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MANAGERIAL DECISION MAKING USING BEST WORST METHOD WITH MULTI-VALUED NEUTROSOPHIC APPROACH

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ABSTRACT. The subject of intrinsic and extrinsic factors constitute to the elements of decision making process characterized by the influence of multi bench mark, diverse consents of the experts on different aspects-at managerial level. Decision makers strive hard to construct consensus in formulating decisions by minimizing the complexity in the process of decision making by applying various approaches of decision making. The efficiency of the decision making methods depends on the time and cost efficiency. In the research area of decision making, best worst method is being explored presently and this method is modified with the integration of various kinds of fuzzy numbers and single valued neutrosophic fuzzy number is one such instance. The efficacy of best worst method with single valued neutrosophic fuzzy number has motivated us to extend the same decision making method with multi valued neutrosophic fuzzy number. It is proposed to formulate a decision making model using best worst method with multi valued neutrosophic approach and to present a comparative analysis of single and multi-valued neutrosophic fuzzy number. The formulated model is validated with real life application and it will certainly benefit the decision makers in framing optimal decisions.

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1. Introduction

In the last decade, concealed by uncertain atmosphere, many algorithms in fuzzy andintutionistic fuzzy have been deliberated intensely for uncertain data processing. The neutrosophic set is a simplification of fuzzy sets and intutionistic fuzzy sets. Neutrosophic set has latent of being a wide-ranging structure for indecision investigation in big data sets.Fuzzy logic was generalized in 2001 by Florentin Smarandache. Currently neutrosophic Fuzzy Sets becomes a recent research area in various appliances of optimization fields. Decision-making(DM) is considered as the cognitive process consequence in the collection of a certainty or a strategy among numerous alternative possibilities. The neutrosophic aggregation operation may be combined with the existing Multi-bench mark decision-making (MCDM) process such as best-worst method (BWM) to solve several types of problems in different fields of decision-making in our everyday life. The multibench mark optimization and compromise solution method have investigated by P.J.G. Pineda, et al. (2018) [5],in which he applied the technique to decide on the appropriate enhancement choice goals with the equivalent weights offerd by the DANP method. The human brain is not capable to review the effects of transforms in one factor on more than four interconnected controling factor. Scherz and Vafadarnikjoo (2019) [7], have worked with various MCDA methods which were applied for single issues inside the turf of prolongable erection.

Jafar Rezaei, (2015) [2] proposed an application of robust optimization technique to a recently developed model named Best-Worst method and the resulted robust approach is formulated as a linear programming. Solairaju et al, (2018) [8] presented the knowledge organization tactic of renovate ingneutrosophic fuzzy values into fuzzy values by means of impreciaion techniques in defuzzyfication. Juan-juan Peng,(2015) [3] presented an approach for solving MCGDM problems and explored by applying the power aggregation operators. Rezaei, J. (2016) [6] derived the final scores of the alternatives by aggregating the weights from different sets of bench mark and alternatives, based on which the best alternative is selected. Several researchers like Abdel (2020) [1] have discussed in his various works for how to make decisions.

In this paper, we have formulate a decision making model using best and worst method with multi valued neutrosophic approach by converting the multi valued neutrosophic representations of the data to the single valued neutrosophic representation using optimistic approach.The paper is organized as follows, section 1 gives a brief introduction and literature survey; preliminaries are presented in section 2; section 3 consists of the methodology, a numerical example has been drafted to validate the proposed model in section 4. Finally the research work is summarized in section 5.

2. PRELIMINARIES

This section comprises of fundamental definitions and methods used in this research work.

2.1. **Neutrosophic Fuzzy Set.** Let ξ^* be the universe. A neutrosophic fuzzy set A_{NF} in ξ^* which is described by a truth membership value T_A, Indeterminacy membership value I_A and a falsity membership value F_A where T_A, I_A and F_A are real set of components of $[0,1]$. It can be denoted by $A_{NF} = \{\langle x, T_A, I_A, F_A \rangle : x \in \xi^*\}$. There is no restriction on the sum of $T_A, I_A, F_A 0^- \leq \sup T_A + \sup I_A + \sup F_A \leq 3^+$.

2.2. **Multi valued Neutrosophic set(MVNS).** Let ξ^* be the universe, with a nonspecific factor in ξ^* denoted by x. A MVNS A in ξ^* is specified by the following functions $\bar{T_A}, \bar{I_A}$ and $\bar{F_A}$ in the structure of subset of $[0,1]$, which can be defined by $A_{NF} = \{\langle x, \bar{T_A}, \bar{I_A}, \bar{F_A} \rangle : x \in \xi^*\}$, where $\bar{T_A}, \bar{I_A}$ and $\bar{F_A}$ denotes truth-membership degree, indeterminacy-membership degree and falsity membership degree respectively, with the below condition: $0 \leq \bar{T_A}, \bar{I_A}, \bar{F_A} \leq 1$.

2.3. **Transformation of neutrosophic fuzzy.** [8] There are two methods for transforming neutrosophic fuzzy values (sets) into fuzzy values (sets).

2.3.1. *Method I (Imprecision membership):* Any neutrosophic fuzzy set $N_A = (T_A, I_A, F_A)$ including neutrosophic fuzzy values are transformed into intuitionistic fuzzy values or vague values as $\eta(A) = (T_A, f_A)$ where f_A is estimated the formula stated below which is called as Impression membership method:

$$f_A = \begin{cases} F_A + \frac{[1-F_A-I_A][1-F_A]}{[F_A+I_A]} & \text{if} \quad F_A = 0 \\ F_A + \frac{[1-F_A-I_A][F_A]}{[F_A+I_A]} & \text{if} \quad 0 < F_A \leq 0.5 \\ F_A + [1-F_A-I_A]\left[0.5 + \frac{F_A-0.5}{F_A+I_A}\right] & \text{if} \quad 0.5 < F_A \leq 1. \end{cases}$$

2.3.2. *Method II (Defuzzification):* After Method I (Median membership), intuitionistic (vague) fuzzy values of the form $\eta(A) = (T_A, f_A)$ are transformed into fuzzy set including fuzzy values as $\langle \Delta(A) \rangle = \left\langle \frac{T_A}{[T_A + f_A]} \right\rangle$.

3. METHODOLOGY

3.1. **Decision Matrix.** The multivalued neutrosophic fuzzy matrics is constructed based on the expert's opinion with reference to the benchmarks of resolution. The following steps are used to convert the multivalued neutrosophic fuzzy in to single fuzzy decision matrix $\mathbb{D}(M)$.

Step 1 : Limited Neutrosophic fuzzy values are shaped into a neutrosophic fuzzy decision matrix having R rows and C columns, such that Rneutrosophic fuzzy attributes (each row) corresponding to C neutrosophic fuzzy alternatives (each column). There are n number of fuzzy decision matrices $\mathbb{D}(M_i)[i = 1 \ to \ n]$ are considered.

Step 2: The exceeding fuzzy matrix (M_i) is modified into a single fuzzy matric $\eta(M_i)$ having two membership functions in which T_{Ai} is unaltered, and f_{Ai} is calculated from I_{Ai}, and F_{Ai} using impression membership (method I).

Step 3: After step 2, and for each $i = 1$ to n, the single fuzzy decision matrix $\mathbb{D}(M_i)$ is defuzzified as a single fuzzy decision matrix $\mathbb{D}(M_i)$ having only one membership by the method II.

3.2. **Best Worst method.** [6] A MCDM problem is formed of some alternatives $(a_1, a_2, ..., a_m)$ and multiple bench mark $(b_1, b_2, ..., b_n)$ and each alternative has a score with respect to each criterion $(\rho_{11}, \rho_{12}, ..., \rho_{mn})$. The MCDM problem can be described as the following matrix:

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \rho_{m1} & \rho_{m2} & \cdots & \rho_{mn} \end{pmatrix}.$$

The key function of an MCDM problem is to get the best option with the best overall value (ν_i).

There are various methods to determine the overall score for every alternative and the easy approach is to apply an additive weighted value function as the following formula (Keeney & Raiffa,1993) [4]:

$$\nu_i = \sum_{j=1}^{n} \omega_j \rho_{ij}, \quad \omega_j \geq 0 \quad \text{and} \quad \sum_{j=1}^{n} \omega_j = 1.$$

The best-worst method (BWM)(Rezaei,2015) is used to discover the heft of each touch stone (ω_j) and we have the score of each alternative with respect to each criterion (ρ_{ij}), and the overall score can be easily obtained.

To summarise the following steps of BWM that can be applied to obtain the weights of the bench mark.

Step 1. Determine a set of decision bench marks. Let us consider the bench mark $(b_1, b_2, ..., b_n)$ that should be utilized to reach at a conclusion. For Example, in the case of Teaching and Learning process the decision bench mark can be listed below,Learner centric (b_1), Activity Based (b_2), Space for creativity (b_3), Interactive (b_4), Time efficiency (b_5).

Step 2. Determine the best (e.g.fascinating,most excellent) and the worst (e.g. unsuitable, least essential) bench mark.

In this step, the decision-maker identifies the best and the worst bench mark. In general,no association is made at this stage. For example, for a particular decision-maker identifies, Learner centric (b_1) and Time Efficiency (b_5) may be the best and the worst bench mark, respectively.

Step 3. Find out the importance of the best bench mark over all the other bench mark using an integer between 1 and 9. The resulting Best-to-Others vector would be:

$$A_B = (a_{B1}, a_{B2}, ..., a_{Bn}).$$

Where a_{Bj} represents the importance of the best bench mark B over bench mark j. That is $a_{BB} = 1$. For our example, the vector explains the importance of Learner centric (b_1) over all the other bench mark.

Step 4 Decide the importance of all the bench mark over the worst bench mark using an integer between 1 and 9. The resulting others-to-Worst vector would be $A_W = (a_{1w}, a_{2w}, ..., a_{nw})^T$, where a_{jw} represents the importance of the bench mark j over the worst bench mark W. That is $a_{ww} = 1$. For our example, the vector indicates the importance of all the bench mark over Time Efficiency (b_5).

Step 5. Find the optimal weights $(\omega_1^*, \omega_2^*, ..., \omega_n^*)$. The optimal weight for the bench mark is the one where, for each pair of ω_B/ω_j and ω_j/ω_w. Then

$\omega_B/\omega_j = a_{Bj}$ and $\omega_j/\omega_w = a_{jw}$. To satisfies the following conditions for all j, we should find a solution where the maximum absolute differences $|\frac{\omega_B}{\omega_j} - a_{Bj}|$ and $|\frac{\omega_j}{\omega_w} - a_{jw}|$ for all j is minimized, which is transferred to the following LPP model,

$$\min \max_j \left\{ |\frac{\omega_B}{\omega_j} - a_{Bj}|, |\frac{\omega_j}{\omega_w} - a_{jw}| \right\}$$

s.t

$$\sum_{j=1}^{n} \omega_j = 1$$

$$\omega_j \geq 0, \quad \text{for all} \quad j \tag{3.1}$$

Model (3.1) is equivalent to the following model:

$\min \psi$ s.t

$$|\frac{\omega_B}{\omega_j} - a_{Bj}| \leq \psi, \quad \text{for all} \quad j$$

$$|\frac{\omega_j}{\omega_w} - a_{jw}| \leq \psi \quad \text{for all} \quad j$$

$$\sum_{j=1}^{n} \omega_j = 1$$

$$\omega_j \geq 0, \quad \text{for all} \quad j \tag{3.2}$$

solving the model (3.2) the optimal weights $(\omega_1^*, \omega_2^*, ..., \omega_n^*)$ and ψ^* are obtained.

The consistency ratio is determined to check the feasibility of the values obtained using the Table 1.

Consistency Ratio $= \frac{\psi^*}{consistency\ index}$

TABLE 1. Consistency index (CI) table.

a_{Bw}	1	2	3	4	5	6	7	8	9
Consistency index $(\max \psi)$	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

4. NUMERICAL EXAMPLE

In this section, an example of MCDM problem is used to demonstrate the proposed decision making method.

Education is important to every person and is rated by every nation. Umoh (2006) has rightly observed education helps the human being to build up in the emotionally, psychologically, ethically,mentally and sensitively by providing suitable atmosphere, teaching new information, thoughts and talents that will facilitate us to be valuable and to society. Generally we can define five bench mark that will help us to measure the Teaching and Learning process. The decision bench mark can be listed below, Learner centric (b_1), Activity Based (b_2), Space for creativity (b_3), Interactive (b_4), Time efficiency (b_5) and the various methods are denoted by M_1, M_2, M_3, M_4, M_5

The initial Decision matrix is

	Learner centric	Activity Based	Space for creativity	Interactive	Time Efficiency
M_1	{(0.9),(0.2,0.3), (0.2)}	{(0.6), (0.1), (0.5,0.2)}	{(0.8), (0.3), (0.5)}	{(0.5), (0.1), (0.4)}	{(0.5), (0.2), (0.7,0.8)}
M_2	{(0.7),(0.4,0.1), (0.5,0.1)}	{(0.9),(0.3,0.7), (0.5,0.2)}	{(0.5), (0.7), (0.4,0.2)}	{(0.4), (0.1), (0.2)}	{(0.4), (0.3), (0.1,0.4)}
M_3	{(0.5,0.8), (0.2,0.5), (0.6)}	{(0.7,0,0.1),(0.6), (0.2,0.6)}	{(0.9,1),(0.1,0.3), (0.2,0.4)}	{(0.6),(0.3), (0.2,0.1)}	{(0.3), (0.2,0.3), (0.4)}
M_4	{(0.8,0.4,0.1), (0.2,0.3),(0.6)}	{(1,0,0), (0.7), (0.4,0.7)}	{(0.7), (0.3), (0.3)}	{(0.6), (0.3), (0.4)}	{(0.4), (0.1), (0.1,0.2)}
M_5	{(1,0,0), (0.4,0.5), (0.5,0.1)}	{(0.1,0.7), (0.5), (0.2,0.3)}	{(0.6), (0.1), (0.2,0.5)}	{(0.8),(0.5), (0.2,0.3)}	{(0.5), (0.2), (0.2)}

Multivalued neutrosophic fuzzy values are converted into single valued fuzzy values using optimistic approach by selecting the max(Truth membership) and min(Indeterminacy and Falsity).

The modified Decision matrix using optimistic approach is

Criterion/ methods	Learner centric	Activity Based	Space for creativity	Interactive	Time Efficiency
M_1	{(0.9),(0.2), (0.2)}	{(0.6), (0.1), (0.2)}	{(0.8), (0.3), (0.5)}	{(0.5),(0.1), (0.4)}	{(0.5), (0.2),(0.7)}
M_2	{(0.7),(0.1), (0.1)}	{(0.9), (0.3), (0.2)}	{(0.5), (0.7), (0.2)}	{(0.4),(0.1), (0.2)}	{(0.4),(0.3), (0.1)}
M_3	{(0.8),(0.2), (0.6)}	{(0.7), (0.6), (0.2)}	{(1), (0.1), (0.2)}	{(0.6),(0.3), (0.1)}	{(0.3),(0.2), (0.4)}
M_4	{(0.8),(0.2), (0.6)}	{(1), (0.7), (0.4)}	{(0.7), (0.3), (0.3)}	{(0.6),(0.3), (0.4)}	{(0.4),(0.1), (0.1)}
M_5	{(1), (0.4), (0.1)}	{(0.7), (0.5), (0.2)}	{(0.6), (0.1), (0.2)}	{(0.8),(0.5), (0.2)}	{(0.5),(0.2), (0.2)}

The single valued fuzzy decision matrix is

$$\mathbb{D}(M) = \begin{pmatrix} (0.6429) & (0.4736) & (0.5614) & (0.3846) & (0.4091) \\ (0.5833) & (0.6923) & (0.6925) & (0.3749) & (0.6154) \\ (0.5246) & (0.7368) & (0.5999) & (0.7059) & (0.3103) \\ (0.5246) & (0.733) & (0.5833) & (0.5122) & (0.444) \\ (0.833) & (0.7102) & (0.4736) & (0.7369) & (0.5) \end{pmatrix}$$

The Nine point scale for Five Bench marks are presented below

(1) Equal relative importance
(2) Equally to moderately more important
(3) Moderately more Important
(4) Moderately to strongly Important
(5) Strongly Important
(6) Strongly to very strongly more Important
(7) Very strongly more Important
(8) Very strongly to extremely more Important
(9) Extremely Important (High priority)

Neutrosophic fuzzy values for Nine point scale value and it is transformed in to Single valued fuzzy number by Imprecision and Defuzzification method

NFN	Method I $\eta(A)$	Fuzzy number	Method II $\Delta(A)$
(1,0,0.1)	1	(1,1)	0.5=1 (Equal importance)
(0.8,0.1,0.3)	0.75	(0.8, 0.75)	0.516
(0.7,0.3,0.4)	0.5714	(0.7, 0.5714)	0.551
(0.8,0.1,0.1)	0.5	(0.8, 0.5)	0.6154
(1,0.4,0.4)	0.5	(1, 0.5)	0.6667
(0.7,0.2,0.1)	0.333	(0.7, 0.333)	0.6776
(0.9,0.3,0.2)	0.4	(0.9, 0.4)	0.6923
(0.8,0.4,0.1)	0.2	(0.8, 0.2)	0.778
(0.8,0.7,0.2)	0.222	(0.8, 0.222)	0.7828

Pairwise comparision for Best and Worst bench mark is as follows

B O /Bench mark	b_1	b_2	b_3	b_4	b_5
Best Bench mark : b_1(Learner centric)	1	3	2	4	7
O W/ Worst Bench mark : b_5					
b_1					7
b_2					4
b_3					2
b_4					3
b_5					1

By linear programming approach we can find optimal solution ψ^*

$\min \psi^*$ s.t

$$\omega_1 - 3\omega_2 \leq \psi, \quad \omega_1 - 2\omega_3 \leq \psi,$$
$$\omega_1 - 4\omega_4 \leq \psi, \quad \omega_1 - 7\omega_5 \leq \psi,$$
$$\omega_2 - 4\omega_5 \leq \psi, \quad \omega_3 - 2\omega_5 \leq \psi,$$
$$\omega_4 - 3\omega_5 \leq \psi$$

$\omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 = 1$ and $\omega_j \geq 0$, for all j.

solving the above constraints we get the following weights

$\omega_1 = 0.6468, \omega_2 = 0.0931, \omega_3 = 0.0994, \omega_4 = 0.09718, \omega_5 = 0.0635.$

The optimal solution $\psi^* = 0.0667$.

Consistency Ratio $= \frac{\psi^*}{consistency\ index} = \frac{0.0667}{2.3} = 0.029 \in (0, 1)$.

The Normalized aggregate values are

Bench marks /methods	Learner centric	Activity Based	Space for creativity	Interactive	Time Efficiency
M1	0.6429	0.4736	0.5614	0.5614	0.4091
M2	0.5833	0.6923	0.6925	0.6925	0.6154
M3	0.5246	0.7368	0.59999	0.59999	0.3103
M4	0.5246	0.7333	0.5833	0.5833	0.4444
M5	0.8333	0.7102	0.4736	0.4736	0.5

The weight and ranking of each bench mark is

Bench mark	b_1	b_2	b_3	b_4	b_5
ω^*	0.6468	0.0931	0.0994	0.09718	0.0635
Rank	1	4	2	3	5
ψ^*	0.0667				

Over all score and Ranking of each alternative methods:

Alternative methods:	M_1	M_2	M_3	M_4	M_5
Overall aggregated value	0.5791	0.5861	0.5558	0.5435	0.7553
Rank	3	2	4	5	1

5. CONCLUSION

A decision making model using best and worst method with multi valued neutrosophic approach is presented in this research work. Using optimistic approach we can calculate the feasible weights for every bench mark by converting mutli valued neutrosophic sets to single valued neutrosophic sets. This optimistic approach is introduced to handle the decision making process characterized by multi neutrosophic fuzzy sets. In future, we recommend that the same model can be applied in various real life circumstances. This method suggested to give an effective aggregated value and optimal weights and also the opinion of several decision makers are considered to make decisions.

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