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Multi-Objective Neutrosophic Optimization Technique and its Application to Structural Design

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ABSTRACT
In this paper, a multi-objective non-linear neutrosophic optimization (NSO) approach for optimizing the design of plane truss structure with multiple objectives subject to a specified set of constraints has been developed. In this optimum design formulation, the objective functions are the weight of the truss and the deflection of loaded joint; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. A classical truss optimization example is presented here in to demonstrate the efficiency of the neutrosophic optimization approach. The test problem includes a three-bar planar truss subjected to a single load condition. This multi-objective structural optimization model is solved by neutrosophic optimization approach with linear and non-linear membership function. Numerical example is given to illustrate our NSO approach..

Keywords
Neutrosophic Set, Single Valued Neutrosophic Set, Neutrosophic Optimization, Structural model.

1. INTRODUCTION
The research area of optimal structural design has been receiving increasing attention from both academia and industry over the past three decades in order to improve structural performance and to reduce design costs. However, in the real world, uncertainty or vagueness is prevalent in the Engineering Computations. In the context of structural design the uncertainty is connected with lack of accurate data of design factors. This tendency has been changing due to the increase in the use of fuzzy mathematical algorithm for dealing with this class of problems. Fuzzy set (FS) theory has long been introduced to handle inexact and imprecise data by Zadeh [1], Later on Bellman and Zadeh [2] used the fuzzy set theory to the decision making problem. The fuzzy set theory also found application in structural design. Several researchers like Wang et al. [3] first applied α-cut method to structural designs where the non-linear problems were solved with various design levels α, and then a sequence of solutions were obtained by setting different level-cut value of α. Rao [4] applied the same α-cut method to design a four–bar mechanism for function generating problem. Structural optimization with fuzzy parameters was developed by Yeh et al. [5]. Xu [6] used two-phase method for fuzzy optimization of structures. Shih et al. [7] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources. Shih et al. [8] developed an alternative α-level-cuts method for optimum structural design with fuzzy resources. Dey et al. [9] used generalized fuzzy number in context of a structural design. Dey et al.[10]used basic t-norm based fuzzy optimization technique for optimization of structure. In such extension, Atanassov [11] introduced Intuitionistic fuzzy set (IFS) which is one of the generalizations of fuzzy set theory and is characterized by a membership function, a non- membership function and a hesitancy function. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. A transportation model was solved by Jana et al.[12]using multi-objective intuitionistic fuzzy linear programming. Dey et al. [13] solved two bar truss non-linear problem by using intuitionistic fuzzy optimization problem. Dey et al. [14] used intuitionistic fuzzy optimization technique for multi objective optimum structural design. Intuitionistic fuzzy sets consider both truth membership and falsity membership. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information. In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership which are independent. Neutrosophic theory was introduced by Smarandache [15]. The motivation of the present study is to give computational algorithm for solving multi-objective structural problem by single valued neutrosophic optimization approach. Neutrosophic optimization technique is very rare in application to structural optimization. Here it is aimed to study the impact of truth exponential membership, indeterminacy exponential membership and falsity hyperbolic membership function in such optimization process. The results are compared numerically linear and nonlinear neutrosophic optimization technique. From our numerical result, it has been seen that there is no change between the result of linear and non-linear neutrosophic optimization technique in the perspective of structural optimization technique.

2. MULI-OBJECTIVE STRUCTURAL MODEL
In the design problem of the structure i.e. lightest weight of the structure and minimum deflection of the loaded joint that satisfies all stress constraints in members of the structure. In truss structure system ,the basic parameters (including allowable stress ,etc) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight with minimum nodes displacement in a given load conditions .

The multi-objective structural model can be expressed as

$$Minimize\ WT(A) \qquad (1)$$

$$minimize\ \delta(A)$$

subject to $\sigma(A) \le [\sigma]$

$A^{\min} \le A \le A^{\max}$

where $A = [A_1, A_2, ...A_n]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar , $WT(A) = \sum_{i=1}^{n} \rho_i A_i L_i$ is the total weight of the structure , $\delta(A)$ is the deflection of the loaded joint ,where L_i, A_i and ρ_i are the bar length, cross section area and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $[\sigma]$ is allowable stress of the group bars under various conditions, $A^{\min}$ and $A^{\max}$ are the lower and upper bounds of cross section area A respectively.

3. MATHEMATICAL PRELIMINARIES

3.1 Fuzzy Set

Let X be a fixed set. A fuzzy set A in X is an object having the form $\tilde{A} = \{(x, T_A(x)) : x \in X\}$ where the function $T_A : X \to [0,1]$ defined the truth membership of the element $x \in X$ to the set A.

3.2 Intuitionistic Fuzzy Set

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form $\tilde{A}^i = \{< X, T_A(x), F_A(x) > | x \in X\}$ where $T_A : X \to [0,1]$ and $F_A : X \to [0,1]$ define the truth membership and falsity membership respectively,for every element of $x \in X$, $0 \le T_A(x) + F_A(x) \le 1$.

3.3 Neutrosophic Set

Let a set X be a space of points (objects) and $x \in X$ .A neutrosophic set $\tilde{A}^n$ in X is defined by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity membership function $F_A(x)$ and having the form $\tilde{A}^n = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ .That is $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, $F_A(x) : X \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ *so*

$0^- \le \sup T_A(x) + I_A(x) + \sup F_A(x) \le 3^+$.

3.4 Single Valued Neutrosophic Set

Let a set X be the universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$ where

$T_A : X \to [0,1]$, $I_A : X \to [0,1]$ and $F_A : X \to [0,1]$ with $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$.

3.5 Complement of Neutrosophic Set

Complement of a single valued neutrosophic set A is denoted by $c(A)$ and is defined by

$T_{c(A)}(x) = F_A(x),\ I_{c(A)}(x) = 1 - F_A(x),\ F_{c(A)}(x) = T_A(x).$

3.6 Union of Neutrosophic Set

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cup B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$T_{c(A)}(x) = \max(T_A(x), T_B(x)),\ I_{c(A)}(x) = \max(I_A(x), I_B(x)),$

$F_{c(A)}(x) = \min(F_A(x), F_B(x))$ *for all* $x \in X$.

3.7 Intersection of Neutrosophic Set

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C , written as $C = A \cap B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$T_{c(A)}(x) = \min(T_A(x), T_B(x)),\ I_{c(A)}(x) = \min(I_A(x), I_B(x)),$

$F_{c(A)}(x) = \max(F_A(x), F_B(x))$ *for all* $x \in X$

4. MATHEMATICAL ANALYSIS

4.1 Neutrosophic Optimization Technique to Solve Minimization type Multi-Objective Non-linear Programming Problem

Decision making is a process of solving the problem involving the goals under constraints. The outcome is a decision which should in an action .Decision making plays an important role in engineering science .It is difficult process due to factors like incomplete and imprecise information which tend to presented real life situations. In the decision making process ,our main target is to find the value from the selected set with the highest degree of membership in the decision set and these values support the goals under constraints only. But there may be situations where some values from selected set cannot support i.e such values strongly against the goals under constraints which are non-admissible. In this case such values are found from selected set with last degree of non –membership in the decision sets. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief systems .In neutrosophic set ,indeterminacy is quantified explicitely and truth membership ,indeterminacy –membership and falsity membership are independent.So it is natural to adopt the purpose the value from the selected set with highest degree of truth-membership, indeterminacy-membership and least degree of falsity membership on the decision set. These factors indicate that a decision making process takes place in neutrosophic environment.

A nonlinear multi-objective optimization of the problem is of the form

$$Minimize \{f_1(x), f_2(x), ..., f_p(x)\} \quad (2)$$

$$g_j(x) \le b_j \quad j = 1, 2, ..., q$$

Now the decision set $\tilde{D}^n$, a conjunction of Neutrosophic objectives and constraints is defined

$$\tilde{D}^n = \left(\bigcap_{k=1}^{p} \tilde{G}_k^n\right) \bigcap \left(\bigcap_{j=1}^{q} \tilde{C}_j^n\right) = \left\{(x, T_{\tilde{D}^n}(x)) I_{\tilde{D}^n}(x), F_{\tilde{D}^n}(x)\right\}$$

here

$$T_{\tilde{D}^n}(x) = \min\begin{Bmatrix} T_{\tilde{G}_1^n}(x), T_{\tilde{G}_2^n}(x), T_{\tilde{G}_3^n}(x), \dots\dots\dots T_{\tilde{G}_p^n}(x); \\ T_{\tilde{C}_1^n}(x), T_{\tilde{C}_2^n}(x), T_{\tilde{C}_3^n}(x), \dots\dots\dots T_{\tilde{C}_q^n}(x) \end{Bmatrix} \; for\, all\;\, x \in X$$

$$I_{\tilde{D}^n}(x) = \min\begin{Bmatrix} I_{\tilde{G}_1^n}(x), I_{\tilde{G}_2^n}(x), I_{\tilde{G}_3^n}(x), \dots\dots\dots I_{\tilde{G}_p^n}(x); \\ I_{\tilde{C}_1^n}(x), I_{\tilde{C}_2^n}(x), I_{\tilde{C}_3^n}(x), \dots\dots\dots I_{\tilde{C}_q^n}(x) \end{Bmatrix} \; for\, all\;\, x \in X$$

$$F_{\tilde{D}^n}(x) = \min\begin{Bmatrix} F_{\tilde{G}_1^n}(x), F_{\tilde{G}_2^n}(x), F_{\tilde{G}_3^n}(x), \dots\dots\dots F_{\tilde{G}_p^n}(x); \\ F_{\tilde{C}_1^n}(x), F_{\tilde{C}_2^n}(x), F_{\tilde{C}_3^n}(x), \dots\dots\dots F_{\tilde{C}_q^n}(x) \end{Bmatrix} \; for\, all\;\, x \in X$$

Where $T_{\tilde{D}^n}(x),\ I_{\tilde{D}^n}(x),\ F_{\tilde{D}^n}(x)$ are truth-membership function, indeterminacy membership function, falsity membership function of neutrosophic decision set respectively .Now using the neutrosophic optimization, problem (2) is transformed to the non-linear programming problem as

$$Max\ \alpha,\ Max\ \gamma,\ Min\ \beta \qquad (3)$$

$$such\ that\ T_{\tilde{G}_k^n}(x) \ge \alpha;\ T_{\tilde{C}_j^n}(x) \ge \alpha;$$

$$I_{\tilde{G}_k^n}(x) \ge \gamma;\ I_{\tilde{C}_j^n}(x) \ge \gamma;$$

$$F_{\tilde{G}_k^n}(x) \le \beta;\ F_{\tilde{C}_j^n}(x) \le \beta;$$

$$\alpha + \beta + \gamma \le 3;$$

$$\alpha \ge \beta; \alpha \ge \gamma;$$

$$\alpha, \beta, \upsilon \in [0,1]$$

Now this non-linear programming problem (3) can be easily solved by an appropriate mathematical programming to give solution of multi-objective non-linear programming problem (1) by neutrosophic optimization approach

4.1.1 Computational Algorithm

Step-1: Solve the MONLP problem (2) as a single objective non-linear problem p times for each problem by taking one of the objectives at a time and ignoring the others. These solution are known as ideal solutions. Let x^k be the respective optimal solution for the k^{th} different objective and evaluate each objective values for all these k^{th} optimal solution.

Step-2: From the result of step-1, determine the corresponding values for every objective for each derived solution, pay-off matrix can be formulated as follows

$$\begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \dots\dots\dots\dots\dots\dots & f_p(x^1) \\ f_1(x^2) & f_2^*(x^2) & \dots\dots\dots\dots\dots\dots & f_p(x^2) \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots\dots\dots\dots & \dots\dots\dots \\ f_1(x^p) & f_2(x^p) & \dots\dots\dots\dots\dots & f_p^*(x^p) \end{bmatrix}$$

Step-3: For each objective $f_k(x)$ find lower bound L_k^μ and the upper bound U_k^μ

$$U_k^T = \max\left\{ f_k\left(x^{r*}\right) \right\}$$

and

$$L_k^T = \min\left\{ f_k\left(x^{r*}\right) \right\}\ \ where\ r = 1,2,\dots,k$$

For truth membership of objectives.

Step-4: upper and lower bounds for indeterminacy and falsity membership of objectives can be presented as follows :

$$for\ k = 1,2,\dots\dots p$$

$$U_k^F = U_k^T\ \ and\ \ L_k^F = L_k^T + t\left(U_k^T - L_k^T\right);$$

$$L_k^I = L_k^T\ \ and\ \ U_k^I = L_k^T + s\left(U_k^T - L_k^T\right)$$

Here t, s are predetermined real numbers in $(0,1)$

Step-5: Define truth membership, indeterminacy membership and falsity membership functions as follows

$$for\ k = 1,2,\dots,p$$

$$T_{f_k(x)}\left(f_k(x)\right) = \begin{cases} 1 & if\ \ f_k(x) \le L^T_{f_k(x)} \\ 1 - \exp\left\{ -\psi\left(\dfrac{U^T_{f_k(x)} - f_k(x)}{U^T_{f_k(x)} - L^T_{f_k(x)}} \right) \right\} & if\ \ L^T_{f_k(x)} \le f_k(x) \le U^T_{f_k(x)} \\ 0 & if\ \ f(x) \ge U^T_{f_k(x)} \end{cases}$$

$$I_{f_k(x)}\left(f_k(x)\right) = \begin{cases} 1 & if\ \ f_k(x) \le L^I_{f(x)} \\ \exp\left\{ \dfrac{U^I_{f_k(x)} - f_k(x)}{U^I_{f_k(x)} - L^I_{f_k(x)}} \right\} & if\ \ L^I_{f_k(x)} \le f_k(x) \le U^I_{f_k(x)} \\ 0 & if\ \ f_k(x) \ge U^I_{f_k(x)} \end{cases}$$

$$F_{f(x)}\left(f(x)\right) = \begin{cases} 0 & if\ \ f(x) \le L^F_{f(x)} \\ \dfrac{1}{2} + \dfrac{1}{2}\tanh\left\{ \left(f(x) - \dfrac{U^F_{f(x)} + L^F_{f(x)}}{2} \right) \tau_{f(x)} \right\} & if\ \ L^F_{f(x)} \le f(x) \le U^F_{f(x)} \\ 1 & if\ \ f(x) \ge U^F_{f(x)} \end{cases}$$

Step-6: Now neutosophic optimization method for MONLP problem gives a equivalent nonlinear programming problem as

$$Maximize\ \left(\alpha - \beta + \gamma\right) \qquad (4)$$

such that

$$T_k\left(f_k(x)\right) \ge \alpha;\ I_k\left(f_k(x)\right) \ge \gamma;\ F_k\left(f_k(x)\right) \le \beta;$$

$$\alpha + \beta + \upsilon \le 3;\ \alpha \ge \beta; \alpha \ge \gamma;\ \alpha, \beta, \gamma \in [0,1];$$

$$g_j(x) \le b_j\ \ x \ge 0,$$

$$k = 1,2,\dots,p;\ \ j = 1,2,\dots,q$$

which is reduced to equivalent non-linear programming problem as

$$Maximize\ \left(\theta - \eta + \kappa\right) \qquad (5)$$

such that

$$f_k(x) + \frac{\theta\left(U_k^T - L_k^T\right)}{4} \le L_k^T;$$

$$f_k(x)+\frac{\eta}{\tau_{f_k}} \le \frac{U_k^T+L_k^T+\varepsilon_{f_k}}{2};$$

$$f_k(x)+\kappa\xi_{f_k} \le L_k^T+\xi_{f_k};\quad for\ k=1,2,....,p$$

Where $\theta=-\log(1-\alpha),\kappa=\log\gamma,$

$$\eta=-\tanh^{-1}(2\beta-1),\psi=4,\tau_{f_k}=\frac{6}{U_k^F-L_k^F}$$

$$\theta+\kappa+\eta\le 3;\ \theta\ge\kappa;\theta\ge\eta;\ \theta,\kappa,\eta\in[0,1];\ g_j(x)\le b_j;\quad x\ge 0,$$

This crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

4.2 Solution of Multi-Objective Structural Optimization Problem (MOSOP) by Neutrosophic Optimization Technique

To solve the MOSOP (1), step 1 of 4.1.1 is used .After that according to step to pay off matrix is formulated.

$$\begin{array}{c} \\ A^1 \\ A^2 \end{array}\begin{array}{c} \begin{array}{cc} WT(A) & \delta(A) \end{array} \\ \begin{bmatrix} WT^*(A^1) & \delta(A^1) \\ WT(A^2) & \delta^*(A^2) \end{bmatrix} \end{array}$$

According to step-2 the bound of weight objective $U_{WT}^T,\ L_{WT}^T;$ $U_{WT}^I,\ L_{WT}^I$ and $U_{WT}^F,\ L_{WT}^F$ for truth, indeterminacy and falsity membership function respectively. Then $L_{WT}^T\le WT(A)\le U_{WT}^T;\ L_{WT}^I\le WT(A)\le U_{WT}^I;$ $L_{WT}^F\le WT(A)\le U_{WT}^F$ .Similarly the bound of deflection objective are $U_\delta^T,L_\delta^T;\ U_\delta^I,L_\delta^I$ and U_δ^F,L_δ^F are respectively for truth, indeterminacy and falsity membership function. Then $L_\delta^T\le\delta(A)\le U_\delta^T;\quad L_\delta^I\le\delta(A)\le U_\delta^I;\quad L_\delta^F\le\delta(A)\le U_\delta^F$ .Where $U_{WT}^F=U_{WT}^T,\ \ L_{WT}^F=L_{WT}^T+\varepsilon_{WT};\ \ L_{WT}^I=L_{WT}^T,\ \ U_{WT}^I=L_{WT}^T+\varepsilon_{WT}$ and $U_\delta^F=U_\delta^T,\ \ L_\delta^F=L_\delta^T+\xi_\delta;\ \ L_\delta^I=L_\delta^T,\ \ U_\delta^I=L_\delta^T+\xi_\delta$ such that

$0<\varepsilon_{WT}<\left(U_{WT}^T-L_{WT}^T\right)$ and $0<\xi_\delta<\left(U_\delta^T-L_\delta^T\right)$.

Therefore the truth, indeterminacy and falsity membership functions for objectives are

$$T_{WT(A)}\left(WT(A)\right)=\begin{cases} 1 & if\ \ WT(A)\le L_{WT(A)}^T \\ 1-\exp\left\{-\psi\left(\dfrac{U_{WT(A)}^T-WT(A)}{U_{WT(A)}^T-L_{WT(A)}^T}\right)\right\} & if\ \ L_{WT(A)}^T\le WT(A)\le U_{WT(A)}^T \\ 0 & if\ \ WT(A)\ge U_{WT(A)}^T \end{cases}$$

$$I_{WT(A)}\left(WT(A)\right)=\begin{cases} 1 & if\ \ WT(A)\le L_{WT(A)}^T \\ \exp\left\{\dfrac{\left(L_{WT(A)}^T+\xi_{WT}\right)-WT(A)}{\xi_{WT}}\right\} & if\ \ L_{WT(A)}^T\le WT(A)\le L_{WT(A)}^T+\xi_{WT} \\ 0 & if\ \ WT(A)\ge L_{WT(A)}^T+\xi_{WT} \end{cases}$$

$$F_{WT(A)}\left(WT(A)\right)=\begin{cases} 0 & if\ WT(A)\le L_{WT(A)}^T+\varepsilon_{WT} \\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(WT(A)-\dfrac{\left(U_{WT(A)}^T+L_{WT(A)}^T\right)+\varepsilon_{WT}}{2}\right)\tau_{WT}\right\} & if\ L_{WT(A)}^T+\varepsilon_{WT}\le WT(A)\le U_{WT(A)}^T \\ 1 & if\ WT(A)\ge U_{WT(A)}^T \end{cases}$$

where $0<\varepsilon_{WT},\xi_{WT}<\left(U_{WT}^T-L_{WT}^T\right)$

and

$$T_{\delta(A)}\left(\delta(A)\right)=\begin{cases} 1 & if\ \ \delta(A)\le L_\delta^T \\ 1-\exp\left\{-\psi\left(\dfrac{U_\delta^T-\delta(A)}{U_\delta^T-L_\delta^T}\right)\right\} & if\ \ L_\delta^T\le\delta(A)\le U_\delta^T \\ 0 & if\ \ \delta(A)\ge U_\delta^T \end{cases}$$

$$I_{\delta(A)}\left(\delta(A)\right)=\begin{cases} 1 & if\ \ \delta(A)\le L_\delta^T \\ \exp\left\{\dfrac{\left(L_\delta^T+\xi_\delta\right)-\delta(A)}{\xi_\delta}\right\} & if\ \ L_\delta^T\le\delta(A)\le L_\delta^T+\xi_\delta \\ 0 & if\ \ \delta(A)\ge L_\delta^T+\xi_\delta \end{cases}$$

$$F_{\delta(A)}\left(\delta(A)\right)=\begin{cases} 0 & if\ \delta(A)\le L_\delta^T+\varepsilon_\delta \\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(\delta(A)-\dfrac{\left(U_\delta^T+L_\delta^T\right)+\varepsilon_\delta}{2}\right)\tau_\delta\right\} & if\ L_\delta^T+\varepsilon_\delta\le\delta(A)\le U_\delta^T \\ 1 & if\ \delta(A)\ge U_\delta^T \end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker and for *where* $0<\varepsilon_\delta,\xi_\delta<\left(U_\delta^T-L_\delta^T\right)$

According to neutrosophic optimization technique considering truth, indeterminacy and falsity membership function for MOSOP (1), crisp non-linear programming problem can be formulated as

$$Maximize\ \left(\alpha+\gamma-\beta\right) \tag{6}$$

Subject to

$$T_{WT}\left(WT(A)\right)\ge\alpha;\ \ T_\delta\left(\delta(A)\right)\ge\alpha;$$

$$I_{WT}\left(WT(A)\right)\ge\gamma;\ I_\delta\left(\delta(A)\right)\ge\gamma;$$

$$F_{WT}\left(WT\left(A\right)\right)\le\beta;\ \ F_{\delta}\left(\delta\left(A\right)\right)\le\beta;$$

$$\sigma\left(A\right)\le\left[\sigma\right];$$

$$\alpha+\beta+\gamma\le3;\ \alpha\ge\beta;\ \alpha\ge\gamma;$$

$$\alpha,\beta,\gamma\in\left[0,1\right],\ A^{\min}\le A\le A^{\max}$$

which is reduced to equivalent non-linear programming problem as

$$\text{Maximize}\ \left(\theta+\kappa-\eta\right) \qquad (7)$$

Such that

$$WT\left(A\right)+\theta\frac{\left(U_{WT}^{T}-L_{WT}^{T}\right)}{\psi}\le U_{WT}^{T};$$

$$WT\left(A\right)+\frac{\eta}{\tau_{WT}}\le\frac{U_{WT}^{T}+L_{WT}^{T}+\varepsilon_{WT}}{2};$$

$$WT\left(A\right)+\kappa\xi_{WT}\le L_{WT}^{T}+\xi_{WT};$$

$$\delta\left(A\right)+\theta\frac{\left(U_{\delta}^{T}-L_{\delta}^{T}\right)}{\psi}\le U_{\delta}^{T};$$

$$\delta\left(A\right)+\kappa\xi_{\delta}\le L_{\delta}^{T}+\xi_{\delta};\ \delta\left(A\right)+\frac{\eta}{\tau_{\delta}}\le\frac{U_{WT}^{T}+L_{WT}^{T}+\varepsilon_{\delta}}{2};$$

$$\theta+\kappa-\eta\le3;\ \theta\ge\kappa;\theta\ge\eta;\ \theta,\kappa,\eta\in\left[0,1\right]$$

where $\theta=-\ln\left(1-\alpha\right);\ \psi=4;\ \tau_{WT}=\frac{6}{\left(U_{WT}^{F}-L_{WT}^{F}\right)};$

$$\tau_{\sigma_i}=\frac{6}{\left(U_{\sigma_i}^{F}-L_{\sigma_i}^{F}\right)};\ \kappa=\ln\gamma;\ \eta=-\tanh^{-1}\left(2\beta-1\right).$$

Solving the above crisp model (7) by an appropriate mathematical programming algorithm optimal solution will be obtained and hence objective functions i.e structural weight and deflection of the loaded joint will attain Pareto optimal solution.

5. NUMERICAL ILLUSTRATION

A well known three bar planer truss is considered to minimize weight of the structure $WT\left(A_1,A_2\right)$ and minimize the deflection $\delta\left(A_1,A_2\right)$ at a loading point of a statistically loaded three bar planer truss subject to stress constraints on each of the truss members

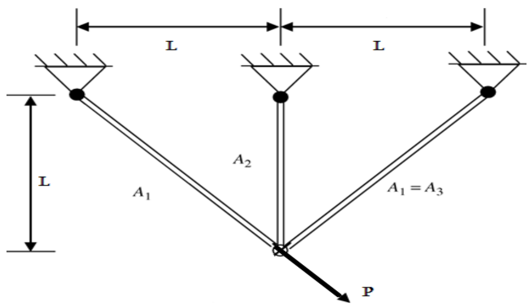

Fig 1: Design of the three-bar planar truss

The multi-objective optimization problem can be stated as follows

$$\text{Minimize}\quad WT\left(A_1,A_2\right)=\rho L\left(2\sqrt{2}A_1+A_2\right) \qquad (8)$$

$$\text{Minimize}\quad \delta\left(A_1,A_2\right)=\frac{PL}{E\left(A_1+\sqrt{2}A_2\right)}$$

Subject to

$$\sigma_1\left(A_1,A_2\right)=\frac{P\left(\sqrt{2}A_1+A_2\right)}{\left(2A_1^2+2A_1A_2\right)}\le\left[\sigma_1^T\right];$$

$$\sigma_2\left(A_1,A_2\right)=\frac{P}{\left(A_1+\sqrt{2}A_2\right)}\le\left[\sigma_2^T\right];$$

$$\sigma_3\left(A_1,A_2\right)=\frac{PA_2}{\left(2A_1^2+2A_1A_2\right)}\le\left[\sigma_3^C\right];$$

$$A_i^{\min}\le A_i\le A_i^{\max}\quad i=1,2$$

where $P=$ applied load ; $\rho=$ material density ; $L=$ length ; $E=$ Young's modulus ; $A_1=$ Cross section of bar-1 and bar-3; $A_2=$ Cross section of bar-2; δ is deflection of loaded joint. $\left[\sigma_1^T\right]$ and $\left[\sigma_2^T\right]$ are maximum allowable tensile stress for bar 1 and bar 2 respectively, σ_3^C is maximum allowable compressive stress for bar 3.The input numeric data are given in Table 1.

Solution : According to step 2 of 4.1.1, pay-off matrix is formulated as follows

$$\begin{array}{c} \\ A^1 \\ A^2 \end{array}\begin{array}{cc} WT\left(A_1,A_2\right) & \delta\left(A_1,A_2\right) \\ \left[\begin{array}{cc} 2.638958 & 14.64102 \\ 19.14214 & 1.656854 \end{array}\right. & \left.\right] \end{array}.$$

Here

$$U_{WT}^{F}=U_{WT}^{T}=19.14214,L_{WT}^{F}=L_{WT}^{T}+\varepsilon_1=2.638958+\varepsilon_1;$$

$$L_{WT}^{I}=L_{WT}^{T}=2.638958,U_{WT}^{I}=L_{WT}^{T}+\xi_1=2.638958+\xi_1$$

such that $0<\varepsilon_1,\xi_1<\left(19.14214-2.638958\right)$;

$$U_{\delta}^{F}=U_{\delta}^{T}=14.64102,L_{\delta}^{F}=L_{\delta}^{T}+\varepsilon_2=1.656854+\varepsilon_2;$$

$$L_{\delta}^{I}=L_{\delta}^{T}=1.656854,U_{\delta}^{I}=L_{\delta}^{T}+\xi_2=1.656854+\xi_2$$

such that $0<\varepsilon_2,\xi_2<\left(14.64102-1.656854\right)$

Here truth, indeterminacy, and falsity membership function for objective functions $WT\left(A_1,A_2\right)$, $\delta\left(A_1,A_2\right)$ are defined as follows

$$T_{WT(A_1,A_2)}\left(WT(A_1,A_2)\right)=\begin{cases}1 & if\ WT(A_1,A_2)\le 2.638958\\ 1-\exp\left\{-4\left(\dfrac{19.14214-WT(A_1,A_2)}{16.503182}\right)\right\} & if\ \ 2.638958\le WT(A_1,A_2)\le 19.14214\\ 0 & if\ \ WT(A_1,A_2)\ge 19.14214\end{cases}$$

$$I_{WT(A_1,A_2)}\left(WT(A_1,A_2)\right)=\begin{cases}1 & if\ WT(A_1,A_2)\le 2.638958\\ \exp\left\{\dfrac{(2.638958+\xi_1)-WT(A_1,A_2)}{\xi_1}\right\} & if\ \ 2.638958\le WT(A_1,A_2)\le 2.638958+\xi_1\\ 0 & if\ \ WT(A_1,A_2)\ge 2.638958+\xi_1\end{cases}$$

$$F_{WT(A_1,A_2)}\left(WT(A_1,A_2)\right)=\begin{cases}0 & if\ WT(A_1,A_2)\le 2.638958\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(WT(A_1,A_2)-\dfrac{21.781098+\varepsilon_1}{2}\right)\dfrac{6}{(16.503182-\varepsilon_1)}\right\} & if\ 2.638958\le WT(A_1,A_2)\le 19.14214\\ 1 & if\ \ WT(A_1,A_2)\ge 19.14214\end{cases}$$

$$0<\varepsilon_1,\xi_1<16.503182$$

and

$$T_{\delta(A_1,A_2)}\left(\delta(A_1,A_2)\right)=\begin{cases}1 & if\ \ \delta(A_1,A_2)\le 1.656854\\ 1-\exp\left\{-4\left(\dfrac{14.64102-\delta(A_1,A_2)}{12.984166}\right)\right\} & if\ \ 1.656854\le \delta(A_1,A_2)\le 14.64102\\ 0 & if\ \ \delta(A_1,A_2)\ge 14.64102\end{cases}$$

$$I_{\delta(A_1,A_2)}\left(\delta(A_1,A_2)\right)=\begin{cases}1 & if\ \ \delta(A_1,A_2)\le 1.656854\\ \exp\left\{\dfrac{(1.656854+\xi_2)-\sigma_T(A_1,A_2)}{\xi_2}\right\} & if\ \ 130\le \delta(A_1,A_2)\le 1.656854+\xi_2\\ 0 & if\ \ \delta(A_1,A_2)\ge 1.656854+\xi_2\end{cases}$$

$$F_{\delta(A_1,A_2)}\left(\delta(A_1,A_2)\right)=\begin{cases}0 & if\ \ \delta(A_1,A_2)\le 1.656854+\varepsilon_2\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(\delta(A_1,A_2)-\dfrac{16.297874+\varepsilon_2}{2}\right)\dfrac{6}{12.984166-\varepsilon_2}\right\} & if\ \ 1.656854+\varepsilon_2\le \delta(A_1,A_2)\le 14.64102\\ 1 & if\ \ \delta(A_1,A_2)\ge 14.64102\end{cases}$$

$$0<\varepsilon_2,\xi_2<12.9842$$

According to neutrosophic optimization technique the MOSOP (8) can be formulated as

Maximize $(\theta+\kappa-\eta)$

$$\left(2\sqrt{2}A_1+A_2\right)+4.1257\theta\le 19.14214;$$

$$\left(2\sqrt{2}A_1+A_2\right)+\frac{\eta(16.503182-\varepsilon_1)}{6}\le\frac{(21.781098+\varepsilon_1)}{2};$$

$$\left(2\sqrt{2}A_1+A_2\right)+\kappa\xi_1\le(2.638958+\xi_1);$$

$$\frac{20}{\left(A_1+\sqrt{2}A_2\right)}+3.2460415\theta\le 14.64102;$$

$$\frac{20}{\left(A_1+\sqrt{2}A_2\right)}+\frac{\eta(12.984166-\varepsilon_2)}{6}\le\frac{(16.297874+\varepsilon_2)}{2};$$

$$\frac{20}{\left(A_1+\sqrt{2}A_2\right)}+\kappa\xi_2\le(1.656854+\xi_2);$$

$$\frac{20\left(\sqrt{2}A_1+A_2\right)}{\left(2A_1^2+2A_1A_2\right)}\le 20;\ \frac{20}{\left(A_1+\sqrt{2}A_2\right)}\le 20;$$

$$\frac{20A_2}{\left(2A_1^2+2A_1A_2\right)}\le 15;$$

$$\theta+\kappa+\eta\le 3;\theta\ge\kappa;\theta\ge\eta$$

$$0.1\le A_1,A_2\le 5$$

Now , using above mentioned truth, indeterminacy and falsity membership function NLP (8) can be solved by NSO technique for different values of $\varepsilon_1,\varepsilon_2$ and ξ_1,ξ_2. The optimum solution of MOSOP(8) is given in table (2) .

Here solutions for the different tolerance ξ_1,ξ_2 and ξ_3 for indeterminacy membership function of objective functions is shown in the table (2). From the table 2, it shows that NSO technique gives same Pareto optimal result for linear and non-linear membership functions in the perspective of Structural Optimization.

6. CONCLUSIONS

The main objective of this work is to illustrate how neutrosophic optimization technique can be utilized to solve a nonlinear structural problem. The concept of neutrosophic optimization technique allows one to define a degree of truth membership, which is not a complement of degree of falsity; rather, they are independent with degree of indeterminacy. In this problem actually it is investigated the effect of non-linear truth, indeterminacy and falsity membership function of neuotrosophic set in perspective of multi-objective structural optimization. Here a non-linear three bar truss design problem has been considered. In this test problem, minimum weight of the structure as well as minimum deflection of loaded joint are minimized. The results of this study may lead to the development of effective neutrosophic technique for solving other model of nonlinear programming problem in different field.

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Table 1: Input data for crisp model (8)

Applied load P (KN)	Volume density ρ (KN/m^3)	Length L (m)	Maximum allowable tensile stress $[\sigma_1^T]$ (KN/m^2)	Maximum allowable compressive stress $[\sigma_3^C]$ (KN/m^2)	Young's modulus E (KN/m^2)	A_i^{min} and A_i^{max} of cross section of bars $(10^{-4}m^2)$
20	100	1	20	15	2×10^7	$A_1^{min}=0.1$ $A_1^{max}=5$ $A_2^{min}=0.1$ $A_2^{max}=5$

Table 2: Optimal solution of MOSOP (8)

Methods	A_1 $\times10^{-4}m^2$	A_2 $\times10^{-4}m^2$	$WT(A_1,A_2)$ $\times10^2 KN$	$\delta(A_1,A_2)$ $\times10^{-7}m$
Neutosophic optimization (NSO) with nonlinear membership function $\varepsilon_1=3.30064, \varepsilon_2=2.59696$ $\xi_1=1.65032, \xi_2=1.29848$	.5777658	2.655110	**4.289278**	2.955334

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