Applied Mathematics,* 77(4): 549-563.

Alhazaymeh, K. and Hassan, N. (2013a). Generalized interval-valued vague soft set.*Applied Mathematical Sciences,* 7(140): 6983-6988.

Alhazaymeh, K. and Hassan, N. (2013b). Generalized vague soft expert set theory. *AIP Conference Proceedings,* 1571: 970-974.

Alhazaymeh, K. and Hassan, N. (2013c). Possibility interval-valued vague soft set. *Applied Mathematical Sciences,* 7(140): 6989-6994.

Alhazaymeh, K. and Hassan, N. (2014a). Application of generalized vague soft expert set in decision making. *International Journal of Pure and Applied Mathematics*, 93(3): 361-367.

Alhazaymeh, K. and Hassan, N. (2014b). Generalized vague soft expert set. *International Journal of Pure and Applied Mathematics*, 93(3): 351-360.

Alhazaymeh, K. and Hassan, N. (2014c). Mapping on generalized vague soft expert set. *International Journal of Pure and Applied Mathematics*, 93(3): 369-376.

Alhazaymeh, K. and Hassan, N. (2014d). Vague soft multiset theory. *International Journal of Pure and Applied Mathematics*, 93(4): 511-523.

Alhazaymeh, K. and Hassan, N. (2015). Vague soft set relations and functions. *Journal of Intelligent and Fuzzy Systems*, 3(28): 1205-1212.

Alkhazaleh, S., Salleh, A. R., and Hassan, N. (2011). Fuzzy parameterized interval-valued fuzzy soft set. *Applied Mathematical Sciences*, 5(67): 3335-3346.

Ali, M.I, Feng, F., Lui, X., Min, W. K and Shabir, M. (2009). On some new operations in soft set theory. *Computers and Mathematics with Applications*, 57 (9): 1547-1553.

Al-Quran, A. and Hassan, N. (2016). Neutrosophic vague soft expert theory. *Journal of Intelligent and Fuzzy Systems*, 30 (6): 3691-3702.

Babitha, K.V. and Sunil, J.J. (2010). Soft set relations and functions. *Computers and Mathematics with Applications*, 60 (7): 1840-1849.

Babitha, K.V., and Sunil, J.J. (2011). Transitive closure and ordering on soft sets. *Computers and Mathematics with Applications*, 62(5): 2235-2239.

Cagman, N., and Enginoglu, S. (2010). Soft matrix theory and its decision making. *Computers and Mathematics with Applications*, 59 (10): 3308-3314.

Cagman, N., Citak, F., and Enginoglu, S. (2010) Fuzzy parameterized fuzzy soft set theory and its application. *Turkish Journal of Fuzzy System*, 1 (1), 21-35.

Cagman. N., Enginoglu, S., and Citak, F. (2011). Fuzzy soft set theory and its applications. *Iranian Journal of Fuzzy System*, 8(3): 137-147.

Cagman, N. (2014). Contributions to the theory of soft sets. *Journal of New Results in Science*, 4: 33-41.

Chen, D., Tsang, E.C.C., Yeung, D.S., and Wang, X. (2005).The parametrization reduction of soft sets and its applications. *Computers and Mathematics with Applications,* 49: 757-763.

Ge, X., and Yang, S. (2011). Investigations on some operations of soft sets. *World Academy of Science, Engineering and Technology,* 51: 1112-1115.

Hassan, N. and Alhazaymeh, K. (2013). Vague soft expert set theory. *AIP Conference Proceedings,* 1522: 953–958.

Maji, P.K., Biswas, R., and Roy, A.R. (2003). Soft set theory. *Computers and Mathematics with Applications,* 45: 555-562.

Majumdar, P., and Samanta, S. K. (2013). Softness of a soft set: Soft set entropy. *Annals of Fuzzy Mathematics and Informatics,* 6 (1): 59-68.

Molodtsov, D. (1999). Soft set theory - First results, *Computers and Mathematics with Applications,* 37: 19-31.

Pawlak, Z. (1982). Rough sets. *International Journal of Computer and Information Sciences,* 11(5): 341-356.

Salleh, A. R., Alkhazaleh, S., Hassan, N. and Ahmad, A. G. (2012). Multiparameterized soft set, *Journal of Mathematics and Statistics,* 8(1): 92-97.

Sezgin, A., and Atagun, A.O. (2011). On operations of soft sets, *Computers and Mathematics with Applications,* 61(5): 1457-1467.

Singh, D., and Onyeozili, I. A. (2012). Some results on distributive and absorption properties on soft operations, *IOSR Journal of Mathematics,* 4(2): 18-30.

Varnamkhasti, M. J. and Hassan, N. (2012). Neurogenetic algorithm for solving combinatorial engineering problems. *Journal of Applied Mathematics,* volume 2012, Article ID 253714.

Varnamkhasti, M. J. and Hassan, N. (2013). A hybrid of adaptive neuro-fuzzy inference system and genetic algorithm. *Journal of Intelligent and Fuzzy Systems,* 25(3):793-796.

Yang, X., Lin, T. Y., Yang, J., Li, Y., and Yu, D. (2009). Combination of interval-valued fuzzy set and soft set. *Computers and Mathematics with Applications,* 58(3): 521-527.

Zhu, P., and Wen, Q. (2013). Operations on soft sets revisited. *Journal of Applied Mathematics,* Article ID 105752.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control,* 8(3): 338-353.