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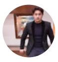

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Linear and Non-Linear Decagonal Neutrosophic numbers: Alpha Cuts, Representation, and solution of large MCDM problems

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Abstract

The postulation of neutrosophic numbers has been analyzed from different angles in this paper. In this current era, our main purpose is to mention Decagonal Neutrosophic numbers. The types of linear and non-linear generalized decagonal neutrosophic numbers play a very critical role in the theory related to uncertainty This approach is helpful in getting a crisp number from a neutrosophic number. The definitions regarding Linear, Non-Linear, symmetry, Asymmetry, alpha cuts have been introduced and large decision-making problems using fuzzy TOPSIS have been solved.

Keywords: Accuracy Functions, Neutrosophic number, Decagonal Neutrosophic numbers (DNN), MCDM, TOPSIS.

1.Introduction

In the line of remarkable researches from fuzzy to neutrosophic, each concept has its unique importance and flexibility. The traditional mathematics based on crisp (e.g. Yes or No) has a well defined [1] property. Fuzzy Set was first established by Zadeh [2], and further extended by Zadeh [3]. In fuzzy set, each element has its corresponding membership function. Molodtsov established soft sets [4], which opened new possibilities for researches and soft sets have been used widely in engineering, medical, economics. Moreover, we introduced ''The best technique to lose weight'' [5], by using soft sets.

The techniques to deal with vagueness and uncertainty were introduced by Smarandache [6] and the generalization of soft to hyper soft sets was also introduced by him. Smarandache [7-9] also discussed the extensions of neutrosophic sets in MCDM and TOPSIS and in other researches it is also mentioned [10-15]. The applicability of these applications is also found in the fields of operational research [16-17].

The neutrosophic numbers from triangular to nonagonal have been published and have established their use in real-life. Triangular and pentagonal have membership function [18-20]. Wang [21] introduced that single-valued neutrosophic sets are an extension of NSs. Ye [22] developed its aggregate operations and Peng [23-24] introduced the applications of neutrosophic sets . The other remarkable MCDM researches have been presented by Abdel-Basset [25] and Riaz [26-27].

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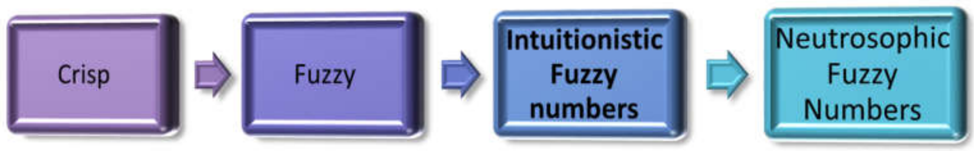
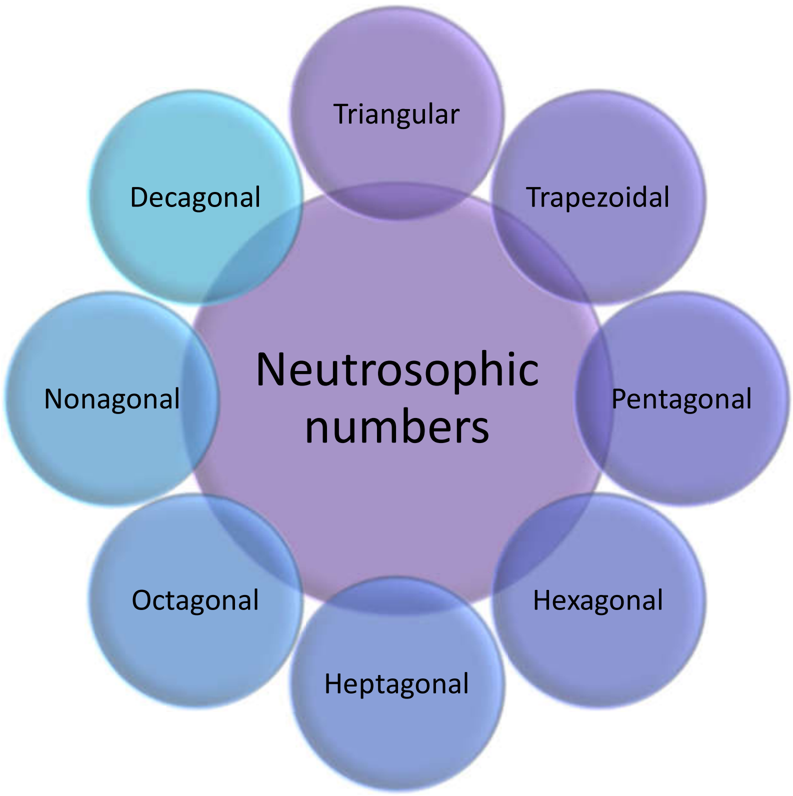


Motivation

The motivation for writing this article arose keeping in perspective the mutli dimensional problems associated with decision-making. So what makes the decognal approach different? This approach can be useful in solving mutli criteria decision-making problems associated with uncertain conditon in a neutrosophic environment. Already triangular neutrosophic numbers to Nonagonal neutrosophic numbers are being used in the fields of medical, engineering, accounting, cryptography. But they carry with them some limitions in their functions. The limitations of using Triangular to nonagonal neutrosophic numbers in solving MCDM are low edges. They solve less complex promblems, such as decision making based on less than ten edges. In order to overcome these limitations, Decagonal Neutrosophic numbers are introduced to deal with big and complex problems pertaining to decision-making.

Each neutrosophic numbers have its edges and capability to deal with fluctuations e.g. triangular has three edges, pentagonal have five, octagonal has eight, and nonagonal have nine edges for truthiness, indeterminacy, and falsity. With decagonal we have ten edges. So, it is suitable to solve decision-making problems in a better way by having ten edges as it gives us a slight edge.



1.1 Contribution: From the beginning of human life, decision-making is a common activity and the complication arises when we have to decide multi-criteria. For this purpose, we give some researches (e.g. octagonal and

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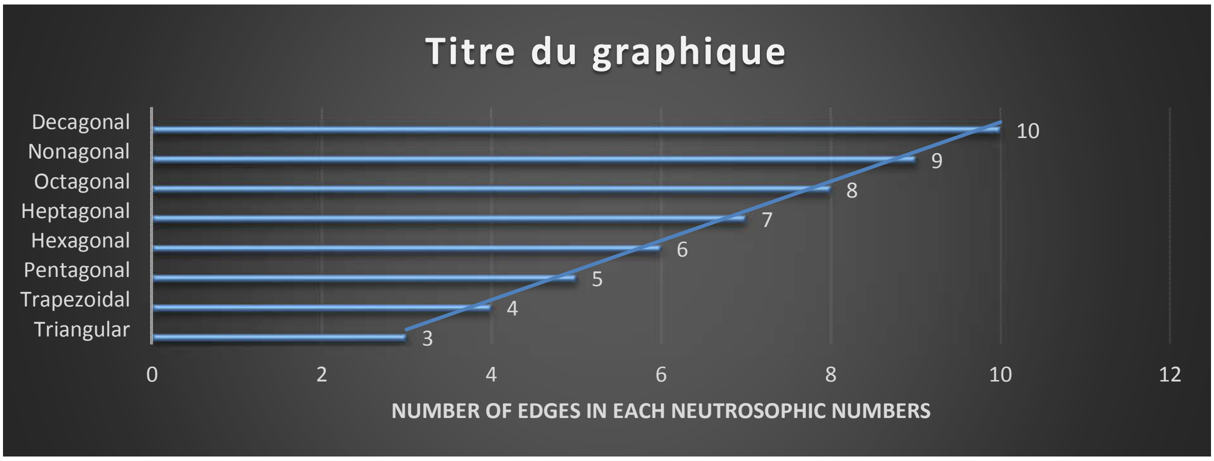
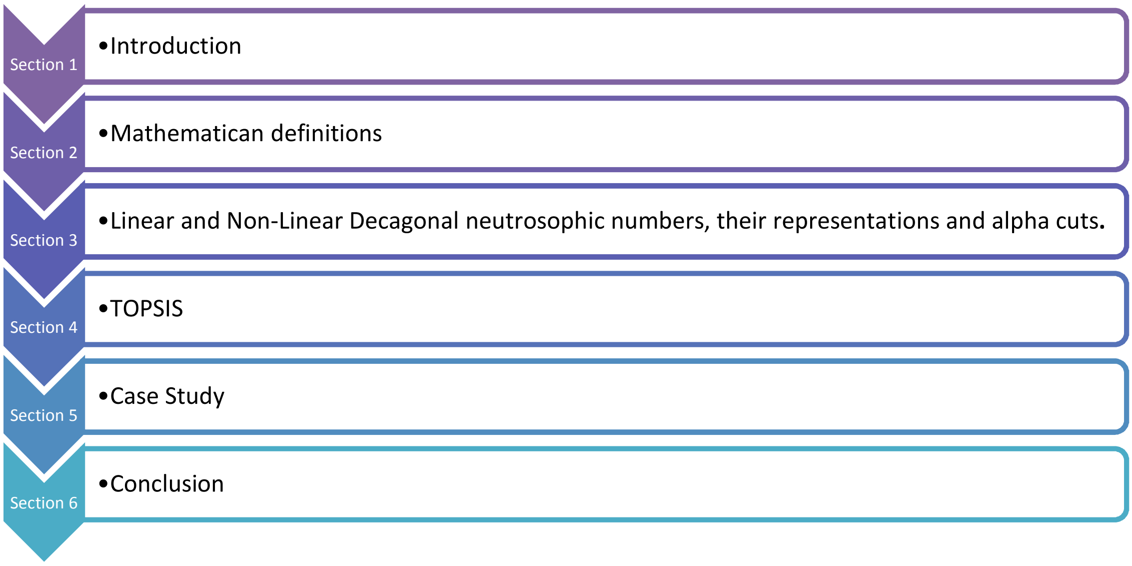
nonagonal neutrosophic numbers), but these have limitations. Now with decagonal, we can deal with maximum large and multi-criteria problems. Moreover, we present representations, alpha cuts, linear, and nonlinear. Now the MCDM problems solve much better and in a decent way. The decagonal is extremely handy, effective, accurate, and can deal with more fluctuations, mentioned below.



Structure of Article:

The article based on these point



2. Definition

In this section we proposed necessary definitions, which will further use in article.

Definition 2.1: Soft sets: Let $\vec{\xi}$ as universal set and the set of attributes is $\vec{\text{€}}$ and P ($\vec{\xi}$) as power set and Ą ⊆ $\vec{\text{€}}$. A pair ($\vec{F}$, Ą) is soft set over $\vec{\xi}$ and mapping is defined as:

$$\vec{F}, \text{Ą} \rightarrow \text{P}\ (\vec{\xi})$$

Moreover,

$$(\vec{F}, Ą) = \{\vec{F}(e) \in P\,(ξ): e \in \vec{€}, \vec{F}(e) = \emptyset \text{ if } e \neq Ą\}$$

Definition 2.2 Neutrosophic sets: Set $n\grave{\acute{A}}$ as neutrosophic if $n\grave{\acute{A}} = \{\dot{x}; \langle [T_{n\grave{\acute{A}}}(\dot{x}), I_{n\grave{\acute{A}}}(\dot{x}), F_{n\grave{\acute{A}}}(\dot{x})\rangle : \dot{x}\epsilon\dot{X}\}$ where, for membership of truthiness $T_{n\grave{\acute{A}}}(\dot{x}) \to [0,1]$, for membership of indeterminacy $I_{n\grave{\acute{A}}}(\dot{x})$, for membership of falsity $F_{n\grave{\acute{A}}}(\dot{x})$ and the relation given following,

$$0^- \leq T_{n\grave{\acute{A}}}(\dot{x}) + I_{n\grave{\acute{A}}}(\dot{x}) + F_{n\grave{\acute{A}}}(\dot{x}) \leq 3^+$$

Definition 2.3: Triangular neutrosophic numbers: Triangular single value neutrosophic number is given as: $A_{N\grave{e}u} = (\acute{P}_1, \acute{P}_2, P_3: \acute{r}_1, \acute{r}_2, \acute{r}_3,)$ moreover, truthiness, indeterminacy and falsity are given as:

$$T_{A_{N\grave{e}u}}(\dot{x}) = \begin{cases} \dfrac{\dot{x} - \acute{p}_1}{\acute{p}_2 - \acute{p}_1} & for\ p_1{'} \leq \dot{x} < \acute{p}_2 \\ 1 & when\ \ \dot{x} = \acute{p}_2 \\ \dfrac{\acute{p}_3 - \dot{x}}{\acute{p}_3 - \acute{p}_2} & for\ p_2 \acute{<} \dot{x} \leq \acute{p}_3 \\ & 0\ otherwise \end{cases}$$

$$I_{A_{N\grave{e}u}}(\dot{x}) = \begin{cases} \dfrac{\acute{q}_2 - \dot{x}}{\acute{q}_2 - \acute{q}_1} & for\ q_1{'} \leq \dot{x} < \acute{q}_2 \\ 0 & when\ \ \dot{x} = \acute{q}_2 \\ \dfrac{\dot{x} - \acute{q}_2}{\acute{q}_3 - \acute{q}_2} & for\ \ q_2 \acute{<} \dot{x} \leq \acute{q}_3 \\ & 1\ otherwise \end{cases}$$

$$F_{A_{N\grave{e}u}}(\dot{x}) = \begin{cases} \dfrac{\dot{x} - \acute{p}_1}{\acute{p}_1 - \acute{p}_2} & for\ p_1{'} \leq \dot{x} < \acute{p}_2 \\ 1 & when\ \ \dot{x} = \acute{p}_2 \\ \dfrac{\acute{p}_3 - \dot{x}}{\acute{p}_3 - \acute{p}_2} & for\ p_2 \acute{<} \dot{x} \leq \acute{p}_3 \\ & 0\ otherwise \end{cases}$$

Where,

$0^- \leq T_{A_{N\grave{e}u}}(\dot{x}) + I_{A_{N\grave{e}u}}(\dot{x}) + F_{A_{N\grave{e}u}}(\dot{x}) \leq 3^+; \dot{x} \in A_{N\grave{e}u}$

Parameter type: $(A_{N\grave{e}u})_{\dot{\alpha},\dot{\beta},\acute{\gamma}} = \left[T_{\cdot N\grave{e}u1}(\dot{\alpha}), T_{\cdot N\grave{e}u2}(\dot{\alpha}); I_{\cdot N\grave{e}u1}\left(\dot{\beta}\right), I_{\cdot N\grave{e}u2}\left(\dot{\beta}\right); F_{\cdot N\grave{e}u1}(\acute{\gamma}), F_{\cdot N\grave{e}u2}(\acute{\gamma})\right]$, where, $T_{\cdot N\grave{e}u1}(\dot{\alpha}) = \acute{p}_1 + \dot{\alpha}(\acute{p}_2 - \acute{p}_1), T_{\cdot N\grave{e}u2}(\dot{\alpha}) = \acute{p}_3 - \dot{\alpha}(\acute{p}_3 - \acute{p}_2), I_{\cdot N\grave{e}u1}\left(\dot{\beta}\right) = \acute{q}_2 - \dot{\beta}(\acute{q}_2 - \acute{q}_1), T_{\cdot N\grave{e}u1}\left(\dot{\beta}\right) = \acute{q}_2 + \dot{\beta}(\acute{q}_3 - \acute{q}_2)$, $F_{\cdot N\grave{e}u1}(\acute{\gamma}) = \acute{r}_2 - \acute{\gamma}(\acute{r}_2 - \acute{r}_1), T_{\cdot N\grave{e}u1}(\acute{\gamma}) = \acute{r}_2 + \acute{\gamma}(\acute{r}_3 - \acute{r}_2)$, where, $0 < \dot{\alpha} \leq 1, 0 < \dot{\beta} \leq 1\ 0 < \acute{\gamma} \leq 1$ and $0 < \dot{\alpha} + \dot{\beta} + \acute{\gamma} < 3$.

Definition 2.4: Trapezoidal neutrosophic numbers: If $\dot{X}$ be universe of discourse, define as: $\acute{N} = \{\langle \dot{X}, T_N(\acute{\dot{x}}), I_N(\acute{\dot{x}}), F_N(\acute{\dot{x}})\rangle \dot{x} \in \dot{X}\}$ where $T_N(\acute{\dot{x}}) \subset [0,1]$, $I_N(\acute{\dot{x}}) \subset [0,1]$, $F_N(\acute{\dot{x}}) \subset [0,1]$ as three trapezoidal numbers $T_N(\acute{\dot{x}}) = \left(\acute{t}_N^1(\dot{x}), \acute{t}_N^2(\dot{x}), \acute{t}_N^3(\dot{x}), \acute{t}_N^4(\dot{x})\right): \dot{x} \to [0,1]$, $I_N(\acute{\dot{x}}) = \left(\acute{\imath}_N^1(\dot{x}), \acute{\imath}_N^2(\dot{x}), \acute{\imath}_N^3(\dot{x}), \acute{\imath}_N^4(\dot{x})\right): \dot{x} \to [0,1]$, $F_N(\acute{\dot{x}}) = \left(\acute{f}_N^1(\dot{x}), \acute{f}_N^2(\dot{x}), \acute{f}_N^3(\dot{x}), \acute{f}_N^4(\dot{x})\right): \dot{x} \to [0,1]$

With condition $0 \leq \acute{t}_N^1(\dot{x}) + \acute{\imath}_N^1(\dot{x}) + \acute{f}_N^1(\dot{x}) \leq 3$, $0 \leq \acute{t}_N^2(\dot{x}) + \acute{\imath}_N^2(\dot{x}) + \acute{f}_N^2(\dot{x}) \leq 3\ \dot{x} \in \dot{X}$.

Definition 2.5: Pentagonal neutrosophic numbers: For single value it is given as:

$\acute{S} = \langle [\acute{m}^1, \acute{n}^1, \acute{o}^1, \acute{p}^1, \acute{q}^1; \dot{\pi}], [\acute{m}^2, \acute{n}^2, \acute{o}^2, \acute{p}^2, \acute{q}^2; \dot{\xi}], [\acute{m}^3, \acute{n}^3, \acute{o}^3, \acute{p}^3, \acute{q}^3, \dot{\partial}] \rangle$ where, $\dot{\pi}, \dot{\xi}, \dot{\partial} \epsilon [0,1]$. The Truth membership function $(\acute{T_S}): \mathbb{R} \rightarrow [0, \dot{\pi}]$, indeterminacy $(\acute{I_S}): \mathbb{R} \rightarrow [0, \dot{\xi}]$, and falsity $(\acute{F_S}): \mathbb{R} \rightarrow [0, \dot{\partial}]$ and given as:

$$T_{\dot{S}}(\dot{x}) = \begin{cases} T_{\dot{S}1}'(\dot{x}) & \dot{m}^1 \le \dot{x} < \dot{n}^1 \\ T_{\dot{S}2}'(\dot{x}) & \dot{n}^1 \le \dot{x} < \dot{o}^1 \\ \dot{\mu} & \dot{x} = \dot{o}^1 \\ T_{\dot{S}2}'(\dot{x}) & \dot{o}^1 \le \dot{x} < \dot{p}^1 \\ T_{\dot{S}1}'(\dot{x}) & \dot{p}^1 \le \dot{x} < \dot{q}^1 \\ 0 & otherwise \end{cases} \qquad I_{\dot{S}}(\dot{x}) = \begin{cases} I_{\dot{S}1}'(\dot{x}) & \dot{m}^2 \le \dot{x} < \dot{n}^2 \\ I_{\dot{S}2}'(\dot{x}) & \dot{n}^2 \le \dot{x} < \dot{o}^2 \\ \dot{\theta} & \dot{x} = \dot{o}^2 \\ I_{\dot{S}2}'(\dot{x}) & \dot{o}^2 \le \dot{x} < \dot{p}^2 \\ I_{\dot{S}1}'(\dot{x}) & \dot{p}^2 \le \dot{x} < \dot{q}^2 \\ 1 & otherwise \end{cases}$$

$$F_{\dot{S}}(\dot{x}) = \begin{cases} F_{\dot{S}1}'(\dot{x}) & \dot{m}^3 \le \dot{x} < \dot{n}^3 \\ F_{\dot{S}2}'(\dot{x}) & \dot{n}^3 \le \dot{x} < \dot{o}^3 \\ \dot{€} & \dot{x} = \dot{o}^3 \\ F_{\dot{S}2}'(\dot{x}) & \dot{o}^3 \le \dot{x} < \dot{p}^3 \\ F_{\dot{S}1}'(\dot{x}) & \dot{p}^3 \le \dot{x} < \dot{q}^3 \\ 1 & otherwise \end{cases}$$

Where, $\langle [(\dot{m}^1 < \dot{n}^1 < \dot{o}^1 < \dot{p}^1 < \dot{q}^1): \dot{\mu}], [(\dot{m}^2 < \dot{n}^2 < \dot{o}^2 < \dot{p}^2 < \dot{q}^2): \dot{\theta}], [(\dot{m}^3 < \dot{n}^3 < \dot{o}^3 < \dot{p}^3 < \dot{q}^3): \dot{€}] \rangle$

Definition 2.6: Octagonal neutrosophic numbers: A neutrosophic number denoted by $\check{S}$ and defined as:

$\check{S} = \langle [(\ , \ , \ , \ , \ , \ , \ , \ ): \dot{\mu}], [(\ ^1, \ ^1, \ ^1, \ ^1, \ ^1, \ ^1, \ ^1, \ ^1): \dot{\theta}], [(\ ^2, \ ^2, \ ^2, \ ^2, \ ^2, \ ^2, \ ^2, \ ^2): \dot{€}] \rangle$ where, $\dot{\mu}, \dot{€}, \dot{\theta} \in [0,1]$.

Truth membership function as, $(\dot{\mu}_S)$:$\mathbb{R} \rightarrow [0,1]$, Indeterminacy membership function as, $(\dot{\theta}_S)$:$\mathbb{R} \rightarrow [0,1]$.

Falsity membership function as, $(\dot{€}_S)$:$\mathbb{R} \rightarrow [0,1]$.

$$\dot{\mu}_{\check{S}}(\dot{x}) = \begin{cases} \dot{\mu}_{\overline{S0}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S1}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S2}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S3}}(\dot{x}) & \le \dot{x} < \\ \partial & \dot{x} = \mathcal{E} \\ \dot{\mu}_{\overline{S3}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S2}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S1}}(\dot{x}) & \le \dot{x} < \\ 0 & otherwise \end{cases} \quad \dot{\theta} = \begin{cases} \dot{\theta}_{\overline{S0}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S1}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S2}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S3}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \tilde{\Omega} & \dot{x} = \mathcal{E}^1 \\ \dot{\theta}_{\overline{S3}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S2}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S1}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ 1 & otherwise \end{cases} \quad \dot{€} = \begin{cases} \dot{€}_{\overline{S0}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S1}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S2}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S3}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \ddot{\upsilon} & \dot{x} = \mathcal{E}^2 \\ \dot{€}_{\overline{S3}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S2}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S1}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ 1 & otherwise \end{cases}$$

$$\check{S} = \langle [(\ < \ < \ < \ < \ < \ < \ < \ ): \dot{\mu}], [(\ ^1 < \ ^1 < \ ^1 < \ ^1 < \ ^1 < \ ^1 < \ ^1 < \ ^1): \dot{\theta}], [(\ ^2 < \ ^2 < \ ^2 < \ ^2 < \ ^2 < \ ^2 < \ ^2 < \ ^2): \dot{€}] \rangle$$

Definition 2.7: Nonagonal neutrosophic numbers: A neutrosophic number denoted by $\check{S}$ and defined as:

$\check{S} = \langle [(\ , \ , \ , \ , \ , \ , \ , \ , \text{Ɋ}): \dot{\mu}], [(\ ^1, \ ^1, \ ^1, \ ^1, \ ^1, \ ^1, \ ^1, \ ^1, \text{Ɋ}^1): \dot{\theta}], [(\ ^2, \ ^2, \ ^2, \ ^2, \ ^2, \ ^2, \ ^2, \ ^2, \text{Ɋ}^2): \dot{€}] \rangle$

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where, $\dot{\mu}, \dot{€}, \dot{\theta} \in [0,1]$. $\check{S} = \langle[(< < < < < < < < ᑫ): \dot{\mu}], [(^1 < ^1 < ^1 < ^1 < ^1 < ^1 < ^1 < ^1 < ᑫ^1): \dot{\theta}], [(^2 < ^2 < ^2 < ^2 < ^2 < ^2 < ^2 < ^2 < ᑫ^2): \dot{€}]\rangle$

Truth membership function as, $(\dot{\mu}_S): \mathbb{R} \to [0,1]$.

Indeterminacy membership function as, $(\dot{\theta}_S): \mathbb{R} \to [0,1]$.

Falsity membership function as, $(\dot{€}_S): \mathbb{R} \to [0,1]$.

$$\dot{\mu}_{\check{S}}(\dot{x}) = \begin{cases} \dot{\mu}_{\overline{S0}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S1}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S2}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S3}}(\dot{x}) & \le \dot{x} < \\ \partial & \dot{x} = \mathcal{E} \\ \dot{\mu}_{\overline{S3}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S2}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S1}}(\dot{x}) & \le \dot{x} < \\ \dot{\mu}_{\overline{S0}}(\dot{x}) & \le \dot{x} < ᑫ \\ 0 \ \ otherwise & \end{cases} \quad \dot{\theta} = \begin{cases} \dot{\theta}_{\overline{S0}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S1}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S2}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S3}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \tilde{\Omega} & \dot{x} = \mathcal{E}^1 \\ \dot{\theta}_{\overline{S3}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S2}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\theta}_{\overline{S1}}(\dot{x}) & ^1 \le \dot{x} < \ ^1 \\ \dot{\mu}_{\overline{S0}}(\dot{x}) & ^1 \le \dot{x} < ᑫ^1 \\ 1 \ \ otherwise & \end{cases} \quad \dot{€} = \begin{cases} \dot{€}_{\overline{S0}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S1}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S2}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S3}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{\mho} & \dot{x} = \mathcal{E}^2 \\ \dot{€}_{\overline{S3}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S2}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{€}_{\overline{S1}}(\dot{x}) & ^2 \le \dot{x} < \ ^2 \\ \dot{\mu}_{\overline{S0}}(\dot{x}) & ^2 \le \dot{x} < ᑫ^2 \\ 1 \ \ otherwise & \end{cases}$$

3. Representation and Properties.

3.1 Linear Decagonal neutrosophic numbers with symmetry

$A_{LS} = (\dot{a}, \dot{b}, \dot{c}, \acute{\dot{d}}, \dot{e}, \dot{f}, \dot{g}, \dot{h}, i, j)$ as linear DNN with these membership function:

$$Truth = T_L(\acute{\dot{X}}) = \begin{cases} 0 & \dot{x} < \dot{a} \\ \dot{k}\left(\frac{\dot{x}-\dot{a}}{\dot{b}-\dot{a}}\right) & \dot{a} < \dot{x} < \dot{b} \\ \dot{k}\left(\frac{\dot{x}-\dot{b}}{\dot{c}-\dot{b}}\right) & \dot{b} < \dot{x} < \dot{c} \\ \dot{k} & \dot{c} < \dot{x} < \dot{d} \\ \dot{k} + (1 \dot{-} k)\left(\frac{\dot{x}-\dot{d}}{\dot{e}-\dot{d}}\right) & \dot{d} < \dot{x} < \dot{e} \\ 1 & \dot{e} < \dot{x} < \dot{f} \\ \dot{k} + (1 \dot{-} k)\left(\frac{\dot{g}-\dot{x}}{\dot{g}-\dot{f}}\right) & \dot{f} < \dot{x} < \dot{g} \\ \dot{k} & \dot{g} < \dot{x} < \dot{h} \\ \dot{k}\left(\frac{i-\dot{x}}{i-\dot{h}}\right) & \dot{h} < \dot{x} < i \\ \dot{k}\left(\frac{j-\dot{x}}{j-i}\right) & i < \dot{x} < j \\ 0 & \dot{x} > j \end{cases}$$

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$$Falsity = F_L(\acute{X}) = \begin{cases} 0 & \dot{x} < \dot{a}^1 \\ \dot{k}\left(\dfrac{\dot{x} - \dot{a}^1}{\dot{b}^1 - \dot{a}^1}\right) & \dot{a} < \dot{x} < \dot{b}^1 \\ \dot{k}\left(\dfrac{\dot{x} - \dot{b}^1}{\dot{c}^1 - \dot{b}^1}\right) & \dot{b}^1 < \dot{x} < \dot{c}^1 \\ \dot{k} & \dot{c}^1 < \dot{x} < \dot{d}^1 \\ \dot{k} + (1 \dot{-} k)\left(\dfrac{\dot{x} - \dot{d}^1}{\dot{e}^1 - \dot{d}^1}\right) & \dot{d}^1 < \dot{x} < \dot{e}^1 \\ 1 & \dot{e}^1 < \dot{x} < \dot{f}^1 \\ \dot{k} + (1 \dot{-} k)\left(\dfrac{\dot{g}^1 - \dot{x}}{\dot{g}^1 - \dot{f}^1}\right) & \dot{f}^1 < \dot{x} < \dot{g}^1 \\ \dot{k} & \dot{g}^1 < \dot{x} < \dot{h}^1 \\ \dot{k}\left(\dfrac{\dot{i}^1 - \dot{x}}{\dot{i}^1 - \dot{h}^1}\right) & \dot{h}^1 < \dot{x} < \dot{i}^1 \\ \dot{k}\left(\dfrac{\dot{j}^1 - \dot{x}}{\dot{j}^1 - \dot{i}^1}\right) & \dot{i}^1 < \dot{x} < \dot{j}^1 \\ 1 & \dot{x} > \dot{j}^1 \end{cases}$$

$$Indeterminacy = I_L(\acute{X}) = \begin{cases} 0 & \dot{x} < \dot{a}^2 \\ \dot{k}\left(\dfrac{\dot{x} - \dot{a}^2}{\dot{b}^2 - \dot{a}^2}\right) & \dot{a}^2 < \dot{x} < \dot{b}^2 \\ \dot{k}\left(\dfrac{\dot{x} - \dot{b}^2}{\dot{c}^2 - \dot{b}^2}\right) & \dot{b}^2 < \dot{x} < \dot{c}^2 \\ \dot{k} & \dot{c}^2 < \dot{x} < \dot{d}^2 \\ \dot{k} + (1 \dot{-} k)\left(\dfrac{\dot{x} - \dot{d}^2}{\dot{e}^2 - \dot{d}^2}\right) & \dot{d}^2 < \dot{x} < \dot{e}^2 \\ 1 & \dot{e}^2 < \dot{x} < \dot{f}^2 \\ \dot{k} + (1 \dot{-} k)\left(\dfrac{\dot{g}^2 - \dot{x}}{\dot{g}^2 - \dot{f}^2}\right) & \dot{f}^2 < \dot{x} < \dot{g}^2 \\ \dot{k} & \dot{g}^2 < \dot{x} < \dot{h}^2 \\ \dot{k}\left(\dfrac{\dot{i}^2 - \dot{x}}{\dot{i}^2 - \dot{h}^2}\right) & \dot{h}^2 < \dot{x} < \dot{i}^2 \\ \dot{k}\left(\dfrac{\dot{j}^2 - \dot{x}}{\dot{j}^2 - \dot{i}^2}\right) & \dot{i}^2 < \dot{x} < \dot{j}^2 \\ 1 & \dot{x} > \dot{j}^2 \end{cases}$$

As, 0<$\dot{k} < 1$

$\acute{A}_\alpha = \{\dot{x} \in \dot{X} | T_L(\acute{X}), F_L(\acute{X}), I_L(\acute{X}) \geq \dot{\alpha}\}$

3.2 $\alpha - \acute{cut}\ of$ Linear DNN with symmetry: We can express as:

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$$Truth = \acute{T}_L(\dot{X}) = \begin{cases} \acute{A}_{1L}(\dot{\alpha}) = \dot{a} + \frac{\dot{\alpha}}{\dot{b}_1}\left(\dot{b} - \dot{a}\right) & for\ \dot{\alpha} \in [0, \grave{b}_1] \\ \acute{A}_{2L}(\dot{\alpha}) = \dot{b} + \frac{1-\dot{\alpha}}{1-\dot{b}_2}\left(\dot{c} - \dot{b}\right) & for\ \dot{\alpha} \in [\grave{b}_2, 1] \\ \acute{A}_{3L}(\dot{\alpha}) = \dot{c} + \frac{1-\dot{\alpha}}{1-\dot{b}_3}\left(\dot{d} - \dot{c}\right) & for\ \dot{\alpha} \in [\grave{b}_3, 1] \\ \acute{A}_{4L}(\dot{\alpha}) = \dot{d} + \frac{1-\dot{\alpha}}{1-\dot{b}_4}\left(\dot{e} - \dot{d}\right) & for\ \dot{\alpha} \in [\grave{b}_4, 1] \\ \acute{A}_{5L}(\dot{\alpha}) = \dot{e} + \frac{1-\dot{\alpha}}{1-\dot{b}_5}\left(\dot{f} - \dot{e}\right) & for\ \dot{\alpha} \in [\grave{b}_5, 1] \\ \acute{A}_{4R}(\dot{\alpha}) = \dot{f} - \frac{\dot{\alpha}}{\dot{b}_4}\left(\dot{g} - \dot{f}\right) & for\ \dot{\alpha} \in [0, \grave{b}_4] \\ \acute{A}_{3R}(\dot{\alpha}) = \dot{g} - \frac{\dot{\alpha}}{\dot{b}_3}\left(\dot{h} - \dot{g}\right) & for\ \dot{\alpha} \in \left[0, \grave{b}_3\right] \\ \acute{A}_{2R}(\dot{\alpha}) = \dot{h} - \frac{\dot{\alpha}}{\dot{b}_2}\left(i - \dot{h}\right) & for\ \dot{\alpha} \in \left[0, \grave{b}_2\right] \\ \acute{A}_{1R}(\dot{\alpha}) = i - \frac{\dot{\alpha}}{\dot{b}_1}(j - i) & for\ \dot{\alpha} \in [0, \grave{b}_1] \end{cases}$$

$$Falsity = \acute{F}_L(\dot{X}) = \begin{cases} \acute{A}_{1L}(\dot{\alpha}) = \dot{a}^1 + \frac{\dot{\alpha}}{\dot{b}_1}\left(\dot{b}^1 - \dot{a}^1\right) & for\ \dot{\alpha} \in [0, \grave{b}_1] \\ \acute{A}_{2L}(\dot{\alpha}) = \dot{b}^1 + \frac{1-\dot{\alpha}}{1-\dot{b}_2}\left(\dot{c}^1 - \dot{b}^1\right) & for\ \dot{\alpha} \in [\grave{b}_2, 1] \\ \acute{A}_{3L}(\dot{\alpha}) = \dot{c}^1 + \frac{1-\dot{\alpha}}{1-\dot{b}_3}\left(\dot{d}^1 - \dot{c}^1\right) & for\ \dot{\alpha} \in [\grave{b}_3, 1] \\ \acute{A}_{4L}(\dot{\alpha}) = \dot{d}^1 + \frac{1-\dot{\alpha}}{1-\dot{b}_4}\left(\dot{e}^1 - \dot{d}^1\right) & for\ \dot{\alpha} \in [\grave{b}_4, 1] \\ \acute{A}_{5L}(\dot{\alpha}) = \dot{e}^1 + \frac{1-\dot{\alpha}}{1-\dot{b}_5}\left(\dot{f}^1 - \dot{e}^1\right) & for\ \dot{\alpha} \in [\grave{b}_5, 1] \\ \acute{A}_{4R}(\dot{\alpha}) = \dot{f}^1 - \frac{\dot{\alpha}}{\dot{b}_4}\left(\dot{g}^1 - \dot{f}^1\right) & for\ \dot{\alpha} \in [0, \grave{b}_4] \\ \acute{A}_{3R}(\dot{\alpha}) = \dot{g}^1 - \frac{\dot{\alpha}}{\dot{b}_3}\left(\dot{h}^1 - \dot{g}^1\right) & for\ \dot{\alpha} \in \left[0, \grave{b}_3\right] \\ \acute{A}_{2R}(\dot{\alpha}) = \dot{h}^1 - \frac{\dot{\alpha}}{\dot{b}_2}\left(i^1 - \dot{h}^1\right) & for\ \dot{\alpha} \in \left[0, \grave{b}_2\right] \\ \acute{A}_{1R}(\dot{\alpha}) = i^1 - \frac{\dot{\alpha}}{\dot{b}_1}(j^1 - i^1) & for\ \dot{\alpha} \in [0, \grave{b}_1] \end{cases}$$

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$$Indeterminacy = \acute{I}_L(\dot{X}) = \begin{cases} \acute{A}_{1L}(\dot{\alpha}) = \dot{a}^2 + \frac{\dot{\alpha}}{\grave{b}_1}(\dot{b}^2 - \dot{a}^2) & for\ \dot{\alpha} \in [0, \grave{b}_1] \\ \acute{A}_{2L}(\dot{\alpha}) = \dot{b}^2 + \frac{1-\dot{\alpha}}{1-\grave{b}_2}(\dot{c}^2 - \dot{b}^2) & for\ \dot{\alpha} \in [\grave{b}_2, 1] \\ \acute{A}_{3L}(\dot{\alpha}) = \dot{c}^2 + \frac{1-\dot{\alpha}}{1-\grave{b}_3}(\dot{d}^2 - \dot{c}^2) & for\ \dot{\alpha} \in [\grave{b}_3, 1] \\ \acute{A}_{4L}(\dot{\alpha}) = \dot{d}^2 + \frac{1-\dot{\alpha}}{1-\grave{b}_4}(\dot{e}^2 - \dot{d}^2) & for\ \dot{\alpha} \in [\grave{b}_4, 1] \\ \acute{A}_{5L}(\dot{\alpha}) = \dot{e}^2 + \frac{1-\dot{\alpha}}{1-\grave{b}_5}(\dot{f}^2 - \dot{e}^2) & for\ \dot{\alpha} \in [\grave{b}_5, 1] \\ \acute{A}_{4R}(\dot{\alpha}) = \dot{f}^2 - \frac{\dot{\alpha}}{\grave{b}_4}(\dot{g}^2 - \dot{f}^2) & for\ \dot{\alpha} \in [0, \grave{b}_4] \\ \acute{A}_{3R}(\dot{\alpha}) = \dot{g}^2 - \frac{\dot{\alpha}}{\grave{b}_3}(\dot{h}^2 - \dot{g}^2) & for\ \dot{\alpha} \in [0, \grave{b}_3] \\ \acute{A}_{2R}(\dot{\alpha}) = \dot{h}^2 - \frac{\dot{\alpha}}{\grave{b}_2}(i^2 - \dot{h}^2) & for\ \dot{\alpha} \in [0, \grave{b}_2] \\ \acute{A}_{1R}(\dot{\alpha}) = i^2 - \frac{\dot{\alpha}}{\grave{b}_1}(j^2 - i^2) & for\ \dot{\alpha} \in [0, \grave{b}_1] \end{cases}$$

Increasing are $\acute{A}_{1L}(\dot{\alpha})$, $\acute{A}_{2L}(\dot{\alpha})$, $\acute{A}_{3L}(\dot{\alpha})$, $\acute{A}_{4L}(\dot{\alpha})$, $\acute{A}_{5L}(\dot{\alpha})$ and decreasing are $\acute{A}_{1R}(\dot{\alpha})$, $\acute{A}_{2R}(\dot{\alpha})$, $\acute{A}_{3R}(\dot{\alpha})$, $\acute{A}_{4R}(\dot{\alpha})$.

3.3 Non-Linear Decagonal neutrosophic numbers with symmetry:

$A_{LS} = (\dot{a}, \dot{b}, \dot{c}, \dot{d}, \dot{e}, \dot{f}, \dot{g}, \acute{\dot{h}}, i, j)_{(\dot{n}_1, \dot{n}_2, \dot{n}_3, \dot{m}_1, \dot{m}_2, \dot{m}_3)}$ as non-linear DNN with these membership function:

$$Truth = T_L(\acute{X}) = \begin{cases} 0 & \dot{x} < \dot{a} \\ \dot{k}\left(\frac{\dot{x}-\dot{a}}{\dot{b}-\dot{a}}\right)^{\dot{n}_1} & \dot{a} < \dot{x} < \dot{b} \\ \dot{k}\left(\frac{\dot{x}-\dot{b}}{\dot{c}-\dot{b}}\right)^{n_2} & \dot{b} < \dot{x} < \dot{c} \\ \dot{k} & \dot{c} < \dot{x} < \dot{d} \\ \dot{k} + (1 \dot{-} k)\left(\frac{\dot{x}-\dot{d}}{\dot{e}-\dot{d}}\right)^{n_3} & \dot{d} < \dot{x} < \dot{e} \\ 1 & \dot{e} < \dot{x} < \dot{f} \\ \dot{k} + (1 \dot{-} k)\left(\frac{\dot{g}-\dot{x}}{\dot{g}-\dot{f}}\right)^{m_1} & \dot{f} < \dot{x} < \dot{g} \\ \dot{k} & \dot{g} < \dot{x} < \dot{h} \\ \dot{k}\left(\frac{i-\dot{x}}{i-\dot{h}}\right)^{m_2} & \dot{h} < \dot{x} < i \\ \dot{k}\left(\frac{j-\dot{x}}{j-i}\right)^{m_3} & i < \dot{x} < j \\ 0 & \dot{x} > j \end{cases}$$

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$$Falsity = F_L(\acute{X}) = \begin{cases} 0 & \dot{x} < \dot{a}^1 \\ \dot{k}\left(\dfrac{\dot{x}-\dot{a}^1}{\dot{b}^1-\dot{a}^1}\right)^{m_1} & \dot{a}^1 < \dot{x} < \dot{b}^1 \\ \dot{k}\left(\dfrac{\dot{x}-\dot{b}^1}{\dot{c}^1-\dot{b}^1}\right)^{m_2} & \dot{b}^1 < \dot{x} < \dot{c}^1 \\ \dot{k} & \dot{c}^1 < \dot{x} < \dot{d}^1 \\ \dot{k}+(1\dot{-}k)\left(\dfrac{\dot{x}-\dot{d}^1}{\dot{e}^1-\dot{d}^1}\right)^{m_3} & \dot{d}^1 < \dot{x} < \dot{e}^1 \\ 1 & \dot{e}^1 < \dot{x} < \dot{f}^1 \\ \dot{k}+(1\dot{-}k)\left(\dfrac{\dot{g}^1-\dot{x}}{\dot{g}^1-\dot{f}^1}\right)^{n_1} & \dot{f}^1 < \dot{x} < \dot{g}^1 \\ \dot{k} & \dot{g}^1 < \dot{x} < \dot{h}^1 \\ \dot{k}\left(\dfrac{\dot{i}^1-\dot{x}}{\dot{i}^1-\dot{h}^1}\right)^{n_2} & \dot{h}^1 < \dot{x} < \dot{i}^1 \\ \dot{k}\left(\dfrac{\dot{j}^1-\dot{x}}{\dot{j}^1-\dot{i}^1}\right)^{n_3} & \dot{i}^1 < \dot{x} < \dot{j}^1 \\ 1 & \dot{x} > \dot{j}^1 \end{cases}$$

$$Indeterminacy = I_L(\acute{X}) = \begin{cases} 0 & \dot{x} < \dot{a}^2 \\ \dot{k}\left(\dfrac{\dot{x}-\dot{a}^2}{\dot{b}^2-\dot{a}^2}\right)^{\dot{m}_1} & \dot{a}^2 < \dot{x} < \dot{b}^2 \\ \dot{k}\left(\dfrac{\dot{x}-\dot{b}^2}{\dot{c}^2-\dot{b}^2}\right)^{\dot{m}_2} & \dot{b}^2 < \dot{x} < \dot{c}^2 \\ \dot{k} & \dot{c}^2 < \dot{x} < \dot{d}^2 \\ \dot{k}+(1\dot{-}k)\left(\dfrac{\dot{x}-\dot{d}^2}{\dot{e}^2-\dot{d}^2}\right)^{\dot{m}_3} & \dot{d}^2 < \dot{x} < \dot{e}^2 \\ 1 & \dot{e}^2 < \dot{x} < \dot{f}^2 \\ \dot{k}+(1\dot{-}k)\left(\dfrac{\dot{g}^2-\dot{x}}{\dot{g}^2-\dot{f}^2}\right)^{\dot{n}_1} & \dot{f}^2 < \dot{x} < \dot{g}^2 \\ \dot{k} & \dot{g}^2 < \dot{x} < \dot{h}^2 \\ \dot{k}\left(\dfrac{\dot{i}^2-\dot{x}}{\dot{i}^2-\dot{h}^2}\right)^{\dot{n}_2} & \dot{h}^2 < \dot{x} < \dot{i}^2 \\ \dot{k}\left(\dfrac{\dot{j}^2-\dot{x}}{\dot{j}^2-\dot{i}^2}\right)^{\dot{n}_3} & \dot{i}^2 < \dot{x} < \dot{j}^2 \\ 1 & \dot{x} > j^2 \end{cases}$$

As, 0<$\dot{k} < 1$ $\acute{A}_\alpha = \{\dot{x} \in \dot{X} | T_L(\acute{X}), F_L(\acute{X}), I_L(\acute{X}) \geq \dot{\alpha}\}$

3.4 $\alpha - cut\ \acute{o}f\ Non$ –Linear DNN with symmetry:

$\alpha - cut\ \acute{o}f\ Non$ –Linear DNN can be defined as $\acute{A}_\alpha = \{\dot{x} \in \dot{X} | T_L(\acute{X}), F_L(\acute{X}), I_L(\acute{X}) \geq \dot{\alpha}\}$

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$$Truth = \acute{T}_L(\dot{X}) = \begin{cases} \acute{A}_{1L}(\dot{\alpha}) = \dot{a} + \left(\frac{\dot{\alpha}}{\dot{b}_1}\right)^{\dot{n}_1}\left(\dot{b} - \dot{a}\right) & for\ \dot{\alpha} \in [0, \grave{b}_1] \\ \acute{A}_{2L}(\dot{\alpha}) = \dot{b} + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_2}\right)^{\dot{n}_2}\left(\dot{c} - \dot{b}\right) & for\ \dot{\alpha} \in [\grave{b}_2, 1] \\ \acute{A}_{3L}(\dot{\alpha}) = \dot{c} + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_3}\right)^{\dot{n}_3}\left(\dot{d} - \dot{c}\right) & for\ \dot{\alpha} \in [\grave{b}_3, 1] \\ \acute{A}_{4L}(\dot{\alpha}) = \dot{d} + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_4}\right)^{\dot{n}_4}\left(\dot{e} - \dot{d}\right) & for\ \dot{\alpha} \in [\grave{b}_4, 1] \\ \acute{A}_{5L}(\dot{\alpha}) = \dot{e} + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_5}\right)^{\dot{n}_5}\left(\dot{f} - \dot{e}\right) & for\ \dot{\alpha} \in [\grave{b}_5, 1] \\ \acute{A}_{4R}(\dot{\alpha}) = \dot{f} - \left(\frac{\dot{\alpha}}{\dot{b}_4}\right)^{\dot{m}_1}\left(\dot{g} - \dot{f}\right) & for\ \dot{\alpha} \in [0, \grave{b}_4] \\ \acute{A}_{3R}(\dot{\alpha}) = \dot{g} - \left(\frac{\dot{\alpha}}{\dot{b}_3}\right)^{\dot{m}_2}\left(\dot{h} - \dot{g}\right) & for\ \dot{\alpha} \in \left[0, \grave{b}_3\right] \\ \acute{A}_{2R}(\dot{\alpha}) = \dot{h} - \left(\frac{\dot{\alpha}}{\dot{b}_2}\right)^{\dot{m}_3}\left(i - \dot{h}\right) & for\ \dot{\alpha} \in \left[0, \grave{b}_2\right] \\ \acute{A}_{1R}(\dot{\alpha}) = i - \left(\frac{\dot{\alpha}}{\dot{b}_1}\right)^{\dot{m}_4}(j - i) & for\ \dot{\alpha} \in [0, \grave{b}_1] \end{cases}$$

$$Falsity = \acute{F}_L(\dot{X}) = \begin{cases} \acute{A}_{1L}(\dot{\alpha}) = \dot{a}^1 + \left(\frac{\dot{\alpha}}{\dot{b}_1}\right)^{\dot{m}_1}\left(\dot{b}^1 - \dot{a}^1\right) & for\ \dot{\alpha} \in [0, \grave{b}_1] \\ \acute{A}_{2L}(\dot{\alpha}) = \dot{b}^1 + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_2}\right)^{\dot{m}_2}\left(\dot{c}^1 - \dot{b}^1\right) & for\ \dot{\alpha} \in [\grave{b}_2, 1] \\ \acute{A}_{3L}(\dot{\alpha}) = \dot{c}^1 + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_3}\right)^{\dot{m}_3}\left(\dot{d}^1 - \dot{c}^1\right) & for\ \dot{\alpha} \in [\grave{b}_3, 1] \\ \acute{A}_{4L}(\dot{\alpha}) = \dot{d}^1 + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_4}\right)^{\dot{m}_4}\left(\dot{e}^1 - \dot{d}^1\right) & for\ \dot{\alpha} \in [\grave{b}_4, 1] \\ \acute{A}_{5L}(\dot{\alpha}) = \dot{e}^1 + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_5}\right)^{\dot{m}_5}\left(\dot{f}^1 - \dot{e}^1\right) & for\ \dot{\alpha} \in [\grave{b}_5, 1] \\ \acute{A}_{4R}(\dot{\alpha}) = \dot{f}^1 - \left(\frac{\dot{\alpha}}{\dot{b}_4}\right)^{\dot{n}_1}\left(\dot{g}^1 - \dot{f}^1\right) & for\ \dot{\alpha} \in [0, \grave{b}_4] \\ \acute{A}_{3R}(\dot{\alpha}) = \dot{g}^1 - \left(\frac{\dot{\alpha}}{\dot{b}_3}\right)^{\dot{n}_2}\left(\dot{h}^1 - \dot{g}^1\right) & for\ \dot{\alpha} \in \left[0, \grave{b}_3\right] \\ \acute{A}_{2R}(\dot{\alpha}) = \dot{h}^1 - \left(\frac{\dot{\alpha}}{\dot{b}_2}\right)^{\dot{n}_3}\left(i^1 - \dot{h}^1\right) & for\ \dot{\alpha} \in \left[0, \grave{b}_2\right] \\ \acute{A}_{1R}(\dot{\alpha}) = i^1 - \left(\frac{\dot{\alpha}}{\dot{b}_1}\right)^{\dot{n}_4}(j^1 - i^1) & for\ \dot{\alpha} \in [0, \grave{b}_1] \end{cases}$$

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$$Indeterminacy = \acute{I}_L(\dot{X}) = \begin{cases} \acute{A}_{1L}(\dot{\alpha}) = \dot{a}^2 + \left(\frac{\dot{\alpha}}{\dot{b}_1}\right)^{\dot{m}_1} (\dot{b}^2 - \dot{a}^2) & for\ \dot{\alpha} \in [0, \grave{b}_1] \\ \acute{A}_{2L}(\dot{\alpha}) = \dot{b}^2 + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_2}\right)^{\dot{m}_2} (\dot{c}^2 - \dot{b}^2) & for\ \dot{\alpha} \in [\grave{b}_2, 1] \\ \acute{A}_{3L}(\dot{\alpha}) = \dot{c}^2 + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_3}\right)^{\dot{m}_3} (\dot{d}^2 - \dot{c}^2) & for\ \dot{\alpha} \in [\grave{b}_3, 1] \\ \acute{A}_{4L}(\dot{\alpha}) = \dot{d}^2 + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_4}\right)^{\dot{m}_4} (\dot{e}^2 - \dot{d}^2) & for\ \dot{\alpha} \in [\grave{b}_4, 1] \\ \acute{A}_{5L}(\dot{\alpha}) = \dot{e}^2 + \left(\frac{1-\dot{\alpha}}{1-\dot{b}_5}\right)^{\dot{m}_5} (\dot{f}^2 - \dot{e}^2) & for\ \dot{\alpha} \in [\grave{b}_5, 1] \\ \acute{A}_{4R}(\dot{\alpha}) = \dot{f}^2 - \left(\frac{\dot{\alpha}}{\dot{b}_4}\right)^{\dot{n}_1} (\dot{g}^2 - \dot{f}^2) & for\ \dot{\alpha} \in [0, \grave{b}_4] \\ \acute{A}_{3R}(\dot{\alpha}) = \dot{g}^2 - \left(\frac{\dot{\alpha}}{\dot{b}_3}\right)^{\dot{n}_2} (\dot{h}^2 - \dot{g}^2) & for\ \dot{\alpha} \in [0, \grave{b}_3] \\ \acute{A}_{2R}(\dot{\alpha}) = \dot{h}^2 - \left(\frac{\dot{\alpha}}{\dot{b}_2}\right)^{\dot{n}_3} (\dot{i}^2 - \dot{h}^2) & for\ \dot{\alpha} \in [0, \grave{b}_2] \\ \acute{A}_{1R}(\dot{\alpha}) = \dot{i}^2 - \left(\frac{\dot{\alpha}}{\dot{b}_1}\right)^{\dot{n}_4} (\dot{j}^2 - \dot{i}^2) & for\ \dot{\alpha} \in [0, \grave{b}_1] \end{cases}$$

Increasing are $\acute{A}_{1L}(\dot{\alpha})$, $\acute{A}_{2L}(\dot{\alpha})$, $\acute{A}_{3L}(\dot{\alpha})$, $\acute{A}_{4L}(\dot{\alpha})$, $\acute{A}_{5L}(\dot{\alpha})$ and decreasing are $\acute{A}_{1R}(\dot{\alpha})$, $\acute{A}_{2R}(\dot{\alpha})$, $\acute{A}_{3R}(\dot{\alpha})$, $\acute{A}_{4R}(\dot{\alpha})$.

$$Truth = T_L(\acute{\dot{X}}) = \begin{cases} 0 & \dot{x} < \dot{a} \\ \dot{p}\left(\frac{\dot{x}-\dot{a}}{\dot{b}-\dot{a}}\right)^{\dot{n}_1} & \dot{a} < \dot{x} < \dot{b} \\ \dot{p}\left(\frac{\dot{x}-\dot{b}}{\dot{c}-\dot{b}}\right)^{n_2} & \dot{b} < \dot{x} < \dot{c} \\ \dot{k} & \dot{c} < \dot{x} < \dot{d} \\ \dot{k} - (\dot{k} - \dot{p})\left(\frac{\dot{x}-\dot{d}}{\dot{e}-\dot{d}}\right)^{n_3} & \dot{d} < \dot{x} < \dot{e} \\ 1 & \dot{e} < \dot{x} < \dot{f} \\ \dot{k} - (\dot{k} - \dot{r})\left(\frac{\dot{g}-\dot{x}}{\dot{g}-\dot{f}}\right)^{m_1} & \dot{f} < \dot{x} < \dot{g} \\ \dot{k} & \dot{g} < \dot{x} < \dot{h} \\ \dot{r}\left(\frac{\dot{i}-\dot{x}}{\dot{i}-\dot{h}}\right)^{m_2} & \dot{h} < \dot{x} < \dot{i} \\ \dot{r}\left(\frac{\dot{j}-\dot{x}}{\dot{j}-\dot{i}}\right)^{m_3} & \dot{i} < \dot{x} < \dot{j} \\ 0 & \dot{x} > \dot{j} \end{cases}$$

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$$Indeterminacy = I_L\acute{}(\dot{X}) = \begin{cases} 0 & \dot{x} < \dot{a}^1 \\ \dot{y}\left(\dfrac{\dot{x}-\dot{a}^1}{\dot{b}^1-\dot{a}^1}\right)^{m_1} & \dot{a}^1 < \dot{x} < \dot{b}^1 \\ \dot{y}\left(\dfrac{\dot{x}-\dot{b}^1}{\dot{c}^1-\dot{b}^1}\right)^{m_2} & \dot{b}^1 < \dot{x} < \dot{c}^1 \\ X & \dot{c}^1 < \dot{x} < \dot{d}^1 \\ \dot{X}-\left(\dot{X} \dot{-} y\right)\left(\dfrac{\dot{x}-\dot{d}^1}{\dot{e}^1-\dot{d}^1}\right)^{m_3} & \dot{d}^1 < \dot{x} < \dot{e}^1 \\ 1 & \dot{e}^1 < \dot{x} < \dot{f}^1 \\ \dot{X}-\left(X \dot{-} Z\right)\left(\dfrac{\dot{g}^1-\dot{x}}{\dot{g}^1-\dot{f}^1}\right)^{n_1} & \dot{f}^1 < \dot{x} < \dot{g}^1 \\ \dot{z} & \dot{g}^1 < \dot{x} < \dot{h}^1 \\ \dot{z}\left(\dfrac{\dot{i}^1-\dot{x}}{\dot{i}^1-\dot{h}^1}\right)^{n_2} & \dot{h}^1 < \dot{x} < \dot{i}^1 \\ \dot{z}\left(\dfrac{\dot{j}^1-\dot{x}}{\dot{j}^1-\dot{i}^1}\right)^{n_3} & \dot{i}^1 < \dot{x} < \dot{j}^1 \\ 1 & \dot{x} > \dot{j}^1 \end{cases}$$

$$Falsity = F_L\acute{}(\dot{X}) = \begin{cases} 0 & \dot{x} < \dot{a}^2 \\ \dot{q}\left(\dfrac{\dot{x}-\dot{a}^2}{\dot{b}^2-\dot{a}^2}\right)^{\dot{m}_1} & \dot{a}^2 < \dot{x} < \dot{b}^2 \\ \dot{q}\left(\dfrac{\dot{x}-\dot{b}^2}{\dot{c}^2-\dot{b}^2}\right)^{\dot{m}_2} & \dot{b}^2 < \dot{x} < \dot{c}^2 \\ \dot{k} & \dot{c}^2 < \dot{x} < \dot{d}^2 \\ \dot{w}-\left(w \dot{-} q\right)\left(\dfrac{\dot{x}-\dot{d}^2}{\dot{e}^2-\dot{d}^2}\right)^{\dot{m}_3} & \dot{d}^2 < \dot{x} < \dot{e}^2 \\ 1 & \dot{e}^2 < \dot{x} < \dot{f}^2 \\ \dot{w}-\left(w \dot{-} s\right)\left(\dfrac{\dot{g}^2-\dot{x}}{\dot{g}^2-\dot{f}^2}\right)^{\dot{n}_1} & \dot{f}^2 < \dot{x} < \dot{g}^2 \\ \dot{w} & \dot{g}^2 < \dot{x} < \dot{h}^2 \\ \dot{s}\left(\dfrac{\dot{i}^2-\dot{x}}{\dot{i}^2-\dot{h}^2}\right)^{\dot{n}_2} & \dot{h}^2 < \dot{x} < \dot{i}^2 \\ \dot{s}\left(\dfrac{\dot{j}^2-\dot{x}}{\dot{j}^2-\dot{i}^2}\right)^{\dot{n}_3} & \dot{i}^2 < \dot{x} < \dot{j}^2 \\ 1 & \dot{x} > \dot{j}^2 \end{cases}$$

As, 0<$\dot{k} < 1$ $\acute{A}_{\dot{\alpha}} = \{\dot{x} \in \dot{X} | T_L\acute{}(\dot{X}), F_L\acute{}(\dot{X}), I_L\acute{}(\dot{X}) \geq \dot{\alpha}\}$

Accuracy function

The accuracy function is given below:

$$\boldsymbol{D}^{\dot{T}_{NO_N}} = \left(\frac{\boldsymbol{\acute{a}} + \boldsymbol{b} + \boldsymbol{\acute{c}} + \boldsymbol{\acute{d}} + \boldsymbol{\acute{e}} + \boldsymbol{\acute{f}} + \boldsymbol{\grave{g}} + \boldsymbol{\acute{h}} + \boldsymbol{\acute{i}} + \boldsymbol{\acute{j}}}{\mathbf{10}}\right)$$

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$$\boldsymbol{D^{\dot{I}_{NO_N}}} = \left(\frac{\acute{a}^1 + b^1 + \acute{c}^1 + \acute{d}^1 + \acute{e}^1 + \acute{f}^1 + \grave{g}^1 + \acute{h}^1 + \acute{\iota}^1 + \acute{j}^1}{10}\right)$$

$$\boldsymbol{D^{\dot{F}_{NO_N}}} = \left(\frac{\acute{a}^2 + b^2 + \acute{c}^2 + \acute{d}^2 + \acute{e}^2 + \acute{f}^2 + \grave{g}^2 + \acute{h}^2 + \acute{\iota}^2 + \acute{j}^2}{10}\right)$$

4. Case Study

We will estimate the flexibility and outcome of decagonal neutrosophic numbers. We will show the strength of the decagonal by the real-life problem. With ten edges we can handle huge problems easily. Suppose a real-life problem with maximum parameters.

Numerical problem: Here U as the universe. A student wants to study abroad. So, he decided to compare different countries. It's a big multi-criteria decision and the method to avail the solution given below:

The different countries are $\acute{A}$, $\acute{B}$, and $\acute{C}$. Choice parameters are $\acute{C_1}$, $\acute{C_2}$, and $\acute{C_3}$.

$$\begin{array}{c}
\begin{array}{ccc} \acute{A} \qquad\qquad\qquad\qquad & \qquad\qquad \acute{B} \qquad\qquad & \qquad\qquad\qquad\qquad \acute{C} \end{array} \\
\left(\begin{array}{lll}
\{\acute{C}_1(0.69,0.59,0.21,0.35,0.47,0.85,0.32,0,13,0.48,0.64) & \{\acute{C}_1(0.21,0.31,0.41,0.51,0.61,0.71,0.76,0.52,0.36,0.46) & \{\acute{C}_1(0.21,0.31,0.41,0.51,0.61,0.72,0.84,0.33,0.30,0.24) \\
(0.65,0.87,0.33,0.29,0.31,0.48,0.67,0.45,0.63,0.21) & (0.79,0.87,0.67,0.98,0.78,0.58,0.48,0.69,0.24,0.62) & (0.25,0.41,0.67,0.75,0.85,0.47,0.67,0.41,0.31,0.49) \\
(0.28,0.58,0.88,0.38,0.55,0.36,0.78,0.24,0.36,0.95)\} & (0.98,0.95,0.94,0.93,0.90,0.92,0.99,0.88,0.78,0.58) & (0.98,0.97,0.96,0.94,0.87,0.81,0.89,0.48,0.61,0.13) \\
\{\acute{C}_2(0.11,0.21,0.19,0.28,0.52,0.36,0.57.0.36,0.38,0.25) & \{\acute{C}_2(0.41,0.51,0.13,0.23,0.43,0.33,0.17,0.58,0.21,0.67) & \{\acute{C}_2(0.31,0.27,0.47,0.59,0.92,0.50,0.47,0.38,0.14,0.27) \\
(0.74,0.24,0.34,0.21,0.98,0.75,0.41,0.58,0.32,0.27) & (0.25,0.35,0.45,0.55,0.65,0.75,0.85,0.95,0.21,0.34) & (0.17,0.28,0.78,0.71,0.94,0.95,0.68,0.74,0.71,0.92) \\
(0.33,0.44,0.55,0.66,0.45,0.87,0.88,0.97,0.23,0.45)\} & (0.12,0.97,0.85,0.81,0.86,0.79,0.75,0.61,0.66,0.68) & (0.21,0.94,0.92,0.97,0.93,0.82,0.81,0.87,0.24,0.93) \\
\{\acute{C}_3(0.45,0.87,0.36,0.45,0.36,0.54,0.63,0.28,0.13,0.11) & \acute{C}_3(0.27,0.58,0.34,0.13,0.23,0.25,0.34,0.22,0.71,0.62) & \{\acute{C}_3(0.14,0.13,0.17,0.27,0.34,0.57,0.19,0.58,0.19,0.74) \\
(0.51,0.29,0.27,0.19,0.23,0.47,0.18,0.72,0.23,0.22) & (0.67,0.24,0.31,0.45,0.65,0.74,0.39,0.52,0.41,0.31) & (0.37,0.47,0.97,0.24,0.87,0.82,0.92,0.67,0.58,0.71) \\
(0.90,0.96,0.11,0.24,0.34,0.87,0.21,0.36,0.78,0.11)\} & (0.57,0.67,0.37,0.47,0.57,0.67,0.77,0.97,0.27,0.68) & (0.81,0.91,0.96,0.92,0.97,0.82,0.81,0.79,0.88,0.67)
\end{array}\right)
\end{array}$$

In above matrix $(\acute{C_1}, \acute{C_2}, \acute{C_3})$ mentioned as row and countries as $(\acute{A}, \acute{B},\ and\ \acute{C})$ in column.

Step 1: Defuzzification of Decagonal neutrosophic number by using accuracy function:

$\boldsymbol{D^{\dot{T}_{NO_N}}} = \left(\frac{\acute{a}+b+\acute{c}+\acute{d}+\acute{e}+\acute{f}+\grave{g}+\acute{h}+\acute{\iota}+\acute{j}}{10}\right)$, $\boldsymbol{D^{\dot{I}_{NO_N}}} = \left(\frac{\acute{a}^1+b^1+\acute{c}^1+\acute{d}^1+\acute{e}^1+\acute{f}^1+\grave{g}^1+\acute{h}^1+\acute{\iota}^1+\acute{j}^1}{10}\right)$, **and**

$\boldsymbol{D^{\dot{F}_{NO_N}}} = \left(\frac{\acute{a}^2+b^2+\acute{c}^2+\acute{d}^2+\acute{e}^2+\acute{f}^2+\grave{g}^2+\acute{h}^2+\acute{\iota}^2+\acute{j}^2}{10}\right)$

By using these formulas, Neutrosophic soft Matrix given below:

Enrollment in local colleges, 2005

Criteria	$\boldsymbol{\grave{A}}$	$\boldsymbol{\grave{B}}$	$\boldsymbol{\grave{C}}$
$\grave{C}_1$	(0.4,0.4,0.5)	(0.4,0.6,0.8)	(0.4,0.5,0.7)
$\grave{C}_1$	(0.3,0.4,0.5)	(0.3,0.5,0.7)	(0.4,0.6,0.7)
$\boldsymbol{\grave{C}_3}$	(0.4,0.3,0.4)	(0.3,0.4,0.6)	(0.3,0.6,0.8)

Step 2: For normalized aggregate fuzzy decision matrix.

$$\widetilde{r_{ij}} = \left(\frac{\widetilde{a_{ij}}}{\widetilde{c_{ij}}}, \frac{\widetilde{b_{ij}}}{\widetilde{c_{ij}}}, \frac{\widetilde{c_{ij}}}{\widetilde{c_{ij}}}\right)$$

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Criteria	$\grave{A}$	$\grave{B}$	$\grave{C}$
$\grave{C}_1$	(0.8,0.8,1.0)	(0.5,0.7,1.0)	(0.5,0.7,1.0)
$\grave{C}_1$	(0.6,0.8,1.0)	(0.4,0.7,1.0)	(0.5,0.8,1.0)
$\grave{C}_3$	(1.0,0.7,1.0)	(0.5,0.6,1.0)	(0.3,0.7,1.0)

For criteria weighting, aggregate decision matrix given below:

$\widetilde{W_1} = (0.3,0.4,0.5)$, $\widetilde{W_2} = (0.5,0.6,0.7)$, and $\widetilde{W_3} = (0.1,0.2,0.3)$

Step 3: $\widetilde{p_{ij}} = \widetilde{r_{ij}}$ will multiply by $\widetilde{w_j}$ moreover, weighted normalized decision matrix.

Criteria	$\grave{A}$	$\grave{B}$	$\grave{C}$
$\grave{C}_1$	(0.2,0.3,0.5)	(0.1,0.2,0.5)	(0.1,0.2,0.5)
$\grave{C}_1$	(0.3,0.4,0.7)	(0.2,0.4,0.7)	(0.2,0.4,0.7)
$\grave{C}_3$	(0.1,0.1,0.3)	(0.5,0.1,0.3)	(0.3,0.1,0.3)

Step 4: Find $\tilde{F}NIS\ and\ \tilde{F}PIS$

$\grave{A}^+ = (\grave{P}_1^+, \grave{P}_2^+, \grave{P}_3^+, \ldots, \grave{P}_n^+)$

$\grave{P}_j^+ = \max(\grave{P}_{ij})\ i = 1,2, \ldots, m\ and\ j = 1,2, \ldots, n$

$\grave{A}^- = (\grave{P}_1^-, \grave{P}_2^-, \grave{P}_3^-, \ldots, \grave{P}_n^-)$

$\grave{P}_j^- = \max(\grave{P}_{ij})\ i = 1,2, \ldots, m\ and\ j = 1,2, \ldots, n$

$\grave{A}^+ = \grave{P}_1^+(0.5,0.5,0.5), \grave{P}_2^+(0.7,0.7,0.7), \grave{P}_3^+(0.3,0.3,0.3)$

$\grave{A}^- = \grave{P}_1^-(0.2,0.1,0.1), \grave{P}_2^-(0.3,0.2,0.2), \grave{P}_3^-(0.1,0.1,0.1)$

$$\acute{d}(\grave{x},\grave{y}) = \sqrt{\frac{1}{3}(\grave{a}_1 - \grave{a}_2)^2 + (\grave{b}_1 - \grave{b}_2)^2 + (\grave{c}_1 - \grave{c}_2)^2}$$

Ideal positive solution

Criteria	$\grave{A}$	$\grave{B}$	$\grave{C}$
$\grave{C}_1$	(0.2)	(0.4)	(0.2)
$\grave{C}_1$	(0.2)	(0.1)	(0.4)
$\grave{C}_3$	(0.3)	(0.3)	(0.2)

Negative Ideal solution

Criteria	$\grave{A}$	$\grave{B}$	$\grave{C}$
$\grave{C}_1$	(0.3)	(0.3)	(0.3)
$\grave{C}_1$	(0.2)	(0.5)	(0.3)
$\grave{C}_3$	(0.1)	(0.1)	(0.2)

Now calculate the distance between every weighted alternative.

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$\grave{d}_i^* = \Sigma_{j=1}^n \grave{d}(\grave{v}_{ij}, \grave{v}_j^*)$, $\grave{d}_i^- = \Sigma_{j=1}^n \grave{d}(\grave{v}_{ij}, \grave{v}_j^-)$

$\grave{d}_1^+ = 0.7$ $\grave{d}_1^- = 0.6$

$\grave{d}_2^+ = 0.8$ $\grave{d}_2^- = 0.9$

$\grave{d}_3^+ = 0.8$ $\grave{d}_3^- = 0.8$

Closeness coefficient

$\grave{C}C_i = \frac{\grave{d}_i^-}{\grave{d}_i^- + \grave{d}_i^+}$

$\grave{C}\grave{C}_1 = 0.4615$

$\grave{C}\grave{C}_2 = 0.5294$

$\grave{C}\grave{C}_3 = 0.5000$

Strategy	Results	Rank
$\grave{C}\grave{C}_1$	(0.4615)	(3)
$\grave{C}\grave{C}_1$	(0.5294)	(1)
$\grave{C}\grave{C}_3$	**(0.5000)**	**(2)**

Clearly $\grave{B} > \grave{C} > \grave{A}$, The best country is $\grave{B}$.

5. Conclusion

In this article, we introduce decagonal neutrosophic numbers (Linear, Non-Linear, Symmetric, Asymmetric). In the environment of MCDM, decagonal neutrosophic numbers will be very effective because of their ten edges. By using decagonal neutrosophic numbers we can deal with daily-life problems more effectively. Decagonal neutrosophic numbers have ten edges to deal with more fluctuations. In order to show the reliability and the working of this tool, we introduce an application based on MCDM. We solve the problem with the TOPSIS technique of MCDM. Moreover, we present aggregate operators of decagonal neutrosophic numbers with matrix notations and operations.

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