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Research Article

Multicriteria Decision-Making Method and Application in the Setting of Trapezoidal Neutrosophic Z-Numbers

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The information expression and modeling of decision-making are critical problems in the fuzzy decision theory and method. However, existing trapezoidal neutrosophic numbers (TrNNs) and neutrosophic Z-numbers (NZNs) and their multicriteria decision-making (MDM) methods reveal their insufficiencies, such as without considering the reliability measures in TrNN and continuous Z-numbers in NZN. To overcome the insufficiencies, it is necessary that one needs to propose trapezoidal neutrosophic Z-numbers (TrNZNs), their aggregation operations, and an MDM method for solving MDM problems with TrNZN information. Hence, this study first proposes a TrNZN set, some basic operations of TrNZNs, and the score and accuracy functions of TrNZN and their ranking laws. Then, the TrNZN weighted arithmetic averaging (TrNZNWAA) and TrNZN weighted geometric averaging (TrNZNWGA) operators are presented based on the operations of TrNZNs. Next, an MDM approach using the proposed aggregation operators and score and accuracy functions is established to carry out MDM problems under the environment of TrNZNs. In the end, the established MDM approach is applied to an MDM example of software selection for revealing its rationality and efficiency in the setting of TrNZNs. The main advantage of this study is that the established approach not only makes assessment information continuous and reliable but also strengthens the decision rationality and efficiency in the setting of TrNZNs.

1. Introduction

In fuzzy decision-making problems, various new fuzzy decision-making methods [1–3] have received many applications under neutrosophic, simplified neutrosophic hesitant fuzzy, and bipolar neutrosophic environments. Then, triangular and trapezoidal fuzzy numbers are usually used for real decision-making problems because they can be depicted by the continuous fuzzy numbers of membership functions rather than exact/discrete fuzzy values. Hence, some researchers extended triangular fuzzy numbers to intuitionistic fuzzy sets (IFSs) and presented triangular intuitionistic fuzzy sets (TIFSs), where the values of the membership and nonmembership functions are triangular fuzzy numbers, and some triangular intuitionistic fuzzy aggregation operators for multicriteria decision-making (MDM) problems with triangular intuitionistic fuzzy information [4–7]. As the extension of TIFSs, Ye [8] introduced a trapezoidal intuitionistic fuzzy set (TrIFS), in which the values of its membership and nonmembership functions are trapezoidal fuzzy numbers rather than triangular fuzzy numbers, and some prioritized weighted aggregation operators of trapezoidal intuitionistic fuzzy numbers (TrIFNs) for MDM problems with TrIFNs. However, TIFSs and TrIFSs cannot depict inconsistence and indeterminacy information. Hence, Ye [9] generalized TrIFS and proposed a trapezoidal neutrosophic set (TrNS), in which the values of its truth, falsity, and indeterminacy membership functions are trapezoidal fuzzy numbers, to express incomplete, indeterminate, and inconsistent information, and then he presented some basic operations of trapezoidal neutrosophic numbers (TrNNs), score and accuracy functions of TrNNs, and TrNN weighted arithmetic averaging (TrNNWAA) and TrNN weighted geometric averaging (TrNNWGA) operators for MDM problems in the setting of TrNNs. Then, some researchers utilized the integrated approach [10] and defuzzification method [11] for the evaluation and MDM problems with interval-valued TrNNs. Further, Giri et al. [12] applied TOPSIS method in MDM problems with interval-valued TrNNs. Also, Jana et al. [13] and Khatter [14] presented some basic operations of interval-valued TrNNs, score and accuracy functions of an interval-valued TrNN, and the interval-valued TrNNWAA and TrNNWGA operators for MDM problems in the setting of interval-valued TrNNs.

The notion of a Z-number introduced by Zadeh [15] is described by a fuzzy number and its reliability measure to strengthen the reliability of the fuzzy information. After that, Z-numbers have been used for many areas [16–22]. Based on the truth, falsity, and indeterminacy Z-numbers, Du et al. [23] extended the Z-number concept and proposed neutrosophic Z-numbers (NZNs) to enhance the reliability of the neutrosophic information, and then they presented basic operations of NZNs, score and accuracy functions of NZN, and the NZN weighted geometric averaging (NZNWGA) and NZN weighted arithmetic averaging (NZNWAA) operators and further established their MDM method under the environment of NZNs.

However, TrNN is described only by the trapezoidal fuzzy numbers of its truth, falsity, and indeterminacy membership functions without considering their reliability measures, while NZN is depicted only by exact/discrete truth, falsity, and indeterminacy Z-numbers rather than continuous Z-numbers. Hence, TrNN and NZN and their MDM methods reveal their insufficiencies in their information expressions and applications. To express both the continuous Z-numbers of truth, falsity, and indeterminacy membership functions and the reliability measures in MDM problems, it is necessary that this study needs to propose an MDM method based on trapezoidal neutrosophic Z-numbers (TrNZNs) to make up such insufficiencies of existing information expressions and MDM methods in the environments of TrNNs and NZNs. To do so, the main aims of this article are (1) to propose a TrNZN set and some basic operations of TrNZNs, (2) to introduce score and accuracy functions of TrNZN for ranking TrNZNs, (3) to put forward the TrNZNWAA and TrNZNWGA operators for aggregating TrNZNs, (4) to develop a MDM approach using the proposed aggregation operators and score and accuracy functions for solving MDM problems under the environment of TrNZNs, and (5) to apply the established MDM approach to an MDM example of software selection for revealing its efficiency in the setting of TrNZNs.

The rest of the article is composed of the following sections. Section 2 introduces some basic notions of TrNNs as preliminaries of this study. Section 3 proposes a TrNZN set, basic operations of TrNZNs, the score and accuracy functions of TrNZN, and their ranking laws of TrNZNs. Then, the TrNZNWAA and TrNZNWGA operators and their relative properties are presented in section 4. Section 5 develops an MDM approach using the TrNZNWAA and TrNZNWGA operators and score and accuracy functions of TrNZNs. In Section 6, the developed MDM approach is applied to an MDM example of software selection to indicate its efficiency in the setting of TrNZNs. In the end, conclusions and further study are contained in Section 7.

2. Preliminaries of TrNSs

In this section, we introduce preliminaries of TrNSs, including TrNNs, operations of TrNNs, two TrNN weighted aggregation operators, and score and accuracy functions of TrNNs for ranking TrNNs.

Ye [9] first proposed TrNS in a universe set U, which is denoted as

$$\widetilde{Y}=\left\{\left\langle u,\mathrm{TN}_{\widetilde{Y}}(u),\mathrm{IN}_{\widetilde{Y}}(u),\mathrm{FN}_{\widetilde{Y}}(u)\right\rangle,\quad u\in U\right\},\tag{1}$$

where $\mathrm{TN}_{\widetilde{Y}}(u)\subseteq[0,1]$, $\mathrm{IN}_{\widetilde{Y}}(u)\subseteq[0,1]$, and $\mathrm{FN}_{\widetilde{Y}}(u)\subseteq[0,1]$ are the truth, indeterminacy, and falsity membership functions; then their values are three trapezoidal fuzzy numbers $\mathrm{TN}_{\widetilde{Y}}(u)=(\mathrm{TN}_1(u),\mathrm{TN}_2(u),\mathrm{TN}_3(u),\ \mathrm{TN}_4(u))$: U $\longrightarrow$ [0, 1], $\mathrm{IN}_{\widetilde{Y}}(u)=(\mathrm{IN}_1(u),\mathrm{IN}_2(u),\ \mathrm{IN}_3(u),\mathrm{IN}_4(u))$: U $\longrightarrow$ [0, 1], and $\mathrm{FN}_{\widetilde{Y}}(u)=(\mathrm{FN}_1(u),\mathrm{FN}_2(u),\mathrm{FN}_3(u),\mathrm{FN}_4(u))$: U $\longrightarrow$ [0, 1] with the condition $0\le\mathrm{TN}_4(u)+\mathrm{IN}_4(u)+\mathrm{FN}_4(u)\le 3$ for $u\in U$. For convenience, a TrNN in $\widetilde{Y}$ is simply denoted by $\widetilde{y}$ = <(TN1, TN2, TN3, TN4), (IN1, IN2, IN3, IN4), (FN1, FN2, FN3, FN4)>.

Regarding two TrNNs $\widetilde{y}_1$ = <(TN_{11}, TN_{12}, TN_{13}, TN_{14}), (IN_{11}, IN_{12}, IN_{13}, IN_{14}), (FN_{11}, FN_{12}, FN_{13}, FN_{14})> and $\widetilde{y}_2$ = <(TN_{21}, TN_{22}, TN_{23}, TN_{24}), (IN_{21}, IN_{22}, IN_{23}, IN_{24}), (FN_{21}, FN_{22}, FN_{23}, FN_{24})>, Ye [14] defined the following basic operations:

(1) $\widetilde{y}_1\oplus\widetilde{y}_2=\left\langle (TN_{11}+TN_{21}-TN_{11}TN_{21},\ TN_{12}+TN_{22}-TN_{12}TN_{22},\ TN_{13}+TN_{23}-TN_{13}TN_{23},\ TN_{14}+TN_{24}-TN_{14}TN_{24}),\ (IN_{11}IN_{21},IN_{12}IN_{22},IN_{13}IN_{23},IN_{14}IN_{24}),\ (FN_{11}FN_{21},FN_{12}FN_{22},FN_{13}FN_{23},FN_{14}FN_{24})\right\rangle$

(2) $\widetilde{y}_1\otimes\widetilde{y}_2=\left\langle (TN_{11}TN_{21},TN_{12}TN_{22},\ TN_{13}TN_{23},TN_{14}TN_{24}),\ (IN_{11}+IN_{21}-IN_{11}IN_{21},IN_{12}+IN_{22}-IN_{12}IN_{22},IN_{13}+IN_{23}-IN_{13}IN_{23},IN_{14}+IN_{24}-IN_{14}\ IN_{24}),\ (FN_{11}+FN_{21}-FN_{11}FN_{21},FN_{12}+FN_{22}-FN_{12}FN_{22},FN_{13}+FN_{23}-FN_{13}\ FN_{23},FN_{14}+FN_{24}-FN_{14}FN_{24})\right\rangle$

(3) $\lambda\widetilde{y}_1=\left\langle (1-(1-TN_{11})^{\lambda},1-(1-TN_{12})^{\lambda},\ 1-(1-TN_{13})^{\lambda},1-(1-TN_{14})^{\lambda}),\ (IN_{11}^{\lambda},IN_{12}^{\lambda},\ IN_{13}^{\lambda},IN_{14}^{\lambda}),\ (FN_{11}^{\lambda},FN_{12}^{\lambda},FN_{13}^{\lambda},FN_{14}^{\lambda})\right\rangle,\ \lambda>0$

(4) $\widetilde{y}_1^{\lambda}=\left\langle (TN_{11}^{\lambda},TN_{12}^{\lambda},TN_{13}^{\lambda},TN_{14}^{\lambda}),\ (1-(1-\ IN_{11})^{\lambda},1-(1-IN_{12})^{\lambda},1-(1-IN_{13})^{\lambda},\ 1-(1-IN_{14})^{\lambda}),\ (1-(1-FN_{11})^{\lambda},1-(1-FN_{12})^{\lambda},1-(1-FN_{13})^{\lambda},1-(1-FN_{14})^{\lambda})\right\rangle,\ \lambda\ge 0$

Regarding a group of TrNNs $\widetilde{y}_j$ = <(TN_{j1}, TN_{j2}, TN_{j3}, TN_{j4}), (IN_{j1}, IN_{j2}, IN_{j3}, IN_{j4}), (FN_{j1}, FN_{j2}, FN_{j3}, FN_{j4})> ($j=1,2,\ldots,n$) with their weights λ_j ($j=1,2,\ldots,n$) for $\lambda_j\in[0,1]$ and $\sum_{j=1}^{n}\lambda_j=1$, Ye [9] proposed the TrNNWAA and TrNNWGA operators:

$$\begin{aligned}
&\text{TrNNWAA}(\tilde{y}_1,\tilde{y}_2,\ldots,\tilde{y}_n)=\mathop{\oplus}_{j=1}^{n}\lambda_j\tilde{y}_j\\
&=\left\langle\left(1-\prod_{j=1}^{n}\left(1-TN_{j1}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-TN_{j2}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-TN_{j3}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-TN_{j4}\right)^{\lambda_j}\right),\right.\\
&\left.\left(\prod_{j=1}^{n}IN_{j1}^{\lambda_j},\prod_{j=1}^{n}IN_{j2}^{\lambda_j},\prod_{j=1}^{n}IN_{j3}^{\lambda_j},\prod_{j=1}^{n}IN_{j4}^{\lambda_j}\right),\left(\prod_{j=1}^{n}FN_{j1}^{\lambda_j},\prod_{j=1}^{n}FN_{j2}^{\lambda_j},\prod_{j=1}^{n}FN_{j3}^{\lambda_j},\prod_{j=1}^{n}FN_{j4}^{\lambda_j}\right)\right\rangle,
\end{aligned}\tag{2}$$

$$\begin{aligned}
&\text{TrNNWGA}\left(\tilde{y}_1,\tilde{y}_2,\ldots,\tilde{y}_n\right)=\mathop{\oplus}_{j=1}^{n}\tilde{y}_j^{\lambda_j}\\
&=\left\langle\left(\prod_{j=1}^{n}TN_{j1}^{\lambda_j},\prod_{j=1}^{n}TN_{j2}^{\lambda_j},\prod_{j=1}^{n}TN_{j3}^{\lambda_j},\prod_{j=1}^{n}TN_{j4}^{\lambda_j}\right),\right.\\
&\left(1-\prod_{j=1}^{n}\left(1-IN_{j1}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-IN_{j2}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-IN_{j3}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-IN_{j4}\right)^{\lambda_j}\right),\\
&\left.\left(1-\prod_{j=1}^{n}\left(1-FN_{j1}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-FN_{j2}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-FN_{j3}\right)^{\lambda_j},1-\prod_{j=1}^{n}\left(1-FN_{j4}\right)^{\lambda_j}\right)\right\rangle.
\end{aligned}\tag{3}$$

Then, the score and accuracy functions of the TrNN $\tilde{y} = <(TN_1, TN_2, TN_3, TN_4), (IN_1, IN_2, IN_3, IN_4), (FN_1, FN_2, FN_3, FN_4)>$ were defined as follows [9]:

$$S(\tilde{y})=\frac{1}{3}\left(2+\frac{TN_1+TN_2+TN_3+TN_4}{4}-\frac{IN_1+IN_2+IN_3+IN_4}{4}-\frac{FN_1+FN_2+FN_3+FN_4}{4}\right),\quad S(\tilde{y})\in[0,1],\tag{4}$$

$$H(\tilde{y})=\frac{TN_1+TN_2+TN_3+TN_4}{4}-\frac{FN_1+FN_2+FN_3+FN_4}{4},\quad H(\tilde{y})\in[-1,1].\tag{5}$$

Based on the score and accuracy functions of TrNNs, the ranking relations between two TrNNs $\tilde{y}_1 = <(TN_{11}, TN_{12}, TN_{13}, TN_{14}), (IN_{11}, IN_{12}, IN_{13}, IN_{14}), (FN_{11}, FN_{12}, FN_{13}, FN_{14})>$ and $\tilde{y}_2 = <(TN_{21}, TN_{22}, TN_{23}, TN_{24}), (IN_{21}, IN_{22}, IN_{23}, IN_{24}), (FN_{21}, FN_{22}, FN_{23}, FN_{24})>$ were defined as follows [9]:

(1) $\tilde{y}_1 \succ \tilde{y}_2$ for $S(\tilde{y}_1) = S(\tilde{y}_2)$

(2) $\tilde{y}_1 \succ \tilde{y}_2$ for $S(\tilde{y}_1) = S(\tilde{y}_2)$ and $H(\tilde{y}_1) > H(\tilde{y}_2)$

(3) $\tilde{y}_1 \cong \tilde{y}_2$ for $S(\tilde{y}_1) = S(\tilde{y}_2)$ and $H(\tilde{y}_1) = H(\tilde{y}_2)$

3. Trapezoidal Neutrosophic Z-Number (TrNZN) Sets

To make trapezoidal neutrosophic information reliable, this section gives the following definitions of a TrNZN set, operations of TrNZNs, score and accuracy functions of TrNZN, and ranking laws of TrNZNs.

Definition 1. Set U as a universe set; then, a TrNZN set in U is defined as the following mathematical representation:

$$\tilde{Z}=\left\{u,\left\langle\left(TZ_{\tilde{V}}(u),TZ_{\tilde{R}}(u)\right),\left(IZ_{\tilde{V}}(u),IZ_{\tilde{R}}(u)\right),\left(FZ_{\tilde{V}}(u),FZ_{\tilde{R}}(u)\right)\right\rangle|u\in U\right\},\tag{6}$$

where $(TZ_{\tilde{V}}(u), TZ_{\tilde{R}}(u))$, $(IZ_{\tilde{V}}(u), IZ_{\tilde{R}}(u))$, and $(FZ_{\tilde{V}}(u), FZ_{\tilde{R}}(u))$ are the truth, indeterminacy, and falsity trapezoidal Z-numbers that are composed of the truth, indeterminacy, and falsity trapezoidal fuzzy numbers and their reliability measures, denoted as $(TZ_{\tilde{V}}(u), TZ_{\tilde{R}}(u)) = ((T_{V1}(u), T_{V2}(u), T_{V3}(u), T_{V4}(u)), (T_{R1}(u), T_{R2}(u), T_{R3}(u), T_{R4}(u)))$: $U \longrightarrow [0, 1] \times [0, 1]$, $((IZ_{\tilde{V}}(u), IZ_{\tilde{R}}(u) = ((I_{V1}(u), I_{V2}(u), I_{V3}(u), I_{V4}(u)), (I_{R1}(u), I_{R2}(u), I_{R3}(u), I_{R4}(u)))$: U $\longrightarrow$ [0, 1]×[0, 1], and $(FZ_{\tilde{V}}(u), FZ_{\tilde{R}}(u)) = ((F_{V1}(u), F_{V2}(u), F_{V3}(u), F_{V4}(u)), (F_{R1}(u), F_{R2}(u), F_{R3}(u), F_{R4}(u)))$: U $\longrightarrow$ [0, 1]×[0, 1] with the conditions $0 \le T_{V4}(u) + I_{V4}(u) + F_{V4}(u) \le 3$ and $0 \le T_{R4}(u) + I_{R4}(u) + F_{R4}(u) \le 3$ for $u \in U$.

For convenience, the three trapezoidal Z-numbers in $\tilde{Z}$ are simply denoted as $(TZ_{\tilde{V}}(u), TZ_{\tilde{R}}(u)) = ((T_{V1},$

$T_{V2}, T_{V3}, T_{V4}), (T_{R1}, T_{R2}, T_{R3}, T_{R4})), (IZ_{\widetilde{V}}(u), IZ_{\widetilde{R}}(u)) = ((I_{V1}, I_{V2}, I_{V3}, I_{V4}), (I_{R1}, I_{R2}, I_{R3}, I_{R4}))$, and $(FZ_{\widetilde{V}}(u), FZ_{\widetilde{R}}(u)) = ((F_{V1}, F_{V2}, F_{V3}, F_{V4}), (F_{R1}, F_{R2}, F_{R3}, F_{R4}))$. Thus, a TrNZN in $\widetilde{Z}$ is simply denoted as $\widetilde{z} = <((T_{V1}, T_{V2}, T_{V3}, T_{V4}), (T_{R1}, T_{R2}, T_{R3}, T_{R4})), ((I_{V1}, I_{V2}, I_{V3}, I_{V4}), (I_{R1}, I_{R2}, I_{R3}, T_{R4})), ((F_{V1}, F_{V2}, F_{V3}, F_{V4}), (F_{R1}, F_{R2}, F_{R3}, F_{R4}))>$.

If $T_{V2} = T_{V3}$, $T_{R2} = T_{R3}$, $I_{V2} = I_{V3}$, $I_{R2} = I_{R3}$, and $F_{V2} = F_{V3}$, $F_{R2} = F_{R3}$ hold in the TrNZN $\widetilde{z}$; it is reduced to the triangular neutrosophic Z-number, which is a special case of TrNZN.

Definition 2. Set $\widetilde{z}_1 = <((T_{V11}, T_{V12}, T_{V13}, T_{V14}), (T_{R11}, T_{R12}, T_{R13}, T_{R14})), ((I_{V11}, I_{V12}, I_{V13}, I_{V14}), (I_{R11}, I_{R12}, I_{R13}, T_{R14})), ((F_{V11}, F_{V12}, F_{V13}, F_{V14}), (F_{R11}, F_{R12}, F_{R13}, F_{R14}))>$ and $\widetilde{z}_2 = <((T_{V21}, T_{V22}, T_{V23}, T_{V24}), (T_{R21}, T_{R22}, T_{R23}, T_{R24})), ((I_{V21}, I_{V22}, I_{V23}, I_{V24}), (I_{R21}, I_{R22}, I_{R23}, T_{R24})), ((F_{V21}, F_{V22}, F_{V23}, F_{V24}), (F_{R21}, F_{R22}, F_{R23}, F_{R24}))>$ as two TrNZNs. Then they are defined as the following basic operations:

(1) $\widetilde{z}_1 \oplus \widetilde{z}_2 = \langle ((T_{V11} + T_{V21} - T_{V11}T_{V21}, T_{V12} + T_{V22} - T_{V12}T_{V22}, T_{V13} + T_{V23} - T_{V13}T_{V23}, T_{V14} + T_{V24} - T_{V14}T_{V24}), (T_{R11} + T_{R21} - T_{R11}T_{R21}, T_{R12} + T_{R22} - T_{R12}T_{R22}, T_{R13} + T_{R23} - T_{R13}T_{R23}, T_{R14} + T_{R24} - T_{R14}T_{R24})), ((I_{V11}I_{V21}, I_{V12}I_{V22}, I_{V13}I_{V23}, I_{V14}I_{V24}), (I_{R11}I_{R21}, I_{R12}I_{R22}, I_{R13}I_{R23}, I_{R14}I_{R24})), ((F_{V11}F_{V21}, F_{V12}F_{V22}, F_{V13}F_{V23}, F_{V14}F_{V24}), (F_{R11}F_{R21}, F_{R12}F_{R22}, F_{R13}F_{R23}, F_{R14}F_{R24}))\rangle$

(2) $\widetilde{z}_1 \otimes \widetilde{z}_2 = \langle ((T_{V11}T_{V21}, T_{V12}T_{V22}, T_{V13}T_{V23}, T_{V14}T_{V24})(T_{R11}T_{R21}, T_{R12}T_{R22}, T_{R13}T_{R23}, T_{R14}T_{R24})), ((I_{V11} + I_{V21} - I_{V11}I_{V21}, I_{V12} + I_{V22} - I_{V12}I_{V22}, I_{V13} + I_{V23} - I_{V13}I_{V23}, I_{V14} + I_{V24} - I_{V14}I_{V24}), (I_{R11} + I_{R21} - I_{R11}I_{R21}, I_{R12} + I_{R22} - I_{R12}I_{R22}, I_{R13} + I_{R23} - I_{R13}I_{R23}, I_{R14} + I_{R24} - I_{R14}I_{R24})), ((F_{V11} + F_{V21} - F_{V11}F_{V21}, F_{V12} + F_{V22} - F_{V12}F_{V22}, F_{V13} + F_{V23} - F_{V13}F_{V23}, F_{V14} + F_{V24} - F_{V14}F_{V24}), (F_{R11} + F_{R21} - F_{R11}F_{R21}, F_{R12} + F_{R22} - F_{R12}F_{R22}, F_{R13} + F_{R23} - F_{R13}F_{R23}, F_{R14} + F_{R24} - F_{R14}F_{R24}))\rangle$

(3) $\lambda\widetilde{z}_1 = \langle ((1 - (1 - T_{V11})^{\lambda}, 1 - (1 - T_{V12})^{\lambda}, 1 - (1 - T_{V13})^{\lambda}, 1 - (1 - T_{V14})^{\lambda}), (1 - (1 - T_{R11})^{\lambda}, 1 - (1 - T_{R12})^{\lambda}, 1 - (1 - T_{R13})^{\lambda}, 1 - (1 - T_{R14})^{\lambda})), ((I_{V11}^{\lambda}, I_{V12}^{\lambda}, I_{V13}^{\lambda}, I_{V14}^{\lambda}), (I_{R11}^{\lambda}, I_{R12}^{\lambda}, I_{R13}^{\lambda}, I_{R14}^{\lambda})), ((F_{V11}^{\lambda}, F_{V12}^{\lambda}, F_{V13}^{\lambda}, F_{V14}^{\lambda}), (F_{R11}^{\lambda}, F_{R12}^{\lambda}, F_{R13}^{\lambda}, F_{R14}^{\lambda}))\rangle, \lambda > 0$

(4) $\widetilde{z}_1^{\lambda} = \langle ((T_{V11}^{\lambda}, T_{V12}^{\lambda}, T_{V13}^{\lambda}, T_{V14}^{\lambda}), (T_{R11}^{\lambda}, T_{R12}^{\lambda}, T_{R13}^{\lambda}, T_{R14}^{\lambda})), ((1 - (1 - I_{V11})^{\lambda}, 1 - (1 - I_{V12})^{\lambda}, 1 - (1 - I_{V13})^{\lambda}, 1 - (1 - I_{V14})^{\lambda}), (1 - (1 - I_{R11})^{\lambda}, 1 - (1 - I_{R12})^{\lambda}, 1 - (1 - I_{R13})^{\lambda}, 1 - (1 - I_{R14})^{\lambda})), ((1 - (1 - F_{V11})^{\lambda}, 1 - (1 - F_{V12})^{\lambda}, 1 - (1 - F_{V13})^{\lambda}, 1 - (1 - F_{V14})^{\lambda}), (1 - (1 - F_{R11})^{\lambda}, 1 - (1 - F_{R12})^{\lambda}, 1 - (1 - F_{R13})^{\lambda}, 1 - (1 - F_{R14})^{\lambda}))\rangle, \lambda > 0$

For ranking TrNZNs, the score and accuracy functions of TrNZN are defined according to the expected value of a trapezoidal fuzzy number and score and accuracy functions of TrNN [9].

Definition 3. Set $\widetilde{z}_1 = <((T_{V11}, T_{V12}, T_{V13}, T_{V14}), (T_{R11}, T_{R12}, T_{R13}, T_{R14})), ((I_{V11}, I_{V12}, I_{V13}, I_{V14}), (I_{R11}, I_{R12}, I_{R13}, T_{R14})), ((F_{V11}, F_{V12}, F_{V13}, F_{V14}), (F_{R11}, F_{R12}, F_{R13}, F_{R14}))>$ as TrNZN. Then the score and accuracy functions of the TrNZN $\widetilde{z}_1$ can be defined as follows:

$$S(\widetilde{z}_1) = \frac{1}{3}\left(\begin{array}{c} 2 + \dfrac{T_{V11} + T_{V12} + T_{V13} + T_{V14}}{4} \times \dfrac{T_{R11} + T_{R12} + T_{R13} + T_{R14}}{4} \\ -\dfrac{I_{V11} + I_{V12} + I_{V13} + I_{V14}}{4} \times \dfrac{I_{R11} + I_{R12} + I_{R13} + I_{R14}}{4} \\ -\dfrac{F_{V11} + F_{V12} + F_{V13} + F_{V14}}{4} \times \dfrac{F_{R11} + F_{R12} + F_{R13} + F_{R14}}{4} \end{array}\right), \quad S(\widetilde{z}_1) \in [0, 1], \tag{7}$$

$$\begin{aligned} H(\widetilde{z}_1) = {} & \frac{T_{V11} + T_{V12} + T_{V13} + T_{V14}}{4} \times \frac{T_{R11} + T_{R12} + T_{R13} + T_{R14}}{4} - \frac{F_{V11} + F_{V12} + F_{V13} + F_{V14}}{4} \\ & \times \frac{F_{R11} + F_{R12} + F_{R13} + F_{R14}}{4}, \quad H(\widetilde{z}) \in [-1, 1]. \end{aligned} \tag{8}$$

Based on equations (7) and (8), ranking laws between two TrNZNs are given by the following definition.

Definition 4. Set $\widetilde{z}_1 = <((T_{V11}, T_{V12}, T_{V13}, T_{V14}), (T_{R11}, T_{R12}, T_{R13}, T_{R14})), ((I_{V11}, I_{V12}, I_{V13}, I_{V14}), (I_{R11}, I_{R12}, I_{R13}, T_{R14})), ((F_{V11}, F_{V12}, F_{V13}, F_{V14}), (F_{R11}, F_{R12}, F_{R13}, F_{R14}))>$ and $\widetilde{z}_2 = <((T_{V21}, T_{V22}, T_{V23}, T_{V24}), (T_{R21}, T_{R22}, T_{R23}, T_{R24})), ((I_{V21}, I_{V22}, I_{V23}, I_{V24}), (I_{R21}, I_{R22}, I_{R23}, T_{R24})), ((F_{V21}, F_{V22}, F_{V23}, F_{V24}), (F_{R21}, F_{R22}, F_{R23}, F_{R24}))>$ as two TrNZNs. Then, the ranking laws between two TrNZNs are defined as follows:

(1) If $S(\widetilde{z}_1) > S(\widetilde{z}_2)$, then $\widetilde{z}_1 \succ \widetilde{z}_2$

(2) If $S(\widetilde{z}_1) = S(\widetilde{z}_2)$ and $H(\widetilde{z}_1) > H(\widetilde{z}_2)$, then $\widetilde{z}_1 \succ \widetilde{z}_2$

(3) If $S(\widetilde{z}_1) = S(\widetilde{z}_2)$ and $H(\widetilde{z}_1) = H(\widetilde{z}_2)$, then $\widetilde{z}_1 \cong \widetilde{z}_2$

4. Weighted Aggregation Operators of TrNZNs

Regarding information aggregation in MDM problems, one usually utilizes the weighted arithmetic and geometric averaging operators as the most basic information aggregation approaches. To aggregate TrNZNs, therefore, this section proposes the two following weighted aggregation operators of TrNZNs based on the basic operations of TrNZNs in Definition 2.

4.1. Weighted Arithmetic Averaging Operator of TrNZNs

Definition 5. Set $\tilde{z}_j = <((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1}, F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4}))>$ $(j = 1, 2, \ldots, n)$ as a series of TrNZNs. Then, the TrNZNWAA operator is defined as

$$\mathrm{TrNZNWAA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) = \mathop{\oplus}_{j=1}^{n} \lambda_j \tilde{z}_j, \tag{9}$$

where λ_j $(j = 1, 2, \ldots, n)$ is the weight of the jth TrNZN $\tilde{z}_j$ $(j = 1, 2, \ldots, n)$ for $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

Based on the basic operations of TrNZNs in Definition 2 and equation (9), we have the following theorem.

Theorem 1. *Set $\tilde{z}_j = <((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1}, F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4}))>$ $(j = 1, 2, \ldots, n)$ as a series of TrNZNs. Then, the aggregated value of equation (9) is also TrNZN, which is yielded by the following equation:*

$$\begin{aligned}
&\mathrm{TrNZNWAA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n)\\
&= \left\langle \begin{array}{c}
\left(\left(1-\prod_{j=1}^{n}(1-T_{Vj1})^{\lambda_j}, 1-\prod_{j=1}^{n}(1-T_{Vj2})^{\lambda_j}, 1-\prod_{j=1}^{n}(1-T_{Vj3})^{\lambda_j}, 1-\prod_{j=1}^{n}(1-T_{Vj4})^{\lambda_j}\right), \left(1-\prod_{j=1}^{n}(1-T_{Rj1})^{\lambda_j}, 1-\prod_{j=1}^{n}(1-T_{Rj2})^{\lambda_j}, 1-\prod_{j=1}^{n}(1-T_{Rj3})^{\lambda_j}, 1-\prod_{j=1}^{n}(1-T_{Rj4})^{\lambda_j}\right)\right),\\
\left(\left(\prod_{j=1}^{n} I_{Vj1}^{\lambda_j}, \prod_{j=1}^{n} I_{Vj2}^{\lambda_j}, \prod_{j=1}^{n} I_{Vj3}^{\lambda_j}, \prod_{j=1}^{n} I_{Vj4}^{\lambda_j}\right), \left(\prod_{j=1}^{n} I_{Rj1}^{\lambda_j}, \prod_{j=1}^{n} I_{Rj2}^{\lambda_j}, \prod_{j=1}^{n} I_{Rj3}^{\lambda_j}, \prod_{j=1}^{n} I_{Rj4}^{\lambda_j}\right)\right), \left(\left(\prod_{j=1}^{n} F_{Vj1}^{\lambda_j}, \prod_{j=1}^{n} F_{Vj2}^{\lambda_j}, \prod_{j=1}^{n} F_{Vj3}^{\lambda_j}, \prod_{j=1}^{n} F_{Vj4}^{\lambda_j}\right), \left(\prod_{j=1}^{n} F_{Rj1}^{\lambda_j}, \prod_{j=1}^{n} F_{Rj2}^{\lambda_j}, \prod_{j=1}^{n} F_{Rj3}^{\lambda_j}, \prod_{j=1}^{n} F_{Rj4}^{\lambda_j}\right)\right)
\end{array}\right\rangle,
\end{aligned} \tag{10}$$

where λ_j $(j = 1, 2, \ldots, n)$ is the weight of the jth TrNZN $\tilde{z}_j$ $(j = 1, 2, \ldots, n)$ for $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

Proof. The proof of equation (10) can be given by mathematical induction.

(1) Set $n = 2$. Then there is the following result:

$$\begin{aligned}
\mathrm{TrNZNWAA}(\tilde{z}_1, \tilde{z}_2) &= \lambda_1 \tilde{z}_1 \oplus \lambda_2 \tilde{z}_2\\
&= \Big\langle\Big(\Big(1-(1-T_{V11})^{\lambda_1} + 1-(1-T_{V21})^{\lambda_2} - \left(1-(1-T_{V11})^{\lambda_1}\right)\left(1-(1-T_{V21})^{\lambda_2}\right),\\
&\quad 1-(1-T_{V12})^{\lambda_1} + 1-(1-T_{V22})^{\lambda_2} - \left(1-(1-T_{V12})^{\lambda_1}\right)\left(1-(1-T_{V22})^{\lambda_2}\right),\\
&\quad 1-(1-T_{V13})^{\lambda_1} + 1-(1-T_{V23})^{\lambda_2} - \left(1-(1-T_{V13})^{\lambda_1}\right)\left(1-(1-T_{V23})^{\lambda_2}\right),\\
&\quad 1-(1-T_{V14})^{\lambda_1} + 1-(1-T_{V24})^{\lambda_2} - \left(1-(1-T_{V14})^{\lambda_1}\right)\left(1-(1-T_{V24})^{\lambda_2}\right)\Big),\\
&\quad \Big(1-(1-T_{R11})^{\lambda_1} + 1-(1-T_{R21})^{\lambda_2} - \left(1-(1-T_{R11})^{\lambda_1}\right)\left(1-(1-T_{R21})^{\lambda_2}\right),\\
&\quad 1-(1-T_{R12})^{\lambda_1} + 1-(1-T_{R22})^{\lambda_2} - \left(1-(1-T_{R12})^{\lambda_1}\right)\left(1-(1-T_{R22})^{\lambda_2}\right),\\
&\quad 1-(1-T_{R13})^{\lambda_1} + 1-(1-T_{R23})^{\lambda_2} - \left(1-(1-T_{R13})^{\lambda_1}\right)\left(1-(1-T_{R23})^{\lambda_2}\right),\\
&\quad 1-(1-T_{V14})^{\lambda_1} + 1-(1-T_{R24})^{\lambda_2} - \left(1-(1-T_{R14})^{\lambda_1}\right)\left(1-(1-T_{R24})^{\lambda_2}\right)\Big)\Big),\\
&\quad \left(\left(I_{V11}^{\lambda_1} I_{V21}^{\lambda_2}, I_{V12}^{\lambda_1} I_{V22}^{\lambda_2}, I_{V13}^{\lambda_1} I_{V23}^{\lambda_2}, I_{V14}^{\lambda_1} I_{V24}^{\lambda_2}\right), \left(I_{R11}^{\lambda_1} I_{R21}^{\lambda_2}, I_{R12}^{\lambda_1} I_{R22}^{\lambda_2}, I_{R13}^{\lambda_1} I_{R23}^{\lambda_2}, I_{R14}^{\lambda_1} I_{R24}^{\lambda_2}\right)\right),\\
&\quad \left(\left(F_{V11}^{\lambda_1} F_{V21}^{\lambda_2}, F_{V12}^{\lambda_1} F_{V22}^{\lambda_2}, F_{V13}^{\lambda_1} F_{V23}^{\lambda_2}, F_{V14}^{\lambda_1} F_{V24}^{\lambda_2}\right), \left(F_{R11}^{\lambda_1} F_{R21}^{\lambda_2}, F_{R12}^{\lambda_1} F_{R22}^{\lambda_2}, F_{R13}^{\lambda_1} F_{R23}^{\lambda_2}, F_{R14}^{\lambda_1} F_{R24}^{\lambda_2}\right)\right)\Big\rangle\\
&= \Big\langle\Big(\Big(1-(1-T_{V11})^{\lambda_1}(1-T_{V21})^{\lambda_2}, 1-(1-T_{V12})^{\lambda_1}(1-T_{V22})^{\lambda_2},\\
&\quad 1-(1-T_{V13})^{\lambda_1}(1-T_{V23})^{\lambda_2}, 1-(1-T_{V14})^{\lambda_1}(1-T_{V24})^{\lambda_2}\Big)\\
&\quad \Big(1-(1-T_{R11})^{\lambda_1}(1-T_{R21})^{\lambda_2}, 1-(1-T_{R12})^{\lambda_1}(1-T_{R22})^{\lambda_2},\\
&\quad 1-(1-T_{R13})^{\lambda_1}(1-T_{R23})^{\lambda_2}, 1-(1-T_{R14})^{\lambda_1}(1-T_{R24})^{\lambda_2}\Big)\Big),\\
&\quad \left(\left(\prod_{j=1}^{2} I_{Vj1}^{\lambda_j}, \prod_{j=1}^{2} I_{Vj2}^{\lambda_j}, \prod_{j=1}^{2} I_{Vj3}^{\lambda_j}, \prod_{j=1}^{2} I_{Vj4}^{\lambda_j}\right), \left(\prod_{j=1}^{2} I_{Rj1}^{\lambda_j}, \prod_{j=1}^{2} I_{Rj2}^{\lambda_j}, \prod_{j=1}^{2} I_{Rj3}^{\lambda_j}, \prod_{j=1}^{2} I_{Rj4}^{\lambda_j}\right)\right),\\
&\quad \left(\left(\prod_{j=1}^{2} F_{Vj1}^{\lambda_j}, \prod_{j=1}^{2} F_{Vj2}^{\lambda_j}, \prod_{j=1}^{2} F_{Vj3}^{\lambda_j}, \prod_{j=1}^{2} F_{Vj4}^{\lambda_j}\right), \left(\prod_{j=1}^{2} F_{Rj1}^{\lambda_j}, \prod_{j=1}^{2} F_{Rj2}^{\lambda_j}, \prod_{j=1}^{2} F_{Rj3}^{\lambda_j}, \prod_{j=1}^{2} F_{Rj4}^{\lambda_j}\right)\right)\Big\rangle.
\end{aligned} \tag{11}$$

(2) Set $n = k$. Then, equation (10) can hold in the following equation:

$$
\begin{aligned}
&\operatorname{TrNZNWAA}\left(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_k\right) = \mathop{\oplus}_{j=1}^{k} \lambda_j \tilde{z}_j \\
&= \Bigg\langle \Bigg(\Bigg(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Vj1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k}\left(1 - T_{Vj2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k}\left(1 - T_{Vj3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k}\left(1 - T_{Vj4}\right)^{\lambda_j} \right), \\
&\quad \left(1 - \prod_{j=1}^{k}\left(1 - T_{Rj1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k}\left(1 - T_{Rj2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k}\left(1 - T_{Rj3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k}\left(1 - T_{Rj4}\right)^{\lambda_j} \right) \Bigg), \\
&\quad \left(\left(\prod_{j=1}^{k} I_{Vj1}^{\lambda_j}, \prod_{j=1}^{k} I_{Vj2}^{\lambda_j}, \prod_{j=1}^{k} I_{Vj3}^{\lambda_j}, \prod_{j=1}^{k} I_{Vj4}^{\lambda_j} \right), \left(\prod_{j=1}^{k} I_{Rj1}^{\lambda_j}, \prod_{j=1}^{k} I_{Rj2}^{\lambda_j}, \prod_{j=1}^{k} I_{Rj3}^{\lambda_j}, \prod_{j=1}^{k} I_{Rj4}^{\lambda_j} \right) \right), \\
&\quad \left(\left(\prod_{j=1}^{k} F_{Vj1}^{\lambda_j}, \prod_{j=1}^{k} F_{Vj2}^{\lambda_j}, \prod_{j=1}^{k} F_{Vj3}^{\lambda_j}, \prod_{j=1}^{k} F_{Vj4}^{\lambda_j} \right), \left(\prod_{j=1}^{k} F_{Rj1}^{\lambda_j}, \prod_{j=1}^{k} F_{Rj2}^{\lambda_j}, \prod_{j=1}^{k} F_{Rj3}^{\lambda_j}, \prod_{j=1}^{k} F_{Rj4}^{\lambda_j} \right) \right) \Bigg\rangle.
\end{aligned} \tag{12}
$$

(3) Set $n = k + 1$. By equations (11) and (12), we can obtain

$$
\begin{aligned}
&\operatorname{TrNZNWAA}\left(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_k, \tilde{z}_{k+1}\right) = \mathop{\oplus}_{j=1}^{k} \lambda_j \tilde{z}_j \oplus \lambda_{k+1} \tilde{z}_{k+1} \\
&= \Bigg\langle \Bigg(\Bigg(1 - \prod_{j=1}^{k}\left(1 - T_{Vj1}\right)^{\lambda_j} + 1 - \left(1 - T_{V(k+1)1}\right)^{\lambda_{k+1}} - \left(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Vj1}\right)^{\lambda_j}\right) \left(1 - \left(1 - T_{V(k+1)1}\right)\right)^{\lambda_{k+1}} \right), 1 - \prod_{j=1}^{k}\left(1 - T_{Vj2}\right)^{\lambda_j} + 1 - \left(1 - T_{V(k+1)2}\right)^{\lambda_{k+1}} - \left(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Vj2}\right)^{\lambda_j}\right) \left(1 - \left(1 - T_{V(k+1)2}\right)\right)^{\lambda_{k+1}} \right), \\
&\quad 1 - \prod_{j=1}^{k}\left(1 - T_{Vj3}\right)^{\lambda_j} + 1 - \left(1 - T_{V(k+1)3}\right)^{\lambda_{k+1}} - \left(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Vj3}\right)^{\lambda_j}\right) \left(1 - \left(1 - T_{V(k+1)3}\right)\right)^{\lambda_{k+1}} \right), 1 - \prod_{j=1}^{k}\left(1 - T_{Vj4}\right)^{\lambda_j} + 1 - \left(1 - T_{V(k+1)4}\right)^{\lambda_{k+1}} - \left(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Vj4}\right)^{\lambda_j}\right) \left(1 - \left(1 - T_{V(k+1)4}\right)\right)^{\lambda_{k+1}} \right) \Bigg) \\
&\quad \Bigg(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Rj1}\right)^{\lambda_j} + 1 - \left(1 - T_{R(k+1)1}\right)^{\lambda_{k+1}} - \left(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Rj1}\right)^{\lambda_j}\right) \left(1 - \left(1 - T_{R(k+1)1}\right)\right)^{\lambda_{k+1}} \right), 1 - \prod_{j=1}^{k}\left(1 - T_{Rj2}\right)^{\lambda_j} + 1 - \left(1 - T_{R(k+1)2}\right)^{\lambda_{k+1}} - \left(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Rj2}\right)^{\lambda_j}\right) \left(1 - \left(1 - T_{R(k+1)2}\right)\right)^{\lambda_{k+1}} \right), \\
&\quad 1 - \prod_{j=1}^{k}\left(1 - T_{Rj3}\right)^{\lambda_j} + 1 - \left(1 - T_{R(k+1)3}\right)^{\lambda_{k+1}} - \left(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Rj3}\right)^{\lambda_j}\right) \left(1 - \left(1 - T_{R(k+1)3}\right)\right)^{\lambda_{k+1}} \right), 1 - \prod_{j=1}^{k}\left(1 - T_{Rj4}\right)^{\lambda_j} + 1 - \left(1 - T_{R(k+1)4}\right)^{\lambda_{k+1}} - \left(\left(1 - \prod_{j=1}^{k}\left(1 - T_{Rj4}\right)^{\lambda_j}\right) \left(1 - \left(1 - T_{R(k+1)4}\right)\right)^{\lambda_{k+1}} \right), \Bigg) \Bigg) \\
&\quad \left(\left(\prod_{j=1}^{k+1} I_{Vj1}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Vj2}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Vj3}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Vj4}^{\lambda_j} \right), \left(\prod_{j=1}^{k+1} I_{Rj1}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Rj2}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Rj3}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Rj4}^{\lambda_j} \right) \right), \left(\left(\prod_{j=1}^{k+1} F_{Vj1}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Vj2}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Vj3}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Vj4}^{\lambda_j} \right), \left(\prod_{j=1}^{k+1} F_{Rj1}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Rj2}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Rj3}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Rj4}^{\lambda_j} \right) \right) \Bigg\rangle \\
&= \Bigg\langle \Bigg(\left(1 - \prod_{j=1}^{k+1}\left(1 - T_{Vj1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k+1}\left(1 - T_{Vj2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k+1}\left(1 - T_{Vj3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k+1}\left(1 - T_{Vj4}\right)^{\lambda_j}, \right) \left(1 - \prod_{j=1}^{k+1}\left(1 - T_{Rj1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k+1}\left(1 - T_{Rj2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k+1}\left(1 - T_{Rj3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{k+1}\left(1 - T_{Rj4}\right)^{\lambda_j} \right) \Bigg), \\
&\quad \left(\left(\prod_{j=1}^{k+1} I_{Vj1}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Vj2}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Vj3}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Vj4}^{\lambda_j} \right), \left(\prod_{j=1}^{k+1} I_{Rj1}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Rj2}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Rj3}^{\lambda_j}, \prod_{j=1}^{k+1} I_{Rj4}^{\lambda_j} \right) \right), \left(\left(\prod_{j=1}^{k+1} F_{Vj1}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Vj2}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Vj3}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Vj4}^{\lambda_j} \right), \left(\prod_{j=1}^{k+1} F_{Rj1}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Rj2}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Rj3}^{\lambda_j}, \prod_{j=1}^{k+1} F_{Rj4}^{\lambda_j} \right) \right) \Bigg\rangle.
\end{aligned} \tag{13}
$$

Regarding the above results, equation (10) can hold for any n. Thus, the proof is completed.

Especially when $\lambda_j = 1/n$ ($j = 1, 2, \ldots, n$), the TrNZNWAA operator is reduced to the TrNZN arithmetic averaging operator. □

Theorem 2. *The TrNZNWAA operator contains the three following properties:*

(P1) Idempotency: set $\tilde{z}_j = \langle((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1}, F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4}))\rangle$ *($j = 1, 2, \ldots, n$) as a series of TrNZNs. If* $\tilde{z}_j = \tilde{z}$ *for* $j = 1, 2, \ldots, n$*, then there exists* $\operatorname{TrNZNWAA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) = \tilde{z}$.

(P2) Set $\tilde{z}_j = \langle((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1},$

F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4})) > (j = 1, 2, . . ., n) as a series of TrNZNs; then, set the minimum and maximum TrNZNs as

$$\tilde{z}^{-}=\left\langle \begin{array}{c} \left(\left(\min_j T_{Vj1},\min_j T_{Vj2},\min_j T_{Vj3},\min_j T_{Vj4}\right),\left(\min_j T_{Rj1},\min_j T_{Rj2},\min_j T_{Rj3},\min_j T_{Rj4}\right)\right),\\ \left(\left(\max_j I_{Vj1},\max_j I_{Vj2},\max_j I_{Vj3},\max_j I_{Vj4}\right),\left(\max_j I_{Rj1},\max_j I_{Rj2},\max_j I_{Rj3},\max_j I_{Rj4}\right)\right),\\ \left(\left(\max_j F_{Vj1},\max_j F_{Vj2},\max_j F_{Vj3},\max_j F_{Vj4}\right),\left(\max_j F_{Rj1},\max_j F_{Rj2},\max_j F_{Rj3},\max_j F_{Rj4}\right)\right) \end{array}\right\rangle,$$

$$\tilde{z}^{+}=\left\langle \begin{array}{c} \left(\left(\max_j T_{Vj1},\max_j T_{Vj2},\max_j T_{Vj3},\max_j T_{Vj4}\right),\left(\max_j T_{Rj1},\max_j T_{Rj2},\max_j T_{Rj3},\max_j T_{Rj4}\right)\right),\\ \left(\left(\min_j I_{Vj1},\min_j I_{Vj2},\min_j I_{Vj3},\min_j I_{Vj4}\right),\left(\min_j I_{Rj1},\min_j I_{Rj2},\min_j I_{Rj3},\min_j I_{Rj4}\right)\right),\\ \left(\left(\min_j F_{Vj1},\min_j F_{Vj2},\min_j F_{Vj3},\min_j F_{Vj4}\right),\left(\min_j F_{Rj1},\min_j F_{Rj2},\min_j F_{Rj3},\min_j F_{Rj4}\right)\right) \end{array}\right\rangle. \tag{14}$$

Then, there is $\tilde{z}^{-} \le TrNZNWAA(\tilde{z}_1,\tilde{z}_2,\ldots,\tilde{z}_n) \le \tilde{z}^{+}$.

(P3) Monotony: set $\tilde{z}_j$= <((TVj1, TVj2, TVj3, TVj4), (TRj1, TRj2, TRj3, TRj4)), ((IVj1, IVj2, IVj3, IVj4), (IRj1, IRj2, IRj3, IRj4)), ((FVj1, FVj2, FVj3, FVj4), (FRj1, FRj2, FRj3, FRj4))> (j = 1, 2, . . ., n) as a series of TrNZNs. If $\tilde{z}_j \le \tilde{z}_j^{*}$ for j = 1, 2, . . ., n, then there is $TrNZNWAA(\tilde{z}_1,\tilde{z}_2,\ldots,\tilde{z}_n) \le TrNZNWAA(\tilde{z}_1^{*},\tilde{z}_2^{*},\ldots,\tilde{z}_n^{*})$.

Proof.

(P1) Owing to $\tilde{z}_j=\tilde{z}$ for $j=1, 2, \ldots, n$, there is the following result:

$$\begin{aligned}
\mathrm{TrNZNWAA}(\tilde{z}_1,\tilde{z}_2,\ldots,\tilde{z}_n) &= \mathop{\oplus}_{j=1}^{n}\lambda_j\tilde{z}_j\\
&=\Bigg\langle\Bigg(\Bigg(1-\prod_{j=1}^{n}(1-T_{Vj1})^{\lambda_j},1-\prod_{j=1}^{n}(1-T_{Vj2})^{\lambda_j},1-\prod_{j=1}^{n}(1-T_{Vj3})^{\lambda_j},1-\prod_{j=1}^{n}(1-T_{Vj4})^{\lambda_j}\Bigg),\\
&\quad\Bigg(1-\prod_{j=1}^{n}(1-T_{Rj1})^{\lambda_j},1-\prod_{j=1}^{n}(1-T_{Rj2})^{\lambda_j},1-\prod_{j=1}^{n}(1-T_{Rj3})^{\lambda_j},1-\prod_{j=1}^{n}(1-T_{Rj4})^{\lambda_j}\Bigg)\Bigg),\\
&\quad\Bigg(\Bigg(\prod_{j=1}^{n}I_{Vj1}^{\lambda_j},\prod_{j=1}^{n}I_{Vj2}^{\lambda_j},\prod_{j=1}^{n}I_{Vj3}^{\lambda_j},\prod_{j=1}^{n}I_{Vj4}^{\lambda_j}\Bigg),\Bigg(\prod_{j=1}^{n}I_{Rj1}^{\lambda_j},\prod_{j=1}^{n}I_{Rj2}^{\lambda_j},\prod_{j=1}^{n}I_{Rj3}^{\lambda_j},\prod_{j=1}^{n}I_{Rj4}^{\lambda_j}\Bigg)\Bigg),\\
&\quad\Bigg(\Bigg(\prod_{j=1}^{n}F_{Vj1}^{\lambda_j},\prod_{j=1}^{n}F_{Vj2}^{\lambda_j},\prod_{j=1}^{n}F_{Vj3}^{\lambda_j},\prod_{j=1}^{n}F_{Vj4}^{\lambda_j}\Bigg),\Bigg(\prod_{j=1}^{n}F_{Rj1}^{\lambda_j},\prod_{j=1}^{n}F_{Rj2}^{\lambda_j},\prod_{j=1}^{n}F_{Rj3}^{\lambda_j},\prod_{j=1}^{n}F_{Rj4}^{\lambda_j}\Bigg)\Bigg)\Bigg\rangle\\
&=\Bigg\langle\Bigg(\Bigg(1-(1-T_{V1})\sum_{j=1}n\lambda_j,1-(1-T_{V2})\sum_{j=1}n\lambda_j,1-(1-T_{V3})\sum_{j=1}n\lambda_j,1-(1-T_{V4})\sum_{j=1}n\lambda_j\Bigg),\\
&\quad\Bigg(1-(1-T_{R1})\sum_{j=1}n\lambda_j,1-(1-T_{R2})\sum_{j=1}n\lambda_j,1-(1-T_{R3})\sum_{j=1}n\lambda_j,1-(1-T_{R4})\sum_{j=1}n\lambda_j\Bigg)\Bigg),
\end{aligned}$$

$$\left(\left(I_{V1}\sum j=1^n w_j, I_{V2}\sum j=1^n w_j, I_{V3}\sum j=1^n w_j, I_{V4}\sum j=1^n w_j\right),\right.$$
$$\left.\left(I_{R1}\sum j=1^n w_j, I_{R2}\sum j=1^n w_j, I_{R3}\sum j=1^n w_j, I_{R4}\sum j=1^n w_j\right)\right),$$
$$\left(\left(F_{V1}\sum j=1^n w_j, F_{V2}\sum j=1^n w_j, F_{V3}\sum j=1^n w_j, F_{V4}\sum j=1^n w_j\right),\right.$$
$$\left.\left.\left(F_{R1}\sum j=1^n w_j, F_{R2}\sum j=1^n w_j, F_{R3}\sum j=1^n w_j, F_{R4}\sum j=1^n w_j\right)\right)\right\rangle$$
$$=\langle(((T_{V1},T_{V2},T_{V3},T_{V4}),(T_{R1},T_{R2},T_{R3},T_{R4})),((I_{V1},I_{V2},I_{V3},I_{V4}),(I_{R1},I_{R2},I_{R3},I_{R4})),$$
$$((I_{V1},I_{V2},I_{V3},I_{V4}),(I_{R1},I_{R2},I_{R3},I_{R4}))\rangle=\tilde{z}. \tag{15}$$

(P2) Due to $\tilde{z}^- \le \tilde{z}_j \le \tilde{z}^+$ for $j = 1, 2, \ldots, n$, there exists $\oplus_{j=1}^n \lambda_j \tilde{z}^- \le \oplus_{j=1}^n \lambda_j \tilde{z}_j \le \oplus_{j=1}^n \lambda_j \tilde{z}^+$. So, the inequality $\tilde{z}^- \le \oplus_{j=1}^n \lambda_j \tilde{z}_j \le \tilde{z}^+$ can hold according to (P1); that is, $\tilde{z}^- \le \mathrm{TrNZNWAA}(\tilde{z}_1, \tilde{\tilde{n}}_2, \ldots, \tilde{\tilde{n}}_n) \le \tilde{z}^+$.

(P3) Due to $\tilde{z}_j \le \tilde{z}_j^*$ for $j = 1, 2, \ldots, n$, there is $\oplus_{j=1}^n \lambda_j \tilde{z}_j \le \oplus_{j=1}^n \lambda_j \tilde{z}_j^*$; that is, $\mathrm{TrNZNWAA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) \le \mathrm{TrNZNWAA}(\tilde{z}_1^*, \tilde{z}_2^*, \ldots, \tilde{z}_n^*)$.

Thus, the proof of these properties is completed. □

4.2. Weighted Geometric Averaging Operator of TrNZNs

Definition 6. Set $\tilde{z}_j$= <((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1}, F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4}))> ($j = 1, 2, \ldots, n$) as a series of TrNZNs. Then, the TrNZNWGA operator is defined as

$$\mathrm{TrNZNWGA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) = \otimes_{j=1}^{n} \tilde{z}_j^{\lambda_j}, \tag{16}$$

where λ_j ($j = 1, 2, \ldots, n$) is the weight of the jth TrNZN $\tilde{z}_j$ for $\lambda_j \in [0, 1]$ and $\sum_{j=1}^n \lambda_j = 1$.

Regarding the basic operations of TrNZNs in Definition 2 and equation (16), we can give the theorem below.

Theorem 3. *Set* $\tilde{z}_j$= <((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1}, F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4}))> *($j = 1, 2, \ldots, n$) as a series of TrNZNs. Then, the aggregated value of the TrNZNWGA operator is also TrNZN, which is obtained by*

$$\begin{aligned}
\mathrm{TrNZNWGA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) &= \otimes_{j=1}^{n} \tilde{z}_j^{\lambda_j} \\
&= \left\langle\left(\left(\prod_{j=1}^n T_{Vj1}^{\lambda_j}, \prod_{j=1}^n T_{Vj2}^{\lambda_j}, \prod_{j=1}^n T_{Vj3}^{\lambda_j}, \prod_{j=1}^n T_{Vj4}^{\lambda_j}\right), \left(\prod_{j=1}^n T_{Rj1}^{\lambda_j}, \prod_{j=1}^n T_{Rj2}^{\lambda_j}, \prod_{j=1}^n T_{Rj3}^{\lambda_j}, \prod_{j=1}^n T_{Rj4}^{\lambda_j}\right)\right),\right. \\
&\left(\left(1-\prod_{j=1}^n\left(1-I_{Vj1}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-I_{Vj2}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-I_{Vj3}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-I_{Vj4}\right)^{\lambda_j}\right),\right. \\
&\left.\left(1-\prod_{j=1}^n\left(1-I_{Rj1}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-I_{Rj2}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-I_{Rj3}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-I_{Rj4}\right)^{\lambda_j}\right)\right), \\
&\left(\left(1-\prod_{j=1}^n\left(1-F_{Vj1}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-F_{Vj2}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-F_{Vj3}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-F_{Vj4}\right)^{\lambda_j}\right),\right. \\
&\left.\left.\left(1-\prod_{j=1}^n\left(1-F_{Rj1}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-F_{Rj2}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-F_{Rj3}\right)^{\lambda_j}, 1-\prod_{j=1}^n\left(1-F_{Rj4}\right)^{\lambda_j}\right)\right)\right\rangle.
\end{aligned} \tag{17}$$

where λ_j *(j = 1, 2, ..., n) is the weight of the jth TrNZN* $\tilde{z}_j$ *for* $\lambda_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$.

Based on the similar proof process of Theorem 1, we can verify Theorem 3, which is omitted.

In particular, the TrNZNWGA operator is reduced to the TrNZN geometric averaging operator when $\lambda_j = 1/n$ *(j = 1, 2, ..., n).*

Theorem 4. *The TrNZNWGA operator also contains the three following properties:*

(P1) Idempotency: set $\tilde{z}_j = <((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1}, F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4}))>$ *(j = 1, 2,...,n) as a series of TrNZNs. If* $\tilde{z}_j = \tilde{z}$ *for j = 1, 2, ..., n, then there exists* $\text{TrNZNWGA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) = \tilde{z}$.

(P2) Boundedness: set $\tilde{z}_j = <((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1}, F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4}))>$ *(j = 1, 2, ..., n) as a series of TrNZNs; then set the minimum and maximum TrNZNs as*

$$\tilde{z}^- = \left\langle \begin{array}{c} \left(\left(\min_j T_{Vj1}, \min_j T_{Vj2}, \min_j T_{Vj3}, \min_j T_{Vj4}\right), \left(\min_j T_{Rj1}, \min_j T_{Rj2}, \min_j T_{Rj3}, \min_j T_{Rj4}\right)\right), \\ \left(\left(\max_j I_{Vj1}, \max_j I_{Vj2}, \max_j I_{Vj3}, \max_j I_{Vj4}\right), \left(\max_j I_{Rj1}, \max_j I_{Rj2}, \max_j I_{Rj3}, \max_j I_{Rj4}\right)\right), \\ \left(\left(\max_j F_{Vj1}, \max_j F_{Vj2}, \max_j F_{Vj3}, \max_j F_{Vj4}\right), \left(\max_j F_{Rj1}, \max_j F_{Rj2}, \max_j F_{Rj3}, \max_j F_{Rj4}\right)\right) \end{array} \right\rangle,$$
$$\tilde{z}^+ = \left\langle \begin{array}{c} \left(\left(\max_j T_{Vj1}, \max_j T_{Vj2}, \max_j T_{Vj3}, \max_j T_{Vj4}\right), \left(\max_j T_{Rj1}, \max_j T_{Rj2}, \max_j T_{Rj3}, \max_j T_{Rj4}\right)\right), \\ \left(\left(\min_j I_{Vj1}, \min_j I_{Vj2}, \min_j I_{Vj3}, \min_j I_{Vj4}\right), \left(\min_j I_{Rj1}, \min_j I_{Rj2}, \min_j I_{Rj3}, \min_j I_{Rj4}\right)\right), \\ \left(\left(\min_j F_{Vj1}, \min_j F_{Vj2}, \min_j F_{Vj3}, \min_j F_{Vj4}\right), \left(\min_j F_{Rj1}, \min_j F_{Rj2}, \min_j F_{Rj3}, \min_j F_{Rj4}\right)\right) \end{array} \right\rangle \quad (18)$$

Then, there is $\tilde{z}^- \leq \text{TrNZNWGA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) \leq \tilde{z}^+$.

(P3) Monotony: set $\tilde{z}_j = <((T_{Vj1}, T_{Vj2}, T_{Vj3}, T_{Vj4}), (T_{Rj1}, T_{Rj2}, T_{Rj3}, T_{Rj4})), ((I_{Vj1}, I_{Vj2}, I_{Vj3}, I_{Vj4}), (I_{Rj1}, I_{Rj2}, I_{Rj3}, I_{Rj4})), ((F_{Vj1}, F_{Vj2}, F_{Vj3}, F_{Vj4}), (F_{Rj1}, F_{Rj2}, F_{Rj3}, F_{Rj4}))>$ *(j = 1, 2, ..., n) as a series of TrNZNs. If* $\tilde{z}_j \leq \tilde{z}_j^*$ *for j = 1, 2, ..., n, then there exists* $\text{TrNZNWGA}(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) \leq \text{TrNZNWGA}(\tilde{z}_1^*, \tilde{z}_2^*, \ldots, \tilde{z}_n^*)$.

By the same proof process of Theorem 2, the properties of the TrNZNWGA operator can be also verified, which are not repeated here.

5. MDM Approach Using the TrNZNWAA and TrNZNWGA Operators and Score and Accuracy Functions

This section establishes an MDM approach by using the TrNZNWAA and TrNZNWGA operators and score and accuracy functions to handle MDM problems with TrNZN information.

Regarding an MDM problem with TrNZN information, a set of alternatives $Q = \{Q_1, Q_2, \ldots, Q_m\}$ are commonly presented and satisfactorily assessed by a set of criteria $S = \{s_1, s_2, \ldots, s_n\}$. Each alternative over criteria is assessed by decision makers and then their given assessment values are expressed in the form of TrNZNs $\tilde{z}_{ij} = <((T_{Vij1}, T_{Vij2}, T_{Vij3}, T_{Vij4}), (T_{Rij1}, T_{Rij2}, T_{Rij3}, T_{Rij4})), ((I_{Vij1}, I_{Vij2}, I_{Vij3}, I_{Vij4}), (I_{Rij1}, I_{Rij2}, I_{Rij3}, I_{Rij4})), ((F_{Vij1}, F_{Vij2}, F_{Vij3}, F_{Vij4}), (F_{Rij1}, F_{Rij2}, F_{Rij3}, F_{Rij4}))>$ $(j = 1, 2, \ldots, n;\ i = 1, 2, \ldots, m)$, where $(T_{Vij1}, T_{Vij2}, T_{Vij3}, T_{Vij4}) \subseteq [0, 1]$ and $(T_{Rij1}, T_{Rij2}, T_{Rij3}, T_{Rij4}) \subseteq [0, 1]$ indicate the truth degrees and reliability measures of the alternative Q_i over the criteria s_j, $(I_{Vij1}, I_{Vij2}, I_{Vij3}, I_{Vij4}) \subseteq [0, 1]$ and $(I_{Rij1}, I_{Rij2}, I_{Rij3}, I_{Rij4}) \subseteq [0, 1]$ indicate the indeterminate degrees and reliability measures of the alternative Q_i over the criteria s_j, and $(F_{Vij1}, F_{Vij2}, F_{Vij3}, F_{Vij4}) \subseteq [0, 1]$ and $(F_{Rij1}, F_{Rij2}, F_{Rij3}, F_{Rij4}) \subseteq [0, 1]$ indicate the falsity degrees and reliability measures of the alternative Q_i over the criteria s_j, along with $0 \leq T_{Vij4} + I_{Vij4} + F_{Vij4} \leq 3$ and $0 \leq T_{Rij4} + I_{Rij4} + F_{Rij4} \leq 3$ for $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, m$. Then, all the specified TrNZNs are constructed as their decision matrix $\tilde{Z} = (\tilde{z}_{ij})_{m \times n}$.

Thus, the TrNZNWAA and TrNZNWGA operators and the score and accuracy functions can be applied to MDM problems with TrNZN information, and then their MDM approach can be indicated by the following procedures:

Step 1: the aggregated TrNZN $\tilde{z}_i$ for Q_i $(i = 1, 2, \ldots, m)$ is obtained by applying the TrNZNWAA or TrNZNWGA operator:

$$\begin{aligned}
\tilde{z}_i &= \mathrm{TrNZNWAA}\left(\tilde{z}_{i1}, \tilde{z}_{i2}, \ldots, \tilde{z}_{in}\right) = \mathop{\oplus}_{j=1}^{n} \lambda_j \tilde{z}_{ij}\\
&= \Bigg\langle \Bigg(\Bigg(\Bigg(1 - \prod_{j=1}^{n}\left(1 - T_{Vij1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - T_{Vij2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - T_{Vij3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - T_{Vij4}\right)^{\lambda_j} \Bigg),\\
&\quad \Bigg(1 - \prod_{j=1}^{n}\left(1 - T_{Rij1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - T_{Rij2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - T_{Rij3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - T_{Rij4}\right)^{\lambda_j} \Bigg) \Bigg),\\
&\quad \Bigg(\Bigg(\prod_{j=1}^{n} I_{Vij1}^{\lambda_j}, \prod_{j=1}^{n} I_{Vij2}^{\lambda_j}, \prod_{j=1}^{n} I_{Vij3}^{\lambda_j}, \prod_{j=1}^{n} I_{Vij4}^{\lambda_j} \Bigg), \Bigg(\prod_{j=1}^{n} I_{Rij1}^{\lambda_j}, \prod_{j=1}^{n} I_{Rij2}^{\lambda_j}, \prod_{j=1}^{n} I_{Rij3}^{\lambda_j}, \prod_{j=1}^{n} I_{Rij4}^{\lambda_j} \Bigg) \Bigg),\\
&\quad \Bigg(\Bigg(\prod_{j=1}^{n} F_{Vij1}^{\lambda_j}, \prod_{j=1}^{n} F_{Vij2}^{\lambda_j}, \prod_{j=1}^{n} F_{Vij3}^{\lambda_j}, \prod_{j=1}^{n} F_{Vij4}^{\lambda_j} \Bigg), \Bigg(\prod_{j=1}^{n} F_{Rij1}^{\lambda_j}, \prod_{j=1}^{n} F_{Rij2}^{\lambda_j}, \prod_{j=1}^{n} F_{Rij3}^{\lambda_j}, \prod_{j=1}^{n} F_{Rij4}^{\lambda_j} \Bigg) \Bigg) \Bigg\rangle,
\end{aligned} \tag{19}$$

$$\begin{aligned}
\tilde{z}_i &= \mathrm{TrNZNWGA}\left(\tilde{z}_{i1}, \tilde{z}_{i2}, \ldots, \tilde{z}_{\mathrm{in}}\right) = \mathop{\otimes}_{j=1}^{n} \tilde{z}_{ij}^{\lambda_j}\\
&= \Bigg\langle \Bigg(\Bigg(\prod_{j=1}^{n} T_{Vij1}^{\lambda_j}, \prod_{j=1}^{n} T_{Vij2}^{\lambda_j}, \prod_{j=1}^{n} T_{Vij3}^{\lambda_j}, \prod_{j=1}^{n} T_{Vij4}^{\lambda_j} \Bigg), \Bigg(\prod_{j=1}^{n} T_{Rij1}^{\lambda_j}, \prod_{j=1}^{n} T_{Rij2}^{\lambda_j}, \prod_{j=1}^{n} T_{Rij3}^{\lambda_j}, \prod_{j=1}^{n} T_{Rij4}^{\lambda_j} \Bigg) \Bigg),\\
&\quad \Bigg(\Bigg(1 - \prod_{j=1}^{n}\left(1 - I_{Vij1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - I_{Vij2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - I_{Vij3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - I_{Vij4}\right)^{\lambda_j} \Bigg),\\
&\quad \Bigg(1 - \prod_{j=1}^{n}\left(1 - I_{Rij1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - I_{Rij2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - I_{Rij3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - I_{Rij4}\right)^{\lambda_j} \Bigg) \Bigg),\\
&\quad \Bigg(\Bigg(1 - \prod_{j=1}^{n}\left(1 - F_{Vij1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - F_{Vij2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - F_{Vij3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - F_{Vij4}\right)^{\lambda_j} \Bigg),\\
&\quad \Bigg(1 - \prod_{j=1}^{n}\left(1 - F_{Rij1}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - F_{Rij2}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - F_{Rij3}\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1 - F_{Rij4}\right)^{\lambda_j} \Bigg) \Bigg) \Bigg\rangle.
\end{aligned} \tag{20}$$

Step 2: by equation (7), we calculate the score values of $S(\tilde{z}_i)$. If necessary, we calculate the accuracy values of $H(\tilde{z}_i)$ ($i = 1, 2, \ldots, m$) by equation (8).

Step 3: all the alternatives Q_i ($i = 1, 2, \ldots, m$) are ranked corresponding to the score values (the accuracy values) and the best one(s) is chosen in the set of alternatives.

Step 4: end.

6. MDM Example and Comparison with Existing MDM Approaches

6.1. MDM Example of Software Selection. This section indicates an MDM example of software selection adapted from [9] to reveal the usability and efficiency of the established MDM approach under the environment of TrNZNs.

In an MDM example, an investment company needs to select a suitable software system from potential software systems, where five candidate software systems are provided preliminarily and denoted as a set of five alternatives $Q = \{Q_1, Q_2, Q_3, Q_4, Q_5\}$. Then, these alternatives must satisfy the requirements of the four criteria: s_1 (the contribution to organization performance), s_2 (the effort to transform from current system), s_3 (the costs of hardware/software investment), and s_4 (the outsourcing software developer reliability). Regarding the importance of the four criteria, the weight values of the four criteria are specified as the weight vector $\lambda = (0.25, 0.25, 0.3, 0.2)$. Thus, decision makers/experts assess the satisfiability of the five alternatives over the four criteria by TrNZNs $\tilde{z}_{ij} = <((T_{Vij1}, T_{Vij2}, T_{Vij3}, T_{Vij4}), (T_{Rij1}, T_{Rij2}, T_{Rij3}, T_{Rij4})), ((I_{Vij1}, I_{Vij2}, I_{Vij3}, I_{Vij4}), (I_{Rij1}, I_{Rij2}, I_{Rij3}, I_{Rij4})), ((F_{Vij1}, F_{Vij2}, F_{Vij3}, F_{Vij4}), (F_{Rij1}, F_{Rij2}, F_{Rij3}, F_{Rij4}))>$ ($j = 1, 2, 3, 4$; $i = 1, 2, 3, 4, 5$), where $(T_{Vij1}, T_{Vij2}, T_{Vij3}, T_{Vij4}) \subseteq [0, 1]$ and $(T_{Rij1}, T_{Rij2}, T_{Rij3}, T_{Rij4}) \subseteq [0, 1]$ indicate that the alternative Q_i satisfies the degrees and reliability measures of the criteria s_j, $(I_{Vij1}, I_{Vij2}, I_{Vij3}, I_{Vij4}) \subseteq [0, 1]$ and $(I_{Rij1}, I_{Rij2}, I_{Rij3}, I_{Rij4}) \subseteq [0, 1]$ indicate the indeterminate degrees and reliability measures of the alternative Q_i over the criteria s_j, and $(F_{Vij1}, F_{Vij2}, F_{Vij3}, F_{Vij4}) \subseteq [0, 1]$ and $(F_{Rij1}, F_{Rij2}, F_{Rij3}, F_{Rij4}) \subseteq [0, 1]$ indicate that the alternative A_i does not satisfy the degrees and reliability measures of the criteria s_j, along with $0 \le T_{Vij4} + I_{Vij4} + F_{Vij4} \le 3$ and $0 \le T_{Rij4} + I_{Rij4} + F_{Rij4} \le 3$. Hence, all the specified TrNZNs can be constructed as the following decision matrix $\tilde{Z} = (\tilde{z}_{ij})_{5\times 4}$:

$$
\widetilde{Z}=\left[\begin{array}{l}
\langle((0.4,0.5,0.6,0.7),(0.4,0.5,0.6,0.7)),((0.0,0.1,0.2,0.3),(0.3,0.4,0.5,0.6)),((0.1,0.1,0.1,0.1),(0.3,0.4,0.5,0.6)\rangle\\
\langle((0.3,0.4,0.5,0.5),(0.5,0.6,0.7,0.8)),((0.1,0.2,0.3,0.4),(0.4,0.5,0.6,0.7)),((0.0,0.1,0.1,0.1),(0.5,0.6,0.7,0.8))\rangle\\
\langle((0.1,0.1,0.1,0.1),(0.5,0.6,0.7,0.8)),((0.1,0.1,0.1,0.1),(0.6,0.7,0.8,0.9)),((0.6,0.7,0.8,0.9),(0.5,0.6,0.7,0.8))\rangle\\
\langle((0.7,0.7,0.7,0.7),(0.4,0.5,0.6,0.7)),((0.0,0.1,0.2,0.3),(0.4,0.5,0.6,0.7)),((0.1,0.1,0.1,0.1),(0.3,0.4,0.5,0.6))\rangle\\
\langle((0.0,0.1,0.2,0.2),(0.4,0.5,0.6,0.7)),((0.1,0.1,0.1,0.1),(0.3,0.4,0.5,0.6)),((0.5,0.6,0.7,0.8),(0.5,0.6,0.7,0.8))\rangle\\
\langle((0.0,0.1,0.2,0.3),(0.3,0.4,0.5,0.6)),((0.0,0.1,0.2,0.3),(0.4,0.5,0.6,0.7)),(0.2,0.3,0.4,0.5),(0.3,0.4,0.5,0.6)\rangle\\
\langle((0.2,0.3,0.4,0.5),(0.6,0.7,0.8,0.9)),((0.0,0.1,0.2,0.3),(0.5,0.6,0.7,0.8)),((0.0,0.1,0.2,0.3),(0.4,0.5,0.6,0.7))\rangle\\
\langle((0.0,0.1,0.1,0.2),(0.5,0.6,0.7,0.8)),((0.0,0.1,0.2,0.3),(0.5,0.6,0.6,0.7)),((0.3,0.4,0.5,0.6),(0.3,0.4,0.5,0.6))\rangle\\
\langle((0.4,0.5,0.6,0.7),(0.5,0.6,0.7,0.8)),((0.1,0.1,0.1,0.1),(0.6,0.7,0.7,0.8)),((0.0,0.1,0.2,0.2),(0.5,0.6,0.7,0.8))\rangle\\
\langle((0.4,0.4,0.4,0.4),(0.3,0.4,0.5,0.6)),((0.0,0.1,0.2,0.3),(0.4,0.5,0.6,0.7)),((0.0,0.1,0.2,0.3),(0.5,0.6,0.7,0.8))\rangle\\
\langle((0.3,0.4,0.5,0.6),(0.4,0.5,0.6,0.7)),((0.0,0.1,0.2,0.3),(0.4,0.5,0.5,0.6)),((0.1,0.1,0.1,0.1),(0.4,0.5,0.5,0.6))\rangle\\
\langle((0.0,0.1,0.1,0.2),(0.5,0.6,0.7,0.8)),((0.1,0.1,0.1,0.1),(0.4,0.5,0.6,0.7)),((0.5,0.6,0.7,0.8),(0.3,0.4,0.5,0.6))\rangle\\
\langle((0.2,0.3,0.4,0.5),(0.3,0.4,0.5,0.6)),((0.0,0.1,0.2,0.3),(0.5,0.6,0.7,0.8)),((0.1,0.2,0.2,0.3),(0.4,0.5,0.6,0.7))\rangle\\
\langle((0.2,0.3,0.4,0.5),(0.5,0.6,0.6,0.7)),((0.0,0.1,0.2,0.3),(0.5,0.6,0.7,0.8)),((0.1,0.2,0.3,0.3),(0.3,0.4,0.5,0.6))\rangle\\
\langle((0.6,0.7,0.7,0.8),(0.5,0.5,0.5,0.5)),((0.1,0.1,0.1,0.1),(0.5,0.6,0.7,0.8)),((0.0,0.1,0.1,0.2),(0.4,0.5,0.5,0.6))\rangle\\
\langle((0.3,0.4,0.5,0.6),(0.3,0.4,0.5,0.6)),((0.1,0.1,0.1,0.1),(0.4,0.5,0.6,0.7)),((0.1,0.2,0.3,0.4),(0.5,0.6,0.6,0.7))\rangle\\
\langle((0.3,0.4,0.5,0.5),(0.5,0.6,0.6,0.7)),((0.0,0.1,0.2,0.3),(0.3,0.4,0.5,0.6)),((0.0,0.1,0.1,0.2),(0.4,0.5,0.6,0.7))\rangle\\
\langle((0.1,0.2,0.3,0.4),(0.6,0.7,0.8,0.9)),((0.1,0.1,0.1,0.1),(0.5,0.6,0.7,0.8)),((0.3,0.4,0.5,0.6),(0.4,0.5,0.6,0.7))\rangle\\
\langle((0.1,0.2,0.3,0.4),(0.5,0.6,0.7,0.8)),((0.1,0.1,0.1,0.1),(0.4,0.5,0.5,0.6)),((0.4,0.5,0.6,0.6),(0.3,0.4,0.5,0.6))\rangle\\
\langle((0.1,0.2,0.3,0.3),(0.4,0.5,0.6,0.7)),((0.1,0.2,0.3,0.4),(0.4,0.5,0.5,0.6)),((0.2,0.3,0.4,0.5),(0.4,0.5,0.6,0.7))\rangle
\end{array}\right]. \tag{21}
$$

Thus, we utilize the established MDM approach to obtain the most suitable software system(s), which can be depicted by the following decision process.

First, by equation (19) or equation (20), we obtain the following aggregated TrNZNs $\widetilde{z}_i$ ($i = 1, 2, 3, 4, 5$):

$\widetilde{z}_1$= <((0.2636, 0.3656, 0.4682, 0.5719), (0.3569, 0.4572, 0.5577, 0.6585)), ((0, 0.1000, 0.1741, 0.2408), (0.3722, 0.4729, 0.5428, 0.6431)), ((0.1189, 0.1512, 0.1762, 0.1973), (0.3622, 0.4638, 0.5186, 0.6188))>

$\widetilde{z}_2$= <((0.1945, 0.2958, 0.3758, 0.4243), (0.5271, 0.6278, 0.7129, 0.8176)), ((0, 0.1189, 0.1798, 0.2319), (0.3993, 0.5005, 0.6012, 0.7018)), ((0, 0.1712, 0.2132, 0.2821), (0.3880, 0.4894, 0.5904, 0.6911))>

$\widetilde{z}_3$= <((0.1081, 0.1848, 0.2421, 0.3245), (0.4710, 0.5735, 0.6776, 0.7856)), ((0, 0.1000, 0.1464, 0.1830), (0.5233, 0.6236, 0.6964, 0.7969)), ((0.2566, 0.3737, 0.4272, 0.5393), (0.3936, 0.4949, 0.5958, 0.6964))>

$\widetilde{z}_4$= <((0.4035, 0.4652, 0.5298, 0.5983), (0.4767, 0.5771, 0.6486, 0.7500)), ((0, 0.1000, 0.1464, 0.1830), (0.4733, 0.5745, 0.6297, 0.7305)), ((0, 0.1699, 0.2366, 0.2366), (0.3409, 0.4427, 0.5439, 0.6447))>

$\widetilde{z}_5$= <((0.3454, 0.4287, 0.4599, 0.5218), (0.4096, 0.4767, 0.5478, 0.6242)), ((0, 0.1149, 0.1481, 0.1737), (0.3980, 0.4995, 0.5789, 0.6798)), ((0, 0.1950, 0.2552, 0.3760), (0.4472, 0.5477, 0.6136, 0.7145))>

Or we obtain the following aggregated TrNZNs $\widetilde{z}_i$ ($i = 1, 2, 3, 4, 5$):

$\widetilde{z}_1$= <((0, 0.2991, 0.4162, 0.5244), (0.3514, 0.4522, 0.5527, 0.6531)), ((0.0209, 0.1000, 0.1809, 0.2639), (0.3764, 0.4767, 0.5478, 0.6486)), ((0.1261, 0.1745, 0.2266, 0.2835), (0.3751, 0.4762, 0.5218, 0.6224))>

$\widetilde{z}_2$= <((0, 0.2456, 0.2918, 0.3798), (0.5233, 0.6236, 0.7018, 0.8022)), ((0.0563, 0.1261, 0.1984, 0.2737), (0.4088, 0.5096, 0.6108, 0.7129)), ((0.1877, 0.2944, 0.3715, 0.4743), (0.3996, 0.5005, 0.6020, 0.7045))>

$\widetilde{z}_3$= <((0, 0.1597, 0.1888, 0.2543), (0.4449, 0.5479, 0.6499, 0.7513)), ((0.0463, 0.1000, 0.1565, 0.2162), (0.5271, 0.6278, 0.7087, 0.8139)), ((0.3437, 0.4500, 0.5422, 0.6655), (0.4042, 0.5051, 0.6064, 0.7087))>

$\widetilde{z}_4$= <((0.2832, 0.3885, 0.4807, 0.5658), (0.4729, 0.5733, 0.6431, 0.7434)), ((0.0463, 0.1000, 0.1565, 0.2162), (0.4867, 0.5884, 0.6430, 0.7458)), ((0.1480, 0.2276, 0.3109, 0.3109), (0.3565, 0.4578, 0.5599, 0.6636))>

$\widetilde{z}_5$= <((0, 0.2912, 0.3756, 0.3910), (0.3980, 0.4729, 0.5428, 0.6089)), ((0.0760, 0.1210, 0.1690, 0.2206), (0.4096, 0.5106, 0.5943, 0.6976)), ((0.1958, 0.3012, 0.3877, 0.5020), (0.4523, 0.5528, 0.6296, 0.7330))>

Then, the results of the MDM approach based on the TrNZNWAA and TrNNWGA operators and the score function are shown in Table 1.

From the results of Table 1, the ranking orders based on the TrNZNWAA and TrNZNWGA operators are identical and the best one indicates the same selection as the software system Q_4.

6.2. Comparison with Existing MDM Approaches. For convenient comparison with existing MDM approach in the setting of TrNNs [9], we may ignore the reliability measures in TrNZNs and only contain the decision matrix of TrNNs in the MDM example as its special case. Thus, existing MDM approach in the setting of TrNNs [9] can be used for the special case of the MDM example. In this case, the decision results based on the TrNNWAA and TrNNWGA operators

Table 1: Results of the MDM approach based on the TrNZNWAA and TrNZNWGA operators and the score function.

Aggregation operator	Score value	Ranking
TrNZNWAA	0.6892, 0.6845, 0.6154, 0.7207, 0.6824	Q4 > Q1 > Q2 > Q5 > Q3
TrNZNWGA	0.6607, 0.6257, 0.5750, 0.6848, 0.6158	Q4 > Q1 > Q2 > Q5 > Q3

Table 2: Results of the MDM approach based on the TrNNWAA and TrNNWGA operators and the score function [9].

Aggregation operator	Score value	Ranking
TrNNWAA	0.7092, 0.6744, 0.5694, 0.7437, 0.7077	Q4 > Q1 > Q5 > Q2 > Q3
TrNNWGA	0.6553, 0.5779, 0.5069, 0.6835, 0.5904	Q4 > Q1 > Q5 > Q2 > Q3

(equations (2) and (3)) and the score function of TrNNs (equation (4)) are introduced from [9], which are shown in Table 2.

Based on the decision results in Tables 1 and 2, we can see that the ranking orders based on the established MDM approach and the existing MDM approach [9] reveal their difference, but the best alternative Q_4 (the best software system) is identical. Then, the reason for their ranking difference is that decision information in the existing MDM approach [9] only contains TrNNs without considering the reliability measures of TrNNs in this MDM example, while decision information in the established MDM approach contains both TrNNs and their reliability measures. Hence, different decision information can result in different ranking results. It is obvious that the reliability measures in this example can affect the ranking order of alternatives, which shows the efficiency and rationality of the established MDM approach under the environment of TrNZNs.

However, the different decision information and decision methods can have an impact on the ranking of alternatives in the MDM problem, which reveals their importance in MDM applications. Thus, existing MDM methods [11–14, 23] only contain the TrNN or NZN information without considering the reliability measures in TrNNs or continuous Z-numbers in NZNs; they may lose some useful decision information so as to result in decision distortion/unreasonable decision results, which reveal some insufficiencies, while the new established approach can contain much more information than existing MDM methods and overcome the insufficiencies. Furthermore, existing methods [11–14, 23] also cannot deal with such MDM problems with TrNZNs.

Based on the above comparative analysis, the new established approach in setting of TrNZNs not only makes assessment information of TrNNs more reliable but also strengthens the effectiveness and continuity of decision information by comparison with existing MDM methods with TrNN and NZN information [9, 11–14, 23], which reveals the highlighting advantages of the new established approach in the information representation and MDM applications. Therefore, the new established approach not only extends existing methods but also demonstrates its superiority over them.

7. Conclusion

To make TrNN reliable, this paper presented a TrNZN set based on the truth, falsity, and indeterminacy trapezoidal Z-numbers as the generalization of the Z-number concept and then defined basic operations of TrNZNs, score and accuracy functions of TrNZNs, and ranking laws of TrNZNs. Next, the TrNZNWAA and TrNZNWGA operators were proposed to aggregate the TrNZN information. Furthermore, an MDM approach based on the two aggregation operators and score and accuracy functions was established in the setting of TrNZNs, in which the assessment values of alternatives over the criteria take the form of TrNZNs containing TrNNs and their reliability measures. Finally, an MDM example of software selection was provided to reveal the suitability and efficiency of the established MDM approach in the setting of TrNZNs.

The main advantage of this study is that the established method not only makes assessment information of TrNNs more reliable but also strengthens the decision rationality and efficiency in solving MDM problems with TrNZN information. However, the established method only uses the basic aggregation algorithms of TrNZNWAA and TrNZNWGA for MDM problems without considering the interactions of some evaluation criteria with each other, which implies the limitation of the proposed method in MDM applications. For capturing these relationships, the future study is to develop other aggregation algorithms and to use them for some other MDM problems including slope design schemes, energy and environmental managements, and medicine options.

Data Availability

There are no underlying data supporting the results of your study.

Conflicts of Interest

The authors declare no conflicts of interest.

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