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METHODOLOGIES AND APPLICATION

Modeling and stability analysis methods of neutrosophic transfer functions

Jun Ye[1] · Wenhua Cui[1]

Abstract

Uncertainty is inherent property in actual control systems because parameters in actual control systems are no constants and changeable under some environments. Therefore, actual systems imply their indeterminate parameters, which can affect the control behavior and performance. Then, a neutrosophic number (NN) presented by Smarandache is very easy expressing determinate and/or indeterminate information because a NN $p = c + dI$ is composed of its determinate term c and its indeterminate term dI for $c, d \in R$ (R is all real numbers), where the symbol "I" denotes indeterminacy. Unfortunately, all uncertain modeling and analysis of practical control systems in existing literature do not provide any concept of NN models and analysis methods till now. Hence, this study firstly proposes a neutrosophic modeling method and defines a neutrosophic transfer function and a neutrosophic characteristic equation. Then, two stability analysis methods of neutrosophic linear systems are established based on the bounded range of all possible characteristic roots and the neutrosophic Routh stability criterion. Finally, the proposed methods are used for two practical examples on the RLC circuit and mass–spring–damper systems with NN parameters. The analysis results demonstrate the effectiveness and feasibility of the proposed methods.

Keywords Neutrosophic transfer function · Neutrosophic characteristic equation · Neutrosophic Routh stability criterion · Neutrosophic characteristic root

1 Introduction

Uncertainty is inherent property in actual control systems because parameters in actual control systems are no constants and changeable under some environments. Therefore, actual systems imply their indeterminate parameters, which can affect the control behavior and performance. In fact, in traditional control problems, the coefficients of the plant are always treated as determinate/nominal values in common control systems. Variations or indeterminacies of system parameters are produced due to many reasons, such as manufacturing tolerances, aging of main components, and environmental changes, which present an uncertain threat to the system. Therefore, such a system needs special modeling and analysis methods of a control system to grantee the robust stability and control performance to the control system with unconcern parameters. For instance, many research methods for modeling uncertain systems were proposed, and the robust stability for dynamic electrical and mechanical systems was assessed in interval linear time-invariant systems (Kolev 1988; Hussein 2005, 2010, 2011, 2015). Kharitonov's theorem (Kharitonov 1979) introduced a significant result in the field of robust stability of systems with parametric uncertainty/interval parameters, which indicated that the strict Hurwitz property of the entire family is equivalent to the strict Hurwitz property of four specifically constructed vertex polynomials. This theorem has been widely used in various control system applications (Czarkowski et al. 1995; Meressi et al. 1993; Precup and Preitl 2006; Hote et al. 2009; Elkaranshawy et al. 2009; Hote et al. 2010).

In indeterminate problems, a neutrosophic number (NN) presented by Smarandache (1998, 2013, 2014) is very easy

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expressing determinate and/or indeterminate information because a NN $p = c + dI$ is composed of its determinate term c and its indeterminate term dI for $c, d \in R$ (R is all real numbers), where the symbol "I" denotes indeterminacy. Hence, NNs have been successfully applied in multiple attribute group decision-making problems (Ye 2016a, 2017a), fault diagnosis problems (Kong et al. 2015; Ye 2016b), expression and analysis problems of rock joint roughness coefficient (Ye et al. 2016; 2017; Chen et al. 2017a, b), linear and nonlinear optimization problems (Jiang and Ye 2016, Ye 2018), and traffic flow problems (Ye 2017b). Unfortunately, all uncertain modeling and analysis of practical control systems in existing literature do not provide any concept of NN models and analysis methods till now. Hence, it is necessary to firstly propose a neutrosophic modeling method and to define a neutrosophic transfer function and a characteristic equation. Then, we establish stability analysis methods of neutrosophic linear time-invariant systems. To do so, the main objectives of this paper are: (1) to propose a neutrosophic modeling method based on actual physical systems with indeterminate parameters; (2) to define a neutrosophic transfer function and its neutrosophic characteristic equation; (3) to present two stability analysis methods of neutrosophic linear systems.

The main contribution of this study is that the modeling and stability analysis methods of neutrosophic transfer functions are proposed for the first time to provide the necessary preliminary basis for the analysis and design of neutrosophic control systems in incomplete, uncertain, and indeterminate environments.

The rest of this paper is structured as the following arrangement. Section 2 introduces some basic concepts of NNs used for this study. Section 3 proposes modeling methods of neutrosophic transfer functions, including the modeling of actual physical systems with indeterminate parameters and the definition of the neutrosophic transfer function and characteristic equation. Section 4 presents two stability analysis methods of neutrosophic systems based on the bounded range of all possible characteristic roots and the neutrosophic Routh stability criterion and uses them for two practical examples on the RLC circuit and mass–spring–damper systems with NN parameters. In Sect. 5, some conclusions and future research are given lastly.

2 Some basic concepts of NNs

In indeterminate situations, the concept of NN was originally presented by Smarandache (1998, 2013, 2014). It is defined as $p = c + dI$ for $c, d \in R$ and $I \in [I^L, I^U]$. It is obvious that NN is composed of its determinate term c and its indeterminate part dI. Based on this expression form of NN, it easily expresses the determinate and/or indeterminate information in real world. For instance, some resistor R in a circuit may suffer from uncertainty and deviation from the nominal value $R = 100\ \Omega$ due to several conditions such as aging, temperature, manufacturing tolerance, or other disturbances. Then, the NN of R can be expressed as $p = 100 + I\ \Omega$, which indicates that its determinate term (nominal value) is 100 Ω and its indeterminate term is I. In real situations, some possible interval range of indeterminacy $I \in [I^L, I^U]$ is usually specified to satisfy some actual requirement. When the indeterminacy I belongs to the specified interval [− 10, 10], it is equivalent to $p = [90, 110]\ \Omega$, i.e., p is within the interval [90, 110] Ω. When the indeterminacy I belongs to the specified interval [− 5, 5], then there is $p = [95, 105]\ \Omega$.

Obviously, a NN $p = c + dI$ may be also expressed as a possible interval number $p = [c + dI^L, c + dI^U]$ for $p \in Z$ (Z is all NNs) and $I \in [I^L, I^U]$. Thus, the NN p implies a changeable interval number corresponding to different indeterminate ranges of $I \in [I^L, I^U]$. Especially, the NN p is reduced to the determinate part $p = c$ when $dI = 0$ for the best case, while the NN p is reduced to the indeterminate part $p = dI$ if $c = 0$ for the worst case; then, p is reduced to a real number if $I^L = I^U$. It is obvious that NN is more flexible and suitable for the expression of determinate and/or indeterminate information, which indicates the advantage of expression and analysis convenience and flexibility in indeterminate system.

For two NNs $p_1 = c_1 + d_1I$ and $p_2 = c_2 + d_2I$ for $c_1, d_1, c_2, d_2 \in R$, $p_1, p_2 \in Z$, and $I \in [I^L, I^U]$, they have the following operational laws (Ye 2018):

$$p_1 + p_2 = (c_1 + d_1I) + (c_2 + d_2I) = c_1 + c_2 + (d_1 + d_2)I, \tag{1}$$

$$p_1 - p_2 = (c_1 + d_1I) - (c_2 + d_2I) = c_1 - c_2 + (d_1 - d_2)I, \tag{2}$$

$$p_1 \times p_2 = (c_1 + d_1I) \times (c_2 + d_2I) = c_1c_2 + (c_1d_2 + c_2d_1)I + d_1d_2I^2, \tag{3}$$

$$\frac{p_1}{p_2} = \frac{c_1 + d_1I}{c_2 + d_2I}. \tag{4}$$

3 Modeling method of neutrosophic transfer functions

Indeterminate system analysis needs mathematical models. From the differential or integral–differential equations describing the behavior of an indeterminate system, process, or component, we can also establish the neutrosophic transfer functions based on the Laplace transformation and its properties (Dazzo and Houpis 1995). To establish neutrosophic transfer functions of indeterminate systems, the following two typical examples are presented to show the modeling method.

Example 1 A typical example on a series RLC circuit (Dazzo and Houpis 1995) is presented as a circuit system consisting of a resistor R, an inductor L, and a capacitor C, which is shown in Fig. 1. Then, some variation or indeterminacy is implied in all components R, L, and C of the circuit.

The output voltage u_o of the circuit indicated in Fig. 1 is excited by the input voltage u_i. Then, the parameters (R, L, and C) of the series RLC circuit are suffering from variations or indeterminacies from their nominal values due to several conditions such as aging, temperature, manufacturing tolerances, or other disturbances. According to the Kirchhoff's laws (Dazzo and Houpis 1995), the RLC circuit equations are given as follows:

$$\begin{cases} u_i = Ri + L\dfrac{\mathrm{d}i}{\mathrm{d}t} + u_o \\ u_o = \dfrac{1}{C}\displaystyle\int i\mathrm{d}t \end{cases}. \tag{5}$$

Then, the relationship between u_o and u_i can be obtained as the following second-order differential equation:

$$LC\frac{\mathrm{d}^2u_o}{\mathrm{d}t^2} + RC\frac{\mathrm{d}u_o}{\mathrm{d}t} + u_o = u_i. \tag{6}$$

Based on the Laplace transformation and its properties (Dazzo and Houpis 1995), the transformed equation with zero initial conditions is as follows:

$$LCs^2U_o(s) + RCsU_o(s) + U_o(s) = U_i(s). \tag{7}$$

Solving for the ratio of the transformed output to the transformed input yields the transfer function of the system:

$$G(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{LCs^2 + RCs + 1}. \tag{8}$$

Under the indeterminate environment, since R, L, and C imply some variations or indeterminacies, they are composed of their determinate terms (nominal values) and indeterminate terms (changeable values), and then LC and RC can be expressed as two neutrosophic numbers $p_2 = c_2 + d_2I$ and $p_1 = c_1 + d_1I$, respectively, for $I \in [I^L, I^U]$. Thus, the transfer function of the circuit with NNs can be represented as the following neutrosophic transfer function:

$$G(s,I) = \frac{U_o(s,I)}{U_i(s,I)} = \frac{1}{p_2s^2 + p_1s + 1}, \tag{9}$$

where $G(s, I)$ is a neutrosophic transfer function of the second-order system, which is expressed as a ratio of the output $U_o(s, I)$ to the input $U_i(s, I)$ for $I \in [I^L, I^U]$ in the s (Laplace) domain, and then p_1 and p_2 are the NN coefficients in the denominator polynomial $U_i(s, I)$.

Example 2 A typical mechanical movement system with mass–spring–damper (Dazzo and Houpis 1995) is considered in Fig. 2, whose parameters are suffering from variations or indeterminacies from the nominal values due to several conditions such as aging, temperature, manufacturing tolerances, or other disturbances.

The free body diagram for this system is illustrated in Fig. 3, where m is mass, k is spring constant, μ is damping constant, $f(t)$ is input force, and y_0 is an initial displacement in the y coordinate direction.

Applying Newton's second law (Dazzo and Houpis 1995), we can get the following equation:

$$f(t) - k[y(t) + y_0] - \mu\frac{\mathrm{d}y(t)}{dt} + mg = m\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}. \tag{10}$$

Since $mg = ky_0$, Eq. (10) can be simplified as the second-order differential equation of the system:

$$m\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2} + \mu\frac{\mathrm{d}y(t)}{\mathrm{d}t} + ky(t) = f(t). \tag{11}$$

By the Laplace transformation (Dazzo and Houpis 1995), the transformed equation with zero initial conditions is as follows:

$$ms^2Y(s) + \mu sY(s) + kY(s) = F(s). \tag{12}$$

Solving for the ratio of the transformed output to the transformed input yields the transfer function of the system:

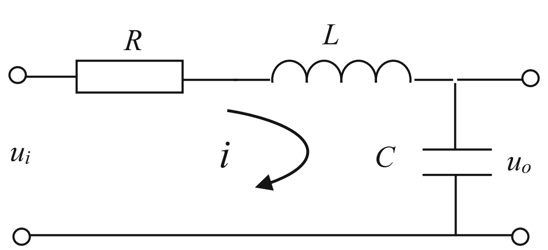

Fig. 1 Series RLC circuit

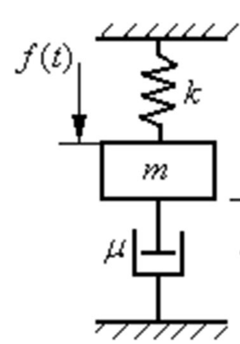

Fig. 2 Mechanical movement system with mass–spring–damper

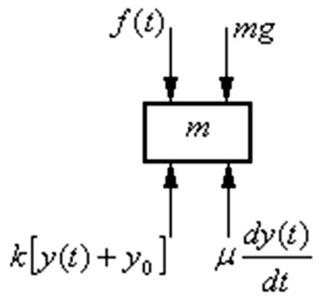

Fig. 3 The free body diagram for mechanical movement system

$$G(s) = \frac{Y(s)}{F(s)} = \frac{1}{ms^2 + \mu s + k}. \quad (13)$$

Under the indeterminate environment, since m, μ, and k imply some variations or indeterminacies, we can consider that m, μ, and k are composed of their determinate terms (nominal values) and indeterminate terms (changeable values), which can be expressed as three neutrosophic numbers $p_2 = c_2 + d_2I$, $p_1 = c_1 + d_1I$, and $p_0 = c_0 + d_0I$, respectively, for indeterminacy $I \in [I^L, I^U]$. Thus, the transfer function of the system with NNs can be represented as the following neutrosophic transfer function:

$$G(s, I) = \frac{Y(s, I)}{F(s, I)} = \frac{1}{p_2 s^2 + p_1 s + p_0}, \quad (14)$$

where $G(s, I)$ is a neutrosophic transfer function of the second-order system, which is expressed as a ratio of the output displacement $Y(s, I)$ to the input force $F(s, I)$ for $I \in [I^L, I^U]$ in the s (Laplace) domain, and then p_0, p_1, and p_2 are the NN coefficients in the denominator polynomial $F(s, I)$.

From the transfer functions of the above two physical systems, we see that different physical systems show the same mathematical models (the same second-order systems). For a linear time-invariant system, we can give the common definition of a neutrosophic transfer function.

Definition 1 For a linear time-invariant system with zero initial conditions, the neutrosophic transfer function itself can be expressed as a ratio of two neutrosophic polynomials with NNs in the complex Laplace variable *s:*

$$G(s, I) = \frac{N(s, I)}{D(s, I)} = \frac{q_m s^m + q_{m-1} s^{m-1} + \cdots q_0}{p_n s^n + p_{n-1} s^{n-1} + \cdots + p_0}, \quad (15)$$

where $q_i = a_i + b_iI$ ($i = 0, 1, \ldots, m$) for $q_j \in Z$ and $I \in [I^L, I^U]$ are the NN coefficients of the numerator polynomial $N(s, I)$ and $p_j = c_j + d_jI$ ($i = 0, 1, \ldots$, n) for $p_i \in Z$ and $I \in [I^L, I^U]$ are the NN coefficients of the denominator polynomial $D(s, I)$. For physical systems, $N(s, I)$ will be of lower order than $D(s, I)$ since nature integrates rather than differentiates.

4 Stability analysis of neutrosophic transfer functions

In Eq. (15), the denominator $D(s, I)$ of the neutrosophic transfer function is called the neutrosophic characteristic equation since it contains all the physical characteristics of the system. The neutrosophic characteristic equation is formed by setting $D(s, I)$ equal to zero. The roots of the neutrosophic characteristic equation (generally neutrosophic characteristic roots) determine the stability of the system and the general nature of the transient response to any input. The numerator neutrosophic polynomial $N(s, I)$ is a function of how the input enters the system. Consequently, $N(s, I)$ does not affect the absolute stability or the number or nature of the transient modes. It does, however, along with the specific input, determine the magnitude and sign of each transient mode and thus establishes the shape of the transient response as well as the steady-state value of the output.

By setting the denominator polynomial equal to zero, the neutrosophic characteristic equation is formed as follows:

$$D(s, I) = p_n s^n + p_{n-1} s^{n-1} + \cdots + p_1 s^1 + p_0 = 0 \quad \text{for } p_j \in Z, \quad (16)$$

and can be also written as the following factored form:

$$D(s, I) = \prod_{j=1}^{n} (s + r_j) = 0 \quad \text{for } r_j \in Z. \quad (17)$$

Based on the stability criterion of conventional systems (Dazzo and Houpis 1995), we can give that a necessary and sufficient condition for the neutrosophic/indeterminate system to be stable is that the NN roots of the neutrosophic characteristic equation have negative real parts. This can ensure that the impulse response will decay exponentially with time.

For higher-order neutrosophic system, it may be difficult to directly solve NN roots of neutrosophic characteristic polynomials. Therefore, this research only proposes two stability analysis methods.

(1) Method 1: Stability criterion based on the bounded range of all possible characteristic roots

Based on the analysis methods of eigenvalues of interval/indeterminate systems introduced by Hussein (2010, 2015), a simple and efficient method can be provided to determine the bounded range of all possible characteristic roots corresponding to a neutrosophic characteristic polynomial for assessing the stability of such a neutrosophic system. By applying the convex hull concept (Hussein 2010, 2015), it can generate a plot (convex hull/bounded edges) that constrains the roots of the entire family of the neutrosophic characteristic polynomials.

Let us consider the following neutrosophic polynomial:

$$D(s,I) = p_n s^n + p_{n-1}s^{n-1} + \cdots + p_1 s^1 + p_0 = 0 \quad for\ p_j \in Z, \tag{18}$$

where $p_j \in [c_j + d_j I^L, c_j + d_j I^U]$ (j = 0, 1, …, n) for $I \in [I^L, I^U]$ are NN coefficients in the neutrosophic characteristic polynomial $D(s, I)$. If the number of NN coefficients in the neutrosophic characteristic polynomial is k, there exists the family of 2^k characteristic polynomials (Hussein 2010, 2015) with all possible coefficient combinations associated with Eq. (18). Thus, the neutrosophic system is robustly stable when the bounded range of all possible roots corresponding to the 2^k characteristic polynomials is located on the left-half s plane (complex plane), in which all possible roots can be constructed as the convex hull (the bounded edges) (Hussein 2010, 2015). In other words, we can also give the following convex hull stability criterion based on the convex hull concept.

Convex hull stability criterion The neutrosophic system is robustly stable as long as the convex hull of all possible characteristic roots is located on the left-half s plane.

(2) Method 2: Neutrosophic Routh stability criterion

The classical Routh stability criterion (Dazzo and Houpis 1995) can be extended to the neutrosophic Routh stability criterion.

Based on the classical Routh array (Dazzo and Houpis 1995), the NN Routh array is defined below:

$$\begin{array}{llllll} s^n & p_n & p_{n-2} & p_{n-4} & p_{n-6} & \cdots \\ s^{n-1} & p_{n-1} & p_{n-3} & p_{n-5} & p_{n-7} & \cdots \\ s^{n-2} & a_1 & a_2 & a_3 & \cdots & \\ s^{n-3} & b_1 & b_2 & \cdots & & \\ \cdots & \cdots & \cdots & & & \\ s^1 & c_1 & & & & \\ s^0 & d_1 & & & & \end{array}. \tag{19}$$

In the NN Routh array (19), s^n, s^{n-1}, s^{n-2}, s^{n-3}, …, s^1, and s^0 are denoted as their rows, p_n, p_{n-1}, p_{n-2}, …, p_0 are the NN coefficients of the neutrosophic characteristic Eq. (18), and then the NNs a_1, a_2, a_3, … and b_1, b_2, … in the s^{n-2}, s^{n-3} rows are calculated by the following formulae:

$$a_1 = \frac{q_{n-1}q_{n-2} - q_n q_{n-3}}{q_{n-1}}, \tag{20}$$

$$a_2 = \frac{q_{n-1}q_{n-4} - q_n q_{n-5}}{q_{n-1}}, \tag{21}$$

$$a_3 = \frac{q_{n-1}q_{n-6} - q_n q_{n-7}}{q_{n-1}}, \tag{22}$$

$$b_1 = \frac{a_1 q_{n-3} - q_{n-1}a_2}{a_1}, \tag{23}$$

$$b_2 = \frac{a_1 q_{n-5} - q_{n-1}a_3}{a_1}, \tag{24}$$

$$b_3 = \frac{a_1 q_{n-7} - q_{n-1}a_4}{a_1}. \tag{25}$$

These calculations are continued until the a's and b's elements are all equal to zeros. Then, the rest of the rows are calculated in this way down to the s^0 row.

Neutrosophic Routh stability criterion A necessary but not sufficient condition for stable roots is that all the NN coefficients in Eq. (18) should be positive. Then, the sufficient condition for stable roots is that all the NN roots of a neutrosophic characteristic Eq. (18) have negative real parts if and only if the elements of the first NN column of the Routh array have the same sign; otherwise, the number of NN roots with positive real parts is equal to the number of changes of sign of the first NN column.

To validate the proposed two methods, we firstly consider the series RLC circuit in Fig. 1 as a typical practical example in the second-order system for convenient analysis and calculation.

Example 3 Consider the series RLC circuit system in Fig. 1, it is assumed that the tolerance in all components of the circuit is to be 10%, such that R = 500 + 500I = [450, 550] Ω, C = 0.01 + 0.01I = [0.009, 0.011] F, L = 0.2 + 0.2I = [0.18, 0.22] H, RC = [4.05, 6.05], and LC = [0.00162, 0.00242] for $I \in [-0.1, 0.1]$.

Based on the neutrosophic transfer function (9) for the series RLC circuit system, there is the following neutrosophic characteristic polynomial:

$$\begin{aligned} D(s,I) &= (0.2+0.2I)(0.01+0.01I)s^2 \\ &\quad + (500+500I)(0.01+0.01I)s + 1 \\ &= (0.00202+0.004I)s^2 + (5.05+10I)s + 1 \\ &= [0.00162, 0.00242]s^2 + [4.05, 6.05]s + 1 = 0. \end{aligned} \tag{26}$$

According to Method 1, the four (2^2 = 4 for two NN coefficients, i.e., k = 2) possible characteristic polynomials are generated from the neutrosophic characteristic polynomial (26) as follows:

$$D_1(s) = 0.00162s^2 + 4.05s + 1 = 0,$$
$$D_2(s) = 0.00162s^2 + 6.05s + 1 = 0,$$
$$D_3(s) = 0.00242s^2 + 4.05s + 1 = 0,$$
$$D_4(s) = 0.00242s^2 + 6.05s + 1 = 0.$$

Then, the roots of the corresponding four characteristic polynomials are calculated by the Matlab software as follows:

$$R = [-2499.7531,\ -0.2469,\ -3734.4026,\ -0.1653,\ -1673.3068,\ -0.247,\ -2499.8347,\ -0.1653].$$

The bounded range of all the roots of the characteristic polynomials is shown in Fig. 4, and this bounded range is a line due to the system having no imaginary poles. Then, all poles of the system neutrosophic transfer function are real and the bounds are located on the real axis of the left-half s plane, so the system is robustly stable.

Corresponding to the above four characteristic equations, the four transfer functions can be constructed as follows:

$$tf1(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{0.00162s^2 + 4.05s + 1},$$
$$tf2(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{0.00162s^2 + 6.05s + 1},$$
$$tf3(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{0.00242s^2 + 4.05s + 1},$$
$$tf4(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{0.00242s^2 + 6.05s + 1}.$$

Hence, their step responses for the RLC system with $I \in [-0.1, 0.1]$ are shown in Fig. 5. Obviously, the neutrosophic system contains indeterminate step responses.

Based on Method 2, it is obvious that the NN coefficients in the neutrosophic characteristic Eq. (26) with $I \in [-0.1, 0.1]$ are positive, which can satisfy the necessary condition. Then, based on the neutrosophic Routh stability criterion for the neutrosophic characteristic Eq. (26), by

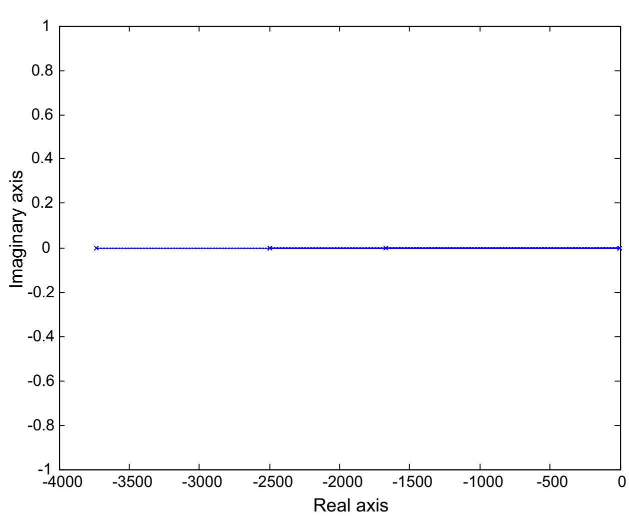

Fig. 4 The bounded range of all roots of characteristic polynomials for $I \in [-0.1, 0.1]$

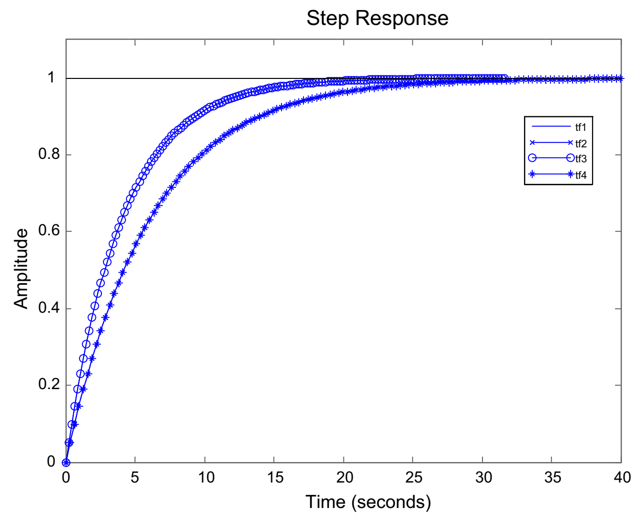

Fig. 5 Step responses of the RLC system with $I \in [-0.1, 0.1]$

using Eqs. (19)–(25), we can construct the following NN Routh array:

$$\begin{array}{lll} s^2 & 0.00202 + 0.004I = [0.00182, 0.00222] & 1 \\ s^1 & 5.05 + 10I = [4.55, 5.55] & 0\,. \\ s^0 & 1 & 0 \end{array}$$

It is obvious that the elements of the first NN column of the NN Routh array have the same sign corresponding to $I \in [-0.1, 0.1]$; hence, the neutrosophic system is robustly stable.

If the indeterminacy of the neutrosophic characteristic Eq. (26) is specified as $I \in [-0.05, 0.05]$, then it can be expressed as the following neutrosophic characteristic equation:

$$\begin{aligned} D(s,I) &= (0.00202{+}0.004I)s^2 + (5.05{+}10I)s + 1 \\ &= [0.00182, 0.00222]s^2 + [4.55, 5.55]s + 1 = 0. \end{aligned} \quad (27)$$

Similarly, the four ($2^2 = 4$) possible characteristic equations are generated from the neutrosophic characteristic polynomial (27) as follows:

$$D_1(s) = 0.00182s^2 + 4.55s + 1 = 0,$$
$$D_2(s) = 0.00182s^2 + 5.55s + 1 = 0,$$
$$D_3(s) = 0.00222s^2 + 4.55s + 1 = 0,$$
$$D_4(s) = 0.00222s^2 + 5.55s + 1 = 0.$$

Thus, the roots of the corresponding four characteristic equations are calculated by the Matlab software as follows:

$$R = [-2499.7802,\ -0.2198,\ -3049.2704,\ -0.1802,\ -2049.3297,\ -0.2198,\ -2499.8198,\ -0.1802].$$

The bounded range of all roots of the characteristic equations is shown in Fig. 6, and then, this bounded range

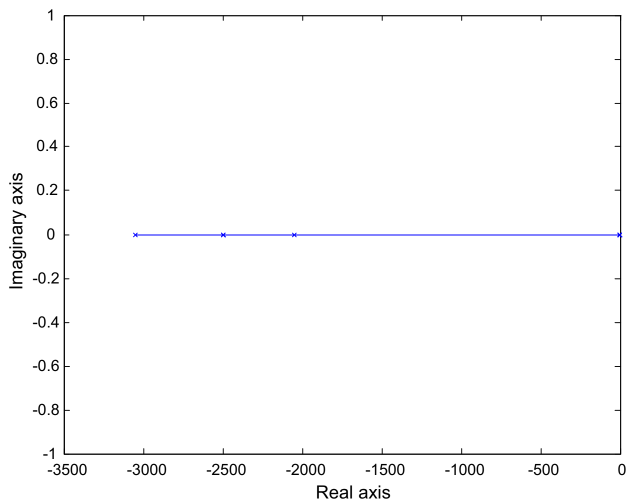

Fig. 6 The roots of all characteristic equations for $I \in [-0.05, 0.05]$

is also a line due to the system having no imaginary poles. Then, all poles of the system neutrosophic transfer function are real. Thus, the bounded range of all the roots is located on the real axis of the left-half s plane, so the system is robustly stable.

Based on the neutrosophic Routh stability criterion and the neutrosophic characteristic Eq. (27), it is obvious that the NN coefficients in the neutrosophic characteristic Eq. (27) with $I \in [-0.05, 0.05]$ are positive, which can also satisfy the necessary condition. Then, by using Eqs. (19)–(25), we can construct the following NN Routh array:

$$\begin{array}{lcc} s^2 & 0.00202 + 0.004I = [0.00182, 0.00222] & 1 \\ s^1 & 5.05 + 10I = [4.55, 5.55] & 0 \\ s^0 & 1 & 0 \end{array}.$$

Obviously, the elements of the first NN column of the NN Routh array have the same sign for $I \in [-0.05, 0.05]$; hence, the neutrosophic system is also robustly stable.

Corresponding to the above four characteristic equations, the four transfer functions can be constructed as follows:

$$tf1(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{0.00182s^2 + 4.55s + 1},$$
$$tf2(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{0.00182s^2 + 5.55s + 1},$$
$$tf3(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{0.00222s^2 + 4.55s + 1},$$
$$tf4(s) = \frac{U_o(s)}{U_i(s)} = \frac{1}{0.00222s^2 + 5.55s + 1}.$$

Hence, their step responses for the mass–spring–damper system with $I \in [-0.05, 0.05]$ are shown in Fig. 7. Generally, the neutrosophic system implies the indeterminate responses, and then, the indeterminate degree of the step responses decreases with narrowing the indeterminate range of I.

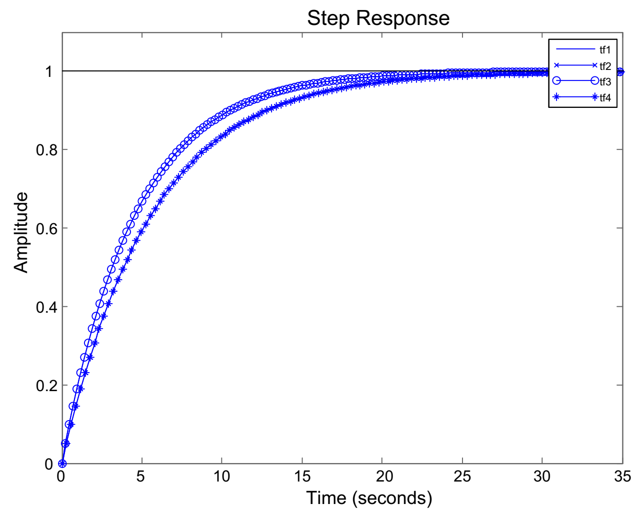

Fig. 7 Step responses the mass–spring–damper system with $I \in [-0.05, 0.05]$

Especially, when the indeterminacy of the neutrosophic characteristic Eq. (27) is specified as $I = 0$, it is reduced to the following determinate/nominal characteristic equation:

$$D(s) = 0.002s^2 + 5s + 1 = 0.$$

Thus, its characteristic roots are -2499.8 and -0.2. It is obvious that the nominal system is stable. Then, the step response of the nominal system is shown in Fig. 8.

Furthermore, we consider the mass–spring–damper system in Fig. 2 as another typical practical example in the second-order system for convenient analysis and calculation.

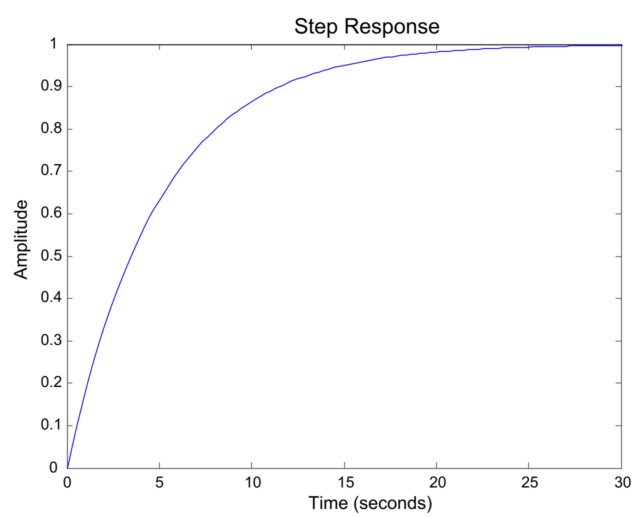

Fig. 8 Step response of the nominal RLC system

Table 1 Physical parameters for the mass–spring–damper system with $I \in [I^L, I^U]$

Parameter	Indeterminate value (NN) of parameters
Mass (m)	$p_2 = 1.0 + I$ kg
Damping coefficient (μ)	$p_1 = 0.2 + 0.2I$ Ns/m
Spring stiffness (k)	$p_0 = 1.0 + I$ Nm
Input force (f)	$f = 1.0$ N

Example 4 Consider that a mass–spring–damper in Fig. 2 as a mechanical movement system, where the parameters suffer from uncertainty and deviations from the nominal values due to several conditions such as aging, temperature, manufacturing tolerances, or other disturbances. Assume that the physical system contains some variation/indeterminacy $I \in [I^L, I^U]$ from the nominal values in mass, spring, and damper parameters, which are given in Table 1.

According to Method 1, from the neutrosophic transfer function (14), there is the following neutrosophic characteristic equation:

$$D(s,I) = (1+I)s^2 + (0.2+0.2I)s + (1+I) = [0.9, 1.1]s^2 + [0.18, 0.22]s + [0.9, 1.1] = 0. \quad (28)$$

Thus, the eight ($2^3 = 8$ for three NN coefficients, i.e., $k = 3$) characteristic equations are generated from the neutrosophic characteristic Eq. (28) as follows:

$$D_1(s) = 0.9s^2 + 0.18s + 0.9 = 0,$$
$$D_2(s) = 0.9s^2 + 0.18s + 1.1 = 0,$$
$$D_3(s) = 0.9s^2 + 0.22s + 0.9 = 0,$$
$$D_4(s) = 0.9s^2 + 0.22s + 1.1 = 0,$$
$$D_5(s) = 1.1s^2 + 0.18s + 0.9 = 0,$$
$$D_6(s) = 1.1s^2 + 0.18s + 1.1 = 0,$$
$$D_7(s) = 1.1s^2 + 0.22s + 0.9 = 0,$$
$$D_8(s) = 1.1s^2 + 0.22s + 1.1 = 0.$$

Then, all the roots of the corresponding eight characteristic polynomials are obtained as follows:

$$R = [-0.1000 - 0.9950i,\ -0.1000 + 0.9950i,\ -0.1000 - 1.1010i,\ -0.1000 + 1.1010i,\ -0.1222 - 0.9925i,\ -0.1222 + 0.9925i,\ -0.1222 - 1.0988i,\ -0.1222 + 1.0988i,\ -0.1000 - 0.9950i,\ -0.1000 + 0.9950i,\ -0.1000 - 1.1010i,\ -0.1000 + 1.1010i,\ -0.1222 - 0.9925i,\ -0.1222 + 0.9925i,\ -0.1222 - 1.0988i,\ -0.1222 + 1.0988i].$$

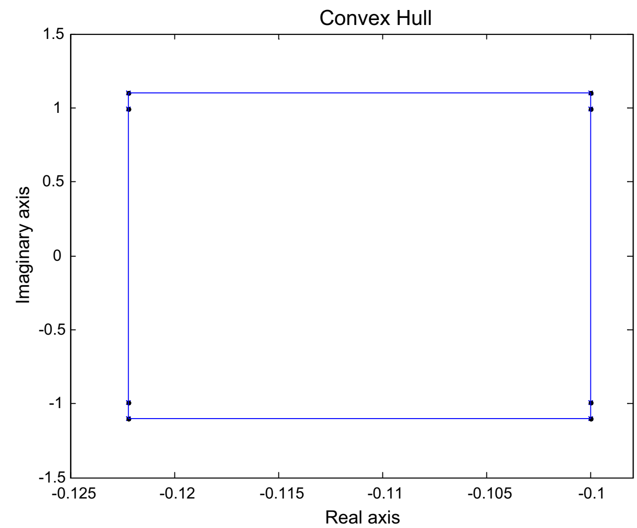

Fig. 9 Convex hull including the roots of all characteristic equations for $I \in [-0.1, 0.1]$

The symmetric bounds of the roots of the characteristic polynomials are constructed as the convex hull in Fig. 9, and then, the convex hull is presented to indicate all the possible roots that are located inside the convex hull. Therefore, the system is robustly stable since the convex hull is located on the left-half plane of complex plane.

Based on Method 2 and the neutrosophic characteristic Eq. (28), it is obvious that the NN coefficients in the neutrosophic characteristic Eq. (28) with $I \in [-0.1, 0.1]$ are positive, which can satisfy the necessary condition. Then, by using Eqs. (19)–(25), we can construct the following NN Routh array:

$$\begin{array}{lcc} s^2 & 1+I = [0.9, 1.1] & 1 \\ s^1 & 0.2+0.2I = [0.18, 0.22] & 0 \\ s^0 & 1+I = [0.9, 1.1] & 0 \end{array}.$$

Obviously, the elements of the first NN column of the NN Routh array have the same sign for $I \in [-0.1, 0.1]$; hence, the neutrosophic system is robustly stable.

Corresponding to the eight characteristic equations, the eight transfer functions can be constructed as follows:

$$tf1(s) = \frac{Y(s)}{F(s)} = \frac{1}{0.9s^2 + 0.18s + 0.9},$$
$$tf2(s) = \frac{Y(s)}{F(s)} = \frac{1}{0.9s^2 + 0.18s + 1.1},$$
$$tf3(s) = \frac{Y(s)}{F(s)} = \frac{1}{0.9s^2 + 0.22s + 0.9},$$
$$tf4(s) = \frac{Y(s)}{F(s)} = \frac{1}{0.9s^2 + 0.22s + 1.1},$$
$$tf5(s) = \frac{Y(s)}{F(s)} = \frac{1}{1.1s^2 + 0.18s + 0.9},$$
$$tf6(s) = \frac{Y(s)}{F(s)} = \frac{1}{1.1s^2 + 0.18s + 1.1},$$
$$tf7(s) = \frac{Y(s)}{F(s)} = \frac{1}{1.1s^2 + 0.22s + 0.9},$$
$$tf8(s) = \frac{Y(s)}{F(s)} = \frac{1}{1.1s^2 + 0.22s + 1.1}.$$

Thus, their step responses for the mass–spring–damper system with $I \in [-0.1, 0.1]$ are shown in Fig. 10. Obviously, the indeterminate system shows the indeterminate responses.

Especially, when the indeterminacy of the neutrosophic characteristic Eq. (28) is specified as $I = 0$, it is reduced to the following nominal characteristic equation:

$$D(s) = s^2 + 0.2s + 1 = 0.$$

Thus, its characteristic roots are $-0.1000 + 0.9950i$ and $-0.1000 - 0.9950i$. It is obvious that the nominal system is stable. Then, the step response of the nominal system is shown in Fig. 11.

These step responses of the neutrosophic and nominal systems above indicate the transient response information of these stable systems, which will provide the necessary preliminary basis for the analysis and design of the neutrosophic control system.

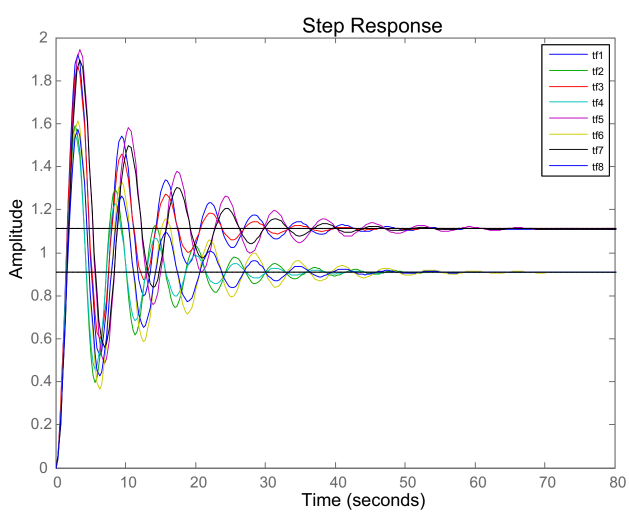

Fig. 10 Step responses of the mass–spring–damper system with $I \in [-0.1, 0.1]$

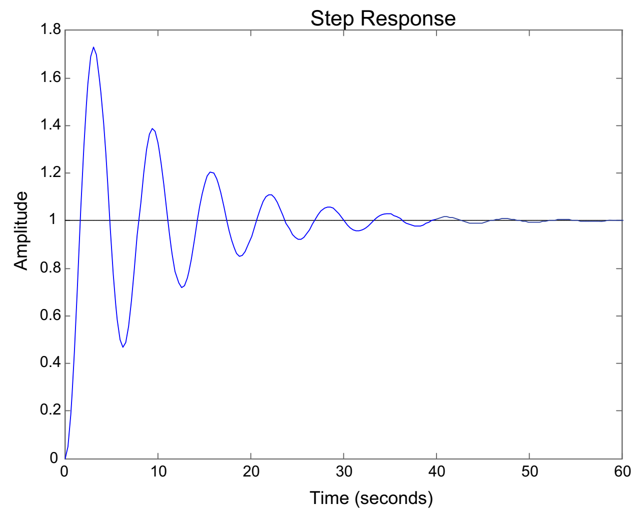

Fig. 11 Step response of the nominal mass–spring–damper system

5 Conclusion

This study firstly presented the concept of neutrosophic transfer functions by modeling of two typical physical systems with indeterminate parameters and then introduced two stability analysis methods: (1) the family of all possible characteristic equations is constructed responding to a neutrosophic characteristic equation, and the robust stability of the neutrosophic system is assessed by the convex hull area including the roots of all possible characteristic equations; (2) the neutrosophic Routh stability criterion is used to assess the robust stability of the neutrosophic system with *n*th-order neutrosophic characteristic equation. Finally, two typical practical examples on the RLC circuit and mass–spring–damper systems were presented for convenient analysis and calculation in the second-order systems to demonstrate the effectiveness and feasibility of the proposed methods. Furthermore, step responses of the neutrosophic systems and nominal systems were given to provide information about the transient response of stable systems.

However, this study proposed the preliminary modeling and stability analysis methods of the neutrosophic/indeterminate system for the first time. Then, regarding the analysis of higher-order/complicated neutrosophic systems, the proposed analysis method may result in the analysis and calculation complexity in this study. Therefore, as the future study, we need to propose new analysis methods in higher-order/complicated neutrosophic systems and to further investigate them to higher-order/complicated neutrosophic systems by more complicated actual examples.

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Compliance with ethical standards

Conflict of interest The authors declare that we have no conflict of interest regarding the publication of this paper.

Ethical approval This article does not contain any studies with human participants performed by any of the authors.

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