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Multi-Valued Neutrosophic Distance-Based QUALIFLEX Method for Treatment Selection

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Abstract: Multi-valued neutrosophic sets (MVNSs) consider the truth-membership, indeterminacy-membership, and falsity-membership simultaneously, which can more accurately express the preference information of decision-makers. In this paper, the normalized multi-valued neutrosophic distance measure is developed firstly and the corresponding properties are investigated as well. Secondly, the normalized multi-valued neutrosophic distance difference is defined and the corresponding partial ordering relation is discussed. Thirdly, based on the developed distances and comparison method, an extended multi-valued neutrosophic QUALItative FLEXible multiple criteria (QUALIFLEX) method is proposed to handle MCDM problems where the weights of criteria are completely unknown. Finally, an example for selection of medical diagnostic plan is provided to demonstrate the proposed method, together with sensitivity analysis and comparison analysis.

1. Introduction

Recently, neutrosophic sets (NSs) [1–3] have become very useful in many areas [4–8] since they collect data and provide some available information. However, because of the complexity and ambiguity of information in the real decision-making process, it is difficult for decision-makers to express their preference accurately by using their extensions, including single-valued neutrosophic sets (SNSs) [9,10] and interval neutrosophic sets (INSs) [11], interval-valued neutrosophic soft sets [12], neutrosophic soft multi-set [13], and neutrosophic refined sets [14]. Then, based on the definitions of SNSs and the hesitant fuzzy sets (HFSs) [15,16], Wang and Li [17] and Ye [18] defined the concept of multi-valued neutrosophic sets (MVNSs) and single-valued neutrosophic hesitant fuzzy sets (SVNHFSs), respectively. MVNSs and SVNHFSs are denoted by truth-membership, indeterminacy-membership, and falsity-membership functions, which comprise of a set of numerical numbers between zero and one. In recent years, MVNSs and SVNHFSs have been extensively studied and applied to different fields. For example, Peng et al. [19–22] defined multi-valued neutrosophic preference relations, outranking relations and aggregation operators. Ji et al. [23] defined an extended an acronym in Portuguese of the Interactive and Multicriteria Decision Making (TODIM) method with multi-valued neutrosophic information. Finally, based on the concept of MVNSs, Peng et al. [24] defined probability MVNSs, and Wu and Wang [25] investigated some cross-entropy measures of MVNSs and applied them to the selection of a middle-level manager.

Furthermore, those aforementioned methods with MVNSs always involve in operations and measures which impact on the final decision-making may be momentous. However, there exist other methods to avoid these defects, namely the relation model. Relation models could rank the alternatives

in terms of priority among the criteria by using outranking relations or priority functions, such as Elimination Et Choix Traduisant la REalité (ELECTRE) [26,27] and QUALItative FLEXible multiple criteria method (QUALIFLEX) [28–30].

The QUALIFLEX method, which was introduced by Paelinck [28–30], is an effective outranking method to handle multi-criteria decision-making (MCDM) problems by arranging a set of preference rankings [31,32]. Moreover, QUALIFLEX method assumes that all possible permutations of the alternatives are determined. Then the best permutation can be identified by maximizing the concordance/discordance index value based on the pair-wise comparisons of alternatives under each criterion [33,34]. The principal advantage of QUALIFLEX method is that it can effectively handle decision-making problems where the criteria numbers are more than the alternative numbers obviously. Recently, several extended QUALIFLEX methods have been developed [35–39]. Moreover, Ji et al. [40] defined a triangular neutrosophic QUALIFLEX-TODIM method for treatment selection. Li and Wang [41] developed a probability hesitant fuzzy QUALIFLEX method to select green suppliers.

Based on the aforementioned studies, some attempts have been made to define outranking relations, preference, aggregation operators, and cross-entropy measures of MVNSs. According to the existing distances between MVNSs, the TODIM method proposed by Wang and Li [17] and Ji et al. [40] forms part of the multi-valued neutrosophic distance, which is a simple extension based on the Hamming distance. Moreover, if the distance of SNSs and HFSs were extended to MVNSs, then this should satisfy the conditions that two MVNSs should be of equal length, i.e., the lengths of the three memberships should be equal and ranked in ascending order; otherwise it needs to add the same element to the shorter one. Thus, in order to address this shortcoming, the main goals of this paper are: (1) provide the improved distance measure of MVNSs and the corresponding distance difference; and (2) extend the QUALIFLEX method to a multi-valued neutrosophic environment based on the proposed distance difference.

The paper is constructed as follows: In Section 2, some definitions and operations of MVNSs are introduced. Then the normalized multi-valued neutrosophic distance measure and corresponding difference distance as well as comparison method are developed in Section 3. The multi-valued distance-based QUALIFLEX method with incomplete weight information is constructed in Section 4. In Section 5, the selection of a medical diagnostic plan is presented to demonstrate the proposed approach. Finally, we summarize the paper with further discussion in Section 6.

2. MVNSs

Some definitions and operations of MVNSs are reviewed in this section, which will be utilized in later analysis.

Definition 1. [17,18] *A MVNS* M *in a space of points (objects)* X *can be expressed as* $M = \left\{ \left\langle x, \dot{T}(x), \dot{I}(x), \dot{F}(x) \right\rangle | x \in X \right\}$. $\dot{T}(x)$, $\dot{I}(x)$, *and* $\dot{F}(x)$ *are denoted by HFSs respectively, i.e., three sets of numerical number in [0,1], denoting the truth-membership degree, indeterminacy-membership function and falsity-membership degree respectively, and satisfying* $0 \leq \gamma, \eta, \xi \leq 1, 0 \leq \gamma^+ + \eta^+ + \xi^+ \leq 3$, *where* $\gamma \in \dot{T}(x)$, $\eta \in \dot{I}(x)$, $\xi \in \dot{F}(x)$, $\gamma^+ = \sup \dot{T}(x)$, $\eta^+ = \sup \dot{I}(x)$ *and* $\xi^+ = \sup \dot{F}(x)$.

If there exists only one element in X, *then* M *is called a multi-valued neutrosophic number (MVNN), denoted by* $M = \left\langle \dot{T}(x), \dot{I}(x), \dot{F}(x) \right\rangle$. *For convenience, a MVNN can be denoted by* $M = \left\langle \dot{T}, \dot{I}, \dot{F} \right\rangle$.

Definition 2. [19] *Let* M $\in$ *MVNSs, the complement of* M *can be defined as* $M^C = \left\{ \left\langle x, \dot{T}^C(x), \dot{I}^C(x), \dot{F}^C(x) \right\rangle | x \in X \right\}$. *Here* $\dot{T}^C(x) = \cup_{\xi \in \dot{F}(x)} \{\xi(x)\}$, $\dot{I}^C(x) = \cup_{\eta \in \dot{I}(x)} \{1 - \eta(x)\}$ *and* $\dot{F}^C(x) = \cup_{\gamma \in \dot{T}(x)} \{\gamma(x)\}$ *for all* $x \in X$.

Definition 3. *Let* $M_1 = \left\{ \left\langle x, \dot{T}_1(x), \dot{I}_1(x), \dot{F}_1(x) \right\rangle \big| x \in X \right\}$ *and* $M_2 = \left\{ \left\langle x, \dot{T}_2(x), \dot{I}_2(x), \dot{F}_2(x) \right\rangle \big| x \in X \right\}$ *be two MVNSs. Then we have* $M_1 \subseteq M_2$ *if* $\gamma_1(x) \leq \gamma_2(x)$, $\eta_1(x) \geq \eta_2(x)$ *and* $\xi_1(x) \geq \xi_2(x)$ *for any* $x \in X$. *Here* $\gamma_i(x) \in \dot{T}_i(x)$, $\eta_i(x) \in \dot{I}_i(x)$ *and* $\xi_i(x) \in \dot{F}_i(x)(i = 1, 2)$.

Definition 4. [19] *Let* $M_1 = \left\{ \left\langle x, \dot{T}_1(x), \dot{I}_1(x), \dot{F}_1(x) \right\rangle \big| x \in X \right\}$ *and* $M_2 = \left\{ \left\langle x, \dot{T}_2(x), \dot{I}_2(x), \dot{F}_2(x) \right\rangle \big| x \in X \right\}$ *be two MVNSs. Also let* $\forall x \in X$, *all values in* $\dot{T}_i(x)$, $\dot{I}_i(x)$ *and* $\dot{F}_i(x)(i = 1, 2)$ *be ranked in ascending order.* $\gamma_i^{\sigma(\cdot)}(x), \eta_i^{\sigma(\cdot)}(x)$ *and* $\xi_i^{\sigma(\cdot)}(x)$ *are the* $(\cdot)$-*th value in* $\dot{T}_i(x)$, $\dot{I}_i(x)$ *and* $\dot{F}_i(x)(i = 1, 2)$ *respectively. Then we have:*

$$M_1 \leq M_2 \text{ if } \gamma_1^{\sigma(j)}(x) \leq \gamma_2^{\sigma(j)}(x) \text{ and } \gamma_1^{\sigma(l_{\dot{T}_1})}(x) \leq \gamma_2^{\sigma(l_{\dot{T}_2})}(x) \left(j = 1, 2, \ldots, l_{\dot{T}(x)}; l_{\dot{T}(x)} = \min\left(l_{\dot{T}_1(x)}, l_{\dot{T}_2(x)} \right) \right),$$

$$\eta_1^{\sigma(k)}(x) \geq \eta_2^{\sigma(k)}(x) \text{ and } \eta_1^{\sigma(l_{\dot{I}_1})}(x) \geq \eta_2^{\sigma(l_{\dot{I}_2})}(x) \left(k = 1, 2, \ldots, l_{\dot{I}(x)}; l_{\dot{I}(x)} = \min\left(l_{\dot{I}(x)}, l_{\dot{I}(x)} \right) \right), \xi_1^{\sigma(m)}(x) \geq \xi_2^{\sigma(m)}(x)$$

$$\text{and } \xi_1^{\sigma(l_{\dot{F}_1})}(x) \geq \xi_2^{\sigma(l_{\dot{F}_2})}(x) \left(m = 1, 2, \ldots, l_{\dot{F}(x)}; l_{\dot{F}(x)} = \min\left(l_{\dot{F}(x)}, l_{\dot{F}(x)} \right) \right).$$

Where $l_{\dot{T}(x)}$, $l_{\dot{I}(x)}$ *and* $l_{\dot{F}(x)}$ *are the number of elements in* $\dot{T}_i(x)$, $\dot{I}_i(x)$ *and* $\dot{F}_i(x)(i = 1, 2)$ *respectively.*

Example 1. *Let* $M_1 = \{x, \langle \{0.1, 0.3\}, \{0.3\}, \{0.2\} \rangle\}$ *and* $M_2 = \{x, \langle \{0.2, 0.6\}, \{0.1, 0.2\}, \{0.1\} \rangle\}$ *be two MVNSs. According to the proposed comparison method in Definition 3, we have* $l_{\dot{T}} = \min\left(l_{\dot{T}_1}, l_{\dot{T}_2} \right) = 2$, $\gamma_1^{\sigma(1)} \leq \gamma_2^{\sigma(1)}$ *and* $\gamma_1^{\sigma(2)} \leq \gamma_2^{\sigma(2)}$, $l_{\tilde{I}} = \min\left(l_{\tilde{I}_1}, l_{\tilde{I}_2} \right) = 1$, $\eta_1^{\sigma(1)} \geq \eta_2^{\sigma(1)}$ *and* $\eta_1^{\sigma(1)} \geq \eta_2^{\sigma(2)}$, $l_{\dot{F}} = \min\left(l_{\dot{F}_1}, l_{\dot{F}_2} \right) = 1$ *and* $\xi_1^{\sigma(1)} \geq \xi_2^{\sigma(1)}$. *Therefore, we have* $M_1 \leq M_2$.

3. Multi-Valued Neutrosophic Distance Measures

Based on the intuitionistic fuzzy H-max distance defined in Ngan et al. [42], the normalized multi-valued neutrosophic distance is proposed. Then the multi-valued neutrosophic distance difference is defined in this section.

3.1. The Normalized Multi-Valued Neutrosophic Distance

Definition 5. *Let* M_1, M_2 *and* M_3 *be three MVNSs on the universe* $X = \{x_1, x_2, \ldots, x_n\}$. *A mapping* $D : MVNS(x) \times MVNS(x) \to R$ *is a normalized distance measure of MVNSs iff it satisfies the following axioms:*

H1. $0 \leq D(M_1, M_2) \leq 1$;
H2. $D(M_1, M_2) = D(M_2, M_1)$;
H3. $D(M_1, M_2) = 0$ *iff* $M_1 = M_2$;
H4. If $M_1 \leq M_2 \leq M_3$, *then* $D(M_1, M_3) \geq D(M_1, M_2)$ *and* $D(M_1, M_3) \geq D(M_2, M_3)$.

Definition 6. *Let* M_1 *and* M_2 *be two MVNSs on the universe* $X = \{x_1, x_2, \ldots, x_n\}$. *The normalized multi-valued neutrosophic measure of* M_1 *and* M_2 *can be defined as:*

$$D_{Gm}(M_1, M_2) = \left(\frac{1}{5n} \sum_{i=1}^{n} \left(\Delta\gamma_{i,12}^{\lambda} + \Delta\eta_{i,12}^{\lambda} + \Delta\xi_{i,12}^{\lambda} + \Delta_{12}^{\lambda}(\gamma_i, \eta_i) + \Delta_{12}^{\lambda}(\gamma_i, \xi_i) \right) \right)^{1/\lambda} (\lambda \geq 1). \quad (1)$$

where $\Delta\gamma_{i,12} = \max_{\substack{\gamma_1(x_i) \in \dot{T}_1(x_i) \\ \gamma_2(x_i) \in \dot{T}_2(x_i)}} |\gamma_1(x_i) - \gamma_2(x_i)|(i = 1, 2, \ldots, n);$

$\Delta\eta_{i,12} = \max_{\substack{\eta_1(x_i) \in \dot{I}_1(x_i) \\ \eta_2(x_i) \in \dot{I}_2(x_i)}} |\eta_1(x_i) - \eta_2(x_i)|(i = 1, 2, \ldots, n);$

$$\Delta\xi_{i,12} = \max_{\substack{\xi_1(x_i) \in \dot{F}_1(x_i) \\ \xi_1(x_i) \in \dot{F}_1(x_i)}} |\xi_1(x_i) - \xi_2(x_i)| (i = 1, 2, \ldots, n);$$

$$\Delta_{12}(\gamma_i, \eta_i) = \left| \max_{\substack{\gamma_1(x_i) \in \dot{T}_1(x_i) \\ \eta_2(x_i) \in \dot{I}_2(x_i)}} \{\gamma_1(x_i), \eta_2(x_i)\} - \max_{\substack{\gamma_2(x_i) \in \dot{T}_2(x_i) \\ \eta_1(x_i) \in \dot{I}_1(x_i)}} \{\gamma_2(x_i), \eta_1(x_i)\} \right| (i = 1, 2, \ldots, n);$$

$$\Delta_{12}(\gamma_i, \xi_i) = \left| \max_{\substack{\gamma_{j_1}(x_i) \in T_1(x_i) \\ \xi_{m_2}(x_i) \in F_2(x_i)}} \{\gamma_{j_1}(x_i), \xi_{m_2}(x_i)\} - \max_{\substack{\gamma_2(x_i) \in \dot{T}_2(x_i) \\ \xi_1(x_i) \in \dot{F}_1(x_i)}} \{\gamma_2(x_i), \xi_1(x_i)\} \right| (i = 1, 2, \ldots, n).$$

(1) *In particular, if* $\lambda = 1$, *then the normalized multi-valued neutrosophic measure reduces to a normalized multi-valued neutrosophic Hausdorff measure, i.e.:*

$$D_{Gm}(M_1, M_2) = \frac{1}{5n} \sum_{i=1}^{n} \left(\Delta\gamma_{i,12} + \Delta\eta_{i,12} + \Delta\xi_{i,12} + \Delta_{12}(\gamma_i, \eta_i) + \Delta_{12}(\gamma_i, \xi_i) \right). \tag{2}$$

(2) *If* $\lambda = 2$, *then the normalized multi-valued neutrosophic measure reduces to a normalized multi-valued neutrosophic Euclidean measure, i.e.:*

$$D_{Gm}(M_1, M_2) = \left(\frac{1}{5n} \sum_{i=1}^{n} \left(\Delta\gamma_{i,12}^2 + \Delta\eta_{i,12}^2 + \Delta\xi_{i,12}^2 + \Delta_{12}^2(\gamma_i, \eta_i) + \Delta_{12}^2(\gamma_i, \xi_i) \right) \right)^{1/2}. \tag{3}$$

Theorem 1. *The normalized multi-valued neutrosophic measure defined in Definition 6 is a normalized distance measure of MVNSs, i.e.,* $D_{Gm}(M_1, M_2)$ *satisfies the following axioms:*

H1. $0 \leq D_{Gm}(M_1, M_2) \leq 1$;
H2. $D_{Gm}(M_1, M_2) = D_{Gm}(M_2, M_1)$;
H3. $D_{Gm}(M_1, M_2) = 0$ *iff* $M_1 = M_2$;
H4. *If* $M_1 \leq M_2 \leq M_3$, *then* $D_{Gm}(M_1, M_3) \geq D_{Gm}(M_1, M_2)$ *and* $D_{Gm}(M_1, M_3) \geq D_{Gm}(M_2, M_3)$.

Proof. H1: Since $0 \leq \Delta\gamma_{i,12} \leq 1$, $0 \leq \Delta\eta_{i,12} \leq 1$, $0 \leq \Delta\xi_{i,12} \leq 1$, $0 \leq \Delta_{12}(\gamma_i, \eta_i) \leq 1$ and $0 \leq \Delta_{12}(\gamma_i, \xi_i) \leq 1$, then we have $0 \leq \frac{1}{5n} \sum_{i=1}^{n} \left(\Delta\gamma_{i,12}^\lambda + \Delta\xi_{i,12}^\lambda + \Delta\eta_{i,12}^\lambda + \Delta_{12}^\lambda(\gamma_i, \eta_i) + \Delta_{12}^\lambda(\gamma_i, \xi_i) \right) \leq 1$. Thus $0 \leq D_{Gm}(M_1, M_2) \leq 1$.

H2: Clearly, we have $D_{Gm}(M_1, M_2) = D_{Gm}(M_2, M_1)$.

H3: If $M_1 = M_2$, then we have $\gamma_1^{\sigma(j)}(x_i) = \gamma_2^{\sigma(j)}(x_i)$, $\gamma_1^{\sigma(l_{\dot{T}_1})}(x_i) = \gamma_2^{\sigma(l_{\dot{T}_2})}(x_i)$ and $\eta_1^{\sigma(k)}(x_i) = \gamma_2^{\sigma(k)}(x_i)$, $\eta_1^{\sigma(l_{l_1})}(x) = \eta_2^{\sigma(l_{l_2})}(x)$ and $\xi_1^{\sigma(m)}(x) = \xi_2^{\sigma(m)}(x)$, $\xi_1^{\sigma(l_{\dot{F}_1})}(x) = \xi_2^{\sigma(l_{\dot{F}_2})}(x)$ for any $x_i \in X$, i.e., $\Delta\gamma_{i,12} = \Delta\eta_{i,12} = \Delta\xi_{i,12} = \Delta_{12}(\gamma_i, \eta_i) = \Delta_{12}(\gamma_i, \xi_i) = 0$.

Therefore, $D_{Gm}(M_1, M_2) = 0$

H4: If $M_1 \leq M_2 \leq M_3$, then we have $0 \leq \gamma_1^{\sigma(j)}(x_i) \leq \gamma_2^{\sigma(j)}(x_i) \leq \gamma_3^{\sigma(j)}(x_i) \leq 1$, $0 \leq \gamma_1^{\sigma(l_{\dot{T}_1})}(x_i) \leq \gamma_2^{\sigma(l_{\dot{T}_2})}(x_i) \leq \gamma_3^{\sigma(l_{\dot{T}_3})}(x_i) \leq 1$ and $1 \geq \eta_1^{\sigma(k)}(x_i) \geq \eta_2^{\sigma(k)}(x_i) \geq \eta_3^{\sigma(k)}(x_i) \geq 0$, $1 \geq \eta_1^{\sigma(l_{l_1})}(x_i) \geq \eta_2^{\sigma(l_{l_2})}(x_i) \geq \eta_3^{\sigma(l_{l_3})}(x_i) \geq 0$, and $1 \geq \xi_1^{\sigma(m)}(x_i) \geq \xi_2^{\sigma(m)}(x_i) \geq \xi_3^{\sigma(m)}(x_i) \geq 0$, $1 \geq \xi_1^{\sigma(l_{\dot{F}_1})}(x_i) \geq \xi_2^{\sigma(l_{\dot{F}_2})}(x_i) \geq \xi_3^{\sigma(l_{\dot{F}_3})}(x_i) \geq 0$ for any $x_i \in X$. Thus, $\Delta\gamma_{i,12}^\lambda \leq \Delta\gamma_{i,13}^\lambda$, $\Delta\eta_{i,12}^\lambda \leq \Delta\eta_{i,13}^\lambda$, $\Delta\xi_{i,12}^\lambda \leq \Delta\xi_{i,13}^\lambda$,

$$1 \geq \max_{\substack{\gamma_3(x_i) \in \dot{T}_3(x_i) \\ \eta_1(x_i) \in \dot{I}_1(x_i)}} \{\gamma_3(x_i), \eta_1(x_i)\} \geq \max_{\substack{\gamma_2(x_i) \in \dot{T}_2(x_i) \\ \eta_1(x_i) \in \dot{I}_1(x_i)}} \{\gamma_2(x_i), \eta_1(x_i)\} \geq \max_{\substack{\gamma_1(x_i) \in \dot{T}_1(x_i) \\ \eta_2(x_i) \in \dot{I}_2(x_i)}} \{\gamma_1(x_i), \eta_2(x_i)\}$$

$$\geq \max_{\substack{\gamma_1(x_i) \in \dot{T}_1(x_i) \\ \eta_3(x_i) \in \dot{I}_3(x_i)}} \{\gamma_1(x_i), \eta_3(x_i)\} \geq 0, \text{ i.e., } \Delta_{12}^{\lambda}(\gamma_i, \eta_i) \leq \Delta_{13}^{\lambda}(\gamma_i, \eta_i) \text{ and } 1 \geq \max_{\substack{\gamma_3(x_i) \in \dot{T}_3(x_i) \\ \xi_1(x_i) \in \dot{F}_1(x_i)}} \{\gamma_3(x_i), \xi_1(x_i)\}$$

$$\geq \max_{\substack{\gamma_2(x_i) \in \dot{T}_2(x_i) \\ \xi_1(x_i) \in \dot{F}_1(x_i)}} \{\gamma_2(x_i), \xi_1(x_i)\} \geq \max_{\substack{\gamma_1(x_i) \in \dot{T}_1(x_i) \\ \xi_2(x_i) \in \dot{F}_2(x_i)}} \{\gamma_1(x_i), \xi_2(x_i)\} \geq \max_{\substack{\gamma_1(x_i) \in \dot{T}_1(x_i) \\ \xi_3(x_i) \in \dot{F}_3(x_i)}} \{\gamma_1(x_i), \xi_3(x_i)\} \geq 0$$

i.e., $\Delta_{12}^{\lambda}(\gamma_i, \xi_i) \leq \Delta_{13}^{\lambda}(\gamma_i, \xi_i)$. Hence, $D_{Gm}(M_1, M_3) \geq D_{Gm}(M_1, M_2)$.

Similarly, $D_{Gm}(M_1, M_3) \geq D_{Gm}(M_2, M_3)$ can be obtained. $\square$

Property 1. *Let* M_1 *and* M_2 *be two MVNSs on the universe* $X = \{x_1, x_2, \ldots, x_n\}$. $\gamma_i^{\sigma(\cdot)}(x)$, $\eta_i^{\sigma(\cdot)}(x)$ *and* $\xi_i^{\sigma(\cdot)}(x)$ *are the* $(\cdot)$-*th value in* $\dot{T}_i(x)$, $\dot{I}_i(x)$ *and* $\dot{F}_i(x)(i = 1, 2)$ *respectively.* *Then* $D_{Gm}(M_1^C, M_2^C) = D_{Gm}(M_1, M_2)$ *iff* $\gamma_1^{\sigma(j)}(x_i) = \xi_1^{\sigma(j)}(x_i)$ *and* $l_{\dot{T}_1}(x_i) = l_{\dot{F}_1}(x_i)$, *and* $\gamma_2^{\sigma(j)}(x_i) = \xi_2^{\sigma(j)}(x_i)$ *and* $l_{\dot{T}_2}(x_i) = l_{\dot{F}_2}(x_i)$ *for any* $x_i \in X$.

Proof. Since $M_1^C = \left\{ \left\langle x, \cup_{\xi_1(x) \in \dot{F}_1(x)} \{\xi_1(x)\}, \cup_{\eta_1(x) \in \dot{I}_1(x)} \{\eta_1(x)\}, \cup_{\gamma_1(x) \in \dot{T}_1(x)} \{\gamma_1(x)\} \right\rangle \right\}$ and $M_2^C = \left\{ \left\langle x, \cup_{\xi_2(x) \in \dot{F}_2(x)} \{\xi_2(x)\}, \cup_{\eta_2(x) \in \dot{I}_2(x)} \{\eta_2(x)\}, \cup_{\gamma_2(x) \in \dot{T}_2(x)} \{\gamma_2(x)\} \right\rangle \right\}$, then $D_{Gm}(M_1^C, M_2^C)$ $= \left(\frac{1}{5n} \sum_{i=1}^{n} \left(\Delta\xi_{i,12}^{\lambda} + \Delta\eta_{i,12}^{\lambda} + \Delta\gamma_{i,12}^{\lambda} + \Delta_{12}^{\lambda}(\xi_i, \gamma_i) + \Delta_{12}^{\lambda}(\xi_i, \eta_i) \right) \right)^{1/\lambda}$. If $\gamma_1^{\sigma(j)}(x_i) = \xi_1^{\sigma(j)}(x_i)$ and $l_{\dot{T}_1}(x_i) = l_{\dot{F}_1}(x_i)$, and $\gamma_2^{\sigma(j)}(x_i) = \xi_2^{\sigma(j)}(x_i)$ and $l_{\dot{T}_2}(x_i) = l_{\dot{F}_2}(x_i)$ for any $x_i \in X$, then $\max_{\substack{\gamma_1(x_i) \in \dot{T}_1(x_i) \\ \eta_2(x_i) \in \dot{I}_2(x_i)}} \{\gamma_1(x_i), \eta_2(x_i)\} = \max_{\substack{\xi_1(x_i) \in \dot{F}_1(x_i) \\ \eta_2(x_i) \in \dot{I}_2(x_i)}} \{\xi_1(x_i), \eta_2(x_i)\}$ and $\max_{\substack{\gamma_2(x_i) \in \dot{T}_2(x_i) \\ \eta_1(x_i) \in \dot{I}_1(x_i)}} \{\gamma_2(x_i), \eta_1(x_i)\} = \max_{\substack{\xi_2(x_i) \in \dot{F}_2(x_i) \\ \eta_1(x_i) \in \dot{I}_1(x_i)}} \{\xi_2(x_i), \eta_1(x_i)\}$, i.e., $\Delta_{12}^{\lambda}(\gamma_i, \eta_i) = \Delta_{12}^{\lambda}(\xi_i, \eta_i)$.

Thus, $D_{Gm}(M_1^C, M_2^C) = \left(\frac{1}{5n} \sum_{i=1}^{n} \left(\Delta\xi_{i,12}^{\lambda} + \Delta\eta_{i,12}^{\lambda} + \Delta\gamma_{i,12}^{\lambda} + \Delta_{12}^{\lambda}(\xi_i, \gamma_i) + \Delta_{12}^{\lambda}(\xi_i, \eta_i) \right) \right)^{1/\lambda} = D_{Gm}(M_1, M_2)$. $\square$

Property 2. *Let* M_1 *and* M_2 *be two MVNSs on the universe* $X = \{x_1, x_2, \ldots, x_n\}$, *then we have the following results:*

(1) if $M_1 \subseteq M_2$, *then* $D_{Gm}(M_1 \cup M_2, M_1 \cap M_2) = D_{Gm}(M_1, M_2)$;
(2) if $M_1 \supseteq M_2$, *then* $D_{Gm}(M_1 \cap M_2, M_1 \cup M_2) = D_{Gm}(M_1, M_2)$.

Proof. *(1) If* $M_1 \subseteq M_2$, *then we have* $\gamma_{M_1}(x_i) \leq \gamma_{M_2}(x_i)$, $\eta_{M_1}(x_i) \geq \eta_{M_2}(x_i)$ *and* $\xi_{M_1}(x_i) \geq \xi_{M_2}(x_i)$ *for any* $x_i \in X$.

From Definition 2, we have:
$$\left| \gamma_{M_1 \cup M_2}(x_i) - \gamma_{M_1 \cap M_2}(x_i) \right| = \left| \max\{\gamma_{M_1}(x_i), \gamma_{M_2}(x_i)\} - \min\{\gamma_{M_1}(x_i), \gamma_{M_2}(x_i)\} \right| = \left| \gamma_{M_2}(x_i) - \gamma_{M_1}(x_i) \right| = \Delta\gamma_{i,12};$$
$$\left| \eta_{M_1 \cup M_2}(x_i) - \eta_{M_1 \cap M_2}(x_i) \right| = \left| \min\{\eta_{M_1}(x_i), \eta_{M_2}(x_i)\} - \max\{\eta_{M_1}(x_i), \eta_{M_2}(x_i)\} \right| = \left| \eta_{M_1}(x_i) - \eta_{M_2}(x_i) \right| = \eta_{i,12};$$
$$\left| \xi_{M_1 \cup M_2}(x_i) - \xi_{M_1 \cap M_2}(x_i) \right| = \left| \min\xi\{v_{M_1}(x_i), \xi_{M_2}(x_i)\} - \max\{\xi_{M_1}(x_i), \xi_{M_2}(x_i)\} \right| = \left| \xi_{M_2}(x_i) - \xi_{M_1}(x_i) \right| = \Delta\xi_{i,12}.$$
Moreover,

$$|\max\{\gamma_{M_1 \cup M_2}(x_i), \eta_{M_1 \cap M_2}(x_i)\} - \max\{\gamma_{M_1 \cap M_2}(x_i), \eta_{M_1 \cup M_2}(x_i)\}|$$
$$= |\max\{\max\{\gamma_{M_1}(x_i), \gamma_{P_2}(x_i)\}, \max\{\eta_{M_1}(x_i), \eta_{M_2}(x_i)\}\} - \max\{\min\{\gamma_{M_1}(x_i), \gamma_{M_2}(x_i)\}, \min\{\eta_{M_1}(x_i), \eta_{M_2}(x_i)\}\}|$$
$$= |\max\{\gamma_{M_2}(x_i), \eta_{M_1}(x_i)\} - \max\{\gamma_{M_1}(x_i), \eta_{M_2}(x_i)\}| = \Delta_{12}(\gamma_i, \eta_i);$$
$$|\max\{\gamma_{M_1 \cup M_2}(x_i), \xi_{M_1 \cap M_2}(x_i)\} - \max\{\gamma_{M_1 \cap M_2}(x_i), \xi_{M_1 \cup M_2}(x_i)\}|$$
$$= |\max\{\max\{\gamma_{M_1}(x_i), \gamma_{M_2}(x_i)\}, \max\{\xi_{M_1}(x_i), \xi_{M_2}(x_i)\}\} - \max\{\min\{\gamma_{M_1}(x_i), \gamma_{M_2}(x_i)\}, \min\{\xi_{M_1}(x_i), \xi_{M_2}(x_i)\}\}|$$
$$= |\max\{\gamma_{M_2}(x_i), \xi_{M_1}(x_i)\} - \max\{\gamma_{M_1}(x_i), \xi_{M_2}(x_i)\}| = \Delta_{12}(\gamma_i, \xi_i).$$

Then $D_{Gm}(M_1 \cup M_2, M_1 \cap M_2) = \left(\frac{1}{5n} \sum\limits_{i=1}^{n} \left(\Delta\gamma_{i,12}^{\lambda} + \Delta\xi_{i,12}^{\lambda} + \Delta\eta_{i,12}^{\lambda} + \Delta_{12}^{\lambda}(\gamma_i, \xi_i) + \Delta_{12}^{\lambda}(\gamma_i, \eta_i) \right) \right)^{1/\lambda} = D_{Gm}(M_1, M_2).$

(2) Similarly, if $M_1 \supseteq M_2$, *then* $D_{Gm}(M_1 \cap M_2, M_1 \cup M_2) = D_{Gm}(M_1, M_2)$ *can be obtained.* $\square$

Property 3. *Let* M, M_1 *and* M_2 *be three MVNSs,* $M^* = \{x, \langle 1, 0, 0 \rangle\}$ *be an ideal MVNS, and* D_{Gm} *be the normalized multi-valued neutrosophic distance. Then we have:*

(1) $\forall M \in MVNSS, 0 \le D_{Gm}(M, M^*) \le 1$;
(2) $D_{Gm}(M^*, M^*) = 0$;
(3) $\forall M \in MVNSS, D_{Gm}(M, M^*) = D_{Gm}(M^*, M)$;
(4) $\forall M_1, M_2 \in MVNSS, if\ M_1 \le M_2, then\ D_{Gm}(M_1, M^*) = D_{Gm}(M_2, M^*)$.

Proof. Based on Theorem 1, the results can be obtained obviously. $\square$

3.2. The Normalized Multi-Valued Neutrosophic Distance Difference

Definition 7. *Let* M, M_1 *and* M_2 *be three MVNSs, and* $M^* = \{x, \langle 1, 0, 0 \rangle\}$ *be an ideal MVNS. Then the multi-valued neutrosophic distance difference between* M_1 *and* M_2 *can be defined as:*

$$Diff(M_1, M_2) = D_{Gm}(M_1, M^*) - D_{Gm}(M_2, M^*). \tag{4}$$

Theorem 2. *The multi-valued neutrosophic distance difference measure defined in Def. 7 satisfies the following properties:*

(1) $-1 \le Diff(M_1, M_2) \le 1$;
(2) if $M_1 = M_2$, *then* $Diff(M_1, M_2) = 0$;
(3) if $M_1 \le M_2$, *then* $Diff(M_1, M_2) \ge 0$;
(4) if $M_1 \ge M_2$, *then* $Diff(M_1, M_2) \le 0$;
(5) if $Diff(M_1, M_2) \ge 0$ *and* $Diff(M_2, M_3) \ge 0$, *then* $Diff(M_1, M_3) \ge 0$.

Proof. (1) Since $0 \le D_{Gm}(M_1, M^*) \le 1$ and $0 \le D_{Gm}(M_2, M^*) \le 1$, so we have $-1 \le D_{Gm}(M_1, M^*) - D_{Gm}(M_2, M^*) \le 1$, i.e., $-1 \le Diff(M_1, M_2) \le 1$.

(2) if $M_1 = M_2$, then $Diff(M_1, M_2) = D_{Gm}(M_1, M^*) - D_{Gm}(M_1, M^*) = 0$.

(3) Since M^* be an ideal MVNS, so $M_1 \le M_2 \le M^*$ can be obtained. According to Theorem 1, we have:

$D_{Gm}(M_1, M^*) \ge D_{Gm}(M_2, M^*)$, i.e., $D_{Gm}(M_1, M^*) - D_{Gm}(M_2, M^*) \ge 0$. Thus, $Diff(M_1, M_2) \ge 0$.

(4) Similarly to the proof in (3), if $M_1 \ge M_2$, then $Diff(M_1, M_2) \le 0$.

(5) Since $Diff(M_1, M_2) = D_{Gm}(M_1, M^*) - D_{Gm}(M_1, M^*) \ge 0$ and $Diff(M_2, M_3) = D_{Gm}(M_2, M^*) - D_{Gm}(M_3, M^*) \ge 0$. So $G_m(M_1, M^*) - D_{Gm}(M_2, M^*) + D_{Gm}(M_2, M^*) - D_{Gm}(M_3, M^*) \ge 0$, i.e., $D_{Gm}(M_1, M^*) - D_{Gm}(M_3, M^*) \ge 0$. Thus, $Diff(M_1, M_3) \ge 0$. $\square$

It is noted that for any two MVNSs M_1 and M_2, the normalized multi-valued neutrosophic distance $D_{Gm}(M_1, M^*)$ and $D_{Gm}(M_2, M^*)$ are real values. Then one of following three conditions should be hold: $D_{Gm}(M_1, M^*) > D_{Gm}(M_2, M^*)$, $D_{Gm}(M_1, M^*) = D_{Gm}(M_2, M^*)$ or $D_{Gm}(M_1, M^*) < D_{Gm}(M_2, M^*)$. It follows that normalized multi-valued neutrosophic distance satisfies the law of trichotomy. Then the partial ordering relation of MVNSs can be drawn via the difference distance.

Definition 8. *Let* M, M_1 *and* M_2 *be three MVNSs, and* $M^* = \{x, \langle 1, 0, 0 \rangle\}$ *be an ideal MVNS. Then the partial ordering relation of MVNSs can be constructed as:*

(1) *If* $Diff(M_1, M_2) > 0$, *i.e.*, $D_{Gm}(M_1, M^*) - D_{Gm}(M_2, M^*) > 0$, *then* M_1 *is inferior to* M_2, *denoted by* $M_1 \prec M_2$;

(2) *If* $Diff(M_1, M_2) = 0$, *i.e.*, $D_{Gm}(M_1, M^*) - D_{Gm}(M_2, M^*) = 0$, *then* M_1 *is indifferent to* M_2, *denoted by* $M_1 \sim M_2$;

(3) *If* $Diff(M_1, M_2) < 0$, *i.e.*, $D_{Gm}(M_1, M^*) - D_{Gm}(M_2, M^*) < 0$, *then* M_1 *is preferred to* M_2, *denoted by* $M_1 \succ M_2$.

Example 2. *Let* $M_1 = \{x, \langle \{0.4, 0.7\}, \{0.2\}, \{0.3\} \rangle\}$ *and* $M_2 = \{x, \langle \{0.5, 0.6\}, \{0.2\}, \{0.3\} \rangle\}$ *be two MVNSs and* $M^* = \{x, \langle 1, 0, 0 \rangle\}$ *be an ideal MVNS.*

(1) *Based on the comparison method in Definition 4, we have* $\gamma_1^{\sigma(1)} \leq \gamma_2^{\sigma(1)}$ *and* $\gamma_1^{\sigma(2)} \nleq \gamma_2^{\sigma(2)}$. $M_1 \nleq M_2$ *can be obtained.*

(2) *According to Definition 10,* $D_{Gm}(M_1, M^*) = 0.366$ *and* $D_{Gm}(M_2, M^*) = 0.374$ *can be obtained. From the comparison method in Definition 10,* $D_{Gm}(M_1, M^*) > D_{Gm}(M_2, M^*)$, *then* M_1 *is inferior to* M_2, *i.e.*, $M_1 \prec M_2$.

4. The Multi-Valued Neutrosophic Distance-Based QUALIFLEX Approach

Assume a group of alternatives denoted by $M = \{M_1, M_2, \ldots, M_n\}$ and corresponding criteria denoted by $C = \{c_1, c_2, \ldots, c_m\}$, and the weight of criterion w_j is completely unknown. $M_{ij} = \left\langle \dot{T}_{M_{ij}}, \dot{I}_{M_{ij}}, \dot{F}_{M_{ij}} \right\rangle$ represents the evaluation value of M_i with respect to criterion c_j, where $\dot{T}_{M_{ij}}$, $\dot{I}_{M_{ij}}$, and $\dot{F}_{M_{ij}}$ are HFNs and indicate the truth-membership, the indeterminacy-membership, and the falsity-membership, respectively. The proposed method consists of the following steps.

Step 1. Transform the evaluation information into MVNNs

According to decision-makers' knowledge and experience, experts provide evaluation values for criteria for each alternative at three levels: high, medium and low. In other words, the option about high, middle, and low in the evaluation process correspond to the three parameters of MVNS, namely, positive membership, neutral membership, and negative membership, respectively. In order to assure the accuracy and effectiveness of the evaluation information, no corresponding information was provided during the evaluation process, and decision-makers were not allowed to communicate with each other. $M_{ij} = \left\langle \dot{T}_{M_{ij}}, \dot{I}_{M_{ij}}, \dot{F}_{M_{ij}} \right\rangle$ is the set of evaluation values for all decision-makers. Then the decision-making matrix can be obtained.

Step 2. Normalize the decision matrix

For each criterion can be divided into two types, including benefit criteria, which means the lager the better, and cost criteria, which means the smaller the better. For the benefit criteria, nothing is done; for the cost criteria, the criterion values can be transformed as $\overline{M}_{ij} = (M_{ij})^c (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$. Here $(M_{ij})^c$ is the complement of M_{ij} as presented in Def. 7.

Step 3. Calculate the weight of criteria

Based on the maximizing deviation method of SVNSs defined by Sahin and Liu [43], the non-linear programming model with MVNNs can be constructed as:

$$(L) \begin{cases} \max P(\omega) = \frac{1}{5^{1/\lambda}} \sum\limits_{j=1}^{m} \sum\limits_{\varsigma=1}^{n} \sum\limits_{\tau=1}^{m} \omega_j (\Delta_{\mu\eta\xi}) \\ \text{subject to } \omega_j \geq 0, \sum\limits_{j=1}^{m} \omega_j^2 = 1. \, j = 1, 2, \ldots, m \end{cases} \tag{5}$$

Then according to the Lagrange function, the weight of criteria can be determined as [43]:

$$\varpi_j \;=\; \frac{\sum\limits_{\varsigma=1}^{n}\sum\limits_{\tau=1}^{n}\left(\Delta_{\mu\eta\xi}\right)}{\sqrt{\sum\limits_{j=1}^{m}\left(\sum\limits_{\varsigma=1}^{n}\sum\limits_{\tau=1}^{n}\left(\Delta_{\mu\eta\xi}\right)\right)^2}}. \tag{6}$$

In order to normalize the weight, then we have:

$$\varpi_j^* \;=\; \frac{\sum\limits_{\varsigma=1}^{n}\sum\limits_{\tau=1}^{n}\left(\Delta_{\mu\eta\xi}\right)}{\sum\limits_{j=1}^{m}\sum\limits_{\varsigma=1}^{n}\sum\limits_{\tau=1}^{n}\left(\Delta_{\mu\eta\xi}\right)}. \tag{7}$$

Here, $\left(\Delta_{\mu\eta\xi}\right) \;=\; \left(\frac{1}{n}\sum\limits_{i=1}^{n}\left(\Delta\gamma_{i,12}^{\lambda}+\Delta\eta_{i,12}^{\lambda}+\Delta\xi_{i,12}^{\lambda}+\Delta_{12}^{\lambda}(\gamma_i,\eta_i)+\Delta_{12}^{\lambda}(\gamma_i,\xi_i)\right)\right)^{1/\lambda}.$

Step 4. Determine the possible permutations

For a group of alternative $M_i(i=1,2,\ldots,n)$, there exist $n!$ permutations of different ranks of alternatives. Assume P^{τ} represents the κ-th permutation as:

$$P^{\kappa} \;=\; (\ldots,M_{\varsigma},\ldots,M_{\tau},\ldots),\ \kappa=1,2,\ldots,n! \tag{8}$$

where $M_{\varsigma}, M_{\tau} \in M$, and M_{ς} is superior than or equal to M_{τ}.

Step 5. Calculate the concordance/discordance index

For each pair of alternatives $(M_{\varsigma}, M_{\tau})$ $(M_{\varsigma}, M_{\tau} \in M)$ respect to the j-th criterion, the corresponding concordance/discordance index $\varphi_j^{\kappa}(M_{\varsigma}, M_{\tau})$ can be defined as:

$$\varphi_j^{\kappa}(M_{\varsigma}, M_{\tau}) \;=\; Diff\left(\overline{M}_{\varsigma j}, \overline{M}_{\tau j}\right). \tag{9}$$

According to the multi-valued neutrosophic distance difference in Def. 10, the following can be true:

(1) If $\varphi_j^{\kappa}(M_{\varsigma}, M_{\tau}) > 0$, i.e., $Diff\left(\overline{M}_{\varsigma j}, \overline{M}_{\tau j}\right) > 0$, then M_{τ} ranks over M_{ς} respect to the j-th criterion under the τ-th permutation;

(2) If $\varphi_j^{\kappa}(M_{\varsigma}, M_{\tau}) = 0$, i.e., $Diff\left(\overline{M}_{\varsigma j}, \overline{M}_{\tau j}\right) = 0$, then both M_{ς} and M_{τ} have the same rank respect to the j-th criterion under the τ-th permutation;

(3) If $\varphi_j^{\kappa}(M_{\varsigma}, M_{\tau}) < 0$, i.e., $Diff\left(\overline{M}_{\varsigma j}, \overline{M}_{\tau j}\right) < 0$, then M_{ς} ranks over M_{τ} respect to the j-th criterion under the τ-th permutation.

Step 6. Determine the weighted concordance/discordance index

Considering the importance weight ϖ_j of each criterion $c_j \in C$ being expressed by MVNNs, the weighted concordance/discordance index $\varphi^{\kappa}(M_{\varsigma}, M_{\tau})$ for each pair of alternatives $(M_{\varsigma}, M_{\tau})(M_{\varsigma}, M_{\tau} \in M)$ can be denoted as:

$$\varphi^{\kappa}(M_{\varsigma}, M_{\tau}) \;=\; \sum_{j=1}^{m} \varphi_j^{\kappa}(M_{\varsigma}, M_{\tau}) \cdot \left(1 - D_{Gm}\left(\varpi_j, M^*\right)\right). \tag{10}$$

Step 7. Calculate the comprehensive concordance/discordance index

For the κ-th permutation, the corresponding comprehensive concordance/discordance index φ^κ can be calculated as:

$$\varphi^\kappa = \sum_{M_\varsigma, M_\tau \in M} \varphi^\kappa (M_\varsigma, M_\tau). \tag{11}$$

Step 8. Rank the alternatives

According to the partial ordering relation of MVNNs, it can be seen that the greater the comprehensive concordance/discordance index value is, the more optimal the final ranking is. Thus, the optimal rank can be obtained with the maximal comprehensive concordance/discordance index φ^κ, i.e.:

$$P^* = \max_{\kappa=1}^{n!} \{\varphi^\kappa\}. \tag{12}$$

5. Illustrative Example

An example for selection of medical diagnostic plan (adapted from Chen et al. [37]) is provided in this section. There is a patient who was a 48 year old female with a history of diabetes mellitus. Her physician made a diagnosis of acute inflammatory demyelinating disease. Then the physician assessed the patient's medical history and her current physical conditions and provided three treatment plans. Thus, how to select a suitable scheme is a MCDM problem. There are three possible schemes $M_i (i = 1, 2, 3)$ to be selected, including steroid therapy M_1, plasmapheresis M_2, and albumin immune therapy M_3. Each scheme can be assessed based on nine criteria, i.e., $c_j (j = 1, 2, \ldots, 9)$: c_1 is the survival rate; c_2 is the seriousness of the side effects; c_3 is the seriousness of the complications; c_4 is the possibility of a cure; c_5 is the uncomfortableness degree of the treatment; c_6 is the cost; c_7 is the number of days of hospitalization; c_8 is the probability of a recurrence and c_9 is the self-care capacity. Three decision-makers could assess three treatment plans under nine criteria in the form of MVNNs. When more than one decision-maker assesses the same value, it is counted once. The weights of criteria are completely unknown.

5.1. Illustration of the Developed Method

The steps of obtaining the optimal alternative, by using the developed approach, are as follows.

Step 1. Transform the evaluation information into MVNNs

Three decision-makers can provide evaluation values for criteria for each alternative at three levels: high, medium and low based on their knowledge and experience. Then sets of high, medium, and low correspond to the three parameters of MVNN, namely, positive membership, neutral membership, and negative membership, respectively. If two or more decision-makers provide the same value, then it is counted only once. Then the final evaluation information are in the form of MVNNs, i.e., $M_{ij} = \left\langle \dot{T}_{M_{ij}}, \dot{I}_{M_{ij}}, \dot{F}_{M_{ij}} \right\rangle$. Thus, the decision matrix can be constructed as described in Table 1.

Table 1. The evaluations of three schemes by decision-makers under criteria.

Criteria	Schemes		
	M_1	M_2	M_3
c_1	<{0.4,0.6,0.7},{0.1},{0.2}>	<{0.4,0.5},{0.1},{0.2}>	<{0.4,0.5,0.7},{0.3},{0.2}>
c_2	<{0.3},{0.3},{0.5,0.7}>	<{0.4},{0.1},{0.4}>	<{0.1,0.3},{0.2},{0.4,0.6}>
c_3	<{0.3},{0.2},{0.4,0.6}>	<{0.4},{0.1},{0.5,0.6,0.7}>	<{0.3},{0.2},{0.5}>
c_4	<{0.5,0.7},{0.2},{0.2}>	<{0.3,0.4},{0.2},{0.5}>	<{0.3,0.6},{0.1},{0.6}>
c_5	<{0.3},{0.2},{0.5}>	<{0.2},{0.3},{0.3,0.5}>	<{0.3},{0.2},{0.4}>
c_6	<{0.3},{0.1},{0.6,0.7}>	<{0.3},{0.2},{0.6}>	<{0.1},{0.2},{0.5,0.6,0.8}>
c_7	<{0.2},{0.1},{0.4,0.6,0.9}>	<{0.2},{0.4},{0.6,0.8}>	<{0.1},{0.2},{0.5,0.7,0.8}>
c_8	<{0.1},{0.3},{0.6,0.8}>	<{0.1},{0.2},{0.7,0.9}>	<{0.3},{0.1},{0.7}>
c_9	<{0.7,0.8,0.9},{0.1},{0.1}>	<{0.6,0.7,0.8},{0.2},{0.3}>	<{0.6,0.9},{0.2},{0.2}>

Step 2. Normalize the decision-making matrix

Since c_1, c_4, and c_9 are benefit types and other criteria are cost types, from Definition 2 the normalized MVNN decision matrix can be determined as presented in Table 2.

Table 2. Normalized decision matrix.

Criteria	Schemes		
	M_1	M_2	M_3
c_1	<{0.4,0.6,0.7},{0.1},{0.2}>	<{0.4,0.5},{0.1},{0.2}>	<{0.4,0.5,0.7},{0.3},{0.2}>
c_2	<{0.5,0.7},{0.3},{0.3}>	<{0.4},{0.1},{0.4}>	<{0.4,0.6},{0.2},{0.1,0.3}>
c_3	<{0.4,0.6},{0.2},{0.3}>	<{0.5,0.6,0.7},{0.1},{0.4}>	<{0.5},{0.2},{0.3}>
c_4	<{0.5,0.7},{0.2},{0.2}>	<{0.3,0.4},{0.2},{0.5}>	<{0.3,0.6},{0.1},{0.6}>
c_5	<{0.5},{0.2},{0.3}>	<{0.3,0.5},{0.3},{0.2}>	<{0.4},{0.2},{0.3}>
c_6	<{0.6,0.7},{0.1},{0.3}>	<{0.6},{0.2},{0.3}>	<{0.5,0.6,0.8},{0.2},{0.1}>
c_7	<{0.4,0.6,0.9},{0.1},{0.2}>	<{0.6,0.8},{0.4},{0.2}>	<{0.5,0.7,0.8},{0.2},{0.1}>
c_8	<{0.6,0.8},{0.3},{0.1}>	<{0.7,0.9},{0.2},{0.1}>	<{0.7},{0.1},{0.3}>
c_9	<{0.7,0.8,0.9},{0.1},{0.1}>	<{0.6,0.7,0.8},{0.2},{0.3}>	<{0.6,0.9},{0.2},{0.2}>

Step 3. Calculate the weight of criteria

From Equation (5), the weight of criteria can be obtained as $\varpi = (0.11, 0.15, 0.09, 0.15, 0.06, 0.10, 0.13, 0.11, 0.10)$.

Step 4. Determine all of the possible permutations

Since $n = 3$, so we have 6(3! = 6) permutations of alternative rankings, i.e., $P^1 = (M_1, M_2, M_3)$, $P^2 = (M_1, M_3, M_2)$, $P^3 = (M_2, M_1, M_3)$, $P^4 = (M_2, M_3, M_1)$, $P^5 = (M_3, M_1, M_2)$, $P^6 = (M_3, M_2, M_1)$.

Step 5. Calculate the concordance/discordance index

From Equation (9), for each pair of alternatives $(M_\varsigma, M_\tau)(M_\varsigma, M_\tau \in M)$ in the permutation P^κ under criterion C_j, the concordance/discordance index $\varphi_j^\kappa(M_\varsigma, M_\tau)$ can be obtained. For simplicity, let $\lambda = 1$ in Equation (1), the normalized multi-valued neutrosophic distance is reduced to the normalized multi-valued neutrosophic Hausdorff distance, i.e., Equation (2), and the results can be founded in Table 3.

Step 6. Calculate the weighted concordance/discordance index.

For simplicity, let $\lambda = 1$ in Equation (1), the weighted concordance/discordance indices $\varphi^\kappa(M_\varsigma, M_\tau)$ can be calculated as presented in Table 4.

Step 7. Calculate the comprehensive concordance/discordance index.

From Equation (11), the comprehensive concordance/discordance index φ^κ can be calculated as shown in Table 5.

Table 3. The concordance/discordance index.

P^1	$\varphi_j^1(M_1,M_2)$	$\varphi_j^1(M_1,M_3)$	$\varphi_j^1(M_2,M_3)$	P^2	$\varphi_j^2(M_1,M_3)$	$\varphi_j^2(M_1,M_2)$	$\varphi_j^2(M_3,M_2)$
c_1	−0.08	−0.04	−0.04	c_1	−0.04	−0.08	−0.04
c_2	−0.12	−0.04	0.08	c_2	−0.04	−0.12	−0.08
c_3	0.06	−0.02	−0.08	c_3	−0.02	0.06	0.08
c_4	−0.22	−0.14	0.08	c_4	−0.14	−0.22	−0.08
c_5	−0.04	−0.06	−0.02	c_5	−0.06	−0.04	0.02
c_6	−0.06	0.04	0.10	c_6	0.04	−0.06	−0.1
c_7	−0.06	−0.02	0.04	c_7	−0.02	−0.06	−0.04
c_8	0.08	−0.02	−0.1	c_8	−0.02	0.08	0.10
c_9	−0.12	−0.06	0.06	c_9	−0.06	−0.12	−0.06
P^3	$\varphi_j^3(M_2,M_1)$	$\varphi_j^3(M_2,M_3)$	$\varphi_j^3(M_1,M_3)$	P^4	$\varphi_j^4(M_2,M_3)$	$\varphi_j^4(M_2,M_1)$	$\varphi_j^4(M_3,M_1)$
c_1	0.08	0.04	−0.04	c_1	0.04	0.08	0.04
c_2	0.12	0.08	−0.04	c_2	0.08	0.12	0.04
c_3	−0.06	−0.08	−0.02	c_3	−0.08	−0.06	0.02
c_4	0.22	0.08	−0.14	c_4	0.08	0.22	0.14
c_5	0.04	−0.02	−0.06	c_5	−0.02	0.04	0.06
c_6	0.06	0.10	0.04	c_6	0.10	0.06	−0.04
c_7	0.06	0.04	−0.02	c_7	0.04	0.06	0.02
c_8	−0.08	−0.10	−0.02	c_8	−0.10	−0.08	0.02
c_9	0.12	0.06	−0.06	c_9	0.06	0.12	0.06
P^5	$\varphi_j^5(M_3,M_1)$	$\varphi_j^5(M_3,M_2)$	$\varphi_j^5(M_1,M_2)$	P^6	$\varphi_j^6(M_3,M_2)$	$\varphi_j^6(M_3,M_1)$	$\varphi_j^6(M_2,M_1)$
c_1	0.04	−0.04	−0.08	c_1	−0.04	0.04	0.08
c_2	0.04	−0.08	−0.12	c_2	−0.08	0.04	0.12
c_3	0.02	0.08	0.06	c_3	0.08	0.02	−0.06
c_4	0.14	−0.08	−0.22	c_4	−0.08	0.14	0.22
c_5	0.06	0.02	−0.04	c_5	0.02	0.06	0.04
c_6	−0.04	−0.10	−0.06	c_6	−0.10	−0.04	0.06
c_7	0.02	−0.04	−0.06	c_7	−0.04	0.02	0.06
c_8	0.02	0.10	0.08	c_8	0.10	0.02	−0.08
c_9	0.06	−0.06	−0.12	c_9	−0.06	0.06	0.12

Table 4. The weighted concordance/discordance index.

P^1	$\varphi^1(M_1,M_2)$ −0.2688	$\varphi^1(M_1,M_3)$ −0.1707	$\varphi^1(M_2,M_3)$ 0.0981	P^2	$\varphi^2(M_1,M_3)$ −0.1707	$\varphi^2(M_1,M_2)$ −0.2688	$\varphi^2(M_3,M_2)$ −0.0981
P^3	$\varphi^3(M_2,M_1)$ 0.2688	$\varphi^3(M_2,M_3)$ 0.0981	$\varphi^3(M_1,M_3)$ −0.1707	P^4	$\varphi^4(M_2,M_3)$ 0.0981	$\varphi^4(M_2,M_1)$ 0.2688	$\varphi^4(M_3,M_1)$ 0.1707
P^5	$\varphi^5(M_3,M_1)$ 0.1707	$\varphi^5(M_3,M_2)$ −0.0981	$\varphi^5(M_1,M_2)$ −0.2688	P^6	$\varphi^6(M_3,M_2)$ −0.0981	$\varphi^6(M_3,M_1)$ 0.1707	$\varphi^6(M_2,M_1)$ 0.2688

Table 5. The comprehensive concordance/discordance index.

φ^1	φ^2	φ^3	φ^4	φ^5	φ^6
−0.3414	−0.5377	0.1963	0.5377	−0.1963	0.3414

Step 8. Rank the alternatives

From the results in Step 5 and Equation (14), $\varphi^4 > \varphi^6 > \varphi^3 > \varphi^5 > \varphi^1 > \varphi^2$ and $P^* = \max_{\kappa=1}^{n!}\{\varphi^\kappa\} = P^4$ can be obtained. Thus, the final order of the three plan is: $M_2 \succ M_3 \succ M_1$. The best treatment plan is M_2 while the worst treatment plan is M_1.

5.2. Sensitivity Analysis

In this subsection, the influence of λ on the ranking of alternatives is discussed. From Figure 1, we can see that the rankings of alternatives are slightly different. If $\lambda = 1, 2, 4$, then the optimal permutation is P^4. The best alternative is always M_2; while the worst alternative is M_1; while if $\lambda = 6, 8, 10$, then the optimal permutation is P^6. The best alternative is M_3. Moreover, the values of comprehensive concordance/discordance index φ^κ become smaller as parameter λ increases. Generally speaking, different values of parameter λ can reflect the decision-makers' preferences and risk attitudes,

which can provide more choices for decision-makers. Moreover, since the evaluation values for three memberships in MVNNs are sets of numerical numbers in [0, 1], so we can see that if the value of parameter is too large, then the difference for the distances of MVNNs will not be distinct.
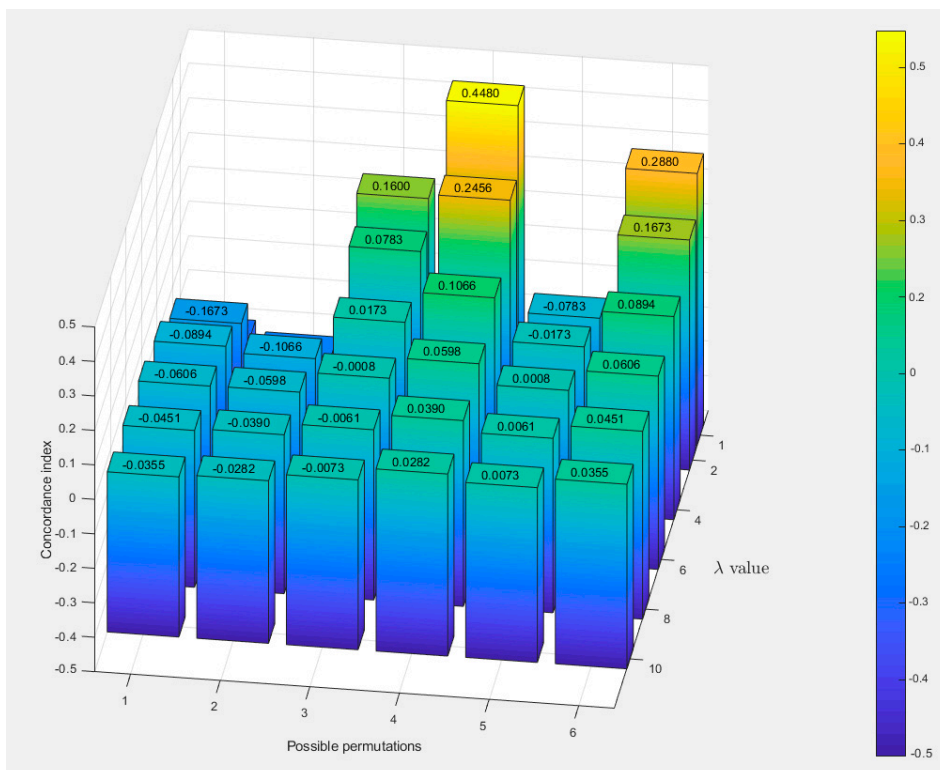

Figure 1. The results of the sensitivity analysis.

5.3. Comparison Analysis

To further validate the practicability of the developed method, a comparison analysis was investigated by utilizing some existing methods with multi-valued neutrosophic information, i.e., Peng et al. [19–21] and Ji et al. [23].

To facilitate a comparison analysis, the same example is used here as well. Since the compared methods presented above cannot handle multi-valued neutrosophic information where the weight is completely unknown, the weights of the criteria was determined as $\varpi = (0.11, 0.15, 0.09, 0.15, 0.06, 0.10, 0.13, 0.11, 0.10)^T$. Then the final results can be calculated as presented in Table 6.

Table 6. The comparison results.

Methods	The Final Ranking	The Best Alternative(s)	The Worst Alternative(s)
Peng et al. [14]	$M_2 \succ M_3 \succ M_1$ or $M_3 \succ M_2 \succ M_1$	M_2 or M_3	M_1
Peng et al. [15]	$M_2 \succ M_3 \succ M_1$	M_2	M_1
Peng et al. [16]	$M_2 \succ M_3 \succ M_1$	M_2	M_1
Ji et al. [23]	$M_2 \succ M_3 \succ M_1$	M_2	M_1
The proposed method	$M_2 \succ M_3 \succ M_1$	M_2	M_1

From the results presented in Table 4, we can see that the results from the proposed approach are consistent with the compared methods in Peng et al. [15,16] and Ji et al. [23]; the optimal treatment plan is M_2, while the worst treatment plan is M_1. For the other compared method in Peng et al. [14], although there is a slight difference in the final rankings of these methods, the optimal treatment plan is M_2 or M_3.

From the comparison analyses presented above, some results can be summarized.

Firstly, if the multi-valued neutrosophic power weighted arithmetic averaging operator and the multi-valued neutrosophic power weighted geometric averaging operator presented in Peng et al. [14] are used respectively, then the different rankings $M_2 \succ M_3 \succ M_1$ and $M_3 \succ M_2 \succ M_1$ can be obtained. However, different aggregation operators are always involved in the operations. Moreover, if the number of elements in MVNNs increases, then the number of elements in the aggregated value will exponentially increase. This will increase the difficulty of decision-making. Secondly, the method of Peng et al. [15] is suitable to solve the MCDM problems where the number of alternatives is more than the number of criteria; while the proposed approach and the method in Peng et al. [16] are preferred to handle MCDM problems where the number of alternatives is fewer than the number of criteria. Thirdly, all of the compared methods developed in Peng et al. [14–16] and Ji et al. [23] cannot deal with some special cases that the weight information is completely unknown. However, the proposed approach can avoid these shortcomings. Therefore, the primary characteristic of the approach developed are not only its ability to availably express the preference information by MVNNs, but also its consideration that the weights' information is completely unknown. It can enlarge the application scope of decision-making methods.

6. Conclusions

In this paper, the normalized multi-valued neutrosophic distance measure is defined, then the normalized multi-valued neutrosophic difference distance is developed as well. Based on the developed distances, a multi-valued neutrosophic distance-based QUALIFLEX approach is proposed to deal with MCDM problems where the weights of criteria are completely unknown. A treatment selection example testified the practicability of the proposed method, and showed that the results are reasonable and credible. The mainly advantages of the developed method over the other methods is that it can handle the MCDM problems where the number of alternatives is fewer than the number of criteria and the weight information is completely unknown, which can be used to obtain the credible and realistic results. However, the limitation of the developed method is that it cannot be suitable for dealing with some problems where the number of alternatives is greater than the number of criteria. In the future, the related distance measures of MVNSs will be further investigated.

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