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Multi Q-Single Valued Neutrosophic Soft Expert Set and its Application in Decision Making

Qaisar Khan[1], Tahir Mahmood[2], Nasruddin Hassan[3]

[1,2] Department of Mathematics, International Islamic university Islamabad Pakistan.
[3] School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, UKM Bangi, Selongar DE, Malaysia.
[1,2,3] Email: qaisarkhan421@gmail.com, tahirbakhat@iiu.edu.pk, nas@ukm.edu.my

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Abstract. Smarandache introduced the concept of neutrosophic set which is the genralistion of fuzzy set, intuitionistic fuzzy set and a better mathematical tool to handle incomplete, inconsistance and vague information. In this work, we propose the concept of multi Q-single valued neutrosophic soft expert set and its basic operations such as, union, intersection, complement, And and OR. Further, we construct an algorithm for decision-making method on multi Q-SVNSES. Finally, an example is provided to show the practicality and effectiveness of the proposed algorithm which consist of indeterminate and inconsistent information. At the end, a comparison has been made through an example with the existing theory.

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1. Introduction

In our daily life most of problems in social sciences, management, engineering and medical sciences usually include data which are not surely crisp, accurate and deterministic in character due to variant uncertainties associated corresponding with these problems. Mostly such problems were solved by using fuzzy set introduced by Zadeh[60]. In Zadeh fuzzy set only membership function was considered and the non-membership function comes automatically. After fuzzy set Attanassov [15] proposed the concept of intuitionistic fuzzy set which is the extension of fuzzy set by introducing membership and non-membership functions and the hesitation function by subtracting the sum of membership and non-membership function from one. But the only limitation in this concept is that the hesitation degree can not be defined independently. To overcome this limitation, Smarandache [49, 50, 51] introduced the concept of neutrosophic set which is the generalization of fuzzy set and intuitionistic fuzzy set. Neutrosophic set is characterized by truth-membership, indeterminacy-membership and falsity-membership functions with the condition that the sum of maximum of the three function is less or equal to three. After the introduction of neutrosophic sets many researcher proposed subclasses of neutrosophic sets and apply them to various areas [58, 59, 54, 55]. Liu et al. [30]and Khan et al.[27, 28] developed some power linguistic Heronian mean operators, power Bonferroni mean operator and power Muirhead mean operators for linguistic neutrosophic sets, interval neutrosophic sets and neutrosophic cubic sets and applied them to multiple attribute decision making and multiple attribute group decision making. Some other generalization of fuzzy sets were studied by many authors and give its applications in different field[47, 40, 48, 22, 26, 7].

The concept of soft set theory was first presented by Molodtsov [37] in 1999, which was a new concept of handling uncertainties and ambiguity.After the introduction of soft set theory Maji et al. [34, 35, 46, 33] presented the concept of fuzzy soft set and and intuitionistic fuzzy soft set and proposed some basic operations and studied their related properties. Further, Fuzzy soft set theory, intuitionistic fuzzy soft set theory and fuzzy soft matrix theory were studied by Cagman et al.and Ahmad et al.[2, 18, 19, 20]. M. Riaz et al. [42, 43, 44] studied certain properties of bipolar fuzzy soft topology via Q-neighborhood, developed measurable soft mappings and fuzzy parametrized soft compact spaces with decision making. S.Roy et al [45] construct soft topology, soft base , soft subase and discussed some related theorems. H. Kamaci et al [24] proposed difference operations for soft matrices and give its application in decision making. Yang et al. [57] combined soft set with interval valued fuzzy set and proposed the concept of interval valued soft set and gave some basic operations such as complement, AND and OR and prove De Morgan law for IVFSSs. Intuitionistic fuzzy soft set and interval valued intuitionistic fuzzy soft set were studied by Xu et al. and Jiang et al.[23, 56]. Maji [36] proposed the concept of neutrosophic soft set by combining soft set with neutrosophic set, proposed basic operational laws for it and studied their related properties. Alkhazaleh et al proposed the concept of possibility fuzzy soft set [9] and proposed operational rules and studied their basic properties. They also give applications of possibility soft set in decision-making. Alkhazaleh et al. and Zhu et al [10, 61] presented the concepts of fuzzy parametrized fuzzy soft set and fuzzy parametrized IVFSSand give its applications in decision-making. Broumi et al. [16] proposed neutrosophic parametrized soft set and give it application in decision-making. Alkhazaleh et al proposed the concept of soft multi-sets [11] which was the generalization of soft set proposed by Molodtsov, followed by the concepts of fuzzy soft multi-set[12], multi Q-fuzzy soft set[3], multi Q-intuitionistic fuzzy soft set [17], multi Q-single valued neutrosophic soft set [31]. Further, multi Q-fuzzy soft expert set was introduced by Adam et al. [6, 4, 5], proposed some operational laws such as, union, complement, intersection, AND and OR operation for it, and also give application in decision making. in recent years Mahmood et al. [32] proposed some generalized aggregation operators for cubic hesitant fuzzy sets and give its application in decision making. Voskoglou [52] give application of fuzzy number to human skills. Abbas et al. [1] proposed upper and lower contra-continuous fuzzy multifunctions.Othman [38] discussed the concept of fuzzy infra-semiopen set. Roa et al. [41] discussed general fixed point theorems in fuzzy metric spaces. Khan et al.[25] discussed Common Fixed Point Theorems for Converse Commuting and OWC Maps in Fuzzy Metric Spaces.

The aim of this article is to proposed the concept of multi Q-single valued neutrosophic soft expert set [13] by extended the concept of expert set, defined some basic operational laws such as complement, union, intersection, AND and OR along with illustrative example. We further give the applications in decision-making. Finally a comparison has been made with the existing theory of multi Q-fuzzy soft expert set through example.

2. Preliminaries

In this section, some basic definitions about soft expert set, multi Q-single valued neutrosophic soft set.

Let U, E, X be respectively, universe of discourse set, set of parameters and set of experts. Let O be the set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 1. [13]*A soft expert set over U is a pair* (F, A) *. Where F is a function given by* $F : A \to P(U)$ *,* $P(U)$ *represent the power set of U.*

Definition 2. [31]*Let X be a universal set and* $Q \neq \emptyset$*. A Q-SVNS* $\tilde{N}_Q$ *in X and Q is defined as,*

$$\tilde{N}_Q = \{(\widehat{\theta}, \hat{u}), \mu_{\tilde{N}}(\widehat{\theta}, \hat{u}), \nu_{\tilde{N}}(\widehat{\theta}, \hat{u}), \xi_{\tilde{N}}(\widehat{\theta}, \hat{u}) : \widehat{\theta} \in X, \hat{u} \in Q\}$$

Where $\mu_{\tilde{N}_Q} : X \times Q \to [0,1], \nu_{\tilde{N}_Q} : X \times Q \to [0,1], \xi_{\tilde{N}_Q} : X \times Q \to [0,1]$, are respectively truth-membership, indeterminacy-membership and falsity membership functions for every $\widetilde{\theta} \in X$ and $\hat{u} \in Q$ and satisfy the condition $0 \leq \mu_{\tilde{N}_Q}(\widehat{\theta}, \hat{u}) + \nu_{\tilde{N}_Q}(\widehat{\theta}, \hat{u}) + \xi_{\tilde{N}_Q}(\widehat{\theta}, \hat{u}) \leq 3$.The set of all Q-SVNS is denoted by $M^K QSVN(X)$.

Definition 3. [31]*Let X be a universal set,* $Q \neq \emptyset$ *and* $\tilde{N}_Q$ *be a Q-SVNS. The complement of* $\tilde{N}_Q$ *is denoted and defined as follows:*

$$\tilde{N}_Q^c = \{(\widehat{\theta}, \hat{u}), \xi_{\tilde{N}_Q}(\widehat{\theta}, \hat{u}), 1 - \nu_{\tilde{N}_Q}(\widehat{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\widehat{\theta}, \hat{u}) : \widehat{\theta} \in X, \hat{u} \in Q\}$$

Definition 4. [31]*Let* $\tilde{A}_Q$ *and* $\tilde{N}_Q$ *be two Q-SVNS. Then the union and intersection is denoted and defined by*

$$\tilde{A}_Q \cup \tilde{N}_Q = \left\{ \begin{array}{c} (\widehat{\theta}, \hat{u}), \max\left(\mu_{\tilde{A}_Q}(\widehat{\theta}, \hat{u}), \mu_{\widetilde{N}_Q}(\widehat{\theta}, \hat{u})\right), \min\left(\nu_{\tilde{A}_Q}(\widehat{\theta}, \hat{u}), \nu_{\widetilde{N}_Q}(\widehat{\theta}, \hat{u})\right), \\ \min\left(\xi_{\tilde{A}_Q}(\widehat{\theta}, \hat{u}), \xi_{\widetilde{N}_Q}(\widehat{\theta}, \hat{u})\right) : \widehat{\theta} \in X, \hat{u} \in Q \end{array} \right\}$$

$$\tilde{A}_Q \cap \tilde{N}_Q = \left\{ \begin{array}{c} (\widehat{\theta}, \hat{u}), \min\left(\mu_{\tilde{A}_Q}(\widehat{\theta}, \hat{u}), \mu_{\widetilde{N}_Q}(\widehat{\theta}, \hat{u})\right), \max\left(\nu_{\tilde{A}_Q}(\widehat{\theta}, \hat{u}), \nu_{\widetilde{N}_Q}(\widehat{\theta}, \hat{u})\right), \\ \max\left(\xi_{\tilde{A}_Q}(\widehat{\theta}, \hat{u}), \xi_{\widetilde{N}_Q}(\widehat{\theta}, \hat{u})\right) : \widehat{\theta} \in X, \hat{u} \in Q \end{array} \right\}$$

Definition 5. [31]*Let X be a non-empty set and Q be any non-empty set ,l be any positive integer and I be a unit interval* $[0, 1]$ *.A multi Q-SVNS* $\tilde{A}_Q$ *in X and Q is a set of ordered sequences*

$$\tilde{A}_Q = \{(\widehat{\theta}, \hat{u}), \mu_j(\widehat{\theta}, \hat{u}), \nu_j(\widehat{\theta}, \hat{u}), \xi_j(\widehat{\theta}, \hat{u}) : \widehat{\theta} \in X, \hat{u} \in Q forall j = 1, 2, .., l\}$$

Where $\mu_j : X \times Q \to I^k, \nu_j : X \times Q \to I^k, \xi_j : X \times Q \to I^k, forall j = 1, 2, \ldots, l$ and are respectively truth-membership, indeterminacy-membership and falsity membership functions for each $\widehat{\theta} \in X$ and $\hat{u} \in Q$ and satisfy the condition $0 \leq \mu_j(\widehat{\theta}, \hat{u}) + \nu_j(\widehat{\theta}, \hat{u}) + \xi_j(\widehat{\theta}, \hat{u}) \leq 3, forall j = 1, 2, \ldots, l$. The functions $\mu_j(\widehat{\theta}, \hat{u}), \nu_j(\widehat{\theta}, \hat{u}), \xi_j(\widehat{\theta}, \hat{u})$for all $j = 1, 2, \ldots, l$ are called the truth-membership , indeterminacy-membership and falsity-membership functions respectively of the multi Q-SVNS $\widetilde{A}_Q$ and satisfy the condition $0 \leq \mu_j(\widehat{\theta}, \hat{u}) + \nu_j(\widehat{\theta}, \hat{u}) + \xi_j(\widehat{\theta}, \hat{u}) \leq 3, forallj = 1, 2, \ldots, l$. l is called the dimension of the Q-SVNS $\widetilde{A}_Q$.The set of all Q-SVNS is denoted by $Z^K QSVNS(X)$.

3. Multi Q-Single Valued Neutrosophic Soft Expert Set

Definition 6. *A multi Q-single valued neutrosophic soft expert set over U is a pair (F_Q, A), where F_Q is a function given by*

$$F_Q : A \to M^K QSVN(U)$$

such that $M^K QSVN(U)$ denotes the set of all multi Q-single valued neutrosophic set over U.

Assume that $F_Q : A \to M^K QSVN(U)$ is a function defined as $F(x) = F(x)(q, u), \forall q \in Q, u \in U$. For each $x_i \in A, F(x_i) = F(x_i)(u, q)$. Where $F(x_i)$ represents the degree of truth-membership, degree of indeterminacy and falsity-membership of the elements of U in $F(x_i)$. Hence $F(x_i)$ can be written as,

$$F(x_i)(u_i, q) = \{(u_i, q), F(x_i)(u_i, q)\} \; for \; i = 1, 2, 3, ...m$$

Where, $F(x_i)(u_i, q) = \langle \mu_j(u_i, q), \nu_j(u_i, q), \xi_j(u_i, q) \rangle$, in which $\mu_j(u_i, q), \nu_j(u_i, q)$ and $\xi_j(u_i, q)$ respectively representing the truth-membership, indeterminacy-membership and falsity-membership degrees of each element $u_i \in U$.

Suppose that a professor wants to select three students for a research project from PhD and MS students, and wants to take the opinion of the two expert professors of there classes about these students. Let $U = \{u_1, u_2, u_3\}$be the set of students and $Q = \{q_1, q_2\}$ be the set of students. Let the decision parameter under consideration is $E = \{e_1 =$ Hardworking, $e_2 =$ Intelligence$\}$ and $X = \{p, q\}$ be the set of expert professors of their classes. Suppose that

$$F_Q(e_1, p, 1) = \left\{ \begin{array}{c} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right\}$$

$$F_Q(e_1, q, 1) = \left\{ \begin{array}{c} \langle (u_1, q_1), (0.3, 0.2, 0.4), (0.4, 0.1, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.2), (0.6, 0.1, 0.1), (0.7, 0.0, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.5), (0.3, 0.2, 0.3), (0.5, 0.1, 0.4) \rangle, \\ \langle (u_2, q_2), (0.9, 0.1, 0.1), (0.7, 0.2, 0.1), (0.5, 0.2, 0.1) \rangle, \\ \langle (u_3, q_1), (0.8, 0.1, 0.1), (0.4, 0.2, 0.1), (0.3, 0.2, 0.4) \rangle, \\ \langle (u_3, q_3), (0.6, 0.2, 0.1), (0.7, 0.2, 0.2), (0.8, 0.0, 0.1) \rangle \end{array} \right\}$$

$$F_Q(e_2,p,1)=\left\{\begin{array}{c}\langle(u_1,q_1),(0.2,0.1,0.4),(0.4,0.2,0.1),(0.5,0.2,0.3)\rangle\,,\\ \langle(u_1,q_2),(0.5,0.2,0.1),(0.6,0.1,0.2),(0.8,0.1,0.1)\rangle\,,\\ \langle(u_2,q_1),(0.1,0.4,0.3),(0.3,0.2,0.4),(0.4,0.1,0.2)\rangle\,,\\ \langle(u_2,q_2),(0.3,0.2,0.1),(0.2,0.1,0.2),(0.4,0.2,0.3)\rangle\,,\\ \langle(u_3,q_1),(0.4,0.1,0.4),(0.5,0.2,0.1),(0.6,0.1,0.1)\rangle\,,\\ \langle(u_3,q_3),(0.2,0.4,0.3),(0.4,0.3,0.4),(0.5,0.2,0.1)\rangle\end{array}\right\}$$

$$F_Q(e_2,q,1)=\left\{\begin{array}{c}\langle(u_1,q_1),(0.1,0.3,0.4),(0.2,0.4,0.4),(0.4,0.2,0.1)\rangle\,,\\ \langle(u_1,q_2),(0.2,0.3,0.4),(0.4,0.1,0.3),(0.6,0.1,0.2)\rangle\,,\\ \langle(u_2,q_1),(0.4,0.1,0.1),(0.6,0.2,0.3),(0.7,0.1,0.2)\rangle\,,\\ \langle(u_2,q_2),(0.3,0.3,0.2),(0.4,0.1,0.3),(0.5,0.1,0.2)\rangle\,,\\ \langle(u_3,q_1),(0.4,0.2,0.1),(0.6,0.1,0.2),(0.7,0.1,0.1)\rangle\,,\\ \langle(u_3,q_3),(0.2,0.3,0.3),(0.5,0.1,0.1),(0.6,0.2,0.3)\rangle\end{array}\right\}$$

$$F_Q(e_1,p,0)=\left\{\begin{array}{c}\langle(u_1,q_1),(0.1,0.2,0.4),(0.3,0.3,0.2),(0.4,0.3,0.1)\rangle\,,\\ \langle(u_1,q_2),(0.2,0.1,0.2),(0.2,0.3,0.4),(0.3,0.1,0.3)\rangle\,,\\ \langle(u_2,q_1),(0.4,0.1,0.3),(0.6,0.1,0.1),(0.7,0.1,0.2)\rangle\,,\\ \langle(u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.3),(0.5,0.1,0.2)\rangle\,,\\ \langle(u_3,q_1),(0.2,0.3,0.4),(0.3,0.4,0.2),(0.4,0.1,0.5)\rangle\,,\\ \langle(u_3,q_3),(0.1,0.4,0.4),(0.3,0.2,0.3),(0.6,0.1,0.3)\rangle\end{array}\right\}$$

$$F_Q(e_1,q,0)=\left\{\begin{array}{c}\langle(u_1,q_1),(0.6,0.1,0.1),(0.7,0.2,0.2),(0.8,0.1,0.1)\rangle\,,\\ \langle(u_1,q_2),(0.4,0.1,0.2),(0.5,0.2,0.3),(0.6,0.2,0.1)\rangle\,,\\ \langle(u_2,q_1),(0.2,0.3,0.2),(0.3,0.3,0.4),(0.5,0.1,0.2)\rangle\,,\\ \langle(u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle\,,\\ \langle(u_3,q_1),(0.2,0.4,0.2),(0.3,0.1,0.1),(0.4,0.2,0.3)\rangle\,,\\ \langle(u_3,q_3),(0.7,0.1,0.2),(0.8,0.0,0.1),(0.9,0.1,0.1)\rangle\end{array}\right\}$$

$$F_Q(e_2,p,0)=\left\{\begin{array}{c}\langle(u_1,q_1),(0.2,0.2,0.4),(0.4,0.2,0.2),(0.5,0.1,0.2)\rangle\,,\\ \langle(u_1,q_2),(0.1,0.2,0.4),(0.2,0.1,0.3),(0.4,0.1,0.1)\rangle\,,\\ \langle(u_2,q_1),(0.3,0.2,0.1),(0.5,0.1,0.1),(0.7,0.2,0.3)\rangle\,,\\ \langle(u_2,q_2),(0.4,0.1,0.3),(0.6,0.1,0.2),(0.7,0.1,0.2)\rangle\,,\\ \langle(u_3,q_1),(0.3,0.2,0.3),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle\,,\\ \langle(u_3,q_3),(0.4,0.1,0.1),(0.5,0.2,0.3),(0.6,0.2,0.3)\rangle\end{array}\right\}$$

$$F_Q(e_2,q,0)=\left\{\begin{array}{c}\langle(u_1,q_1),(0.3,0.2,0.2),(0.4,0.3,0.2),(0.5,0.1,0.1)\rangle\,,\\ \langle(u_1,q_2),(0.4,0.1,0.1),(0.5,0.2,0.1),(0.7,0.2,0.3)\rangle\,,\\ \langle(u_2,q_1),(0.6,0.1,0.3),(0.7,0.1,0.1),(0.8,0.2,0.1)\rangle\,,\\ \langle(u_2,q_2),(0.5,0.2,0.3),(0.6,0.2,0.3),(0.8,0.0,0.1)\rangle\,,\\ \langle(u_3,q_1),(0.2,0.3,0.4),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle\,,\\ \langle(u_3,q_3),(0.3,0.2,0.3),(0.5,0.1,0.1),(0.7,0.2,0.3)\rangle\end{array}\right\}$$

Then the multi Q-SVNSES (F_Q, E) consisting of the following group of approximations.

$$(F_Q, E) =$$

$$\left\{ \left(F_Q(e_1, p, 1), \left\{ \begin{array}{c} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle , \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle , \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle , \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle , \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle , \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right\} \right) , \right.$$

$$\left(F_Q(e_1, p, 0), \left\{ \begin{array}{c} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle , \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle , \\ \langle (u_2, q_1), (0.4, 0.1, 0.3), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle , \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle , \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle , \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right\} \right) ,$$

Each element of the multi Q-SVNSES implies the opinion of each expert professors based on each parameter about the students with their own classes.

Further, we shall define the definitions of agree, disagree, equal and subsets multi Q-SVNSESs.

Definition 7. *Let (F_Q, A) and (G_Q, B) be two multi QSVNSESs. Then (F_Q, A) is said to be multi Q-SVNSE subset of (G_Q, B) , denoted by $(F_Q, A) \subseteq (G_Q, B)$,if $A \subseteq B$ and $F_Q(x) \subseteq G_Q(x)$, $\forall x \in A$.*

Definition 8. *Let (F_Q, A) and (G_Q, B) be two multi QSVNSESs. Then (F_Q, A) is said to be equal to (G_Q, B) , denoted by $(F_Q, A) = (G_Q, B)$,if $(F_Q, A) \subseteq (G_Q, B)$ and $(G_Q, B) \subseteq (F_Q, A)$.*

Definition 9. *An agree-multi Q-SVNSES $(F_Q, A)_1$ over U and Q is a multi Q-SVNSE subset of the multi Q-SVNSES (F_Q, A) defined as foolows:*

$$(F_Q, A)_1 = \{F_{Q_1}(\eta) : \eta \in E \times X \times \{1\}\} .$$

Definition 10. *A disagree-multi Q-SVNSES $(F_Q, A)_0$ over U and Q is a multi Q-SVNSE subset of the multi Q-SVNSES (F_Q, A) defined as follows:*

$$(F_Q, A)_0 = \{F_{Q_1}(\eta) : \eta \in E \times X \times \{0\}\} .$$

Example 1. *Consider Example (3) . The agree-multi Q-SVNSES $(F_Q, E)_1$ over U and Q is,*

$$\left(F_Q(e_1, p, 1), \left\{ \begin{array}{c} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle , \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle , \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle , \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle , \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle , \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right\} \right)$$

$$\left(F_Q(e_1,q,1), \left\{ \begin{array}{c} \langle (u_1,q_1),(0.3,0.2,0.4),(0.4,0.1,0.3),(0.5,0.1,0.2)\rangle, \\ \langle (u_1,q_2),(0.5,0.2,0.2),(0.6,0.1,0.1),(0.7,0.0,0.1)\rangle, \\ \langle (u_2,q_1),(0.1,0.4,0.5),(0.3,0.2,0.3),(0.5,0.1,0.4)\rangle, \\ \langle (u_2,q_2),(0.9,0.1,0.1),(0.7,0.2,0.1),(0.5,0.2,0.1)\rangle, \\ \langle (u_3,q_1),(0.8,0.1,0.1),(0.4,0.2,0.1),(0.3,0.2,0.4)\rangle, \\ \langle (u_3,q_3),(0.6,0.2,0.1),(0.7,0.2,0.2),(0.8,0.0,0.1)\rangle \end{array} \right\} \right)$$

$$\left(F_Q(e_2,p,1), \left\{ \begin{array}{c} \langle (u_1,q_1),(0.2,0.1,0.4),(0.4,0.2,0.1),(0.5,0.2,0.3)\rangle, \\ \langle (u_1,q_2),(0.5,0.2,0.1),(0.6,0.1,0.2),(0.8,0.1,0.1)\rangle, \\ \langle (u_2,q_1),(0.1,0.4,0.3),(0.3,0.2,0.4),(0.4,0.1,0.2)\rangle, \\ \langle (u_2,q_2),(0.3,0.2,0.1),(0.2,0.1,0.2),(0.4,0.2,0.3)\rangle, \\ \langle (u_3,q_1),(0.4,0.1,0.4),(0.5,0.2,0.1),(0.6,0.1,0.1)\rangle, \\ \langle (u_3,q_3),(0.2,0.4,0.3),(0.4,0.3,0.4),(0.5,0.2,0.1)\rangle \end{array} \right\} \right)$$

$$\left(F_Q(e_2,q,1), \left\{ \begin{array}{c} \langle (u_1,q_1),(0.1,0.3,0.4),(0.2,0.4,0.4),(0.4,0.2,0.1)\rangle, \\ \langle (u_1,q_2),(0.2,0.3,0.4),(0.4,0.1,0.3),(0.6,0.1,0.2)\rangle, \\ \langle (u_2,q_1),(0.4,0.1,0.1),(0.6,0.2,0.3),(0.7,0.1,0.2)\rangle, \\ \langle (u_2,q_2),(0.3,0.3,0.2),(0.4,0.1,0.3),(0.5,0.1,0.2)\rangle, \\ \langle (u_3,q_1),(0.4,0.2,0.1),(0.6,0.1,0.2),(0.7,0.1,0.1)\rangle, \\ \langle (u_3,q_3),(0.2,0.3,0.3),(0.5,0.1,0.1),(0.6,0.2,0.3)\rangle \end{array} \right\} \right)$$

And the disagree-multi Q-SVNSES $(F_Q,E)_0$over U and Q is,

$$\left(F_Q(e_1,p,0), \left\{ \begin{array}{c} \langle (u_1,q_1),(0.1,0.2,0.4),(0.3,0.3,0.2),(0.4,0.3,0.1)\rangle, \\ \langle (u_1,q_2),(0.2,0.1,0.2),(0.2,0.3,0.4),(0.3,0.1,0.3)\rangle, \\ \langle (u_2,q_1),(0.4,0.1,0.3),(0.6,0.1,0.1),(0.7,0.1,0.2)\rangle, \\ \langle (u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.3),(0.5,0.1,0.2)\rangle, \\ \langle (u_3,q_1),(0.2,0.3,0.4),(0.3,0.4,0.2),(0.4,0.1,0.5)\rangle, \\ \langle (u_3,q_3),(0.1,0.4,0.4),(0.3,0.2,0.3),(0.6,0.1,0.3)\rangle \end{array} \right\} \right)$$

$$\left(F_Q(e_1,q,0), \left\{ \begin{array}{c} \langle (u_1,q_1),(0.6,0.1,0.1),(0.7,0.2,0.2),(0.8,0.1,0.1)\rangle, \\ \langle (u_1,q_2),(0.4,0.1,0.2),(0.5,0.2,0.3),(0.6,0.2,0.1)\rangle, \\ \langle (u_2,q_1),(0.2,0.3,0.2),(0.3,0.3,0.4),(0.5,0.1,0.2)\rangle, \\ \langle (u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle, \\ \langle (u_3,q_1),(0.2,0.4,0.2),(0.3,0.1,0.1),(0.4,0.2,0.3)\rangle, \\ \langle (u_3,q_3),(0.7,0.1,0.2),(0.8,0.0,0.1),(0.9,0.1,0.1)\rangle \end{array} \right\} \right)$$

$$\left(F_Q(e_2,p,0), \left\{ \begin{array}{c} \langle (u_1,q_1),(0.2,0.2,0.4),(0.4,0.2,0.2),(0.5,0.1,0.2)\rangle, \\ \langle (u_1,q_2),(0.1,0.2,0.4),(0.2,0.1,0.3),(0.4,0.1,0.1)\rangle, \\ \langle (u_2,q_1),(0.3,0.2,0.1),(0.5,0.1,0.1),(0.7,0.2,0.3)\rangle, \\ \langle (u_2,q_2),(0.4,0.1,0.3),(0.6,0.1,0.2),(0.7,0.1,0.2)\rangle, \\ \langle (u_3,q_1),(0.3,0.2,0.3),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle, \\ \langle (u_3,q_3),(0.4,0.1,0.1),(0.5,0.2,0.3),(0.6,0.2,0.3)\rangle \end{array} \right\} \right)$$

$$\left(F_Q(e_2,q,0), \left\{ \begin{array}{c} \langle (u_1,q_1),(0.3,0.2,0.2),(0.4,0.3,0.2),(0.5,0.1,0.1)\rangle, \\ \langle (u_1,q_2),(0.4,0.1,0.1),(0.5,0.2,0.1),(0.7,0.2,0.3)\rangle, \\ \langle (u_2,q_1),(0.6,0.1,0.3),(0.7,0.1,0.1),(0.8,0.2,0.1)\rangle, \\ \langle (u_2,q_2),(0.5,0.2,0.3),(0.6,0.2,0.3),(0.8,0.0,0.1)\rangle, \\ \langle (u_3,q_1),(0.2,0.3,0.4),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle, \\ \langle (u_3,q_3),(0.3,0.2,0.3),(0.5,0.1,0.1),(0.7,0.2,0.3)\rangle \end{array} \right\} \right)$$

Further we shall define the definition of complement of multi Q-SVNSES and it related proposition.

Definition 11. *The complement of muti Q-SVNSES* (F_Q, A) *is denoted by* $(F, A)^c$ *and is defined by* $(F_Q, A)^c = \left(F_Q^c; \rceil A\right)$, *where* $F_Q^c : \rceil A \to M^k QF(U)$ *is a function given by* $F_Q^c(x) = c\left(F_Q(\rceil x)\right), \forall x \in \rceil A$ *such that c is the multi Q-SVN complement.*

Proposition 3.1. *Let* (F_Q, A) *be a multi Q-SVNSES. Then*

$$\begin{aligned}(1)\ \left((F_Q, A)^c\right)^c &= (F_Q, A)\\ (2)\ (F_Q, A)_1^c &= (F_Q, A)_0^c\\ (3)\ (F_Q, A)_0^c &= (F_Q, A)_1^c\end{aligned}$$

Proof. The proof of this proposition is easy by using the properties of multi Q-SVNS. □

4. Union and intersection of multi Q-SVNSESs

In this section we propose the definitions of union and intersection of multi Q-SVNSES, give some examples and discussed their related properties.

Definition 12. *Let* (F_Q, A) *and* (G_Q, B) *be two multi Q-SVNSESs over* U *and* Q. *Then their union is a multi Q-SVNSES denoted by* $(M_Q, C) = (F_Q, A)\,(G_Q, B)$, *where* $C = A \cup B$ *and* $\forall\, x \in C$, *the multi Q-SVNSES* (M_Q, C) *is defined as follows:*

$$M_Q(x) = \begin{cases} F_Q(x) & if\ x \in A - B; \\ G_Q(x) & if\ x \in B - A; \\ F_Q(x) G_Q(x) & if\ x \in A \cap B. \end{cases}$$

Let us suppose that $U = \{u_1, u_2, u_3, u_4\},\ E = \{e_1, e_2, e_3\}$ are respectively, the universe of discourse and the set of parameters. Let $Q = \{q_1, q_2\}$ be a non-empty set and $X = \{p, q\}$be the expert set. Then (F_Q, A) and (G_Q, B) are two multi Q-SVNSESs over U and Q such that

$$\begin{aligned} A &= \{(e_1, p, 1), (e_2, p, 0), (e_1, q, 1), (e_2, q, 1)\};\\ B &= \{(e_1, p, 1), (e_1, q, 1), (e_2, q, 1)\}\end{aligned}$$

$$(F_Q, A) = \left\{ (e_1, p, 1), \begin{pmatrix} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{pmatrix}, \right.$$

$$(e_1, q, 1), \begin{pmatrix} \langle (u_1, q_1), (0.3, 0.2, 0.4), (0.4, 0.1, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.2), (0.6, 0.1, 0.1), (0.7, 0.0, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.5), (0.3, 0.2, 0.3), (0.5, 0.1, 0.4) \rangle, \\ \langle (u_2, q_2), (0.9, 0.1, 0.1), (0.7, 0.2, 0.1), (0.5, 0.2, 0.1) \rangle, \\ \langle (u_3, q_1), (0.8, 0.1, 0.1), (0.4, 0.2, 0.1), (0.3, 0.2, 0.4) \rangle, \\ \langle (u_3, q_3), (0.6, 0.2, 0.1), (0.7, 0.2, 0.2), (0.8, 0.0, 0.1) \rangle \end{pmatrix}$$

$$(e_2, q, 1), \begin{pmatrix} \langle (u_1, q_1), (0.2, 0.1, 0.4), (0.4, 0.2, 0.1), (0.5, 0.2, 0.3) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.1), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.3), (0.3, 0.2, 0.4), (0.4, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.1), (0.2, 0.1, 0.2), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_1), (0.4, 0.1, 0.4), (0.5, 0.2, 0.1), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_3, q_3), (0.2, 0.4, 0.3), (0.4, 0.3, 0.4), (0.5, 0.2, 0.1) \rangle \end{pmatrix}$$

$$\left. (e_2, p, 0), \begin{pmatrix} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{pmatrix} \right\}$$

and

$$(G_Q, B) = \left\{ (e_1, p, 1), \begin{pmatrix} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.3), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{pmatrix} \right.$$

$$(e_1, q, 1), \begin{pmatrix} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.2, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.3, 0.3, 0.4), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.2), (0.3, 0.1, 0.1), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.2), (0.8, 0.0, 0.1), (0.9, 0.0, 0.1) \rangle \end{pmatrix}$$

$$(e_2,q,1),\left(\begin{array}{c}\langle(u_1,q_1),(0.2,0.2,0.4),(0.4,0.2,0.2),(0.5,0.1,0.2)\rangle,\\ \langle(u_1,q_2),(0.1,0.2,0.4),(0.2,0.1,0.3),(0.4,0.1,0.1)\rangle,\\ \langle(u_2,q_1),(0.3,0.2,0.1),(0.5,0.1,0.1),(0.7,0.2,0.3)\rangle,\\ \langle(u_2,q_2),(0.4,0.1,0.3),(0.6,0.1,0.2),(0.7,0.1,0.2)\rangle,\\ \langle(u_3,q_1),(0.3,0.2,0.3),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle,\\ \langle(u_3,q_3),(0.4,0.1,0.1),(0.5,0.2,0.3),(0.6,0.2,0.3)\rangle\end{array}\right)\Bigg\}$$

Then the union of (F_Q,A) and (G_Q,B) is $(M_Q,C)=(F_Q,A)\,(G_Q,B)$, such that

$$\begin{aligned}(M_Q,C) \quad = \quad & \Bigg\{(e_1,p,1),\left(\begin{array}{c}\langle(u_1,q_1),(0.1,0.2,0.3),(0.3,0.3,0.2),(0.4,0.3,0.1)\rangle,\\ \langle(u_1,q_2),(0.3,0.1,0.2),(0.4,0.2,0.3),(0.6,0.1,0.1)\rangle,\\ \langle(u_2,q_1),(0.4,0.1,0.1),(0.6,0.1,0.1),(0.7,0.1,0.2)\rangle,\\ \langle(u_2,q_2),(0.6,0.1,0.2),(0.8,0.1,0.1),(0.5,0.1,0.2)\rangle,\\ \langle(u_3,q_1),(0.7,0.2,0.2),(0.6,0.1,0.1),(0.5,0.1,0.2)\rangle,\\ \langle(u_3,q_3),(0.2,0.3,0.4),(0.3,0.2,0.3),(0.6,0.1,0.3)\rangle\end{array}\right),\\ & (e_1,q,1),\left(\begin{array}{c}\langle(u_1,q_1),(0.6,0.1,0.1),(0.7,0.1,0.2),(0.8,0.1,0.1)\rangle,\\ \langle(u_1,q_2),(0.5,0.1,0.2),(0.6,0.1,0.1),(0.7,0.0,0.1)\rangle,\\ \langle(u_2,q_1),(0.2,0.3,0.2),(0.4,0.2,0.2),(0.5,0.1,0.2)\rangle,\\ \langle(u_2,q_2),(0.9,0.1,0.1),(0.7,0.2,0.1),(0.6,0.1,0.1)\rangle,\\ \langle(u_3,q_1),(0.8,0.1,0.1),(0.4,0.1,0.1),(0.4,0.2,0.3)\rangle,\\ \langle(u_3,q_3),(0.7,0.1,0.1),(0.8,0.0,0.1),(0.9,0.0,0.1)\rangle\end{array}\right),\\ & (e_2,q,1),\left(\begin{array}{c}\langle(u_1,q_1),(0.2,0.1,0.4),(0.4,0.2,0.1),(0.5,0.1,0.2)\rangle,\\ \langle(u_1,q_2),(0.5,0.2,0.1),(0.6,0.1,0.2),(0.8,0.1,0.1)\rangle,\\ \langle(u_2,q_1),(0.3,0.2,0.1),(0.5,0.1,0.1),(0.7,0.1,0.2)\rangle,\\ \langle(u_2,q_2),(0.4,0.1,0.1),(0.6,0.1,0.2),(0.7,0.1,0.2)\rangle,\\ \langle(u_3,q_1),(0.4,0.1,0.3),(0.5,0.2,0.1),(0.6,0.1,0.1)\rangle,\\ \langle(u_3,q_3),(0.4,0.1,0.1),(0.5,0.2,0.3),(0.6,0.2,0.1)\rangle\end{array}\right),\\ & (e_2,p,0),\left(\begin{array}{c}\langle(u_1,q_1),(0.2,0.2,0.4),(0.4,0.2,0.2),(0.5,0.1,0.2)\rangle,\\ \langle(u_1,q_2),(0.1,0.2,0.4),(0.2,0.1,0.3),(0.4,0.1,0.1)\rangle,\\ \langle(u_2,q_1),(0.3,0.2,0.1),(0.5,0.1,0.1),(0.7,0.2,0.3)\rangle,\\ \langle(u_2,q_2),(0.4,0.1,0.3),(0.6,0.1,0.2),(0.7,0.1,0.2)\rangle,\\ \langle(u_3,q_1),(0.3,0.2,0.3),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle,\\ \langle(u_3,q_3),(0.4,0.1,0.1),(0.5,0.2,0.3),(0.6,0.2,0.3)\rangle\end{array}\right)\Bigg\}\end{aligned}$$

Definition 13. *Let (F_Q,A) and (G_Q,B) be two multi Q-SVNSESs over U and Q. Then their intersection is a multi Q-SVNSES denoted by $(M_Q,C)=(F_Q,A)\,(G_Q,B)$, where $C=A\cup B$ and $\forall\, x\in C$, the multi Q-SVNSES (M_Q,C) is defined as follows:*

$$M_Q(x)=\begin{cases} F_Q(x) & if\ x\in A-B;\\ G_Q(x) & if\ x\in B-A;\\ (F_Q,A)\,(G_Q,B) & if\ x\in A\cap B.\end{cases}$$

Example 2. *From Example(4), we have $(M_Q,C)=(F_Q,A)\,(G_Q,B)$, where*

$$(H_Q,C) = \left\{ (e_1,p,1), \begin{pmatrix} \langle(u_1,q_1),(0.1,0.2,0.3),(0.2,0.4,0.4),(0.4,0.3,0.1)\rangle, \\ \langle(u_1,q_2),(0.2,0.1,0.5),(0.2,0.3,0.4),(0.3,0.1,0.3)\rangle, \\ \langle(u_2,q_1),(0.4,0.2,0.3),(0.1,0.4,0.4),(0.6,0.2,0.2)\rangle, \\ \langle(u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.3),(0.4,0.2,0.2)\rangle, \\ \langle(u_3,q_1),(0.2,0.3,0.4),(0.3,0.4,0.2),(0.4,0.3,0.5)\rangle, \\ \langle(u_3,q_2),(0.1,0.4,0.4),(0.1,0.2,0.5),(0.3,0.2,0.4)\rangle \end{pmatrix}, \right.$$

$$(e_1,q,1), \begin{pmatrix} \langle(u_1,q_1),(0.3,0.2,0.4),(0.4,0.2,0.3),(0.5,0.1,0.2)\rangle, \\ \langle(u_1,q_2),(0.4,0.2,0.2),(0.5,0.2,0.3),(0.6,0.2,0.1)\rangle, \\ \langle(u_2,q_1),(0.1,0.4,0.5),(0.3,0.3,0.4),(0.5,0.1,0.4)\rangle, \\ \langle(u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.2),(0.5,0.2,0.3)\rangle, \\ \langle(u_3,q_1),(0.2,0.4,0.2),(0.3,0.2,0.1),(0.3,0.2,0.4)\rangle, \\ \langle(u_3,q_3),(0.6,0.2,0.2),(0.7,0.2,0.2),(0.8,0.0,0.1)\rangle \end{pmatrix}$$

$$(e_2,q,1), \begin{pmatrix} \langle(u_1,q_1),(0.2,0.2,0.4),(0.4,0.2,0.2),(0.5,0.2,0.3)\rangle, \\ \langle(u_1,q_2),(0.1,0.2,0.4),(0.2,0.1,0.3),(0.4,0.1,0.1)\rangle, \\ \langle(u_2,q_1),(0.1,0.4,0.3),(0.3,0.2,0.4),(0.4,0.1,0.2)\rangle, \\ \langle(u_2,q_2),(0.3,0.2,0.3),(0.2,0.2,0.2),(0.4,0.2,0.3)\rangle, \\ \langle(u_3,q_1),(0.3,0.2,0.4),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle, \\ \langle(u_3,q_3),(0.2,0.4,0.3),(0.4,0.3,0.4),(0.5,0.2,0.3)\rangle \end{pmatrix}$$

$$\left. (e_2,p,0), \begin{pmatrix} \langle(u_1,q_1),(0.2,0.2,0.4),(0.4,0.2,0.2),(0.5,0.1,0.2)\rangle, \\ \langle(u_1,q_2),(0.1,0.2,0.4),(0.2,0.1,0.3),(0.4,0.1,0.1)\rangle, \\ \langle(u_2,q_1),(0.3,0.2,0.1),(0.5,0.1,0.1),(0.7,0.2,0.3)\rangle, \\ \langle(u_2,q_2),(0.4,0.1,0.3),(0.6,0.1,0.2),(0.7,0.1,0.2)\rangle, \\ \langle(u_3,q_1),(0.3,0.2,0.3),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle, \\ \langle(u_3,q_3),(0.4,0.1,0.1),(0.5,0.2,0.3),(0.6,0.2,0.3)\rangle \end{pmatrix} \right\}$$

Now from the definitions of union and intersection of multi Q-SVNSESs we have the following proposition.

Proposition 4.1. *Let (F_Q,A), (H_Q,B) and (N_Q,C) are any three multi Q-SVNSESs over U and Q. Then we have,*

$$(1)\ (F_Q,A)\cup(F_Q,A) = (F_Q,A)$$
$$(2)\ (F_Q,A)\cap(F_Q,A) = (F_Q,A)$$

$$(3)\ (F_Q,A)\cup((H_Q,B)\cup(N_Q,C)) = ((F_Q,A)\cup(H_Q,B))\cup(N_Q,C);$$
$$(4)\ (F_Q,A)\cap((H_Q,B)\cap(N_Q,C)) = ((F_Q,A)\cap(H_Q,B))\cap(N_Q,C);$$

$$(5)\ (F_Q,A)\cap((H_Q,B)\cup(N_Q,C)) = ((F_Q,A)\cap(H_Q,B))\cup((F_Q,A)\cap(N_Q,C));$$
$$(6)\ (F_Q,A)\cup((H_Q,B)\cap(N_Q,C)) = ((F_Q,A)\cup(H_Q,B))\cap((F_Q,A)\cup(N_Q,C)).$$

Proof. The proof of this proposition is easy, by using the properties of multi Q-SVNSs. □

5. AND AND OR OPERATIONS OF MULTI Q_SVNSESS

In this section we propose the definitions of AND and OR operations for multi Q-SVNSESs and give some examples.

Definition 14. *Let (F_Q, A) and (G_Q, B) be two multi Q-SVNSESs over U and Q. Then the "AND" operation for (F_Q, A) and (G_Q, B) is denoted by $(F_Q, A) \wedge (G_Q, B)$ and is defined as $(M, A \times B) = (F_Q, A) \wedge (G_Q, B)$ such that $M_Q(\rho, \varphi) = (F_Q(\rho) \cap G_Q(\varphi))$; $\forall\ (\rho, \varphi) \in A \times B$.*

. Suppose Example (4) Let

$$A = \{(e_1, p, 1), (e_2, p, 0)\}, \text{and} B = \{(e_1, p, 1), (e_1, q, 1)\}$$

$$(F_Q, A) = \left\{ \begin{array}{l} (e_1, p, 1), \left(\begin{array}{c} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right), \\ (e_2, p, 0), \left(\begin{array}{c} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right) \end{array} \right\}$$

and

$$(G_Q, B) = \left\{ \begin{array}{l} (e_1, p, 1), \left(\begin{array}{c} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.3), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right) \\ \left(e_1, q, 1), \left(\begin{array}{c} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.2, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.3, 0.3, 0.4), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.2), (0.3, 0.1, 0.1), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.2), (0.8, 0.0, 0.1), (0.9, 0.0, 0.1) \rangle \end{array} \right) \right. \end{array} \right\}$$

Then $(F_Q, A)\, AND\, (G_Q, B) =$

$$\left\{\begin{array}{c}((e_1,p,1),(e_1,p,1)), \begin{pmatrix} \langle (u_1,q_1),(0.1,0.2,0.4),(0.2,0.4,0.4),(0.4,0.3,0.1)\rangle, \\ \langle (u_1,q_2),(0.2,0.1,0.5),(0.2,0.3,0.4),(0.3,0.1,0.3)\rangle, \\ \langle (u_2,q_1),(0.4,0.2,0.3),(0.1,0.4,0.4),(0.6,0.2,0.2)\rangle, \\ \langle (u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.3),(0.5,0.2,0.2)\rangle, \\ \langle (u_3,q_1),(0.2,0.3,0.4),(0.3,0.4,0.2),(0.4,0.1,0.5)\rangle, \\ \langle (u_3,q_3),(0.1,0.4,0.4),(0.1,0.2,0.5),(0.3,0.2,0.4)\rangle \end{pmatrix}, \\ ((e_1,p,1),(e_1,q,1)), \begin{pmatrix} \langle (u_1,q_1),(0.1,0.2,0.3),(0.2,0.4,0.4),(0.4,0.3,0.1)\rangle, \\ \langle (u_1,q_2),(0.3,0.1,0.5),(0.4,0.2,0.3),(0.6,0.2,0.1)\rangle, \\ \langle (u_2,q_1),(0.2,0.3,0.2),(0.1,0.4,0.4),(0.5,0.2,0.2)\rangle, \\ \langle (u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.2),(0.4,0.2,0.3)\rangle, \\ \langle (u_3,q_1),(0.2,0.4,0.2),(0.3,0.1,0.1),(0.4,0.2,0.3)\rangle, \\ \langle (u_3,q_3),(0.2,0.3,0.4),(0.1,0.2,0.5),(0.3,0.2,0.4)\rangle \end{pmatrix}, \\ ((e_1,p,0),(e_1,p,1)) \begin{pmatrix} \langle (u_1,q_1),(0.1,0.2,0.4),(0.3,0.3,0.2),(0.4,0.3,0.2)\rangle, \\ \langle (u_1,q_2),(0.1,0.2,0.4),(0.2,0.3,0.4),(0.3,0.1,0.3)\rangle, \\ \langle (u_2,q_1),(0.3,0.2,0.3),(0.5,0.1,0.1),(0.7,0.2,0.3)\rangle, \\ \langle (u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.3),(0.5,0.1,0.2)\rangle, \\ \langle (u_3,q_1),(0.2,0.3,0.4),(0.3,0.4,0.2),(0.4,0.1,0.5)\rangle, \\ \langle (u_3,q_3),(0.1,0.4,0.4),(0.3,0.2,0.3),(0.6,0.2,0.3)\rangle \end{pmatrix}, \\ ((e_1,p,0),(e_1,q,1)) \begin{pmatrix} \langle (u_1,q_1),(0.2,0.2,0.4),(0.4,0.2,0.2),(0.5,0.1,0.2)\rangle, \\ \langle (u_1,q_2),(0.1,0.2,0.4),(0.2,0.2,0.3),(0.4,0.2,0.1)\rangle, \\ \langle (u_2,q_1),(0.2,0.3,0.2),(0.3,0.3,0.4),(0.5,0.2,0.3)\rangle, \\ \langle (u_2,q_2),(0.3,0.2,0.4),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle, \\ \langle (u_3,q_1),(0.2,0.4,0.3),(0.3,0.2,0.2),(0.4,0.2,0.3)\rangle, \\ \langle (u_3,q_3),(0.4,0.1,0.2),(0.5,0.2,0.3),(0.6,0.2,0.3)\rangle \end{pmatrix} \end{array}\right\}$$

Definition 15. *Let (F_Q, A) and (G_Q, B) be two multi Q-SVNSESs over U and Q. Then the "OR" operation for (F_Q, A) and (G_Q, B) is denoted by $(F_Q, A) \vee (G_Q, B)$ and is defined as $(M, A \times B) = (F_Q, A) \vee (G_Q, B)$ such that $M_Q(\rho, \varphi) = (F_Q(\rho) \cup G_Q(\varphi))\,; \forall\ (\rho, \varphi) \in A \times B$.*

Example 3. *Suppose Example* (5) $(F_Q, A)\,OR\,(G_Q, B)$=

$$\left\{ \begin{array}{l} ((e_1,p,1),(e_1,p,1))\,, \begin{pmatrix} \langle (u_1,q_1),(0.1,0.2,0.3),(0.3,0.3,0.2),(0.4,0.3,0.1)\rangle\,, \\ \langle (u_1,q_2),(0.3,0.1,0.2),(0.4,0.2,0.3),(0.6,0.1,0.1)\rangle\,, \\ \langle (u_2,q_1),(0.4,0.1,0.1),(0.6,0.1,0.1),(0.7,0.1,0.2)\rangle\,, \\ \langle (u_2,q_2),(0.6,0.1,0.2),(0.8,0.1,0.1),(0.5,0.1,0.2)\rangle\,, \\ \langle (u_3,q_1),(0.7,0.2,0.2),(0.6,0.1,0.1),(0.5,0.1,0.2)\rangle\,, \\ \langle (u_3,q_3),(0.2,0.3,0.4),(0.3,0.2,0.3),(0.6,0.1,0.3)\rangle \end{pmatrix}, \\ ((e_1,p,1),(e_1,q,1))\,, \begin{pmatrix} \langle (u_1,q_1),(0.6,0.1,0.1),(0.7,0.2,0.2),(0.8,0.1,0.1)\rangle\,, \\ \langle (u_1,q_2),(0.4,0.1,0.2),(0.5,0.2,0.3),(0.6,0.1,0.1)\rangle\,, \\ \langle (u_2,q_1),(0.4,0.2,0.1),(0.3,0.3,0.4),(0.6,0.1,0.2)\rangle\,, \\ \langle (u_2,q_2),(0.6,0.1,0.2),(0.8,0.1,0.1),(0.6,0.1,0.2)\rangle\,, \\ \langle (u_3,q_1),(0.7,0.2,0.2),(0.6,0.1,0.1),(0.5,0.3,0.2)\rangle\,, \\ \langle (u_3,q_3),(0.7,0.1,0.4),(0.8,0.0,0.1),(0.9,0.0,0.1)\rangle \end{pmatrix}, \\ ((e_2,p,0),(e_1,p,1))\,, \begin{pmatrix} \langle (u_1,q_1),(0.2,0.2,0.4),(0.4,0.2,0.2),(0.5,0.1,0.1)\rangle\,, \\ \langle (u_1,q_2),(0.2,0.1,0.2),(0.6,0.1,0.1),(0.7,0.1,0.1)\rangle\,, \\ \langle (u_2,q_1),(0.4,0.1,0.1),(0.6,0.1,0.1),(0.7,0.1,0.2)\rangle\,, \\ \langle (u_2,q_2),(0.4,0.1,0.3),(0.6,0.1,0.2),(0.7,0.1,0.2)\rangle\,, \\ \langle (u_3,q_1),(0.3,0.2,0.3),(0.4,0.2,0.2),(0.6,0.1,0.3)\rangle\,, \\ \langle (u_3,q_3),(0.4,0.1,0.1),(0.5,0.2,0.3),(0.6,0.1,0.3)\rangle \end{pmatrix}, \\ ((e_2,p,0),(e_1,q,1))\,, \begin{pmatrix} \langle (u_1,q_1),(0.6,0.1,0.1),(0.7,0.2,0.2),(0.8,0.1,0.1)\rangle\,, \\ \langle (u_1,q_2),(0.4,0.1,0.2),(0.5,0.1,0.3),(0.6,0.1,0.1)\rangle\,, \\ \langle (u_2,q_1),(0.3,0.2,0.1),(0.5,0.1,0.1),(0.7,0.1,0.2)\rangle\,, \\ \langle (u_2,q_2),(0.4,0.1,0.3),(0.6,0.1,0.2),(0.7,0.1,0.2)\rangle\,, \\ \langle (u_3,q_1),(0.3,0.2,0.2),(0.4,0.1,0.1),(0.6,0.1,0.3)\rangle\,, \\ \langle (u_3,q_3),(0.7,0.1,0.1),(0.8,0.0,0.1),(0.9,0.0,0.1)\rangle \end{pmatrix} \end{array} \right\}$$

Definition 16. *Let N_Q be a multi Q-single valued neutrosophic soft expert element. Then the the score function of N_Q is defined as follows:*

$$S_c(N_Q) = \frac{1}{k}_j \frac{2 + \mu_j(u_i, q) - \nu_j(u_i, q) - \xi_j(u_i, q)}{3}$$

6. An Application of Multi Q-SVNSESs in Decision Making

In this section, we present an application of multi Q-SVNSES theory in a decision making problem by using the score values of each element in agree multi Q-SVNSESs and disagree multi Q-SVNSESs.

Example 4. *Let us suppose that a university wants to fill the vacancy of assistant professor in the department of mathematics to be chosen by the expert committee. Suppose that the candidates passed the entry test is three. Let the $U = \{u_1, u_2, u_3\}$ be the set of candidates with two types of qualifications $Q = \{q_1 = master\ of\ science,\ q_2 = PhD\}$ and the set of parameters consider by the hiring committee is $E = \{e_1 =$experience$, e_2 =$*

$teaching\ skills\}$. *Let* $X = \{p, q\}$ *be the two expert of the committee. After a lot of discussion the committee formulate the following multi Q-SVNSES.*

$$(M_Q, D) = \left\{ (e_1, p, 1), \begin{pmatrix} (u_1, q_1), (0.3, 0.1, 0.2), (0.3, 0.2, 0.2), (0.4, 0.1, 0.1), \\ (u_1, q_2), (0.2, 0.1, 0.1), (0.2, 0.2, 0.1), (0.6, 0.2, 0.3), \\ (u_2, q_1), (0.2, 0.3, 0.4), (0.4, 0.2, 0.4), (0.4, 0.3, 0.4), \\ (u_2, q_2), (0.1, 0.2, 0.3), (0.4, 0.3, 0.3), (0.5, 0.3, 0.3), \\ (u_3, q_1), (0.2, 0.4, 0.4), (0.3, 0.3, 0.3), (0.4, 0.2, 0.3), \\ (u_3, q_2), (0.2, 0.1, 0.3), (0.2, 0.2, 0.2), (0.5, 0.2, 0.2) \end{pmatrix} \right. ,$$

$$(e_1, p, 1), \begin{pmatrix} (u_1, q_1), (0.2, 0.1, 0.2), (0.3, 0.2, 0.3), (0.5, 0.1, 0.1), \\ (u_1, q_2), (0.1, 0.2, 0.3), (0.2, 0.1, 0.2), (0.3, 0.2, 0.1), \\ (u_2, q_1), (0.2, 0.4, 0.4), (0.3, 0.2, 0.3), (0.5, 0.2, 0.2), \\ (u_2, q_2), (0.2, 0.2, 0.3), (0.3, 0.1, 0.2), (0.5, 0.2, 0.1), \\ (u_3, q_1), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.2, 0.2), \\ (u_3, q_2), (0.2, 0.2, 0.2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3) \end{pmatrix},$$

$$(e_2, p, 1), \begin{pmatrix} (u_1, q_1), (0.1, 0.3, 0.4), (0.3, 0.2, 0.2), (0.5, 0.1, 0.4), \\ (u_1, q_2), (0.2, 0.3, 0.3), (0.3, 0.1, 0.3), (0.4, 0.2, 0.2), \\ (u_2, q_1), (0.2, 0.2, 0.4), (0.3, 0.2, 0.4), (0.5, 0.2, 0.3), \\ (u_2, q_2), (0.1, 0.2, 0.4), (0.3, 0.2, 0.3), (0.5, 0.2, 0.2), \\ (u_3, q_1), (0.2, 0.1, 0.4), (0.3, 0.2, 0.3), (0.4, 0.1, 0.2), \\ (u_3, q_2), (0.3, 0.1, 0.2), (0.3, 0.1, 0.1), (0.4, 0.1, 0.2) \end{pmatrix}$$

$$(e_2, q, 1), \begin{pmatrix} (u_1, q_1), (0.2, 0.2, 0.4), (0.3, 0.1, 0.2), (0.4, 0.1, 0.3), \\ (u_1, q_2), (0.1, 0.1, 0.5), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), \\ (u_2, q_1), (0.3, 0.1, 0.4), (0.4, 0.1, 0.3), (0.4, 0.1, 0.2), \\ (u_2, q_2), (0.2, 0.1, 0.3), (0.3, 0.2, 0.2), (0.5, 0.1, 0.3), \\ (u_3, q_1), (0.1, 0.1, 0.2), (0.2, 0.2, 0.1), (0.3, 0.1, 0.2), \\ (u_3, q_2), (0.1, 0.2, 0.4), (0.2, 0.2, 0.3), (0.3, 0.2, 0.3), \end{pmatrix},$$

$$(e_1, p, 0), \begin{pmatrix} (u_1, q_1), (0.1, 0.2, 0.5), (0.3, 0.1, 0.3), (0.5, 0.1, 0.2), \\ (u_1, q_2), (0.2, 0.3, 0.3), (0.3, 0.2, 0.3), (0.4, 0.1, 0.2), \\ (u_2, q_1), (0.1, 0.1, 0.4), (0.2, 0.2, 0.3), (0.4, 0.2, 0.2), \\ (u_2, q_2), (0.1, 0.1, 0.3), (0.2, 0.1, 0.3), (0.3, 0.2, 0.3), \\ (u_3, q_1), (0.2, 0.2, 0.2), (0.3, 0.2, 0.2), (0.5, 0.1, 0.1), \\ (u_3, q_2), (0.1, 0.3, 0.2), (0.2, 0.1, 0.2), (0.3, 0.1, 0.3) \end{pmatrix},$$

$$(e_1, q, 0), \begin{pmatrix} (u_1, q_1), (0.2, 0.1, 0.1), (0.3, 0.1, 0.1), (0.5, 0.1, 0.1), \\ (u_1, q_2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.2), (0.4, 0.1, 0.3), \\ (u_2, q_1), (0.1, 0.2, 0.3), (0.2, 0.3, 0.4), (0.3, 0.4, 0.4), \\ (u_2, q_2), (0.2, 0.1, 0.3), (0.3, 0.2, 0.1), (0.4, 0.2, 0.3), \\ (u_3, q_1), (0.3, 0.1, 0.1), (0.3, 0.2, 0.1), (0.4, 0.3, 0.3), \\ (u_3, q_2), (0.2, 0.2, 0.2), (0.3, 0.3, 0.4), (0.5, 0.2, 0.2) \end{pmatrix}$$

$$
(e_2,p,0)\,,\left(\begin{array}{c}
(u_1,q_1),(0.3,0.1,0.1)\,,(0.3,0.2,0.3)\,,(0.4,0.2,0.1)\,,\\
(u_1,q_2),(0.2,0.3,0.4)\,,(0.3,0.2,0.3)\,,(0.5,0.2,0.2)\,,\\
(u_2,q_1),(0.1,0.2,0.4)\,,(0.2,0.3,0.3)\,,(0.4,0.2,0.2)\,,\\
(u_2,q_2),(0.2,0.2,0.2)\,,(0.3,0.2,0.1)\,,(0.5,0.2,0.3)\,,\\
(u_3,q_1),(0.2,0.1,0.2)\,,(0.2,0.1,0.3)\,,(0.6,0.2,0.2)\,,\\
(u_3,q_2),(0.1,0.1,0.2)\,,(0.4,0.2,0.2)\,,(0.5,0.2,0.2)
\end{array}\right)
$$

$$
\left.(e_2,q,0)\,,\left(\begin{array}{c}
(u_1,q_1),(0.1,0.3,0.5)\,,(0.2,0.2,0.3)\,,(0.3,0.3,0.4)\,,\\
(u_1,q_2),(0.2,0.2,0.6)\,,(0.2,0.1,0.2)\,,(0.5,0.1,0.2)\,,\\
(u_2,q_1),(0.3,0.1,0.1)\,,(0.3,0.1,0.2)\,,(0.4,0.1,0.2)\,,\\
(u_2,q_2),(0.2,0.2,0.4)\,,(0.4,0.2,0.3)\,,(0.4,0.2,0.2)\,,\\
(u_3,q_1),(0.2,0.1,0.3)\,,(0.3,0.1,0.2)\,,(0.5,0.2,0.3)\,,\\
(u_3,q_2),(0.1,0.4,0.3)\,,(0.3,0.4,0.3)\,,(0.6,0.2,0.3)
\end{array}\right)\right\}
$$

The following algorithm may be followed by the university committee to fill the vacancy of assistant professor.

(1) Input the multi Q-SVNSES (M_Q, D).

(2) Find agree-multi Q-SVNSES and disagree-multi Q-SVNSES.

(3) Find the score values using Definition (16) of each agree-multi Q-SVN soft expert element and disagree-multi Q-SVN soft expert element.

(4) Find $Y_j =_i \widetilde{S}\,(u,q)_{ij}$ for agree-multi Q-SVN soft expert elements.

(5) Find $Z_j =_i \widetilde{S}\,(u,q)_{ij}$ for disagree-multi Q-SVN soft expert elements.

(6) Find $K_j = Y_j - Z_j$.

(7) Find k, for which $K_k = \max K_j$, where $K_k$is the optimal choice object. If k has more than one value, then any one could be selected by the university committee according to their option.

Table 1 and Table 2 respectively represent the agree-multi Q-SVNSES, disagree-multi Q_SNSES and the score values of each multi Q-SVN soft expert elements.

$Table$ 1 presentation of agree-multi Q-SVNSES

$U \times Q$	(u_1,q_1)	(u_1,q_2)	(u_2,q_1)	(u_2,q_2)	(u_3,q_1)	(u_3,q_2)
$(e_1,p,1)$	0.678	0.667	0.556	0.589	0.556	0.633
$(e_1,q,1)$	0.667	0.611	0.589	0.656	0.589	0.600
$(e_2,p,1)$	0.589	0.611	0.589	0.600	0.622	0.689
$(e_2,q,1)$	0.622	0.622	0.656	0.644	0.633	0.556
$Y_j =_i \widetilde{S}\,(u,q)_{ij}$	$y_1 = 2.556$	$y_2 = 2.511$	$y_3 = 2.389$	$y_4 = 2.489$	$y_5 = 2.400$	$y_6 = 2.478$

$Table$ 2 presentation of disagree-multi Q-SVNSES

$U \times Q$	(u_1,q_1)	(u_1,q_2)	(u_2,q_1)	(u_2,q_2)	(u_3,q_1)	(u_3,q_2)
$(e_1,p,0)$	0.611	0.611	0.589	0.589	0.667	0.600
$(e_1,q,0)$	0.711	0.611	0.511	0.633	0.656	0.611
$(e_2,p,0)$	0.667	0.600	0.567	0.644	0.656	0.656
$(e_2,q,0)$	0.511	0.611	0.689	0.611	0.644	0.567
$Z_j =_i \widetilde{S}\,(u,q)_{ij}$	$z_1 = 2.500$	$z_2 = 2.433$	$z_3 = 2.356$	$z_4 = 2.478$	$z_5 = 2.622$	$z_6 = 2.433$

From Table 1 and Table 2 we able to find the value of K_j as shown in Table 3.

$Table.$ 3 $K_j{=}Y_j{-}Z_j$

j	$U \times Q$	Y_j	Z_j	K_j
1	(u_1,q_1)	2.556	2.500	0.056
2	(u_1,q_2)	2.511	2.433	0.078
3	(u_2,q_1)	2.389	2.356	0.033
4	(u_2,q_2)	2.489	2.478	0.011
5	(u_3,q_1)	2.400	2.622	-0.222
6	(u_3,q_2)	2.478	2.433	0.044 height

So from Table 3 the maximum value of $K_j = 0.078$, so $k = 0.078$, hence the university committee will choose the candidate u_1 with PhD degree for the job.

7. Comparison and discussion

In this we propose the concept of multi Q-SVNSES which is the generalization of Q-fuzzy soft expert set defined by F. Adam et al.[6] . The concept of multi Q-fuzzy soft expert set can not explain the universal set U in detail while the concept of multi Q-SVNSES defined in this article can explain the universal set U in more detail with three membership functions namely, truth-membership, indeterminacy-membership and falsity-membership. To illustrate the advantages of our proposed method with that of the method defined by F.Adam et al. [6] let us consider the above Example 4 by neglecting the falsity-membership and indeterminacy-membership functions. That is $\nu_j(u_i, q) = 0$ and $\xi_j(u_i, q) = 0$.Then the multi Q-fuzzy soft expert set can define the above information as follows:

$$(M_Q, D) = \left\{ (e_1, p, 1), \left(\begin{array}{c} ((u_1, q_1), (0.3, 0.3, 0.4)), ((u_1, q_2), (0.2, 0.2, 0.6)), \\ ((u_2, q_1), (0.2, 0.4, 0.4)), ((u_2, q_2), (0.1, 0.4, 0.5)), \\ ((u_3, q_1), (0.2, 0.3, 0.4)), ((u_3, q_2), 0.2, 0.2, 0.5) \end{array} \right), ... \right\}$$

Now using the algorithm defined by F. Adam et al. [6] we have,

Table 4 and Table 5 respectively represent the agree-multi Q-fuzzy soft expert set,disagree-multi Q-fuzzy soft expert sets and the mean of each multi Q-fuzzy soft expert sets.

$Table$ 4 presentation of agree-multi Q-fuzzy soft expert set

$U \times Q$	(u_1, q_1)	(u_1, q_2)	(u_2, q_1)	(u_2, q_2)	(u_3, q_1)	(u_3, q_2)
$(e_1, p, 1)$	0.333	0.333	0.333	0.333	0.333	0.300
$(e_1, q, 1)$	0.333	0.200	0.333	0.333	0.233	0.300
$(e_1, p, 1)$	0.300	0.300	0.333	0.300	0.300	0.333
$(e_2, q, 1)$	0.300	0.333	0.367	0.333	0.200	0.200
$C_j =_{ij} (u, q)_{ij}$	$c_1 = 1.267$	$c_2 = 1.167$	$c_3 = 1.367$	$c_4 = 1.300$	$c_5 = 1.067$	$c_6 = 1.133$ height

$Table$ 5 presentation of disagree-multi Q-fuzzy soft expert set

$U \times Q$	(u_1, q_1)	(u_1, q_2)	(u_2, q_1)	(u_2, q_2)	(u_3, q_1)	(u_3, q_2)
$(e_1, p, 0)$	0.300	0.300	0.233	0.200	0.333	0.2
$(e_1, p, 0)$	0.333	0.300	0.200	0.300	0.333	0.333
$(e_1, p, 0)$	0.333	0.333	0.233	0.333	0.333	0.333
$(e_2, q, 0)$	0.200	0.300	0.333	0.333	0.333	0.333
$K_j =_{ij} (u, q)_{ij}$	$k_1 = 1.167$	$k_2 = 1.233$	$k_3 = 1.000$	$k_4 = 1.167$	$k_5 = 1.333$	$k_6 = 1.200$ height

$Table.$ 3 $S_j = C_j - K_j$

j	$U \times Q$	C_j	K_j	S_j
1	(u_1, q_1)	1.267	1.167	0.100
2	(u_1, q_2)	1.167	1.233	-0.067
3	(u_2, q_1)	1.367	1.000	0.367
4	(u_2, q_2)	1.300	1.167	0.133
5	(u_3, q_1)	1.067	1.333	-0.267
6	(u_3, q_2)	1.133	1.200	-0.067 height

So from Table 3 we can see that the maximum value $r = 0.367$. So the university committee will choose the candidate u_2 with master of science degree.

As we can see that the result obtained through using the method defined by F. Adam et al. [6] and the result obtained by using the method in this article is totally different. The reason is that the multi Q-fuzzy soft expert set can handle a limited information while the concept of multi Q-SVNSES proposed in this article can handle incomplete, inconsistent and indeterminate data. Which makes the concept and method proposed in this article more accurate and realistic then that defined by F. Adam et al.[6].

8. CONCLUSION

In this article we reviewed the basic concept of Q-SVNSESs, proposed some operational laws such as, complement, intersection, union, OR and AND and discussed some basic properties related to these operations. Further, we defined an algorithm and applied to solve a decision making problem of selecting assistant professor. Finally a comparison had been made through example to show the advantages and practicality of the proposed concept and decision making algorithm with the existing concept of multi Q-fuzzy soft expert set. This new concept of multi Q-SVNSES will provide a remarkable addition to the existing theories for handling inconsistent and indeterminate information. This new concept of multi Q-SVNSES will stimulant more progress in research and relevant applications.

REFERENCES

[1] S. E. Abbas, M. A. Hebeshi and I. M. Taha, *On Upper and Lower Contra-Continuous Fuzzy Multifunctions*, Punjab Univ. j. math. **47**, No. 1 (2017) 105-117.
[2] B. Ahmad, & A. Kharal, *On fuzzy soft sets*, Advances in Fuzzy Systems, 2009.
[3] F. Adam and N. Hassan, *Q-fuzzy soft set* Applied Mathematical Sciences, **8**,NO.174 (2014) 8689-8695.
[4] F. Adam and N. Hassan *Operations on Q-fuzzy soft set* Applied Mathematical Sciences, **8**, NO.175 (2014)8697-8701.
[5] F. Adam and N. Hassan, *Multi Q-fuzzy soft set and its application* Far East Journal of Mathematical Sciences, **97**,NO.7(2015) 871.
[6] F. Adam, and N. Hassan, Multi Q-fuzzy soft expert set and its application Journal of Intelligent & Fuzzy Systems, **30**,NO.2,(2016) 943-950.
[7] M. Akram and G. Shahzadi, *Certain Characterization of m-Polar Fuzzy Graphs by Level Graphs*, Punjab Univ. j. math. **49**, No. 1 (2017) 1-12.
[8] *A new capacity for plurisubharmonic functions*, Acta Math. **170**, (1988) 1-21.
[9] S. Alkhazaleh, A. R. Salleh, and N. Hassan *Possibility fuzzy soft set*, Advances in Decision Sciences, 2011.
[10] S. Alkhazaleh, A. R. Salleh, and N. Hassan, *Fuzzy parameterized interval-valued fuzzy soft set*, Applied Mathematical Sciences, **5**NO.67 (2011)3335-3346.
[11] S. Alkhazaleh, A. R. Salleh, A. R., and N. Hassan,*Soft multisets theory*, Applied Mathematical Sciences, **5**,NO.72, (2011) 3561-3573.
[12] S. Alkhazaleh & A. R. Salleh, *Fuzzy soft multiset theory*, In Abstract and Applied Analysis (Vol. 2012). Hindawi Publishing Corporation.
[13] S. Alkhazaleh, and A. R. Salleh, *Soft expert sets* Advances in Decision Sciences, (2012).
[14] *Tranchage et prolongement des courants positifs fermes*, Math. Ann. **507**, (1991) 673-687.
[15] K. T. Atanassov Intuitionistic fuzzy sets. Fuzzy sets and Systems, **20**,NO.1 (1986) 87-96.
[16] S. Broumi, I. Deli and F. Smarandache, *Neutrosophic parametrized soft set theory and its decision making* International Frontier Science Letters, **1**,NO.1 (2014) 1-11.
[17] S. Broumi, *Q-intuitionistic fuzzy soft sets* J New Theory, **5**, (2015) 80-91.
[18] N. Cagman, S. Enginoglu, and F. Citak, *Fuzzy soft set theory and its applications* Iranian Journal of Fuzzy Systems, **8**,NO.3 (2011) 137-1
[19] N. Cagman and S. Karatas, *Intuitionistic fuzzy soft set theory and its decision making* Journal of Intelligent & Fuzzy Systems, **24**,NO.4 (2013)829-836.
[20] N. Cagman, and S. Enginoglu, *Fuzzy soft matrix theory and its application in decision making*, Iranian Journal of Fuzzy Systems, **9**,NO.1 (2012)109-119.
[21] J. P. Domnay, *Complex Analytic and Differential Geometry*, http://www-fourier.ujf-grenoble.fr/demailly/books.html
[22] A. Fahmi, S. Abdullah, F. Amin and A. Ali, *Weighted Average Rating (War) Method for Solving Group Decision Making Problem Using Triangular Cubic Fuzzy Hybrid Aggregation (Tcfha) Operator*, Punjab Univ. j. math. **50**, No. 1 (2018) 23-34.
[23] Y. Jiang, Y. Tang, Q. Chen, H. Liu, and J. Tang, *Interval-valued intuitionistic fuzzy soft sets and their properties*, Computers & Mathematics with Applications, **60**NO.3,(2010) 906-918.

[24] H. Kamaci, A. O. Atagun and E. Aygun, *Difference Operations of Soft Matrices with Applications in Decision Making*, Punjab Univ. j. math. **51**, No. 3 (2019) 1-21.
[25] M. A. Khan and Sumitra, *Common Fixed Point Theorems for Converse Commuting and OWC Maps in Fuzzy Metric Spaces*, Punjab Univ. j. math. **44**, (2012) 57-63.
[26] M. Sajjad Ali Khan, K. Rahman, A. Fahm and M. Shakeel, *Generalized* $(\epsilon,\epsilon)-FuzzyQuasi-IdealsinSemigroups, PunjabUniv.j.math.\mathbf{50}, No.1(2018)35-53.$
[27] Q. Khan, P. Liu, T. Mahmood, F. Smarandache, and K. Ullah, *Some Interval Neutrosophic Dombi Power Bonferroni Mean Operators and Their Application in Multi–Attribute Decision–Making*. Symmetry, **10**, No. 10 (2018) 459.
[28] Q. Khan, N. Hassan, and Mahmood, *Neutrosophic Cubic Power Muirhead Mean Operators with Uncertain Data for Multi-Attribute Decision-Making* Symmetry, **10**NO.10 (2018) 444.
[29] *A characterization of Holomorphic chains*, Ann. Math. **18**, (1974) 253-287.
[30] P. Liu, T. Mahmood, and Q. Khan, *Group decision making based on power Heronian aggregation operators under linguistic neutrosophic environment*, International Journal of Fuzzy Systems, **20**, No. 3 (2018) 970-985.
[31] T. Mahmood, Q. Khan, and M. A. Khan, *Q-SINGLE VALUED NEUTROSOPHIC SOFT SETS* Journal of New Theory, **13**, (2016)10-25.
[32] T. Mahmood, F. Mehmood and Q. Khan, *Some Generalized Aggregation Operators for Cubic Hesitant Fuzzy Sets and Their Applications to Multi Criteria Decision Making*,Punjab Univ. j. math. **49**, No. 1 (2017) 31-49.
[33] P. K. Maji, R. K. Biswas, and A. R. Roy, Intuitionistic fuzzy soft sets. Journal of fuzzy mathematics, **9** NO.3,(2001) 677-692.
[34] P. K. Maji, R. K. Biswas and A. Roy, *Soft set theory* Computers & Mathematics with Applications, **45**, NO.(4-5), (2003) 555-562.
[35] P. K. Maji, R. K. BISWAS, and A. Roy,*Fuzzy soft sets*, (2001) 589-602.
[36] P. K. Maji,*Neutrosophic soft set*, Annals of Fuzzy Mathematics and Informatics, **5** NO.1,(2013) 157-168.
[37] D. Molodtsov, *Soft set theory—first results*. Computers & Mathematics with Applications, **37**NO.(4-5), (1999) 19-31.
[38] H. A. Othman, *ON Fuzzy Infra-Semiopen Sets*, Punjab Univ. j. math. **48**, No. 2 (2016) 1-10.
[39] *Geometric Measure Theory*, Berlin, New-York, Springer-Verlag, 1989.
[40] K. Rahman, A. Ali and M. S. A. Khan, *Some Interval-Valued Pythagorean Fuzzy Weighted Averaging Aggregation Operators and Their Application to Multiple Attribute Decision Making*, Punjab Univ. j. math. **50**, No. 2 (2018) 113-129.
[41] K. P. R. Rao and K. V. S. Parvathi, *General Common Fixed Point Theorems in Fuzzy Metric Spaces*, Punjab Univ. j. math. **44**, (2012) 51-55.
[42] M. Riaz and S. T. Tehrim, *Certain Properties of Bipolar Fuzzy Soft Topology Via Q-neighborhood*, Punjab Univ. j. math. **51**, No. 2 (2019) 155-172.
[43] M. Riaz and K. Naeem, *Measurable Soft Mappings*, Punjab Univ. j. math. **48**, No. 2 (2016) 19-34.
[44] M. Riaz and M. R. Hashmi, *Fuzzy Parameterized Fuzzy Soft Compact Spaces with Decision-Making*, Punjab Univ. j. math. **50**, No. 2 (2018) 131-145.
[45] S. Roy and T. K. Samanta, *A note on a soft topological space*, Punjab Univ. j. math. **46**, No. 1 (2014) 19-24.
[46] A. R. Roy and P. K. Maji, *A fuzzy soft set theoretic approach to decision making problems* Journal of Computational and Applied Mathematics, **203** NO.2,(2007) 412-418.
[47] M. Shakeel, S. Abdullah and A. Fahmi, *Triangular Cubic Power Aggregation Operators and Their Application to Multiple Attribute Group Decision Making*, Punjab Univ. j. math. **50**, No. 3 (2018) 75-99.
[48] M. Shakeel, S. Abdullah, M. Sajjad Ali Khan and K. Rahman, *Averaging Aggregation Operators with Interval Pythagorean Trapezoidal Fuzzy Numbers and Their Application to Group Decision Making*, Punjab Univ. j. math. Math. **50**, No. 2 (2018) 147-170.
[49] F. Smarandache, Neutrosophy: Neutrosophic Probability, Set, and Logic: Analytic Synthesis & Synthetic Analysis (1998).
[50] F. Smarandache, *A unifying field in logics* (No. math. GM/0101228).
[51] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability: Neutrsophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. Infinite Study (2005).

[52] M. G. Voskoglou, *Application of Fuzzy Numbers to Assessment of Human Skills*, Punjab Univ. j. math. **50**, No. 2 (2018) 85-96.

[53] *Opérateur de Monge Ampère, tranchage et extention de courants positifs fermés, Thèse d'état*, Fac. Sc. Tunis 1996.

[54] H. Wang, F. Smarandache, R. Sunderraman and Y. Q. Zhang, *interval neutrosophic sets and logic: theory and applications in computing: Theory and applications in computing* **5** (2005) Infinite Study.

[55] H. Wang, F. Smarandache, Y., Zhang and R. Sunderraman, *Single valued neutrosophic sets* Review of the Air Force Academy, NO.1, (2010)1-10.

[56] Y. J. Xu, Y. K. Sun and D. F. Li, *Intuitionistic fuzzy soft set. In Intelligent Systems and Applications* (ISA), 2010 2nd International Workshop on (pp. 1-4). IEEE.

[57] X. Yang, T. Y. Lin, J. Yang, Y., Li and D. Yu, Combination of interval-valued fuzzy set and soft set. Computers & Mathematics with Applications, **58**, NO.3 (2009) 521-527.

[58] J. Ye, Multi-criteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, International Journal of General Systems **42**, No. 4 (2013)386-394.

[59] J. Ye, A multi-criteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of Intelligent & Fuzzy Systems **26**, No. 5 (2014) 2459-2466.

[60] L. A Zadeh, *Fuzzy sets*, Information and control, **8**,NO.3 (1965) 338-353.

[61] K. Zhu & J. Zhan, *Fuzzy parametrized fuzzy soft sets and decision making*, International Journal of Machine Learning and Cybernetics, **7**,NO.(6)(2016) 1207-1212.