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INTRODUCTION TO NEUTROSOPHIC HYPERGROUPS

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Abstract The objective of this paper is to introduce the concept of *neutrosophic* hypergroup and present some of its elementary properties.

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1. INTRODUCTION

In 1995, Florentin Smarandache introduced the notion of Neutrosophy as a new branch of philosophy. Neutrosophy is the base of neutrosophic logic which is an extension of the fuzzy logic in which indeterminancy is included. In the neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminancy in a subset I, and the percentage of falsity in a subset F. Since the world is full of indeterminancy, several real world problems involving indeterminancy arising from law, medicine, sociology, psychology, politics, engineering, industry, economics, management and decision making, finance, stocks and share, meteorology, artificial intelligence, IT, communication etc can be solved by neutrosophic logic.

2. NEUTROSOPHIC ALGEBRAIC STRUCTURES

Using Neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache introduced the concept of neutrosophic algebraic structures in [12]. Some of the neutrosophic algebraic structures introduced and studied included neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, neutrosophic bigroupoids and so on. In [13], Vasantha Kandasamy introduced and studied *neutrosophic* rings. In [1], Agboola et al. studied the structure of neutrosophic polynomial rings and in [2], Agboola et al. studied neutrosophic ideals and *neutrosophic* quotient rings. In [3], Agboola et al. studied neutrosophic groups and subgroups.

3. NEUTROSOPHIC GROUPS

Definition 3.1. [12] *Let $(G, \star)$ be any group and let $N(G) = \langle G \cup I \rangle$. The couple $(N(G), \star)$ is called a neutrosophic group generated by G and I under the binary operation $\star$.*

I is called the neutrosophic element with the property $I \star I = I$. I^{-1}, the inverse of I is not defined and hence does not exist.

$N(G)$ is said to be commutative if $a \star b = b \star a$ for all $a, b \in N(G)$.

Theorem 3.1. [12] *Let $N(G)$ be a neutrosophic group. Then,*

(1) $N(G)$ in general is not a group;

(2) $N(G)$ always contain a group.

Definition 3.2. *Let $N(G)$ be a neutrosophic group. Then,*

(1) A proper subset $N(H)$ of $N(G)$, where $H \subset G$, is said to be a neutrosophic subgroup of $N(G)$ if $N(H)$ is a neutrosophic group, that is, $N(H)$ contains a proper subset which is a group;

(2) $N(H)$ is said to be a pseudo neutrosophic subgroup if it does not contain a proper subset which is a group.

Example 3.1. *(1) $(N(\mathbb{Z}), +)$, $(N(\mathbb{Q}), +)$ $(N(\mathbb{R}), +)$ and $(N(\mathbb{C}), +)$ are neutrosophic groups of integer, rational, real and complex numbers, respectively.*

(2) $(\langle\{\mathbb{Q} - \{0\}\} \cup I\rangle, \cdot)$, $(\langle\{\mathbb{R} - \{0\}\} \cup I\rangle, \cdot)$ and $(\langle\{\mathbb{C} - \{0\}\} \cup I\rangle, \cdot)$ are neutrosophic groups of rational, real and complex numbers, respectively.

Example 3.2. [3] *Let $N(G) = \{e, a, b, c, I, aI, bI, cI\}$ be a set, where $a^2 = b^2 = c^2 = e$, $bc = cb = a, ac = ca = b, ab = ba = c$, then $N(G)$ is a commutative neutrosophic group under multiplication since $\{e, a, b, c\}$ is the Klein 4-group. $N(H) = \{e, a, I, aI\}$, $N(K) = \{e, b, I, bI\}$ and $N(P) = \{e, c, I, cI\}$ are neutrosophic subgroups of $N(G)$.*

Theorem 3.2. [3] *Let N(H) be a non-empty proper subset of a neutrosophic group $(N(G), \star)$. Then, N(H) is a neutrosophic subgroup of N(G) if and only if the following conditions hold:*

(1) $a, b \in N(H)$ implies that $a \star b \in N(H)$;

(2) there exists a proper subset A of N(H) such that $(A, \star)$ is a group.

Theorem 3.3. [3] *Let N(H) be a non-empty proper subset of a neutrosophic group $(N(G), \star)$. Then, N(H) is a pseudo neutrosophic subgroup of N(G) if and only if the following conditions hold:*

(1) $a, b \in N(H)$ implies that $a \star b \in N(H)$;

(2) $N(H)$ does not contain a proper subset A such that $(A, \star)$ is a group.

4. HYPERGROUPS

The theory of hyperstructures was introduced in 1934 by Marty [9] at the 8th Congress of Scandinavian Mathematicians. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Several books have been written on this topic, see [4, 5, 5, 8, 10]. Hyperstructure theory both extends some well-known group results and introduce new topics leading us to a wide variety of applications, as well as to a broadening of the investigation fields. In this part, we present the notion of hypergroup and some well-known related concepts. These concepts will be used in the building of neutrosophic hypergroups, for more details we refer the readers to see [4, 5, 5, 8, 9, 10].

Let H be a non-empty set and $\circ : H \times H \rightarrow \mathcal{P}^{\star}(H)$ be a hyperoperation. The couple $(H, \circ)$ is called a *hypergroupoid*. For any two non-empty subsets A and B of H and $x \in H$, we define

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \ A \circ x = A \circ \{x\} \text{ and } x \circ B = \{x\} \circ B.$$

Definition 4.1. *A hypergroupoid $(H, \circ)$ is called a semihypergroup if for all a, b, c of H we have $(a \circ b) \circ c = a \circ (b \circ c)$, which means that*

$$\bigcup_{u \in a \circ b} u \circ c = \bigcup_{v \in b \circ c} a \circ v.$$

A hypergroupoid $(H, \circ)$ is called a quasihypergroup if for all a of H we have $a \circ H = H \circ a = H$. This condition is also called the reproduction axiom.

Definition 4.2. *A hypergroupoid $(H, \circ)$ which is both a semihypergroup and a quasi-hypergroup is called a hypergroup.*

Definition 4.3. *Let $(H, \circ)$ and $(H', \circ')$ be two hypergroupoids. A map $\phi : H \rightarrow H'$, is called*

(1) an inclusion homomorphism if for all x, y of H, we have $\phi(x \circ y) \subseteq \phi(x) \circ' \phi(y)$;

(2) a good homomorphism if for all x, y of H, we have $\phi(x \circ y) = \phi(x) \circ' \phi(y)$.

Let $(H, \circ)$ be a semihypergroup and R be an equivalence relation on H. If A and B are non-empty subsets of H, then

$$\begin{aligned} A\overline{R}B &\text{ means that } \forall a \in A, \exists b \in B \text{ such that } aRb \text{ and} \\ &\qquad \forall b' \in B, \exists a' \in A \text{ such that } a'Rb'; \\ A\overline{\overline{R}}B &\text{ means that } \forall a \in A, \forall b \in B, \text{ we have } aRb. \end{aligned}$$

Definition 4.4. *The equivalence relation ρ is called*

(1) regular on the right (on the left) if for all x of H, from $a\rho b$, it follows that $(a \circ x)\overline{\rho}(b \circ x)$ $((x \circ a)\overline{\rho}(x \circ b)$ respectively);

(2) strongly regular on the right (on the left) if for all x of H, from $a\rho b$, it follows that $(a \circ x)\overline{\overline{\rho}}(b \circ x)$ $((x \circ a)\overline{\overline{\rho}}(x \circ b)$ respectively);

(3) ρ is called regular (strongly regular) if it is regular (strongly regular) on the right and on the left.

Theorem 4.1. . *Let $(H, \circ)$ be a semihypergroup and ρ be an equivalence relation on H.*

(1) *If ρ is regular, then H/ρ is a semihypergroup, with respect to the following hyperoperation: $\overline{x} \otimes \overline{y} = \{\overline{z} \mid z \in x \circ y\}$;*

(2) *If the above hyperoperation is well defined on H/ρ, then ρ is regular.*

Corollary 4.1. *If $(H, \circ)$ is a hypergroup and ρ is an equivalence relation on H, then R is regular if and only if $(H/\rho, \otimes)$ is a hypergroup.*

Theorem 4.2. *Let $(H, \circ)$ be a semihypergroup and ρ be an equivalence relation on H.*

(1) *If ρ is strongly regular, then H/ρ is a semigroup, with respect to the following operation: $\overline{x} \otimes \overline{y} = \{\overline{z} \mid z \in x \circ y\}$;*

(2) *If the above operation is well defined on H/ρ, then ρ is strongly regular.*

Corollary 4.2. *If $(H, \circ)$ is a hypergroup and ρ is an equivalence relation on H, then ρ is strongly regular if and only if $(H/\rho, \otimes)$ is a group.*

Definition 4.5. *Let $(H, \circ)$ is a semihypergroup and A be a non-empty subset of H. We say that A is a complete part of H if for any nonzero natural number n and for all $a_1, \ldots, a_n$ of H, the following implication holds:*

$$A \cap \prod_{i=1}^{n} a_i \neq \emptyset \implies \prod_{i=1}^{n} a_i \subseteq A.$$

Theorem 4.3. *If $(H, \circ)$ is a semihypergroup and R is a strongly regular relation on H, then for all z of H, the equivalence class of z is a complete part of H.*

5. NEUTROSOPHIC HYPERGROUPS

Definition 5.1. *Let $(H, \star)$ be any hypergroup and let $< H \cup I >= \{x = (a, bI) : a, b \in H\}$. The couple $N(H) = (< H \cup I >, \star)$ is called a neutrosophic hypergroup generated by H and I under the hyperoperation $\star$. The part a is called the non-neutrosophic part of x and the part b is called the neutrosophic part of x.*

If $x = (a, bI)$ and $y = (c, dI)$ are any two elements of $N(H)$, where $a, b, c, d \in H$, we define $x \star y = (a, bI) \star (c, dI) = \{(u, vI) \mid u \in a \star c,\ v \in a \star d \cup b \star c \cup b \star d\} = (a \star c, (a \star d \cup b \star c \cup b \star d)I)$. Note that $a \star c \subseteq H$ and $(a \star d \cup b \star c \cup b \star d) \subseteq H$.

Definition 5.2. *Let $N(H)$ be a neutrosophic hypergroup and let $N(K)$ be a proper subset of $N(H)$. Then,*

(1) $N(K)$ is said to be a neutrosophic sub-hypergroup of $N(H)$ if $N(K)$ is a neutrosophic hypergroup, that is, $N(K)$ must contain a proper subset which is a hypergroup;

(2) $N(K)$ is said to be a pseudo neutrosophic sub-hypergroup of $N(H)$ if $N(K)$ is a neutrosophic hypergroup which contains no proper subset which is a hypergroup.

Theorem 5.1. *Let $N(H)$ be a neutrosophic hypergroup. Then, $N(H)$ is a semihypergroup.*

Proof. Let $x = (a, bI),\ y = (c, dI),\ z = (e, fI)$ be arbitrary elements of $N(H)$, where $a, b, c, d, e, f \in H$. Then,

$$\begin{aligned} x \star y &= (a, bI) \star (c, dI) \\ &= \{(u, vI) \mid u \in a \star c,\ v \in a \star d \cup b \star c \cup b \star d\} \\ &= (a \star c, (a \star d \cup b \star c \cup b \star d)I) \\ &\subseteq N(H). \end{aligned}$$

Hence, $(N(H), \star)$ is a hypergroupoid.

Next,

$$\begin{aligned} x \star (y \star z) &= (a, bI) \star ((c, dI) \star (e, fI)) \\ &= (a, bI) \star (c \star e, (c \star f \cup d \star e \cup d \star f)I)) \\ &= (a \star (c \star e), ((a \star (c \star f)) \cup (a \star (d \star e)) \cup (a \star (d \star f)) \cup (b \star (c \star e)) \\ &\quad \cup (b \star (c \star f)) \cup (b \star (d \star e)) \cup (b \star (d \star f)))I) \\ &= ((a \star c) \star e, (((a \star c) \star f) \cup ((a \star d) \star e) \cup ((a \star d) \star f) \cup ((b \star c) \star e) \\ &\quad \cup ((b \star c) \star f) \cup ((b \star d) \star e) \cup ((b \star d) \star f))I) \\ &= ((a, bI) \star (c, dI)) \star (e, fI) \\ &= (x \star y) \star z. \end{aligned}$$

Accordingly, $(N(H), \star)$ is a semihypergroup. ∎

Lemma 5.1. *Let $N(H)$ be a neutrosophic hypergroup. Then, $x \star N(H) = N(H) \star x \subset N(H)$ for all $x = (a, bI) \in N(H)$.*

Proof. We have

$$\begin{aligned} x \star N(H) &= (a, bI) \star N(H) \\ &= (a, bI) \star \{(h_1, h_2 I) : h_1, h_2 \in H\} \\ &= \{(a \star h_1, (a \star h_2 \cup b \star h_1 \cup b \star h_2)I) : a, b, h_1, h_2 \in H\} \\ &= \{(u, vI) : u \in a \star h_1, v \in (a \star h_2 \cup b \star h_1 \cup b \star h_2)\} \\ &\subset N(H) \end{aligned}$$

Similarly, $N(H) \star x \subset N(H)$ and therefore, $x \star N(H) = N(H) \star x \subset N(H)$. ∎

Theorem 5.2. *If $N(H)$ is a neutrosophic hypergroup, then*

(1) $N(H)$ in general is not a hypergroup;

(2) $N(H)$ always contain a hypergroup.

Proof. (1) Follows directly from Theorem 5.3 and Lemma 5.4.
(2) Follows from the definition of a *neutrosophic* hypergroup. ∎

Example 5.1. *Let $H = \{a, b, (a, aI), (a, bI), (b, aI), (b, bI)\}$ be a set and let $\star$ be a hyperoperation on H defined in the table below.*

$\star$	a	b	(a, aI)	(a, bI)	(b, aI)	(b, bI)
a	a	b	(a, aI)	(a, bI)	(b, aI)	(b, bI)
b	b	a b	(b, bI)	(b, aI) (b, bI)	(a, bI) (b, bI)	(a, aI) (a, bI) (b, aI) (b, bI)
(a, aI)	(a, aI)	(b, bI)	(a, aI)	(a, aI) (a, bI)	(b, aI) (b, bI)	(b, bI)
(a, bI)	(a, bI)	(b, aI) (b, bI)	(a, aI) (a, bI)	(a, aI) (a, bI)	(b, aI) (b, bI)	(b, aI) (b, bI)
(b, aI)	(b, aI)	(b, bI) (a, bI)	(b, aI) (b, bI)	(b, aI) (b, bI)	(a, aI) (a, bI) (b, aI) (b, bI)	(a, aI) (a, bI) (b, aI) (b, bI)
(b, bI)	(b, bI)	(a, aI) (a, bI) (b, aI) (b, bI)	(b, bI)	(b, aI) (b, bI)	(a, aI) (a, bI) (b, aI) (b, bI)	(a, aI) (a, bI) (b, aI) (b, bI)

It is clear from the table that $(H, \star)$ is a neutrosophic hypergroup since it contains a proper subset $\{a, b\}$ which is a hypergroup under $\star$.

Theorem 5.3. *Let $(N(H), \star_1)$ and $(N(K), \star_2)$ be any two neutrosophic hypergroups. Then, $(N(H) \times N(K), \star)$ is a neutrosophic hypergroup, where*

$$(x_1, x_2)\star(y_1, y_2) = \{(x, y) : x \in x_1\star_1 y_1, y \in x_2\star_2 y_2, \ \forall\, (x_1, x_2), (y_1, y_2) \in N(H)\times N(K)\}.$$

Theorem 5.4. *Let $(N(H), \star)$ be a neutrosophic hypergroup and let $(K, \circ)$ be a hypergroup. Then, $(N(H) \times K, \star')$ is a neutrosophic hypergroup, where*

$$(h_1, k_1) \star' (h_2, k_2) = \{(h, k) : h \in h_1 \star h_2, k \in k_1 \circ k_2, \ \forall\ (h_1, k_1), (h_2, k_2) \in N(H) \times K\}.$$

Definition 5.3. *Let $(N(H_1), \star_1)$ and $(N(H_2), \star_2)$ be any two neutrosophic hypergroups and let $f : N(H_1) \to N(H_2)$ be a map. Then,*

(a) f is called a homomorphism if:
(1) for all x, y of $N(H_1)$, $f(x \star_1 y) \subseteq f(x) \star_2 f(y)$,
(2) $f(I) = I$.

(b) f is called a good homomorphism if:
(1) for all x, y of $N(H_1)$, $f(x \star_1 y) = f(x) \star_2 f(y)$,
(2) $f(I) = I$.

(c) f is called an isomorphism if f is a homomorphism and f^{-1} is also a homomorphism.

(d) f is called a 2-homomorphism if for all x, y of $N(H_1)$,

$$f^{-1}(f(x) \star_2 f(y)) = f^{-1}(f(x \star_1 y)).$$

(e) f is called an almost strong homomorphism if for all x, y of $N(H_1)$,

$$f^{-1}(f(x) \star_2 f(y)) = f^{-1}(f(x)) \star_1 f^{-1}(f(y)).$$

Definition 5.4. *Let $N(K)$ be a neutrosophic sub-hypergroup of a neutrosophic hypergroup $(N(H), \star)$. Then,*

(1) $N(K)$ is said to be closed on the left (right) if for all $k_1, k_2 \in N(K)$, $x \in N(H)$ we have $k_2 \in x \star k_1$ ($k_2 \in k_1 \star x$) implies that $x \in N(K)$;

(2) $N(K)$ is said to be ultraclosed on the left (right) if for all $x \in N(H)$ we have $x \star N(K) \cap x \star (N(H) \backslash N(K)) = \emptyset (N(K) \star x \cap (N(H) \backslash N(K)) \star x = \emptyset)$;

(3) $N(K)$ is said to be left (right) conjugable if $N(K)$ is left (right) closed and if for all $x \in N(H)$, there exists $h \in N(H)$ such that $x \star h \subseteq N(K)$ ($h \star x \subseteq N(K)$);

(4) $N(K)$ is said to be (closed, ultraclosed, conjugable) if it is left and right (closed, ultraclosed, conjugable).

Lemma 5.2. *Let $N(K)$ be a neutrosophic sub-hypergroup of a neutrosophic hypergroup $(N(H), \star)$. For all $x \in N(H)$, we have*

(1) $x \star N(K) \subset N(H)$;

(2) $x \star (N(H) \backslash N(K)) \subset N(H)$;

(3) $x \star N(K) \cup x \star (N(H)\backslash N(K)) \subset N(H)$.

Lemma 5.3. *Let $N(K)$ be a neutrosophic sub-hypergroup of a neutrosophic hypergroup $(N(H), \star)$. For all $x \in N(K)$, we have*

(1) $x \star N(K) \subset N(K)$,

(2) $x \star (N(H)\backslash N(K)) \subset N(K)$;

(3) $x \star N(K) \cup x \star (N(H)\backslash N(K)) \subset N(H)$.

Theorem 5.5. *Let $N(K)$ be a neutrosophic sub-hypergroup of a neutrosophic hypergroup $(N(H), \star)$. Then,*

(1) $N(K) \star N(K) \subset N(K)$;

(2) $N(K) \star (N(H)\backslash N(K)) \subset N(K)$.

Theorem 5.6. *Let $N(K)$ be a neutrosophic sub-hypergroup of a neutrosophic hypergroup $(N(H), \star)$. If $N(K)$ is conjugable then it is not ultraclosed.*

Proof. Suppose that $N(K)$ is conjugable. Then, $N(K)$ is closed and for all $x \in N(H)$, there exists $y \in N(H)$ such that $x \star y \subseteq N(K)$ and $y \star x \subseteq N(K)$. Let $B = N(K) \cap (N(H)\backslash N(K))$ so that

$$\begin{aligned} x \star B &= x \star N(K) \cap x \star (N(H)\backslash N(K)) \\ &\subset N(H) \cap N(H) \\ &= N(H). \end{aligned}$$

This shows that $B \neq \emptyset$ and thus, $N(K)$ is not left ultraclosed. Similarly, it can be shown that $N(K)$ is not right ultraclosed. Hence, $N(K)$ is not ultraclosed. ∎

Theorem 5.7. *Let $(N(H_1), \star_1)$ and $(N(H_2), \star_2)$ be any two neutrosophic hypergroups and let $f : N(H_1) \to N(H_2)$ be a map.*

(1) If f is a bijective homomorphism, then f is an isomorphism if and only if it is good.

(2) If f is a strong homomorphism, then it is almost strong.

(3) If f is a good homomorphism and $N(K)$ is a neutrosophic(pseudo neutrosophic) sub-hypergroup of $N(H_1)$, then $f(N(K))$ is a neutrosophic(pseudo neutrosophic) sub-hypergroup of $N(H_2)$.

Theorem 5.8. *Let $(N(H), \star)$ be a neutrosophic hypergroup and let ρ be an equivalence relation on $N(H)$.*

(1) If ρ is regular, then $N(H)/\rho$ is a neutrosophic hypergroup.

(2) If ρ is strongly regular, then $N(H)/\rho$ is a neutrosophic group.

Proof. The proof follows from Theorem 4.7 and Corollary 4.8. ∎

Theorem 5.9. *Let $(N(H), \star)$ be a neutrosophic hypergroup and let ρ be a regular equivalence relation on $N(H)$. Then, the map $\phi : N(H) \to N(H)/\rho$ defined by $\phi(x) = \overline{x}$ is not a homomorphism (good homomorphism).*

Proof. It is clear since $I \in N(H)$ but $\phi(I) \neq I$. ∎

Theorem 5.10. *Let $(N(H), \star)$ be a neutrosophic hypergroup and let ρ be a strongly regular equivalence relation on $N(H)$. Then, for all $x \in N(H)$, $\overline{x}$ is a complete part of $N(H)$.*

6. CONCLUSION

In this paper, we have extended *neutrosophic* theory to hypergroup theory. Basic properties of *neutrosophic* hypergroups were presented and it was shown that every hypergroup is contained in a *neutrosophic* hypergroup but generally, a *neutrosophic* hypergroup is not a hypergroup.

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