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Minimal Domination via Neutrosophic Over Graphs

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Abstract. *NOverG* and their types are invented in this article. The idea of domination and minimal domination in *NOverG* are established. Several interesting characterization are discussed along with the examples.

INTRODUCTION

Zadeh in 1965[11] was invent the idea of a fuzzy set as a mathematical frame work for representing vagueness and imprecise information. A fuzzy graph was introduced by A.Rosenfeld[6]. Application of fuzzy graphs play a viral role in the fields of traffic congestion problem, decision analysis, networking, privacy and security, etc, for obtaining solutions.

F. Smarandache[7,8] presented the generalization of *IF* logic and defined as "neutrosophic logic". Also he defined a neutrosophic over set [9]. The graphs in single valued neutrosophic set theory was introduced [1]. R.Narmada Devi[3,4] were introduced the concepts of neutrosophic complex \mathcal{N} -continuity and neutrosophic complex graphs. The definition of domination was defined by Ore[5]. Later many researchers like A.somasundaram[10], Nagoor Gani[2] were discussed the domination in fuzzy graphs. In this article, *NOverG* and their types are introduced. Some interesting characterization of domination and minimal domination of neutrosophic over graph are established. Several interesting characterization are established along with the examples.

PRELIMINARIES

Definition .1 [9] A Single-Valued Neutrosophic over set A is defined as: $A = (x, \langle T(x), I(x), F(x) \rangle), x \in \mathcal{U}$ such that there exist some elements in A that have atleast one neutrosophic component that is > 1 and no element has neutrosophic components that are < 0 and $T(x), I(x), F(x) \in [0, \Omega]$, where Ω is called overlimit such that $0 < 1 < \Omega$.

Definition .2 [9] Let A and B be any two Neutrosophic Over Set's in X . Then

- (i) $A \cup B = \langle x, T_{A \cup B}(x), I_{A \cup B}(x), F_{A \cup B}(x) \rangle$ where $T_{A \cup B}(x) = T_A(x) \vee T_B(x), I_{A \cup B}(x) = I_A(x) \vee I_B(x)$ and $F_{A \cup B}(x) = F_A(x) \wedge F_B(x)$.
- (ii) $A \cap B = \langle x, T_{A \cap B}(x), I_{A \cap B}(x), F_{A \cap B}(x) \rangle$ where $T_{A \cap B}(x) = T_A(x) \wedge T_B(x), I_{A \cap B}(x) = I_A(x) \wedge I_B(x)$ and $F_{A \cap B}(x) = F_A(x) \vee F_B(x)$.
- (iii) $A \subseteq B$ if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)$ and $F_A(x) \geq F_B(x)$, for all $x \in X$.
- (iv) the complement of A is defined as $C(A) = \langle x, T_{C(A)}(x), I_{C(A)}(x), F_{C(A)}(x) \rangle$ where $T_{C(A)}(x) = 1 - T_A(x), I_{C(A)}(x) = 1 - I_A(x)$ and $F_{C(A)}(x) = 1 - F_A(x)$.
- (v) $0_N = \{ \langle x, 0, 0, \Omega \rangle : x \in X \}$ and $\Omega_N = \{ \langle x, \Omega, \Omega, 0 \rangle : x \in X \}$

NEW IDEA ON NEUTROSOPHIC OVER GRAPHS

Definition .3 A *NOverG* $G = (A, B)$ is a *NG* on a crisp graph G^* where A is an neutrosophic vertex over set on V and B is a neutrosophic edge over set on E respectively such that

- (i) $\mathfrak{T}_B(xy) \leq \min[\mathfrak{T}_A(x), \mathfrak{T}_A(y)]$,

- (ii) $\mathfrak{I}_B(xy) \leq \min[\mathfrak{I}_A(x), \mathfrak{I}_A(y)]$ and
- (iii) $\mathfrak{F}_B(xy) \geq \max[\mathfrak{F}_A(x), \mathfrak{F}_A(y)]$, for every $xy \in E \subseteq V \times V$.

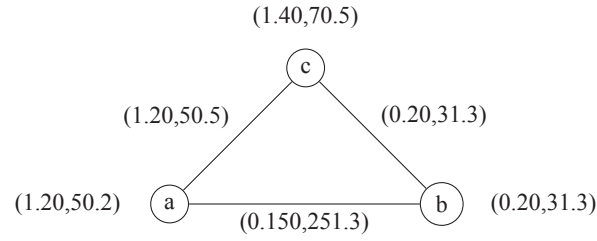


FIGURE 1. *NOverG*

Definition .4 Let G be any a *NOverG* of G^* . Then $H = (A_1, B_1)$ is called *NOverSubG* if $A_1 = A$ and $B_1 = B$.

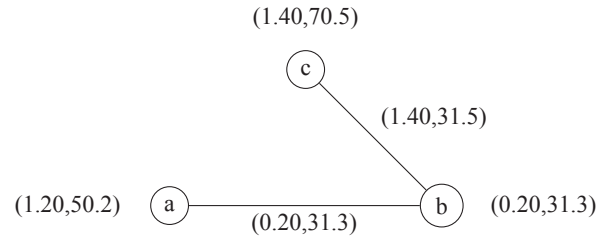


FIGURE 2. *NOverSubG* for FIGURE 1

Definition .5 A complement *NOverG* is $C(G)$ if

- (i) $C(A) = A$,
- (ii) $\mathfrak{I}_{C(B)}(xy) = [\mathfrak{I}_A(x) \wedge \mathfrak{I}_A(y)] - \mathfrak{I}_B(xy)$,
- (iii) $\mathfrak{J}_{C(B)}(xy) = [\mathfrak{J}_A(x) \wedge \mathfrak{J}_A(y)] - \mathfrak{J}_B(xy)$ and
- (iv) $\mathfrak{F}_{C(B)}(xy) = [\mathfrak{F}_A(x) \vee \mathfrak{F}_A(y)] - \mathfrak{F}_B(xy)$, for every $xy \in E$.

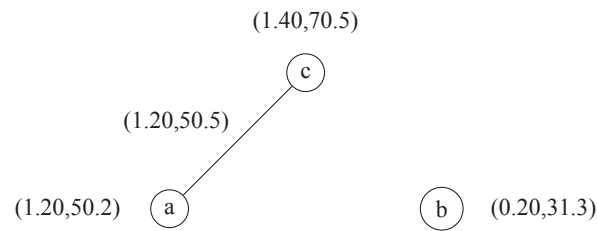


FIGURE 3. complement *NOverG* for FIGURE 2

Definition .6 A complete *NoverG* is a graph if

- (i) $\mathfrak{I}_B(xy) = \min[\mathfrak{I}_A(x), \mathfrak{I}_A(y)]$,
- (ii) $\mathfrak{J}_B(xy) = \min[\mathfrak{J}_A(x), \mathfrak{J}_A(y)]$ and
- (iii) $\mathfrak{F}_B(xy) = \max[\mathfrak{F}_A(x), \mathfrak{F}_A(y)]$, for every $x, y \in V$.

Moreover, it is denoted by K_A where A is a neutrosophic over vertex set in V .

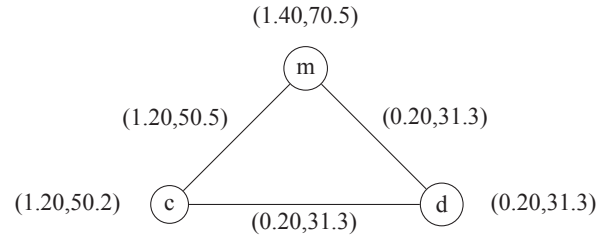


FIGURE 4. completeNoverG

Definition .7 A \mathcal{S} trongNOverG is a graph if

- (i) $\mathfrak{T}_B(xy) = \min[\mathfrak{T}_A(x), \mathfrak{T}_A(y)]$,
- (ii) $\mathfrak{I}_B(xy) = \min[\mathfrak{I}_A(x), \mathfrak{I}_A(y)]$ and
- (iii) $\mathfrak{F}_B(xy) = \max[\mathfrak{F}_A(x), \mathfrak{F}_A(y)]$, for every $xy \in E$.

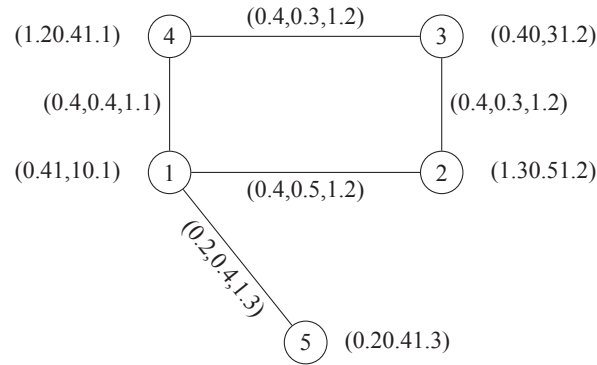


FIGURE 5. \mathcal{S} strongNOverG

Definition .8 Let G be any NOverG. Then the

- (i) **order** of G is given by

$$\mathcal{O} = \langle \mathfrak{T}_{\mathcal{O}}, \mathfrak{I}_{\mathcal{O}}, \mathfrak{F}_{\mathcal{O}} \rangle$$

where $\mathfrak{T}_{\mathcal{O}} = \sum_{u \in V} \mathfrak{T}_A(u)$, $\mathfrak{I}_{\mathcal{O}} = \sum_{u \in V} \mathfrak{I}_A(u)$, and $\mathfrak{F}_{\mathcal{O}} = \sum_{u \in V} \mathfrak{F}_B(u)$.

- (ii) **size** of G is defined by

$$\mathcal{S} = \langle \mathfrak{T}_{\mathcal{S}}, \mathfrak{I}_{\mathcal{S}}, \mathfrak{F}_{\mathcal{S}} \rangle$$

where $\mathfrak{T}_{\mathcal{S}} = \sum_{xy \in E} \mathfrak{T}_B(xy)$, $\mathfrak{I}_{\mathcal{S}} = \sum_{xy \in E} \mathfrak{I}_B(xy)$, and $\mathfrak{F}_{\mathcal{S}} = \sum_{xy \in E} \mathfrak{F}_B(xy)$.

Definition .9 For NOverG,

- (i) a vertex $u \in V$ is **adjacent** to the other vertex $v \in V$ if $\mathfrak{T}_B(uv) = \mathfrak{T}_A(u) \wedge \mathfrak{T}_A(v)$, $\mathfrak{I}_B(uv) = \mathfrak{I}_A(u) \wedge \mathfrak{I}_A(v)$ and $\mathfrak{F}_B(uv) = \mathfrak{F}_A(u) \vee \mathfrak{F}_A(v)$.
- (ii) an edge $uv \in E$ of G is an **effective edge** if $\mathfrak{T}_B(uv) = \mathfrak{T}_A(u) \wedge \mathfrak{T}_A(v)$, $\mathfrak{I}_B(uv) = \mathfrak{I}_A(u) \wedge \mathfrak{I}_A(v)$ and $\mathfrak{F}_B(uv) = \mathfrak{F}_A(u) \vee \mathfrak{F}_A(v)$.

Example .1 From the above FIGURE 5,

- (i) $\mathcal{O} = \langle 3.5, 2.7, 4.9 \rangle$ and $\mathcal{S} = \langle 1.8, 1.9, 7.2 \rangle$.
- (ii) Every edge in E is an effective.
- (iii) Every pair of vertices in V are adjacent.

Proposition .1 Let G be any $NOverG$.

- (i) If every edge is effective then G is $\mathfrak{S}trongNOverG$.
- (ii) For every pair of nodes are adjacent, G is $\mathfrak{S}trongNOverG$.

Definition .10 Let G be any $NOverG$. Then

- (i) an open neighbourhood of $x \in V$ is defined by $\mathcal{N}(x) = \{y \in V / xy \in E \text{ is an effective edge}\}$.
- (ii) $\mathcal{N}[x] = \mathcal{N}(x) \cup \{x\}$.

Definition .11 Let G be any $NOverG$. Then for $x_i \in V$

- (i) **$NOver$ effective degree of a vertex x_i is**
 $\mathcal{E}\mathcal{D}(x_i) = \langle \mathfrak{T}_{\mathcal{E}\mathcal{D}}(x_i), \mathfrak{I}_{\mathcal{E}\mathcal{D}}(x_i), \mathfrak{F}_{\mathcal{E}\mathcal{D}}(x_i) \rangle$ where $\mathfrak{T}_{\mathcal{E}\mathcal{D}}(x_i) = \sum_{y \in N(x_i)} \mathfrak{T}_B(x_i y)$,
 $\mathfrak{I}_{\mathcal{E}\mathcal{D}}(x_i) = \sum_{y \in N(x_i)} \mathfrak{I}_B(x_i y)$ and $\mathfrak{F}_{\mathcal{E}\mathcal{D}}(x_i) = \sum_{y \in N(x_i)} \mathfrak{F}_B(x_i y)$.
- (ii) **$NOver$ min-effective degree of G is**
 $\mathcal{E}\mathcal{D}_\delta(G) = \langle \mathfrak{T}_{\mathcal{E}\mathcal{D}_\delta}(x_i), \mathfrak{I}_{\mathcal{E}\mathcal{D}_\delta}(x_i), \mathfrak{F}_{\mathcal{E}\mathcal{D}_\delta}(x_i) \rangle$ where $\mathfrak{T}_{\mathcal{E}\mathcal{D}_\delta}(x_i) = \min_{1 \leq i \leq n} \sum_{y \in N(x_i)} \mathfrak{T}_B(x_i y)$,
 $\mathfrak{I}_{\mathcal{E}\mathcal{D}_\delta}(x_i) = \min_{1 \leq i \leq n} \sum_{y \in N(x_i)} \mathfrak{I}_B(x_i y)$ and $\mathfrak{F}_{\mathcal{E}\mathcal{D}_\delta}(x_i) = \min_{1 \leq i \leq n} \sum_{y \in N(x_i)} \mathfrak{F}_B(x_i y)$.
- (iii) **$NOver$ max-effective degree of G is**
 $\mathcal{E}\mathcal{D}_\Delta(G) = \langle \mathfrak{T}_{\mathcal{E}\mathcal{D}_\Delta}(x_i), \mathfrak{I}_{\mathcal{E}\mathcal{D}_\Delta}(x_i), \mathfrak{F}_{\mathcal{E}\mathcal{D}_\Delta}(x_i) \rangle$ where $\mathfrak{T}_{\mathcal{E}\mathcal{D}_\Delta}(x_i) = \max_{1 \leq i \leq n} \sum_{y \in N(x_i)} \mathfrak{T}_B(x_i y)$,
 $\mathfrak{I}_{\mathcal{E}\mathcal{D}_\Delta}(x_i) = \max_{1 \leq i \leq n} \sum_{y \in N(x_i)} \mathfrak{I}_B(x_i y)$ and $\mathfrak{F}_{\mathcal{E}\mathcal{D}_\Delta}(x_i) = \max_{1 \leq i \leq n} \sum_{y \in N(x_i)} \mathfrak{F}_B(x_i y)$.
- (iv) **$NOver$ cardinality of $M \subseteq V$ is**

$$\mathfrak{C}ard(M) = \sum_{u \in M} \mathfrak{T}_A(u) + \sum_{u \in M} \mathfrak{I}_A(u) + \sum_{u \in M} \mathfrak{F}_A(u).$$

Example .2 From the above FIGURE 5, $\mathcal{N}(x_i)$, $\mathcal{N}[x_i]$ and $\mathcal{E}\mathcal{D}(x_i)$, for each vertex $x_i \in V$ are given in the below table.

$x_i \in V$	$\mathcal{N}(x_i)$	$\mathcal{N}[x_i]$	$\mathcal{E}\mathcal{D}(x_i)$
1	$\{2, 4, 5\}$	$\{1, 2, 4, 5\}$	$\langle 1.0, 1.3, 3.6 \rangle$
2	$\{1, 3\}$	$\{1, 2, 3\}$	$\langle 0.8, 0.8, 2.4 \rangle$
3	$\{2, 4\}$	$\{2, 3, 4\}$	$\langle 0.8, 0.6, 2.4 \rangle$
4	$\{1, 3\}$	$\{1, 3, 4\}$	$\langle 0.8, 0.7, 2.3 \rangle$
5	$\{1\}$	$\{1, 5\}$	$\langle 0.2, 0.4, 1.3 \rangle$

Moreover, $\mathcal{E}\mathcal{D}_\delta(G) = \langle 0.2, 0.4, 1.3 \rangle$ and $\mathcal{E}\mathcal{D}_\Delta(G) = \langle 1.0, 1.3, 3.6 \rangle$. Also for $M = \{1, 2, 4\} \subseteq V$, $\mathfrak{C}ard(M) = 2.9 + 2.0 + 2.4 = 7.3$.

Definition .12 A $\mathfrak{B}NOverG$ is a graph if V can be partitioned into two nonempty sets V_1 and V_2 such that $\mathfrak{T}_B(xy) = 0, \mathfrak{I}_B(xy) = 0, \mathfrak{F}_B(xy) = 0, \forall x, y \in V_1$ or V_2 .

Moreover, if $\mathfrak{T}_B(xy) = \mathfrak{T}_A(x) \wedge \mathfrak{T}_A(y), \mathfrak{I}_B(xy) = \mathfrak{I}_A(x) \wedge \mathfrak{I}_A(y)$ and $\mathfrak{F}_B(xy) = \mathfrak{F}_A(x) \vee \mathfrak{F}_A(y), \forall x \in V_1, y \in V_2$ and $xy \in E$. Then G is $\mathfrak{C}\mathfrak{B}NOverG$ and it is denoted by K_{A_1, A_2} where A_1 and A_2 are neutrosophic over vertex sets which are restrictions of neutrosophic over vertex set A to V_1 and V_2 respectively.

Example .3 The following graphs are examples $\mathfrak{B}NOverG$ and $\mathfrak{C}\mathfrak{B}NOverG$.

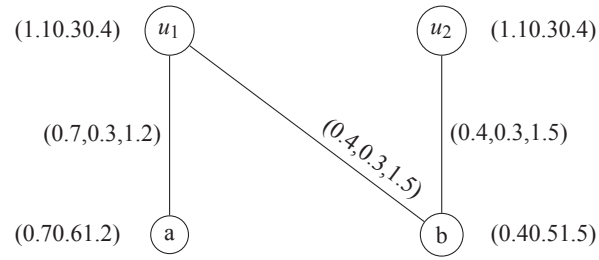


FIGURE 6. $\mathfrak{B}NOverG$

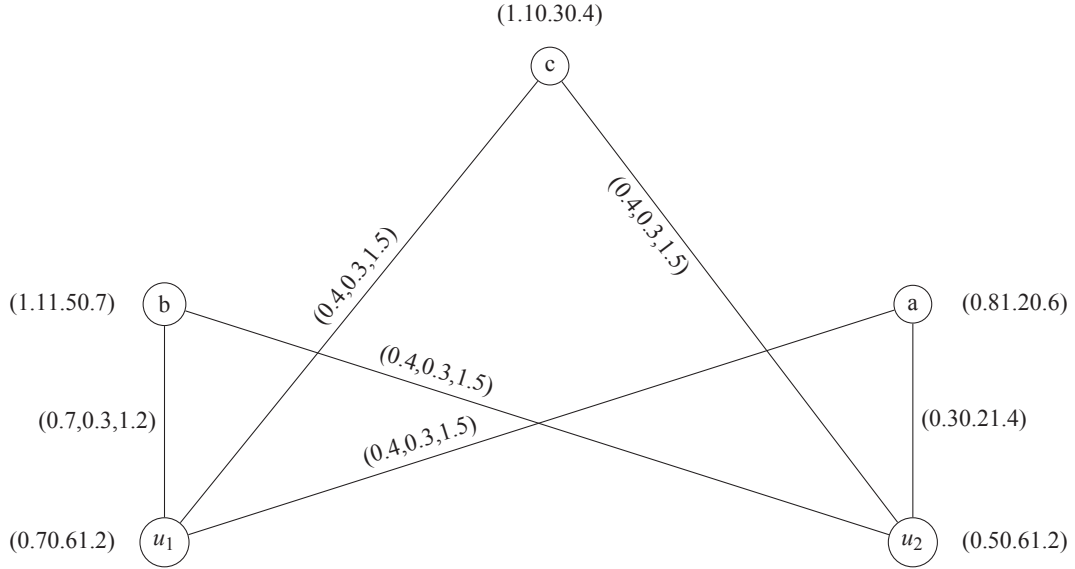


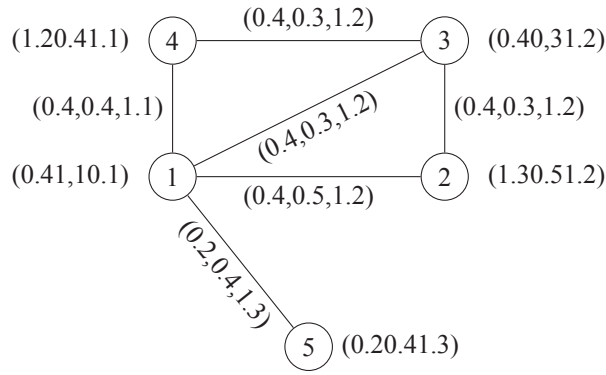
FIGURE 7. $\mathfrak{C}\mathfrak{B}NOverG$

DOMINATION IN $NOverG$

Definition .13 Let G be any a $NOverG$. Let $r, s \in V$. Then, we define r is dominates to s in G (in short., $r \underset{\cong}{D} s$) if $\mathfrak{T}_B(rs) = \mathfrak{T}_A(r) \wedge \mathfrak{T}_A(s)$, $\mathfrak{J}_B(rs) = \mathfrak{J}_A(r) \wedge \mathfrak{J}_A(s)$ and $\mathfrak{F}_B(rs) = \mathfrak{F}_A(r) \vee \mathfrak{F}_A(s)$.

A subset M of V is a dominat set in G if $\forall r \notin M$, there exist $\exists s \in M \ni r \underset{\cong}{D} s$. The min-cardinality of a dominat set in G is called Dominat number of G and it is denoted by D_γ^G .

Example .4 In the graph, $M_1 = \{1, 3\}$, $M_2 = \{1, 3, 4\}$, $M_3 = \{1\}$ are some dominat sets of G . Also $D_\gamma^G = 0.4 + 1.1 + 0.1 = 1.6$.



Remark .1 Let G be any a $NOverG$. Let $x, y \in V$. Then

- (i) If a relation is symmetric on V , G is dominat.
- (ii) For every open neighbourhood of each $x \in V$, then every element in $\mathcal{N}(x) \underset{\cong}{D}$ by x .
- (iii) If $\mathfrak{T}_B(rs) < \mathfrak{T}_A(r) \wedge \mathfrak{T}_A(s)$ (or) $\mathfrak{J}_B(rs) < \mathfrak{J}_A(r) \wedge \mathfrak{J}_A(s)$ (or) $\mathfrak{F}_B(rs) < \mathfrak{F}_A(r) \vee \mathfrak{F}_A(s)$, for $r, s \in V$, then V is only dominat set in G . Moreover, $D_\gamma^G = \sum[\mathfrak{T}_\emptyset + \mathfrak{J}_\emptyset + \mathfrak{F}_\emptyset]$.

Proposition .2 In any $NOverG$,

- (i) $D_\gamma^G + D_\gamma^{C(G)} \leq 2(\sum[\mathfrak{T}_\emptyset + \mathfrak{J}_\emptyset + \mathfrak{F}_\emptyset])$ where $D_\gamma^{C(G)}$ is the dominat number of $C(G)$.
- (ii) Moreover the equality holds iff V is only dominat set in G .

Proof: The inequality is trivial. Let $D_\gamma^G = \sum[\mathfrak{T}_\emptyset + \mathfrak{J}_\emptyset + \mathfrak{F}_\emptyset]$. This implies that $\mathfrak{T}_B(rs) < \mathfrak{T}_A(r) \wedge \mathfrak{T}_A(s)$, $\mathfrak{J}_B(rs) < \mathfrak{J}_A(r) \wedge \mathfrak{J}_A(s)$ and $\mathfrak{F}_B(rs) < \mathfrak{F}_A(r) \vee \mathfrak{F}_A(s)$, for $r, s \in V$.

Assume that $D_\gamma^{C(G)} = \sum[\mathfrak{T}_\emptyset + \mathfrak{J}_\emptyset + \mathfrak{F}_\emptyset]$. This implies that $\mathfrak{T}_A(r) \wedge \mathfrak{T}_A(s) - \mathfrak{T}_B(rs) < \mathfrak{T}_A(r) \wedge \mathfrak{T}_A(s)$, $\mathfrak{J}_A(r) \wedge \mathfrak{J}_A(s) - \mathfrak{J}_B(rs) < \mathfrak{J}_A(r) \wedge \mathfrak{J}_A(s)$ and $\mathfrak{F}_A(r) \vee \mathfrak{F}_A(s) - \mathfrak{F}_B(rs) < \mathfrak{F}_A(r) \vee \mathfrak{F}_A(s)$, for $r, s \in V$. Therefore $\mathfrak{T}_B(rs) > 0$, $\mathfrak{J}_B(rs) > 0$, $\mathfrak{F}_B(rs) > 0$.

Hence, $D_\gamma^G + D_\gamma^{C(G)} \leq 2(\sum[\mathfrak{T}_\emptyset + \mathfrak{J}_\emptyset + \mathfrak{F}_\emptyset])$ iff $0 < \mathfrak{T}_B(rs) < \mathfrak{T}_A(r) \wedge \mathfrak{T}_A(s)$, $0 < \mathfrak{J}_B(rs) < \mathfrak{J}_A(r) \wedge \mathfrak{J}_A(s)$ and $0 < \mathfrak{F}_B(rs) < \mathfrak{F}_A(r) \vee \mathfrak{F}_A(s)$, for $r, s \in V$ iff the V is only only dominat set in G .

Definition .14 A subset $M \subseteq V$ in $NOverG$ is a min-dominat set if $M - \{r\}$ not a dominating set in G , $\forall r \in M$.

Definition .15 A vertex x of a $NOverG$ G is an isolated vertex if $\mathcal{N}(x) = \emptyset$. Moreover, an isolated vertex does not $\underset{\cong}{D}$ to any other vertex in G .

Proposition .3 A dominat set M of $NOverG$ is a min-dominat set iff for each $r \in M$ one of the following two conditions holds:

- (i) $\mathcal{N}(r) \cap M = \emptyset$
- (ii) \exists a vertex $c \in V - M \ni \mathcal{N}(c) \cap M = \{c\}$.

Proof: Consider M is min-dominat set of G . $\forall m \in M$, $M - \{m\}$ is not a dominat set of G . Hence, $\exists r \in V - (M - \{m\})$ which is not a dominat set by any node in $M - \{m\}$.

If $r = m$, we get r is not an adjacent of any node in M .

If $r \neq m$, r is not dominated by $M - \{r\}$ but it is dominated by G . Then node r is adjacent only to $m \in M$. That is, $\mathcal{N}(x) \cap M = \{x\}$.

Conversely, Let M be a dominat set and $\forall m \in M$, one of the two conditions holds.

Assume that M is not min-dominat set. Then $\exists m \in M$, $M - \{m\}$ is dominat set. Hence m is adjacent to atleast one vertex in $M - \{m\}$. This implies that condition (i) does not hold.

If $M - \{m\}$ is dominat set, then every node in $V - M$ is an adjacent to atleast one node in $M - \{m\}$. This implies that condition (ii) does not hold. Hence M is a min-dominat set of G .

Proposition .4 If M is a min-dominat set $NOverG$, then $V - M$ is a dominat set of G .

Proof: Let $M - \{x\}$ be a min-dominat set of G , for all $x \in M$. If M is a min-dominat set, then $M - \{x\}$ is not a dominat set of G . clearly, every vertex $x_i \in V - M$ is an adjacent to the vertex in $M - \{x\}$. That is, $\mathfrak{T}_B(x_i y_j) = \mathfrak{T}_A(x_i) \wedge \mathfrak{T}_A(y_j)$, $\mathfrak{J}_B(x_i y_j) = \mathfrak{J}_A(x_i) \wedge \mathfrak{J}_A(y_j)$ and $\mathfrak{F}_B(x_i y_j) = \mathfrak{F}_A(x_i) \vee \mathfrak{F}_A(y_j)$, for $\{y_j\} \cup M$. Clearly every $y_j \in M$ is an adjacent to vertex in $x_i \in V - M$. This implies that $V - M$ is a dominat set of G .

Remark .2 For any $NOverG$ without isolated node,

$$(i) D_\gamma^G \leq (1/2)(\sum[T_\theta + I_\theta + F_\theta])$$

(ii) If G and $C(G)$ have no isolated vertices, then $D_\gamma^G + D_\gamma^{C(G)} \leq 2(\sum[\mathfrak{T}_\theta + \mathfrak{J}_\theta + \mathfrak{F}_\theta])$ where $D_\gamma^{C(G)}$ is the dominat number of $C(G)$.

Further equality holds iff $D_\gamma^G = D_\gamma^{C(G)}(1/2)(\sum[\mathfrak{T}_\theta + \mathfrak{J}_\theta + \mathfrak{F}_\theta])$

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