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Multi-Criteria Decision-Making Method Based on Simplified Neutrosophic Linguistic Information with Cloud Model

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Received: 25 April 2018; Accepted: 29 May 2018; Published: 1 June 2018

Abstract: This study introduces simplified neutrosophic linguistic numbers (SNLNs) to describe online consumer reviews in an appropriate manner. Considering the defects of studies on SNLNs in handling linguistic information, the cloud model is used to convert linguistic terms in SNLNs to three numerical characteristics. Then, a novel simplified neutrosophic cloud (SNC) concept is presented, and its operations and distance are defined. Next, a series of simplified neutrosophic cloud aggregation operators are investigated, including the simplified neutrosophic clouds Maclaurin symmetric mean (SNCMSM) operator, weighted SNCMSM operator, and generalized weighted SNCMSM operator. Subsequently, a multi-criteria decision-making (MCDM) model is constructed based on the proposed aggregation operators. Finally, a hotel selection problem is presented to verify the effectiveness and validity of our developed approach.

Keywords: simplified neutrosophic linguistic numbers; cloud model; Maclaurin symmetric mean; multi-criteria decision-making

1. Introduction

Nowadays, multi-criteria decision-making (MCDM) problems are attracting more and more attention. Lots of studies suggest that it is difficult to describe decision information completely because the information is usually inconsistent and indeterminate in real-life problems. To address this issue, Smarandache [1] put forward neutrosophic sets (NSs). Now, NSs have been applied to many fields and extended to various forms. Wang et al. [2] presented the concept of single-valued neutrosophic sets (SVNSs) and demonstrated its application, Ye [3] proposed several kinds of projection measures of SVNSs, and Ji et al. [4] proposed Bonferroni mean aggregation operators of SVNSs. Wang et al. [5] used interval numbers to extend SVNSs, and proposed the interval-valued neutrosophic set (IVNS). Ye [6] introduced trapezoidal neutrosophic sets (TrNSs), and proposed a series of trapezoidal neutrosophic aggregation operators. Liang et al. [7] introduced the preference relations into TrNSs. Peng et al. [8] combined the probability distribution with NSs to propose the probability multi-valued neutrosophic sets. Wu et al. [9] further extended this set to probability hesitant interval neutrosophic sets. All of the aforementioned sets are the descriptive tools of quantitative information.

Zhang et al. [10] proposed a method of using NSs to describe online reviews posted by consumers. For example, a consumer evaluates a hotel with the expressions: ‘the location is good’, ‘the service is neither good nor bad’, and ‘the room is in a mess’. Obviously, there is active, neutral, and passive information in this review. According to the NS theory, such review information can be characterized by employing truth, neutrality, and falsity degrees. This information presentation method has been proved to be feasible [11]. However, in practical online reviews, the consumer usually gives a comprehensive evaluation before posting the text reviews. NSs can describe the text reviews, but they cannot represent the comprehensive evaluation. To deal with this issue, many scholars have studied the combination of NSs and linguistic term sets [12,13]. The semantic of linguistic term set provides precedence on a qualitative level, and such precedence is more sensitive for decision-makers than a common ranking due to the expression of absolute benchmarks [14–16]. Based on the concepts of NSs and linguistic term sets, Ye [17] proposed interval neutrosophic linguistic sets (INLSs) and interval neutrosophic linguistic numbers (INLNs). Then, many interval neutrosophic linguistic MCDM approaches were developed [18,19]. Subsequently, Tian et al. [20] introduced the concepts of simplified neutrosophic linguistic sets (SNLSs) and simplified neutrosophic linguistic numbers (SNLNs). Wang et al. [21] proposed a series of simplified neutrosophic linguistic Maclaurin symmetric mean aggregation operators and developed a MCDM method. The existed studies on SNLNs simply used the linguistic functions to deal with linguistic variables in SNLNs. This strategy is simple, but it cannot effectively deal with qualitative information because it ignores the randomness of linguistic variables.

The cloud model is originally proposed by Li [22] in the light of probability theory and fuzzy set theory. It characterizes the randomness and fuzziness of a qualitative concept rely on three numerical characters and makes the conversion between qualitative concepts and quantitative values becomes effective. Since the introduction of the cloud model, many scholars have conducted lots of studies and applied it to various fields [23–25], such as hotel selection [26], data detection [27], and online recommendation algorithms [28]. Currently, the cloud model is considered as the best way to handle linguistic information and it is used to handle multiple qualitative decision-making problems [29–31], such as linguistic intuitionistic problems [32] and Z-numbers problems [33]. Considering the effectiveness of the cloud model in handling qualitative information, we utilize the cloud model to deal with linguistic terms in SNLNs. In this way, we propose a new concept by combining SNLNs and cloud model to solve real-life problems.

The aggregation operator is one of the most important tool of MCDM method [34–37]. Maclaurin symmetric mean (MSM) operator, defined by Maclaurin [38], possess the prominent advantage of summarizing the interrelations among input variables lying between the maximum value and minimum value. The MSM operator can not only take relationships among criteria into account, but it can also improve the flexibility of aggregation operators in application by adding parameters. Since the MSM operator was proposed, it has been expanded to various fuzzy sets [39–43]. For example, Liu and Zhang [44] proposed many MSM operators to deal with single-valued trapezoidal neutrosophic information, Ju et al. [45] proposed a series of intuitionistic linguistic MSM aggregation operators, and Yu et al. [46] proposed the hesitant fuzzy linguistic weighted MSM operator.

From the above analysis, the motivation of this paper is presented as follows:

1. The cloud model is a reliable tool for dealing with linguistic information, and it has been successfully applied to handle multifarious linguistic problems, such as probabilistic linguistic decision-making problems. The existing studies have already proved the effectiveness and feasibility of using the cloud model to process linguistic information. In view of this, this paper introduces the cloud model to process linguistic evaluation information involved in SNLNs.
2. As an efficient and applicable aggregation operator, MSM not only takes into account the correlation among criteria, but also adjusts the scope of the operator through the transformation of parameters. Therefore, this paper aims to accommodate the MSM operator to simplified neutrosophic linguistic information environments.

The remainder of this paper is organized as follows. Some basic definitions are introduced in Section 2. In Section 3, we propose a new concept of SNCs and the corresponding operations and distance. In Section 4, we propose some simplified neutrosophic cloud aggregation operators. In Section 5, we put forward a MCDM approach in line with the proposed operators. Then, in Section 6, we provide a practical example concerning hotel selection to verify the validity of the developed method. In Section 7, a conclusion is presented.

2. Preliminaries

This section briefly reviews some basic concepts, including linguistic term sets, linguistic scale function, NSs, SNSs, and cloud model, which will be employed in the subsequent analyses.

2.1. Linguistic Term Sets and Linguistic Scale Function

Definition 1 ([47]). *Let* $H = \{h_\tau | \tau = 1, 2, \cdots, 2t+1, t \in N^*\}$ *be a finite and totally ordered discrete term set, where* N^* *is a set of positive integers, and* h_τ *is interpreted as the representation of a linguistic variable. Then, the following properties should be satisfied:*

(1) The linguistic term set is ordered: $h_\tau < h_\upsilon$ *if and only if* $\tau < \upsilon$*, where* $(h_\tau, h_\upsilon \in H)$*;*

(2) If a negation operator exists, then $neg(h_\tau) = h_{(2t+1-\tau)}$ $(\tau, \upsilon = 1, 2, \cdots, 2t+1)$*.*

Definition 2 ([48]). *Let* $h_\tau \in H$ *be a linguistic term. If* $\theta_\tau \in [0,1]$ *is a numerical value, then the linguistic scale function* f *that conducts the mapping from* h_τ *to* θ_τ $(\tau = 1, 2, \cdots, 2t+1)$ *can be defined as*

$$f : s_\tau \to \theta_\tau \ (\tau = 1, 2, \cdots, 2t+1), \tag{1}$$

where $0 \le \theta_1 < \theta_2 < \cdots < \theta_{2t+1} \le 1$*.*

Based on the existed studies, three types of linguistic scale functions are described as

$$f_1(h_x) = \theta_x = \frac{x}{2t}, \ (x = 1, 2, \cdots, 2t+1), \ \theta_x \in [0,1]; \tag{2}$$

$$f_2(h_y) = \theta_y = \begin{cases} \dfrac{\alpha^t - \alpha^{t-y}}{2\alpha^t - 2}, & (y = 1, 2, \cdots, t+1), \\ \dfrac{\alpha^t + \alpha^{y-t} - 2}{2\alpha^t - 2}, & (y = t+2, t+3, \cdots, 2t+1); \end{cases} \tag{3}$$

$$f_3(h_z) = \theta_z = \begin{cases} \dfrac{t^\beta - (t-z)^\beta}{2t^\beta}, & (z = 1, \cdots, t+1), \\ \dfrac{t^\gamma + (z-t)^\gamma}{2t^\gamma}, & (z = t+2, \cdots, 2t+1). \end{cases} \tag{4}$$

2.2. SNSs and SNLSs

Definition 3 ([1]). *Let* X *be a space of points (objects), and x be a generic element in* X*. A NS* A *in* X *is characterized by a truth-membership function* $T_A(x)$*, a indeterminacy-membership function* $I_A(x)$*, and a falsity-membership function* $F_A(x)$*.* $T_A(x)$*,* $I_A(x)$*, and* $F_A(x)$ *are real standard or nonstandard subsets* $]0^-, 1^+[$*. That is,* $T_A(x) : x \to]0^-, 1^+[$*,* $I_A(x) : x \to]0^-, 1^+[$*, and* $F_A(x) : x \to]0^-, 1^+[$*. There is no restriction on the sum of* $T_A(x)$*,* $I_A(x)$*, and* $F_A(x)$*, so* $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$*.*

In fact, NSs are very difficult for application without specification. Given this, Ye [34] introduced SNSs by reducing the non-standard intervals of NSs into a kind of standard intervals.

Definition 4 ([17]). *Let* X *be a space of points with a generic element* x*. Then, an SNS* B *in* X *can be defined as* $B = \{(x, T_B(x), I_B(x), F_B(x)) | x \in X\}$*, where* $T_B(x) : X \to [0,1]$*,* $I_B(x) : X \to [0,1]$*, and*

$F_B(x): X \to [0,1]$. *In addition, the sum of* $T_B(x)$, $I_B(x)$, *and* $F_B(x)$ *satisfies* $0 \le T_B(x) + I_B(x) + F_B(x) \le 3$. *For simplicity,* B *can be denoted as* $B = \langle T_B(x), I_B(x), F_B(x) \rangle$, *which is a subclass of NSs.*

Definition 5 ([20]). *Let* X *be a space of points with a generic element x, and* $H = \{ h_\tau \mid \tau = 1, 2, \cdots, 2t+1, t \in N^* \}$ *be a linguistic term set. Then an SNLS* C *in* X *is defined as* $C = \{ \langle x, h_C(x), (T_C(x), I_C(x), F_C(x)) \rangle \mid x \in X \}$, *where* $h_C(x) \in H, T_C(x) \in [0,1], I_C(x) \in [0,1]$, $F_C(x) \in [0,1]$ *and* $0 \le T_C(x) + I_C(x) + F_C(x) \le 3$ *for any* $x \in X$. *In addition,* $T_C(x)$, $I_C(x)$, *and* $F_C(x)$ *represent the degree of truth-membership, indeterminacy-membership, and falsity-membership of the element* x *in* X *to the linguistic term* $h_C(x)$, *respectively. For simplicity, a SNLN is expressed as* $\langle h_C(x), (T_C(x), I_C(x), F_C(x)) \rangle$.

2.3. The Cloud Model

Definition 6 ([22]). *Let* U *be a universe of discourse and* T *be a qualitative concept in* U. $x \in U$ *is a random instantiation of the concept* T, *and* x *satisfies* $x \sim N\left(Ex, \left(En^*\right)^2\right)$, *where* $En^* \sim N\left(En, He^2\right)$, *and the degree of certainty that* x *belongs to the concept* T *is defined as*

$$\mu = e^{-\frac{(x-Ex)^2}{2\left(En^*\right)^2}},$$

then the distribution of x *in the universe* U *is called a normal cloud, and the cloud* C *is presented as* $C = (Ex, En, He)$.

Definition 7 ([33]). *Let* $M(Ex_1, En_1, He_1)$ *and* $N(Ex_2, En_2, He_2)$ *be two clouds, then the operations between them are defined as*

(1) $M + N = \left(Ex_1 + Ex_2, \sqrt{En_1^2 + En_2^2}, \sqrt{He_1^2 + He_2^2}\right)$;

(2) $M - N = \left(Ex_1 - Ex_2, \sqrt{En_1^2 + En_2^2}, \sqrt{He_1^2 + He_2^2}\right)$;

(3) $M \times N = \left(Ex_1 Ex_2, \sqrt{\left(En_1 Ex_2\right)^2 + \left(En_2 Ex_1\right)^2}, \sqrt{\left(He_1 Ex_2\right)^2 + \left(He_2 Ex_1\right)^2}\right)$;

(4) $\lambda M = \left(\lambda Ex_1, \sqrt{\lambda} En_1, \sqrt{\lambda} He_1\right)$; *and*

(5) $M^\lambda = \left(Ex_1^\lambda, \sqrt{\lambda} Ex_1^{\lambda-1} En_1, \sqrt{\lambda} Ex_1^{\lambda-1} He_1\right)$.

2.4. Transformation Approach of Clouds

Definition 8 ([33]). *Let* H_i *be a linguistic term in* $H = \{ H_i \mid i = 1, 2, \ldots, 2t+1 \}$, *and* f *be a linguistic scale function. Then, the procedures for converting linguistic variables to clouds are presented below.*

(1) Calculate θ_i *: Map* H_i *to* θ_i *employing Equation (2) or (3) or (4).*

(2) Calculate Ex_i *:* $Ex_i = X_{\min} + \theta_i \left(X_{\max} - X_{\min}\right)$.

(3) Calculate En_i *: Let* (x, y) *be a cloud droplet. Since* $x \sim N\left(Ex_i, En_i'^2\right)$, *we have* $3En_i' = \max\{X_{\max} - Ex_i, Ex_i - X_{\min}\}$ *in the light of* 3σ *principle of the normal distribution curve. Then,*

$$En_i^{'} = \begin{cases} \dfrac{(1-\theta_i)(X_{\max}-X_{\min})}{3} & 1 \le i \le t+1 \\ \dfrac{\theta_i (X_{\max}-X_{\min})}{3} & t+2 \le i \le 2t+1 \end{cases}$$. *Thus* $En_i = \dfrac{En_{i-1}^{'} + En_i^{'} + En_{i+1}^{'}}{3},\ (1 < i < 2t+1)$,

$En_i = \dfrac{En_i^{'} + En_{i+1}^{'}}{2},\ (i=1)$ *and* $En_i = \dfrac{En_{i-1}^{'} + En_i^{'}}{2},\ (i=2t+1)$ *can be obtained.*

(4) Calculate He_i *:* $He_i = \dfrac{\left(En^{'+} - En_i\right)}{3}$*, where* $En^{'+} = \max\left\{En_i^{'}\right\}$.

3. Simplified Neutrosophic Clouds and the Related Concepts

Based on SNLNs and the cloud transformation method, a novel concept of SNCs is proposed. Motivated by the existing studies, we provide the operations and comparison method for SNCs and investigate the distance measurement of SNCs.

3.1. SNCs and Their Operational Rules

Definition 9. *Let* X *be a space of points with a generic element* x*,* $H = \left\{h_\tau \middle| \tau = 1,2,\cdots,2t+1, t \in N^*\right\}$ *be a linguistic term set, and* $\langle h_C(x),(T_C(x), I_C(x), F_C(x))\rangle$ *be a SNLN. In accordance with the cloud conversion method described in Section 2.4, the linguistic term* $h_C(x) \in H$ *can be converted into the cloud* $\langle Ex, En, He\rangle$ *. Then, a simplified neutrosophic cloud (SNC) is defined as*

$$Y = \left(\langle Ex, En, He\rangle, \langle T, I, F\rangle\right)$$

Definition 10. *Let* $a = \left\langle (Ex_1, En_1, He_1), (T_1, I_1, F_1)\right\rangle$ *and* $b = \left\langle (Ex_2, En_2, He_2), (T_2, I_2, F_2)\right\rangle$ *be two SNCs, then the operations of SNC are defined as*

(1)
$$a \oplus b = \left(\left\langle Ex_1 + Ex_2, \sqrt{En_1^2 + En_2^2}, \sqrt{He_1^2 + He_2^2}\right\rangle, \left\langle \frac{T_1\left(Ex_1 + En_1^2 + He_1^2\right) + T_2\left(Ex_2 + En_2^2 + He_2^2\right)}{Ex_1 + Ex_2 + En_1^2 + He_1^2 + En_2^2 + He_2^2}, \frac{I_1\left(Ex_1 + En_1^2 + He_1^2\right) + I_2\left(Ex_2 + En_2^2 + He_2^2\right)}{Ex_1 + Ex_2 + En_1^2 + He_1^2 + En_2^2 + He_2^2}, \frac{F_1\left(Ex_1 + En_1^2 + He_1^2\right) + F_2\left(Ex_2 + En_2^2 + He_2^2\right)}{Ex_1 + Ex_2 + En_1^2 + He_1^2 + En_2^2 + He_2^2} \right\rangle\right);$$

(2) $a \otimes b = \left(\langle Ex_1Ex_2, En_1En_2, He_1He_2\rangle, \langle\langle T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2\rangle\right);$

(3) $\lambda a = \left(\left\langle \lambda Ex_1, \sqrt{\lambda} En_1, \sqrt{\lambda} He_1\right\rangle, \langle T_1, I_1, F_1\rangle\right);$ *and*

(4) $a^\lambda = \left(\left\langle Ex_1^\lambda, En_1^\lambda, He_1^\lambda\right\rangle, \left\langle T_1^\lambda, 1-(1-I_1)^\lambda, 1-(1-F_1)^\lambda\right\rangle\right).$

Theorem 1. *Let* $a = \left\langle (Ex_1, En_1, He_1), (T_1, I_1, F_1)\right\rangle$ *,* $b = \left\langle (Ex_2, En_2, He_2), (T_2, I_2, F_2)\right\rangle$ *and* $c = \left\langle (Ex_3, En_3, He_3), (T_3, I_3, F_3)\right\rangle$ *be three SNCs. Then, the following properties should be satisfied*

(1) $a + b = b + a$*;*
(2) $(a+b)+c = a+(b+c)$*;*
(3) $\lambda a + \lambda b = \lambda(a+b)$*;*
(4) $\lambda_1 a + \lambda_2 a = (\lambda_1 + \lambda_2)a$ *;*
(5) $a \times b = b \times a$ *;*
(6) $(a \times b) \times c = a \times (b \times c)$ *;*
(7) $a^{\lambda_1} \times a^{\lambda_2} = a^{\lambda_1 + \lambda_2}$ *;*

(8) $\left(a\times b\right)^{\lambda}=a^{\lambda}\times b^{\lambda}$.

3.2. Distance for SNCs

Definition 11. *Let* $a=\left\langle\left(Ex_1,En_1,He_1\right),\left(T_1,I_1,F_1\right)\right\rangle$ *and* $b=\left\langle\left(Ex_2,En_2,He_2\right),\left(T_2,I_2,F_2\right)\right\rangle$ *be two SNCs, then the generalized distance between a and b is defined as*

$$d\left(a,b\right)=\left|\left(1-\beta_1\right)Ex_1-\left(1-\beta_2\right)Ex_2\right|+\left(\frac{1}{3}\left(\left|\left(1-\beta_1\right)Ex_1T_1-\left(1-\beta_2\right)Ex_2T_2\right|^{\lambda}+\left|\left(1-\beta_1\right)Ex_1\left(1-I_1\right)-\left(1-\beta_2\right)Ex_2\left(1-I_2\right)\right|^{\lambda}+\left|\left(1-\beta_1\right)Ex_1\left(1-F_1\right)-\left(1-\beta_2\right)Ex_2\left(1-F_2\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}, \quad (5)$$

where $\beta_1=\frac{\sqrt{En_1^{\,2}+He_1^{\,2}}}{\sqrt{En_1^{\,2}+He_1^{\,2}}+\sqrt{En_2^{\,2}+He_2^{\,2}}}$ *and* $\beta_2=\frac{\sqrt{En_2^{\,2}+He_2^{\,2}}}{\sqrt{En_1^{\,2}+He_1^{\,2}}+\sqrt{En_2^{\,2}+He_2^{\,2}}}$. *When λ = 1 and 2, the generalized distance above becomes the Hamming distance and the Euclidean distance, respectively.*

Theorem 2. *Let* $a=\left\langle\left(Ex_1,En_1,He_1\right),\left(T_1,I_1,F_1\right)\right\rangle$, $b=\left\langle\left(Ex_2,En_2,He_2\right),\left(T_2,I_2,F_2\right)\right\rangle$, *and* $c=\left\langle\left(Ex_3,En_3,He_3\right),\left(T_3,I_3,F_3\right)\right\rangle$ *be three SNCs. Then, the distance given in Definition 11 satisfies the following properties:*

(1) $d\left(a,b\right)\geq 0$;
(2) $d\left(a,b\right)=d\left(b,a\right)$; *and*
(3) *If* $Ex_1\leq Ex_2\leq Ex_3$, $En_1\geq En_2\geq En_3$, $He_1\geq He_2\geq He_3$, $T_1\leq T_2\leq T_3$, $I_1\geq I_2\geq I_3$, , *and* $F_1\geq F_2\geq F_3$, *then* $d\left(a,b\right)\leq d\left(a,c\right)$, *and* $d\left(b,c\right)\leq d\left(a,c\right)$.

Proof. It is easy to prove that (1) and (2) in Theorem 2 are true. The proof of (3) in Theorem 2 is depicted in the following.

Let $\beta_{(a,b)1}=\frac{\sqrt{En_1^{\,2}+He_1^{\,2}}}{\sqrt{En_1^{\,2}+He_1^{\,2}}+\sqrt{En_2^{\,2}+He_2^{\,2}}}$, $\beta_{(a,b)2}=\frac{\sqrt{En_2^{\,2}+He_2^{\,2}}}{\sqrt{En_1^{\,2}+He_1^{\,2}}+\sqrt{En_2^{\,2}+He_2^{\,2}}}$, $\beta_{(a,c)1}=\frac{\sqrt{En_1^{\,2}+He_1^{\,2}}}{\sqrt{En_1^{\,2}+He_1^{\,2}}+\sqrt{En_3^{\,2}+He_3^{\,2}}}$, and $\beta_{(a,c)2}=\frac{\sqrt{En_3^{\,2}+He_3^{\,2}}}{\sqrt{En_1^{\,2}+He_1^{\,2}}+\sqrt{En_3^{\,2}+He_3^{\,2}}}$, then there are

$$\begin{aligned}d\left(a,c\right)=&\left|\left(1-\beta_{(a,c)1}\right)Ex_1-\left(1-\beta_{(a,c)2}\right)Ex_3\right|\\&+\left(\frac{1}{3}\left(\left|\left(1-\beta_{(a,c)1}\right)Ex_1T_1-\left(1-\beta_{(a,c)2}\right)Ex_3T_3\right|^{\lambda}\right.\right.\\&+\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-I_3\right)\right|^{\lambda}\\&\left.\left.+\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-F_3\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}},\end{aligned}$$

$$\begin{aligned}d\left(a,b\right)=&\left|\left(1-\beta_{(a,b)1}\right)Ex_1-\left(1-\beta_{(a,b)2}\right)Ex_2\right|\\&+\left(\frac{1}{3}\left(\left|\left(1-\beta_{(a,b)1}\right)Ex_1T_1-\left(1-\beta_{(a,b)2}\right)Ex_2T_2\right|^{\lambda}+\left|\left(1-\beta_{(a,b)1}\right)\right.\right.\right.\\&\left.+Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-I_2\right)\right|^{\lambda}\\&\left.\left.+\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-F_2\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}.\end{aligned}$$

Thus, we have

$$
\begin{aligned}
d(a,c)-d(a,b) &= \left(1-\beta_{(a,b)1}\right)Ex_1-\left(1-\beta_{(a,c)1}\right)Ex_1 \\
&+\left(1-\beta_{(a,c)2}\right)Ex_3-\left(1-\beta_{(a,b)2}\right)Ex_2 \\
&+\left(\frac{1}{3}\left(\left|\left(1-\beta_{(a,c)1}\right)Ex_1T_1-\left(1-\beta_{(a,c)2}\right)Ex_3T_3\right|^{\lambda}\right.\right. \\
&+\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-I_3\right)\right|^{\lambda} \\
&\left.\left.+\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-F_3\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}} \\
&-\left(\frac{1}{3}\left(\left|\left(1-\beta_{(a,b)1}\right)Ex_1T_1-\left(1-\beta_{(a,b)2}\right)Ex_2T_2\right|^{\lambda}\right.\right. \\
&+\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-I_2\right)\right|^{\lambda} \\
&\left.\left.+\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-F_2\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}.
\end{aligned}
$$

Let

$$
\begin{aligned}
p &= \left(1-\beta_{(a,b)1}\right)Ex_1-\left(1-\beta_{(a,c)1}\right)Ex_1+\left(1-\beta_{(a,c)2}\right)Ex_3-\left(1-\beta_{(a,b)2}\right)Ex_2 \\
&= \left(1-\frac{\sqrt{En_1^2+He_1^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_2^2+He_2^2}}\right)Ex_1-\left(1-\frac{\sqrt{En_1^2+He_1^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_3^2+He_3^2}}\right)Ex_1 \\
&+\left(1-\frac{\sqrt{En_3^2+He_3^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_3^2+He_3^2}}\right)Ex_3-\left(1-\frac{\sqrt{En_2^2+He_2^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_2^2+He_2^2}}\right)Ex_2.
\end{aligned}
$$

$$
\begin{aligned}
q &= \left(\frac{1}{3}\left(\left|\left(1-\beta_{(a,c)1}\right)Ex_1T_1-\left(1-\beta_{(a,c)2}\right)Ex_3T_3\right|^{\lambda}\right.\right. \\
&+\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-I_3\right)\right|^{\lambda} \\
&\left.\left.+\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-F_3\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}} \\
&-\left(\frac{1}{3}\left(\left|\left(1-\beta_{(a,b)1}\right)Ex_1T_1-\left(1-\beta_{(a,b)2}\right)Ex_2T_2\right|^{\lambda}\right.\right. \\
&+\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-I_2\right)\right|^{\lambda} \\
&\left.\left.+\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-F_2\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}},
\end{aligned}
$$

then $d(a,c)-d(a,b)=p+q$.

Simplifying the above equations, the following results can be obtained.

$$
\begin{aligned}
p &= \frac{\sqrt{En_2^2+He_2^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_2^2+He_2^2}}Ex_1-\frac{\sqrt{En_3^2+He_3^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_3^2+He_3^2}}Ex_1 \\
&+\frac{\sqrt{En_1^2+He_1^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_3^2+He_3^2}}Ex_3-\frac{\sqrt{En_1^2+He_1^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_2^2+He_2^2}}Ex_2.
\end{aligned}
$$

Since $Ex_1 \le Ex_2 \le Ex_3$, $En_1 \ge En_2 \ge En_3$, and $He_1 \ge He_2 \ge He_3$, we have

$$
\frac{\sqrt{En_2^2+He_2^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_2^2+He_2^2}}Ex_1-\frac{\sqrt{En_3^2+He_3^2}}{\sqrt{En_1^2+He_1^2}+\sqrt{En_3^2+He_3^2}}Ex_1 \ge 0,
$$

$$\frac{\sqrt{En_1^{\ 2}+He_1^{\ 2}}}{\sqrt{En_1^{\ 2}+He_1^{\ 2}}+\sqrt{En_3^{\ 2}+He_3^{\ 2}}}Ex_3-\frac{\sqrt{En_1^{\ 2}+He_1^{\ 2}}}{\sqrt{En_1^{\ 2}+He_1^{\ 2}}+\sqrt{En_2^{\ 2}+He_2^{\ 2}}}Ex_2\geq 0.$$

Thus, $p \geq 0$ is determined.

According to $p=\left|\left(1-\beta_{(a,c)1}\right)Ex_1-\left(1-\beta_{(a,c)2}\right)Ex_3\right|-\left|\left(1-\beta_{(a,b)1}\right)Ex_1-\left(1-\beta_{(a,b)2}\right)Ex_2\right|\geq 0$, the following inequalities can be deduced.

$$\left|\left(1-\beta_{(a,c)1}\right)Ex_1-\left(1-\beta_{(a,c)2}\right)Ex_3\right|\geq\left|\left(1-\beta_{(a,b)1}\right)Ex_1-\left(1-\beta_{(a,b)2}\right)Ex_2\right|,$$

$$\left|\left(1-\beta_{(a,c)1}\right)Ex_1-\left(1-\beta_{(a,c)2}\right)Ex_3\right|^{\lambda}\geq\left|\left(1-\beta_{(a,b)1}\right)Ex_1-\left(1-\beta_{(a,b)2}\right)Ex_2\right|^{\lambda}.$$

Since $T_1 \leq T_2 \leq T_3$, the following inequality is true.

$$\left|\left(1-\beta_{(a,c)1}\right)Ex_1T_1-\left(1-\beta_{(a,c)2}\right)Ex_3T_3\right|^{\lambda}\geq\left|\left(1-\beta_{(a,b)1}\right)Ex_1T_1-\left(1-\beta_{(a,b)2}\right)Ex_2T_2\right|^{\lambda}.$$

In a similar manner, we can also obtain

$$\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-I_3\right)\right|^{\lambda}\geq\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-I_2\right)\right|^{\lambda},$$

$$\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-F_3\right)\right|^{\lambda}\geq\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-F_2\right)\right|^{\lambda}.$$

Thus, there is

$$\begin{aligned}
q=&\left(\frac{1}{3}\left(\left|\left(1-\beta_{(a,c)1}\right)Ex_1T_1-\left(1-\beta_{(a,c)2}\right)Ex_3T_3\right|^{\lambda}\right.\right.\\
&+\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-I_3\right)\right|^{\lambda}\\
&\left.\left.+\left|\left(1-\beta_{(a,c)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,c)2}\right)Ex_3\left(1-F_3\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}\\
&-\left(\frac{1}{3}\left(\left|\left(1-\beta_{(a,b)1}\right)Ex_1T_1-\left(1-\beta_{(a,b)2}\right)Ex_2T_2\right|^{\lambda}\right.\right.\\
&+\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-I_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-I_2\right)\right|^{\lambda}\\
&\left.\left.+\left|\left(1-\beta_{(a,b)1}\right)Ex_1\left(1-F_1\right)-\left(1-\beta_{(a,b)2}\right)Ex_2\left(1-F_2\right)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}\\
&\geq 0.
\end{aligned}$$

Thus, $d\left(a,c\right)-d(a,b)\geq 0\Rightarrow d\left(a,c\right)\geq d(a,b)$. The inequality $d\left(a,c\right)\geq d(b,c)$ can be proved similarly. Hence, the proof of Theorem 2 is completed. □

Example 1. *Let* $a=\left\langle\left(0.5,0.2,0.1\right),\left(0.7,0.3,0.5\right)\right\rangle$, *and* $b=\left\langle\left(0.6,0.1,0.1\right),\left(0.8,0.2,0.4\right)\right\rangle$ *be two SNCs. Then, according to Definition 11, the Hamming distance* $d_{Hamming}\left(a,b\right)$ *and Euclidean distance* $d_{Euclidean}\left(a,b\right)$ *are calculated as*

$$d_{Hamming}\left(a,b\right)=0.4304\text{, and }d_{Euclidean}\left(a,b\right)=0.3224.$$

4. SNCs Aggregation Operators

Maclaurin [38] introduced the MSM aggregation operator firstly. In this section, the MSM operator is expanded to process SNC information, and the SNCMSM operator and the weighted SNCMSM operator are then proposed.

Definition 12 ([38]**).** *Let* $x_i\ (i=1,2,\cdots,n)$ *be the set of nonnegative real numbers. A MSM aggregation operator of dimension n is mapping* $MSM^{(m)}:\left(R^+\right)^n \to R^+$*, and it can be defined as*

$$MSM^{(m)}(x_1,x_2,\cdots,x_n)=\left(\frac{\sum\limits_{1\le i_1<\cdots<i_m\le n}\prod\limits_{j=1}^{m}x_{i_j}}{C_n^m}\right)^{\frac{1}{m}}, \tag{6}$$

where $(i_1,i_2,\cdots,i_m)$ *traverses all the m-tuple combination of* $(i=1,2,\cdots,n)$, $C_n^m=\dfrac{n!}{m!(n-m)!}$ *is the binomial coefficient. In the subsequent analysis, assume that* $i_1<i_2<,\dots,<i_m$*. In addition,* x_{i_j} *refers to the* i_j *th element in a particular arrangement.*

It is clear that $MSM^{(m)}$ *has the following properties:*

(1) Idempotency. If $x\ge 0$ *and* $x_i=x$ *for all* i*, then* $MSM^{(m)}(x,x,\dots,x)=x$*.*
(2) Monotonicity. If $x_i\le y_i$*, for all* i*,* $MSM^{(m)}(x_1,x_2,\dots,x_n)\le MSM^{(m)}(y_1,y_2,\dots,y_n)$*, where* x_i *and* y_i *are nonnegative real numbers.*
(3) Boundedness. $MIN\{x_1,x_2,\dots,x_n\}\le MSM^{(m)}(x_1,x_2,\dots,x_n)\le MAX\{x_1,x_2,\dots,x_n\}$.

4.1. SNCMSM Operator

In this subsection, the traditional $MSM^{(m)}$ operator is extended to accommodate the situations where the input variables are made up of SNCs. Then, the SNCMSM operator is developed.

Definition 13. *Let* $a_i=\langle(Ex_i,En_i,He_i),(T_i,I_i,F_i)\rangle\ (i=1,2,\dots,n)$ *be a collection of SNCs. Then, the SNCMSM operator can be defined as*

$$SNCMSM^{(m)}(a_1,a_2,\cdots,a_n)=\left(\frac{\underset{1\le i_1<\cdots<i_m\le n}{\oplus}\left(\overset{m}{\underset{j=1}{\otimes}}a_{i_j}\right)}{C_n^m}\right)^{\frac{1}{m}}, \tag{7}$$

where $m=1,2,\dots,n$ *and* $(i_1,i_2,\cdots,i_m)$ *traverses all the m-tuple combination of* $(i=1,2,\cdots,n)$*,* $C_n^m=\dfrac{n!}{m!(n-m)!}$ *is the binomial coefficient.*

In light of the operations of SNCs depicted in Definition 10, Theorem 3 can be acquired.

Theorem 3. *Let* $a_i=\langle(Ex_i,En_i,He_i),(T_i,I_i,F_i)\rangle\ (i=1,2,\dots,n)$ *be a collection of SNCs, the aggregated value acquired by the SNCMSM operator is also a SNC and can be expressed as*

$$SNCMSM^{(m)}(a_1,a_2,\cdots,a_n)$$
$$=\left(\left\langle\left(\frac{\sum_{k=1}^{C_n^m}\prod_{j=1}^{m}Ex_{i_j^{(k)}}}{C_n^m}\right)^{\frac{1}{m}},\left(\frac{\sqrt{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2}}{\sqrt{C_n^m}}\right)^{\frac{1}{m}},\left(\frac{\sqrt{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2}}{\sqrt{C_n^m}}\right)^{\frac{1}{m}}\right\rangle,\right.$$
$$\left\langle\left(\frac{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}T_{i_j^k}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}},\right.$$
$$1-\left(1-\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-I_{i_j^k}\right)\right)\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}}, \tag{8}$$
$$\left.\left.1-\left(1-\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-F_{i_j^k}\right)\right)\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}}\right\rangle\right).$$

Proof.

$$a_{i_j^{(k)}}=\left(\left\langle Ex_{i_j^{(k)}},En_{i_j^{(k)}},He_{i_j^{(k)}}\right\rangle,\left\langle T_{i_j^{(k)}},I_{i_j^{(k)}},F_{i_j^{(k)}}\right\rangle\right),\ \ (j=1,2,\ldots,m).$$

$$\Rightarrow\bigotimes_{j=1}^{m}a_{i_j^{(k)}}=\left(\left\langle\prod_{j=1}^{m}Ex_{i_j^{(k)}},\prod_{j=1}^{m}En_{i_j^{(k)}},\prod_{j=1}^{m}He_{i_j^{(k)}}\right\rangle,\right.$$
$$\left.\left\langle\prod_{j=1}^{m}T_{i_j^{(k)}},1-\prod_{j=1}^{m}\left(1-I_{i_j^{(k)}}\right),1-\prod_{j=1}^{m}\left(1-F_{i_j^{(k)}}\right)\right\rangle\right)$$

$$\Rightarrow\bigoplus_{1\le t_1<\cdots<t_m\le n}\left(\bigotimes_{j=1}^{m}a_{i_j}\right)=\left(\left\langle\sum_{k=1}^{C_n^m}\prod_{j=1}^{m}Ex_{i_j^{(k)}},\sqrt{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2},\sqrt{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2}\right\rangle,\right.$$
$$\left\langle\frac{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}T_{i_j^k}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)},\right.$$

$$\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-I_{i_j^k}\right)\right)\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)},$$

$$\left.\left.\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-F_{i_j^k}\right)\right)\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)}\right\rangle\right)$$

$$\Rightarrow\left(\frac{\underset{1\le i_1<\cdots<i_m\le n}{\oplus}\left(\overset{m}{\underset{j=1}{\otimes}}\left(a_{i_j}\right)\right)}{C_n^m}\right)^{\frac{1}{m}}$$

$$=\left(\left\langle\left(\frac{\sum_{k=1}^{C_n^m}\prod_{j=1}^{m}Ex_{i_j^{(k)}}}{C_n^m}\right)^{\frac{1}{m}},\left(\frac{\sqrt{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2}}{\sqrt{C_n^m}}\right)^{\frac{1}{m}},\left(\frac{\sqrt{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2}}{\sqrt{C_n^m}}\right)^{\frac{1}{m}}\right\rangle,\right.$$

$$\left\langle\left(\frac{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}T_{i_j^k}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}},\right.$$

$$1-\left(1-\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-I_{i_j^k}\right)\right)\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}},$$

$$\left.\left.1-\left(1-\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-F_{i_j^k}\right)\right)\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}}\right\rangle\right).$$

The proof of Theorem 3 is completed. □

Theorem 4. *(Idempotency) If* $a_i = a = \left(\langle Ex_a, En_a, He_a\rangle, \langle T_a, I_a, F_a\rangle\right)$ *for all* $i = 1,2,\ldots,n$ *, then* $SNCMSM^{(m)}(a,a,\cdots,a) = a = \left(\langle Ex_a, En_a, He_a\rangle, \langle T_a, I_a, F_a\rangle\right)$.

Proof. Since $a_i = a$, there are

$$SNCMSM^{(m)}(a,a,\cdots,a)$$

$$= \left(\left\langle \left(\frac{\sum_{k=1}^{C_n^m} \prod_{j=1}^{m} Ex_a}{C_n^m} \right)^{\frac{1}{m}}, \left(\frac{\sqrt{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} En_a \right)^2}}{\sqrt{C_n^m}} \right)^{\frac{1}{m}}, \left(\frac{\sqrt{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} He_a \right)^2}}{\sqrt{C_n^m}} \right)^{\frac{1}{m}} \right\rangle, \right.$$

$$\left\langle \left(\frac{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} T_a \left(\prod_{j=1}^{m} Ex_a + \left(\prod_{j=1}^{m} En_a \right)^2 + \left(\prod_{j=1}^{m} He_a \right)^2 \right) \right)}{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} Ex_a + \left(\prod_{j=1}^{m} En_a \right)^2 + \left(\prod_{j=1}^{m} He_a \right)^2 \right)} \right)^{\frac{1}{m}}, \right.$$

$$1 - \left(1 - \frac{\sum_{k=1}^{C_n^m} \left(\left(1 - \prod_{j=1}^{m} \left(1 - I_{i_j^k} \right) \right) \left(\prod_{j=1}^{m} Ex_a + \left(\prod_{j=1}^{m} En_a \right)^2 + \left(\prod_{j=1}^{m} He_a \right)^2 \right) \right)}{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} Ex_a + \left(\prod_{j=1}^{m} En_a \right)^2 + \left(\prod_{j=1}^{m} He_a \right)^2 \right)} \right)^{\frac{1}{m}},$$

$$\left. \left. 1 - \left(1 - \frac{\sum_{k=1}^{C_n^m} \left(\left(1 - \prod_{j=1}^{m} \left(1 - I_{i_j^k} \right) \right) \left(\prod_{j=1}^{m} Ex_a + \left(\prod_{j=1}^{m} En_a \right)^2 + \left(\prod_{j=1}^{m} He_a \right)^2 \right) \right)}{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} Ex_a + \left(\prod_{j=1}^{m} En_a \right)^2 + \left(\prod_{j=1}^{m} He_a \right)^2 \right)} \right)^{\frac{1}{m}} \right\rangle \right)$$

$$= \left(\langle Ex_a, En_a, He_a \rangle, \langle T_a, I_a, F_a \rangle \right) = a.$$

□

Theorem 5 (Commutativity**).** *Let* $(a'_1, a'_2, \cdots, a'_n)$ *be any permutation of* $(a_1, a_2, \cdots, a_n)$. *Then,* $SNCMSM^{(m)}(a'_1, a'_2, \cdots, a'_n) = SNCMSM^{(m)}(a_1, a_2, \cdots, a_n)$.

Theorem 5 can be proved easily in accordance with Definition 13 and Theorem 3.

Three special cases of the SNCMSM operator are discussed below by selecting different values for the parameter *m*.

(1) If *m* = 1, then the SNCMSM operator becomes the simplest arithmetic average aggregation operator as follows:

$$SNCMSM^{(1)}(a_1, a_2, \cdots, a_n) = \frac{\oplus_{i=1}^{n} a_i}{n}$$

$$= \left(\left\langle \sum_{i=1}^{n} Ex_i, \sqrt{\sum_{i=1}^{n} En_i^2}, \sqrt{\sum_{i=1}^{n} He_i^2} \right\rangle, \left\langle \frac{\sum_{i=1}^{n} T_i \left(Ex_i + En_i^2 + He_i^2 \right)}{\sum_{i=1}^{n} \left(Ex_i + En_i^2 + He_i^2 \right)}, \right. \right. \tag{9}$$

$$\left.\left.\frac{\sum_{i=1}^{n} I_i\left(Ex_i + En_i^2 + He_i^2\right)}{\sum_{i=1}^{n}\left(Ex_i + En_i^2 + He_i^2\right)}, \frac{\sum_{i=1}^{n} F_i\left(Ex_i + En_i^2 + He_i^2\right)}{\sum_{i=1}^{n}\left(Ex_i + En_i^2 + He_i^2\right)}\right\rangle\right).$$

(2) If $m = 2$, then the SNCMSM operator is degenerated to the following form:

$$SNCMSM^{(2)}\left(a_1, a_2, \cdots, a_n\right) = \left(\frac{\oplus_{i,j=1,i\neq j}^{n} a_i \otimes a_j}{n(n-1)}\right)^{\frac{1}{2}}$$

$$= \left(\left\langle\left(\frac{\sum_{\substack{i,j=1\\ i\neq j}}^{n} Ex_i Ex_j}{n(n-1)}\right)^{\frac{1}{2}}, \left(\sqrt{\frac{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left(En_i En_j\right)^2}{n(n-1)}}\right)^{\frac{1}{2}}, \left(\sqrt{\frac{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left(He_i He_j\right)^2}{n(n-1)}}\right)^{\frac{1}{2}}\right\rangle,\right.$$

$$\left\langle\left(\frac{\sum_{\substack{i,j=1\\ i\neq j}}^{n} T_i T_j\left(Ex_i Ex_j + En_i^2 En_j^2 + He_i^2 He_j^2\right)}{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left(Ex_i Ex_j + En_i^2 En_j^2 + He_i^2 He_j^2\right)}\right)^{\frac{1}{2}},\right. \tag{10}$$

$$1-\left(1-\frac{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left[1-\left(1-I_i\right)\left(1-I_j\right)\right]\left(Ex_i Ex_j + En_i^2 En_j^2 + He_i^2 He_j^2\right)}{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left(Ex_i Ex_j + En_i^2 En_j^2 + He_i^2 He_j^2\right)}\right)^{\frac{1}{2}},$$

$$\left.\left.1-\left(1-\frac{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left[1-\left(1-F_i\right)\left(1-F_j\right)\right]\left(Ex_i Ex_j + En_i^2 En_j^2 + He_i^2 He_j^2\right)}{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left(Ex_i Ex_j + En_i^2 En_j^2 + He_i^2 He_j^2\right)}\right)^{\frac{1}{2}}\right\rangle\right).$$

(3) If $m = n$, then the SNCMSM operator becomes the geometric average aggregation operator as follows:

$$SNCMSM^{(n)}\left(a_1, a_2, \cdots, a_n\right) = \left(\otimes_{i=1}^{n} a_i\right)^{\frac{1}{n}}$$

$$= \left(\left\langle\left(\prod_{i=1}^{n} Ex_i\right)^{\frac{1}{n}}, \left(\prod_{i=1}^{n} En_i\right)^{\frac{1}{n}}, \left(\prod_{i=1}^{n} He_i\right)^{\frac{1}{n}}\right\rangle,\right. \tag{11}$$

$$\left.\left\langle\left(\prod_{i=1}^{n} T_i\right)^{\frac{1}{n}}, \left(1-\prod_{i=1}^{n}\left(1-I_i\right)\right)^{\frac{1}{n}}, \left(1-\prod_{i=1}^{n}\left(1-F_i\right)\right)^{\frac{1}{n}}\right\rangle\right).$$

4.2. Weighted SNCMSM Operator

In this subsection, a weighted SNCMSM operator is investigated. Moreover, some desirable properties of this operator are analyzed.

Definition 14. *Let* $a_i = \langle (Ex_i, En_i, He_i), (T_i, I_i, F_i) \rangle (i = 1, 2, \ldots, n)$ *be a collection of SNCs, and* $w = (w_1, w_2, \ldots w_n)^T$ *be the weight vector, with* $w_i \in [0,1]$ *and* $\sum_{i=1}^{n} w_i = 1$. *Then, the weighted simplified neutrosophic clouds Maclaurin symmetric mean (WSNCMSM) operator is defined as*

$$WSNCMSM_w^{(m)}(a_1, a_2, \cdots, a_n) = \left(\frac{\underset{1 \le i_1 < \cdots < i_m \le n}{\oplus} \left(\underset{j=1}{\overset{m}{\otimes}} \left(nw_{i_j} \cdot a_{i_j} \right) \right)}{C_n^m} \right)^{\frac{1}{m}}, \tag{12}$$

where $m = 1, 2, \ldots, n$ *and* $(i_1, i_2, \cdots, i_m)$ *traverses all the m-tuple combination of* $(i = 1, 2, \cdots, n)$, $C_n^m = \frac{n!}{m!(n-m)!}$ *is the binomial coefficient.*

The specific expression of the WSNCMSM operator can be obtained in accordance with the operations provided in Definition 10.

Theorem 6. *Let* $a_i = \langle (Ex_i, En_i, He_i), (T_i, I_i, F_i) \rangle (i = 1, 2, \ldots, n)$ *be a collection of SNCs, and* $m = 1, 2, \ldots, n$*. Then, the aggregated value acquired by the WSNCMSM operator can be expressed as*

$$\begin{aligned}
& WSNCMSM_w^{(m)}(a_1, a_2, \cdots, a_n) \\
&= \left\langle \left\langle \left(\frac{\sum_{k=1}^{C_n^m} \prod_{j=1}^{m} nw_{i_j} Ex_{i_j^{(k)}}}{C_n^m} \right)^{\frac{1}{m}}, \left(\frac{\sqrt{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} En_{i_j^{(k)}} \right)^2}}{\sqrt{C_n^m}} \right)^{\frac{1}{m}}, \left(\frac{\sqrt{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} He_{i_j^{(k)}} \right)^2}}{\sqrt{C_n^m}} \right)^{\frac{1}{m}} \right\rangle, \right. \\
& \left\langle \left(\frac{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} T_{i_j^k} \left(\prod_{j=1}^{m} nw_{i_j} Ex_{i_j^{(k)}} + \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} En_{i_j^{(k)}} \right)^2 + \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} He_{i_j^{(k)}} \right)^2 \right) \right)}{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} nw_{i_j} Ex_{i_j^{(k)}} + \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} En_{i_j^{(k)}} \right)^2 + \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} He_{i_j^{(k)}} \right)^2 \right)} \right)^{\frac{1}{m}}, \right. \\
& 1 - \left(1 - \frac{\sum_{k=1}^{C_n^m} \left(\left(1 - \prod_{j=1}^{m} \left(1 - I_{i_j^k} \right) \right) \left(\prod_{j=1}^{m} nw_{i_j} Ex_{i_j^{(k)}} + \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} En_{i_j^{(k)}} \right)^2 + \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} He_{i_j^{(k)}} \right)^2 \right) \right)}{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} nw_{i_j} Ex_{i_j^{(k)}} + \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} En_{i_j^{(k)}} \right)^2 + \left(\prod_{j=1}^{m} \sqrt{nw_{i_j}} He_{i_j^{(k)}} \right)^2 \right)} \right)^{\frac{1}{m}},
\end{aligned} \tag{13}$$

$$1-\left(1-\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-F_{i_j^k}\right)\right)\left(\prod_{j=1}^{m} nw_{i_j} Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}\sqrt{nw_{i_j}}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}\sqrt{nw_{i_j}}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m} nw_{i_j} Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}\sqrt{nw_{i_j}}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}\sqrt{nw_{i_j}}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}}\Bigg\rangle\Bigg).$$

Theorem 6 can be proved similarly according to the proof procedures of Theorem 3.

Theorem 7. *(Reducibility) Let* $w=\left(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right)^T$*, then,* $WSNCMSM_w^{(m)}(a_1, a_2, \ldots, a_n) = SNCMSM^{(m)}(a_1, a_2, \ldots, a_n)$*.*

Proof.

When $w=\left(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right)^T$,

$$WSNCMSM_w^{(m)}(a_1,a_2,\cdots,a_n)$$

$$=\left(\left\langle\left(\frac{\sum_{k=1}^{C_n^m}\prod_{j=1}^{m} n\cdot\frac{1}{n}Ex_{i_j^{(k)}}}{C_n^m}\right)^{\frac{1}{m}},\left(\frac{\sqrt{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}En_{i_j^{(k)}}\right)^2}}{\sqrt{C_n^m}}\right)^{\frac{1}{m}},\left(\frac{\sqrt{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}He_{i_j^{(k)}}\right)^2}}{\sqrt{C_n^m}}\right)^{\frac{1}{m}}\right\rangle,\right.$$

$$\left\langle\left(\frac{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}T_{i_j^k}\left(\prod_{j=1}^{m} n\cdot\frac{1}{n}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m} n\cdot\frac{1}{n}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}},\right.$$

$$1-\left(1-\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-I_{i_j^k}\right)\right)\left(\prod_{j=1}^{m} n\cdot\frac{1}{n}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m} n\cdot\frac{1}{n}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}},$$

$$1-\left(1-\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-F_{i_j^k}\right)\right)\left(\prod_{j=1}^{m} n\cdot\frac{1}{n}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}He_{i_j^{(k)}}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m} n\cdot\frac{1}{n}Ex_{i_j^{(k)}}+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}En_{i_j^{(k)}}\right)^2+\left(\prod_{j=1}^{m}\sqrt{n\cdot\frac{1}{n}}He_{i_j^{(k)}}\right)^2\right)}\right)^{\frac{1}{m}}\Bigg\rangle\Bigg)$$

$$=SNCMSM^{(m)}(a_1,a_2,\cdots,a_n).$$

The proof of Theorem 7 is completed. □

Definition 15. *Let* $a_i = \langle (Ex_i, En_i, He_i), (T_i, I_i, F_i) \rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of SNCs, and* $w = (w_1, w_2, \ldots, w_n)^T$ *be the weight vector, which satisfies* $\sum_{i=1}^{n} w_i = 1$*, and* $w_i > 0$ $(i = 1, 2, \ldots, n)$*. Then the generalized weighted simplified neutrosophic clouds Maclaurin symmetric mean (GWSNCMSM) operator is defined as*

$$GWSNCMSM^{(m, p_1, p_2, \ldots, p_m)}(a_1, \ldots, a_n) = \left(\frac{\oplus_{1 \le i_1 < \cdots < i_m \le n} \left(\otimes_{j=1}^{m} \left(n w_{i_j} \otimes a_{i_j} \right)^{p_j} \right)}{C_n^m} \right)^{\frac{1}{p_1 + \cdots + p_m}}, \tag{14}$$

where $m = 1, 2, \ldots, n$.

The specific expression of the GWSNCMSM operator can be obtained in accordance with the operations provided in Definition 10.

Theorem 8. *Let* $a_i = \langle (Ex_i, En_i, He_i), (T_i, I_i, F_i) \rangle$ $(i = 1, 2, \ldots, n)$ *be a collection of SNCs, and* $m = 1, 2, \ldots, n$*. Then, the aggregated value acquired by the GWSNCMSM operator can be expressed as*

$$GWSNCMSM^{(m, p_1, p_2, \ldots, p_m)}(a_1, \ldots, a_n) = \left\langle \left(\left(\frac{\sum_{k=1}^{C_n^m} \prod_{j=1}^{m} \left(n w_{i_j} Ex_{i_j^{(k)}} \right)^{P_j}}{C_n^m} \right)^{\frac{1}{p_1 + \cdots + p_m}}, \right. \right.$$

$$\left. \left(\frac{\sqrt{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} En_{i_j^{(k)}} \right)^{P_j} \right)^2}}{\sqrt{C_n^m}} \right)^{\frac{1}{p_1 + \cdots + p_m}}, \left(\frac{\sqrt{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} He_{i_j^{(k)}} \right)^{P_j} \right)^2}}{\sqrt{C_n^m}} \right)^{\frac{1}{p_1 + \cdots + p_m}} \right\rangle,$$

$$\left\langle \left(\frac{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} \left(T_{i_j^k} \right)^{P_j} \left(\prod_{j=1}^{m} \left(n w_{i_j} Ex_{i_j^{(k)}} \right)^{P_j} + \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} En_{i_j^{(k)}} \right)^{P_j} \right)^2 + \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} He_{i_j^{(k)}} \right)^{P_j} \right)^2 \right) \right)}{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} \left(n w_{i_j} Ex_{i_j^{(k)}} \right)^{P_j} + \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} En_{i_j^{(k)}} \right)^{P_j} \right)^2 + \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} He_{i_j^{(k)}} \right)^{P_j} \right)^2 \right)} \right)^{\frac{1}{p_1 + \cdots + p_m}}, \right. \tag{15}$$

$$1 - \left(1 - \frac{\sum_{k=1}^{C_n^m} \left(\left(1 - \prod_{j=1}^{m} \left(1 - I_{i_j^k} \right)^{P_j} \right) \left(\prod_{j=1}^{m} \left(n w_{i_j} Ex_{i_j^{(k)}} \right)^{P_j} + \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} En_{i_j^{(k)}} \right)^{P_j} \right)^2 + \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} He_{i_j^{(k)}} \right)^{P_j} \right)^2 \right) \right)}{\sum_{k=1}^{C_n^m} \left(\prod_{j=1}^{m} \left(n w_{i_j} Ex_{i_j^{(k)}} \right)^{P_j} + \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} En_{i_j^{(k)}} \right)^{P_j} \right)^2 + \left(\prod_{j=1}^{m} \left(\sqrt{n w_{i_j}} He_{i_j^{(k)}} \right)^{P_j} \right)^2 \right)} \right)^{\frac{1}{p_1 + \cdots + p_m}},$$

$$1-\left(1-\frac{\sum_{k=1}^{C_n^m}\left(\left(1-\prod_{j=1}^{m}\left(1-F_{i_j^k}\right)^{P_j}\right)\left(\prod_{j=1}^{m}\left(nw_{i_j}Ex_{i_j^{(k)}}\right)^{P_j}+\left(\prod_{j=1}^{m}\left(\sqrt{nw_{i_j}}En_{i_j^{(k)}}\right)^{P_j}\right)^2+\left(\prod_{j=1}^{m}\left(\sqrt{nw_{i_j}}He_{i_j^{(k)}}\right)^{P_j}\right)^2\right)\right)}{\sum_{k=1}^{C_n^m}\left(\prod_{j=1}^{m}\left(nw_{i_j}Ex_{i_j^{(k)}}\right)^{P_j}+\left(\prod_{j=1}^{m}\left(\sqrt{nw_{i_j}}En_{i_j^{(k)}}\right)^{P_j}\right)^2+\left(\prod_{j=1}^{m}\left(\sqrt{nw_{i_j}}He_{i_j^{(k)}}\right)^{P_j}\right)^2\right)}\right)^{\frac{1}{p_1+\cdots+p_m}}\Bigg\rangle\Bigg).$$

Theorem 8 can be proved similarly according to the proof procedures of Theorem 3.

5. MCDM Approach under Simplified Neutrosophic Linguistic Circumstance

In this section, a MCDM approach is developed on the basis of the proposed simplified neutrosophic cloud aggregation operators to solve real-world problems. Consider a MCDM problem with simplified neutrosophic linguistic evaluation information, which can be converted to SNCs. Then, let $A=\{a_1,a_2,\ldots,a_m\}$ be a discrete set of alternatives, and $C=\{c_1,c_2,\ldots,c_n\}$ be the set of criteria. Suppose that the weight of the criteria is $w=(w_1,w_2,\ldots,w_s)^T$, where $w_k\geq 0$, and $\sum_{k=1}^{s}w_k=1$. The original evaluation of alternative a_i under criterion c_j is expressed as SNLNs $\gamma_{ij}=\langle s_{ij},(T_{ij},I_{ij},F_{ij})\rangle$ (i = 1, 2, …, m; j = 1, 2, …, n). The primary procedures of the developed method are presented in the following.

Step 1: Normalize the evaluation information.

Usually, two kinds of criteria—benefit criteria and cost criteria—exist in MCDM problems. Then, in accordance with the transformation principle of SNLNs [42], the normalization of original evaluation information can be shown as

$$\tilde{\gamma}_{ij}=\begin{cases}\langle s_{ij},(T_{ij},I_{ij},F_{ij})\rangle, & \text{for benifit criterion,}\\ \langle h_{(2t+1-sub(s_{ij}))},(T_{ij},I_{ij},F_{ij})\rangle, & \text{for cost criterion.}\end{cases} \tag{16}$$

Step 2: Convert SNLNs to SNCs.

Based on the transformation method described in Section 2.4 and Definition 9, we can convert SNLNs to SNCs. The SNC evaluation information can be obtained as $a_{ij}=\langle(Ex_{ij},En_{ij},He_{ij}),(T_{ij},I_{ij},F_{ij})\rangle$ (i = 1, 2, …, m; j = 1, 2, …, n).

Step 3: Acquire the comprehensive evaluation for each alternative.

The WSNCMSM operator or the GWSNCMSM operator can be employed to integrate the evaluation of $a_{ij}(j=1,2,\ldots,n)$ under all criteria and acquire the comprehensive evaluation $a_i=\langle(Ex_i,En_i,He_i),(T_i,I_i,F_i)\rangle$ for the alternative a_i.

Step 4: Compute the distance between the comprehensive evaluation of a_i and the PIS/NIS.

First, in accordance with the obtained overall evaluation values, the positive ideal solution (PIS) a^+ and negative ideal solution (NIS) a^- are determined as

$$a^+=\langle(\max_i(Ex_i),\min_i(En_i),\min_i(He_i)),(\max_i(T_i),\min_i(I_i),\min_i(F_i))\rangle,$$

$$a^-=\langle(\min_i(Ex_i),\max_i(En_i),\max_i(He_i)),(\min_i(T_i),\max_i(I_i),\max_i(F_i))\rangle.$$

Second, in accordance with the proposed distance of SNCs, the distance $d\left(a_i, a^+\right)$ between a_i and a^+, and the distance $d\left(a_i, a^-\right)$ between a_i and a^- can be calculated.

Step 5: Compute the relative closeness of each alternative.

In the following, the relative closeness of each alternative can be calculated as

$$I_i = \frac{d\left(a_i, a^+\right)}{d\left(a_i, a^+\right) + d\left(a_i, a^-\right)} \tag{17}$$

where $d\left(a_i, a^+\right)$ and $d\left(a_i, a^-\right)$ are obtained in Step 4.

Step 6: Rank all the alternatives.

In accordance with the relative closeness I_i of each alternative, we can rank all the alternatives. The smaller the value of I_i, the better the alternative a_i is.

6. Illustrative Example

This section provides a real-world problem of hotel selection (adapted from Wang et al. [49]) to demonstrate the validity and feasibility of the developed approach.

6.1. Problem Description

Nowadays, consumers often book hotels online when traveling or on business trip. After they leave the hotel, they may evaluate the hotel and post the online reviews on the website. In this case, the online reviews are regard as the most important reference for the hotel selection decision of potential consumers. In order to enhance the accuracy of hotel recommendation in line with lots of online reviews, this study devotes to applying the proposed method to address hotel recommendation problems effectively. In practical hotel recommendation problems, many hotels (e.g., 10 hotels) need to be recommended for consumers. In order to save space, we select five hotels from a tourism website for recommendation here. The developed approach can be similarly applied to address hotel recommendation problems with many hotels. The five hotels are represented as a_1, a_2, a_3, a_4 and a_5. The employed linguistic term set is described as follows:

S = {s_1, s_2, s_3, s_4, s_5, s_6, s_7} = {extremely poor, very poor, poor, fair good, very good, extremely good}

In this paper, we focus on the four hotel evaluation criteria including, c_1, location (such as near the downtown and is the traffic convenient or not); c_2, service (such as friendly staff and the breakfast); c_3, sleep quality (such as the soundproof effect of the room); and c_4, comfort degree (such as the softness of the bed and the shower). Wang et al. [49] introduced a text conversion technique to transform online reviews to neutrosophic linguistic information. Motivated by this idea, the online reviews of five hotels under four criteria can be described as SNLNs, as shown in Table 1. For simplicity, the weight information of the four criteria is assumed to be $w = (0.25, 0.22, 0.35, 0.18)^T$.

Table 1. Evaluation values in SNLNs.

a_i	c_1	c_2	c_3	c_4
a_1	$\langle s_4, (0.6, 0.6, 0.1) \rangle$	$\langle s_5, (0.6, 0.4, 0.3) \rangle$	$\langle s_4, (0.8, 0.5, 0.1) \rangle$	$\langle s_2, (0.8, 0.3, 0.1) \rangle$
a_2	$\langle s_2, (0.7, 0.5, 0.1) \rangle$	$\langle s_4, (0.6, 0.4, 0.2) \rangle$	$\langle s_3, (0.6, 0.2, 0.4) \rangle$	$\langle s_4, (0.7, 0.4, 0.3) \rangle$
a_3	$\langle s_3, (0.5, 0.1, 0.2) \rangle$	$\langle s_4, (0.6, 0.5, 0.3) \rangle$	$\langle s_6, (0.7, 0.6, 0.1) \rangle$	$\langle s_2, (0.5, 0.5, 0.2) \rangle$
a_4	$\langle s_2, (0.4, 0.5, 0.3) \rangle$	$\langle s_3, (0.5, 0.3, 0.4) \rangle$	$\langle s_4, (0.6, 0.8, 0.2) \rangle$	$\langle s_5, (0.9, 0.3, 0.1) \rangle$

a_5	$\langle s_5,(0.6,0.4,0.4)\rangle$	$\langle s_5,(0.8,0.3,0.1)\rangle$	$\langle s_3,(0.7,0.5,0.1)\rangle$	$\langle s_4,(0.6,0.5,0.2)\rangle$

6.2. Illustration of the Developed Methods

According to the steps of the developed method presented in Section 5, the optimal alternative from the five hotels can be determined.

6.2.1. Case 1—Approach based on the WSNCMSM Operator.

Let linguistic scale function be $f_1(h_x)$, and $m = 2$ in Equation (13) in the subsequent calculation. Then, the hotel selection problem can be addressed according to the following procedures.

Step 1: Normalize the evaluation information.

Obviously, the four criteria are the benefit type in the hotel selection problem above. Thus, the evaluation information does not need to be normalized.

Step 2: Convert SNLNs to SNCs.

Utilize the transformation method presented in Section 2.4, we transform the linguistic term s_i in SNLNs to the cloud model (Ex_i, En_i, He_i). The obtained results are shown as follows:

$$s_1 \to (Ex_1, En_1, He_1) = (0.833, 1.25, 0.231),$$
$$s_2 \to (Ex_2, En_2, He_2) = (1.667, 1.11, 0.278),$$
$$s_3 \to (Ex_3, En_3, He_3) = (2.5, 0.833, 0.37),$$
$$s_4 \to (Ex_4, En_4, He_4) = (3.33, 0.556, 0.463),$$
$$s_5 \to (Ex_5, En_5, He_5) = (4.167, 0.278, 0.556),$$
$$s_6 \to (Ex_6, En_6, He_6) = (5, 0.741, 0.401),$$
$$s_7 \to (Ex_7, En_7, He_7) = (5.833, 0.972, 0.324).$$

Then, according to Definition 9, SNLNs can be converted to SNCs, as presented in Table 2.

Table 2. Evaluation information in SNCs.

a_i	c_1	c_2	c_3	c_4
a_1	$\langle(3.33,0.556,0.463),(0.6,0.6,0.1)\rangle$	$\langle(4.167,0.278,0.556),(0.6,0.4,0.3)\rangle$	$\langle(3.33,0.556,0.463),(0.8,0.5,0.1)\rangle$	$\langle(1.667,1.11,0.278),(0.8,0.3,0.1)\rangle$
a_2	$\langle(1.667,1.11,0.278),(0.7,0.5,0.1)\rangle$	$\langle(3.33,0.556,0.463),(0.6,0.4,0.2)\rangle$	$\langle(2.5,0.833,0.37),(0.6,0.2,0.4)\rangle$	$\langle(3.33,0.556,0.463),(0.7,0.4,0.3)\rangle$
a_3	$\langle(2.5,0.833,0.37),(0.5,0.1,0.2)\rangle$	$\langle(3.33,0.556,0.463),(0.6,0.5,0.3)\rangle$	$\langle(5,0.741,0.401),(0.7,0.6,0.1)\rangle$	$\langle(1.667,1.11,0.278),(0.5,0.5,0.2)\rangle$
a_4	$\langle(1.667,1.11,0.278),(0.4,0.5,0.3)\rangle$	$\langle(2.5,0.833,0.37),(0.5,0.3,0.4)\rangle$	$\langle(3.33,0.556,0.463),(0.6,0.8,0.2)\rangle$	$\langle(4.167,0.278,0.556),(0.9,0.3,0.1)\rangle$
a_5	$\langle(4.167,0.278,0.556),(0.6,0.4,0.4)\rangle$	$\langle(4.167,0.278,0.556),(0.8,0.3,0.1)\rangle$	$\langle(2.5,0.833,0.37),(0.7,0.5,0.1)\rangle$	$\langle(3.33,0.556,0.463),(0.6,0.5,0.2)\rangle$

Step 3: Acquire the comprehensive evaluation for each alternative.

The WSNCMSM operator is employed to integrate the evaluations of alternative a_i under all the criteria. Then, the overall evaluation a_i^* for each alternative are obtained as

$$a_1^* = \langle(3.1311, 0.6228, 0.4509), (0.6866, 0.4765, 0.1589)\rangle,$$
$$a_2^* = \langle(2.5946, 0.7909, 0.3881), (0.642, 0.3621, 0.2638)\rangle,$$
$$a_3^* = \langle(3.1691, 0.801, 0.3835), (0.5986, 0.4584, 0.1895)\rangle,$$
$$a_4^* = \langle(2.6569, 0.727, 0.4159), (0.6231, 0.5308, 0.2358)\rangle,$$
$$a_5^* = \langle(3.4126, 0.5065, 0.4786), (0.6766, 0.4208, 0.2091)\rangle.$$

Step 4: Compute the distance between the comprehensive evaluation of a_i and the PIS/NIS.

First, the PIS a^+ and the NIS a^- are determined as $a^+ = \langle (3.4126, 0.5065, 0.3835), (0.6866, 0.3621, 0.1586) \rangle$, and $a^- = \langle (2.5946, 0.801, 0.4786), (0.5986, 0.5308, 0.2638) \rangle$, respectively. Then, based on Equation (5), the distance $d(a_i^*, a^+)$, and the distance $d(a_i^*, a^-)$ are computed as

$d(a_1^*, a^+) = 0.8324$, $d(a_2^*, a^+) = 1.5966$, $d(a_3^*, a^+) = 1.2447$, $d(a_4^*, a^+) = 1.4864$, and $d(a_5^*, a^+) = 0.3361$; $d(a_1^*, a^-) = 1.0135$, $d(a_2^*, a^-) = 0.2137$, $d(a_3^*, a^-) = 0.6535$, $d(a_4^*, a^-) = 0.3012$, and $d(a_5^*, a^-) = 1.5101$.

Step 5: Calculate the relative closeness of each alternative.

By using Equation (17), the relative closeness of each alternative is computed as

$I_1 = 0.4509$, $I_2 = 0.882$, $I_3 = 0.6557$, $I_4 = 0.8315$, and $I_5 = 0.1821$.

Step 6: Rank all the alternatives.

On the basis of the comparison rule, the smaller the value of I_i, the better the alternative a_i is. We can rank the alternatives as $a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$. The best one is a_5.

When $m = 3$ is used in Equation (13), the overall assessment value for each alternative a_i are derived as follows:

$$a_1^* = \langle (5.2615, 0.454, 0.2915), (0.5675, 0.6174, 0.229) \rangle,$$
$$a_2^* = \langle (4.1045, 0.6629, 0.2384), (0.5177, 0.503, 0.3688) \rangle,$$
$$a_3^* = \langle (5.1405, 0.6986, 0.2307), (0.4449, 0.5936, 0.2832) \rangle,$$
$$a_4^* = \langle (4.0855, 0.5792, 0.2593), (0.468, 0.6791, 0.3475) \rangle,$$
$$a_5^* = \langle (6.2421, 0.3334, 0.328), (0.5531, 0.5645, 0.2977) \rangle.$$

And the positive ideal point is determined as $a^+ = \langle (6.2421, 0.3334, 0.2307), (0.5675, 0.503, 0.229) \rangle$, the negative ideal point is determined as $a^- = \langle (4.0855, 0.6986, 0.328), (0.4449, 0.6791, 0.3688) \rangle$. Then, the results of the distance between a_i^* and a^+, and the distance between a_i^* and a^- are obtained as

$d(a_1^*, a^+) = 2.1919$, $d(a_2^*, a^+) = 4.064$, $d(a_3^*, a^+) = 3.7056$, $d(a_4^*, a^+) = 3.7812$, and $d(a_5^*, a^+) = 0.8571$; $d(a_1^*, a^-) = 2.4095$, $d(a_2^*, a^-) = 0.4656$, $d(a_3^*, a^-) = 1.085$, $d(a_4^*, a^-) = 0.6172$, and $d(a_5^*, a^-) = 3.8179$.

Therefore, the relative closeness of each alternative is calculated as

$I_1 = 0.4764$, $I_2 = 0.8972$, $I_3 = 0.7735$, $I_4 = 0.8597$, and $I_5 = 0.1833$

According to the results of I_i, we can rank the alternatives as $a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$. The best one is a_5, which is the same as the obtained result in the situation $m = 2$.

6.2.2. Case 2—Approach based on the GWSNCMSM Operator.

Let the linguistic scale function be $f_1(h_x)$, and $m=2$, $p_1=1$, $p_2=2$ in Equation (15) in the subsequent calculation. Then, the hotel selection problem can be addressed according to the following procedures.

Step 1: Normalize the evaluation information.

Obviously, the four criteria are the benefit type in the hotel selection problem above. Thus, the evaluation information does not need to normalize.

Step 2: Convert SNLNs to SNCs.

The obtained SNCs are the same as those in Case 1.

Step 3: Acquire the comprehensive evaluation for each alternative.

The GWSNCMSM operator is employed to integrate the evaluations of alternative a_i under all the criteria. Then, the overall evaluation a_i^* for each alternative are obtained as

$$a_1^* = \langle(3.2899, 0.7006, 0.4668), (0.7068, 0.4812, 0.1544)\rangle,$$
$$a_2^* = \langle(2.693, 0.805, 0.3968), (0.6395, 0.3374, 0.29)\rangle,$$
$$a_3^* = \langle(3.7063, 0.8318, 0.3958), (0.6366, 0.5081, 0.1637)\rangle,$$
$$a_4^* = \langle(2.9311, 0.7165, 0.4401), (0.6654, 0.5197, 0.2125)\rangle,$$
$$a_5^* = \langle(3.3078, 0.5638, 0.4675), (0.6871, 0.4227, 0.1846)\rangle$$

Step 4: Compute the distance between the comprehensive evaluation of a_i and the PIS/NIS.

First, the PIS a^+ and the NIS a^- are determined as $a^+ = \langle(3.7063, 0.5638, 0.3958),$ $(0.7068, 0.3374, 0.1544)\rangle$, and $a^- = \langle(2.693, 0.8318, 0.4675),$ $(0.6366, 0.5197, 0.29)\rangle$ respectively. Then, based on Equation (5), the distance $d(a_i^*, a^+)$, and the distance $d(a_i^*, a^-)$ are computed as

$$d(a_1^*, a^+) = 1.0407,\ d(a_2^*, a^+) = 1.6913,\ d(a_3^*, a^+) = 1.0619,\ d(a_4^*, a^+) = 1.371, \text{ and}$$
$$d(a_5^*, a^+) = 0.6054;\ d(a_1^*, a^-) = 0.9235,\ d(a_2^*, a^-) = 0.2183,\ d(a_3^*, a^-) = 0.9925,$$
$$d(a_4^*, a^-) = 0.5323, \text{ and } d(a_5^*, a^-) = 1.2871.$$

Step 5: Calculate the relative closeness of each alternative.

By using Equation (17), the relative closeness of each alternative is calculated as

$$I_1 = 0.5298,\ I_2 = 0.8857,\ I_3 = 0.5169,\ I_4 = 0.7203, \text{ and } I_4 = 0.7203$$

Step 6: Rank all the alternatives.

On the basis of the comparison rule, the smaller the value of I_i, the better the alternative a_i is. We can rank the alternatives as $a_5 \succ a_3 \succ a_1 \succ a_4 \succ a_2$, the best one is a_5.

Using the parameters $m=2$, $p_1=1$, and $p_2=2$ in the aggregation operators, the ranking results acquired by the developed methods with the WSNCMSM operator and the GWSNCMSM operator are almost identical, and these rankings are described in Table 3. The basically identical ranking results indicate that the developed methods in this paper have a strong stability.

Table 3. Ranking results based on different operators.

Proposed Operators	*m*	*p*1	*p*2	Rankings
WSNCMSM	2	\	\	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$

WSNCMSM	3	\	\	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
GWSNCMSM	2	1	2	$a_5 \succ a_3 \succ a_1 \succ a_4 \succ a_2$

6.3. Comparative Analysis and Sensitivity Analysis

This subsection implements a comparative study to verify the applicability and feasibility of the developed method. The developed method aims to improve the effectiveness of handling simplified neutrosophic linguistic information. Therefore, the proposed method can be demonstrated by comparing with the approaches in Wang et al. [21] and Tian et al. [20] that deal with SNLNs merely depend on the linguistic functions. The comparison between the developed method and two existed approaches is feasible because these three methods are based on the same information description tool and the aggregation operators developed in these methods have the same parameter characteristics. Two existing methods are employed to address the same hotel selection problem above, and the ranking results acquired by different approaches are described in Table 4.

Table 4. Ranking results obtained by different methods.

Methods	Rankings
Wang et al.'s method [21] ($m = 2$)	$a_5 \succ a_1 \succ a_3 \succ a_2 \succ a_4$
The proposed approach based on $WSNCMSM_w^{(m)}\,(m=2)$	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
Wang et al.'s method [21] $(m=2, p_1=1, p_2=1)$	$a_5 \succ a_3 \succ a_1 \succ a_2 \succ a_4$
Tian et al.'s method [20] $(m=2, p_1=1, p_2=1)$	$a_5 \succ a_3 \succ a_1 \succ a_4 \succ a_2$
The proposed approach based on $GWSNCMSM^{(m,p_1,p_2,\ldots,p_m)}\,(m=2, p_1=1, p_2=1)$	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$

As described in Table 4, the rankings acquired by the developed approaches and that obtained by the existed approaches have obvious difference. However, the best alternative is always a_5, which demonstrates that the developed approach is reliable and effective for handling decision-making problems under simplified neutrosophic linguistic circumstance. There are still differences between the approaches developed in this paper and the methods presented by Wang et al. [21] and Tian et al. [20], which is that the proposed approaches use the cloud model instead of linguistic function to deal with linguistic information. The advantages of the proposed approaches in handling practical problems are summarized as follows:

First, comparing with the existing methods with SNLNs, the proposed approaches uses the cloud model to process qualitative evaluation information involved in SNLNs. The existing methods handle linguistic information merely depending on the relevant linguistic functions, which may result in loss and distortion of the original information. However, the cloud model depicts the randomness and fuzziness of a qualitative concept with three numerical characteristics perfectly, and it is more suitable to handle linguistic information than the linguistic function because it can reflect the vagueness and randomness of linguistic variables simultaneously.

Second, being compared with the simplified neutrosophic linguistic Bonferroni mean aggregation operator given in Tain et al. [20], the simplified neutrosophic clouds Maclaurin symmetric mean operator provided in this paper take more generalized forms and contain more flexible parameters that facilitate selecting the appropriate alternative.

In addition, being compared with SNLNs, SNCs not only provide the truth, indeterminacy, and falsity degrees for the evaluation object, but also utilize the cloud model to characterize linguistic information effectively.

The ranking results may vary with different values of parameters in the proposed aggregation operators. Thus, a sensitivity analysis will be implemented to analyze the influence of the parameter p_j on ranking results. The obtained results are presented in Table 5.

Table 5. Ranking results with different p_j under $m = 2$.

p_1	p_2	Rankings Based on GWSNCMSM
1	0	$a_5 \succ a_1 \succ a_3 \succ a_2 \succ a_4$

0	1	$a_4 \succ a_5 \succ a_3 \succ a_2 \succ a_1$
1	2	$a_5 \succ a_3 \succ a_1 \succ a_4 \succ a_2$
1	3	$a_3 \succ a_5 \succ a_1 \succ a_4 \succ a_2$
1	4	$a_3 \succ a_5 \succ a_1 \succ a_4 \succ a_2$
1	5	$a_3 \succ a_1 \succ a_5 \succ a_4 \succ a_2$
2	1	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
3	1	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
4	1	$a_1 \succ a_5 \succ a_3 \succ a_4 \succ a_2$
5	1	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$
0.5	0.5	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
1	1	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
2	2	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
3	3	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
4	4	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$
5	5	$a_5 \succ a_1 \succ a_3 \succ a_4 \succ a_2$

The data in Table 5 indicates that the best alternative is a_5 or a_1, and the worst one is a_2 when using the GWSNCMSM operator with different p_j under $m = 2$ to fuse evaluation information. When $p_1 = 0$, we can find the ranking result has obvious differences with other results. Therefore, $p_1 = 0$ is not used in practice. The data in Table 5 also suggests that the ranking vary obviously when the value of p_1 far exceeds the value of p_2. Thus, it can be concluded that the values of p_1 and p_2 should be selected as equally as possible in practical application. The difference of ranking results in Table 5 reveals that the values of p_1 and p_2 have great impact on the ranking results. As a result, selecting the appropriate parameters is a significant action when handling MCDM problems. In general, the values can be set as $p_1 = p_2 = 1$ or $p_1 = p_2 = 2$, which is not only simple and convenient but it also allows the interrelationship of criteria. It can be said that p_1 and p_2 are correlative with the thinking mode of the decision-maker; the bigger the values of p_1 and p_2, the more optimistic the decision-maker is; the smaller the values of p_1 and p_2, the more pessimistic the decision-maker is. Therefore, decision-makers can flexibly select the values of parameters based on the certain situations and their preferences and identify the most precise result.

7. Conclusions

SNLNs take linguistic terms into account on the basis of NSs, and they make the data description more complete and consistent with practical decision information than NSs. However, the cloud model, as an effective way to deal with linguistic information, has never been considered in combination with SNLNs. Motivated by the cloud model, we put forward a novel concept of SNCs based on SNLNs. Furthermore, the operation rules and distance of SNCs were defined. In addition, considering distinct importance of input variables, the WSNCMSM and GWSNCMSM operators were proposed and their properties and special cases were discussed. Finally, the developed approach was successfully applied to handle a practical hotel selection problem, and the validity of this approach was demonstrated.

The primary contributions of this paper can be summarized as follows. First, to process linguistic evaluation information involved in SNLNs, the cloud model is introduced and used. In this way, a new concept of SNCs is presented, and the operations and distance of SNCs are proposed. Being compared with other existing studies on SNLNs, the proposed method is more effective because the cloud model can comprehensively reflect the uncertainty of qualitative evaluation information. Second, based on the related studies, the MSM operator is extended to simplified neutrosophic cloud circumstances, and a series of SNCMSM aggregation operators are proposed. Third, a MCDM method is developed in light of the proposed aggregation operators, and its effectiveness and stability are demonstrated using the illustrative example, comparative analysis, and sensitivity analysis.

In some situations, asymmetrical and non-uniform linguistic information exists in practical problems. For example, customers pay more attention to negative comments when selecting hotels. In future study, we are going to introduce the unbalanced linguistic term sets to depict online linguistic comments and propose the hotel recommendation method.

Author Contributions: J.-Q.W., C.-Q.T., X.Z., H.-Y.Z., and T.-L.W. conceived and worked together to achieve this work; C.-Q.T. and X.Z. wrote the paper.

Conflicts of Interest: The authors declare no conflict of interest.

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