Malaya Journal of Matematik, Vol. 8, No. 4, 1824-1829, 2020

https://doi.org/10.26637/MJM0804/0082

Neutrosophic α-generalized semi homeomorphisms

V. Banu Priya[1], S. Chandrasekar[2]* and M. Suresh[3]

Abstract

In this paper, the concepts of Neutrosophic α-generalized semi homeomorphism and Neutrosophic $i\alpha$ - generalized semi homeomorphism are introduced and also discussed the properties.

Keywords

Neutrosophic α-generalized semi homeomorphisms, Neutrosophic $i\alpha$-generalized semi homeomorphisms.

AMS Subject Classification

34B24, 34B27.

[1] *Department of Mathematics, R.M.K. College of Engineering and Technology, Tiruvallur-601206, Tamil Nadu, India.*
[2] *PG and Research Department of Mathematics, Arignar Anna Government Arts College, Namakkal-637002, Tamil Nadu, India.*
[3] *Department of Mathematics, R.M.D. Engineering College, Tiruvallur-601206, Tamil Nadu, India.*
***Corresponding author**: [1] spriya.maths@gmail.com; [2]chandrumat@gmail.com; [3]sureshmaths2209@gmail.com
Article History: Received **24** September **2020**; Accepted **30** October **2020**

Contents

1. Introduction

A.A. Salama [21] introduced Neutrosophic topological spaces by using Smarandache's Neutrosophic sets [9]. V. Banu Priya et al., [5, 6] introduced Neutrosophic αgs closed sets and its continuity. Md. Hanif Page et al., [10] introduced Neutrosophic Generalized Homeomorphism, M. Parimala et al., [13] introduced Neutrosophic $\alpha\psi$ Homeomorphism in Neutrosophic Topological Spaces. In this paper, we introduce the concepts of Neutrosophic α homeomorphism, Neutrosophic α generalized homeomorphism and followed by Neutrosophic α-generalized semi homeomorphism and Neutrosophic $i\alpha$-generalized semi homeomorphism. We discussed their properties and relationships.

2. Preliminaries

In this section, we recall some definitions and operations of Neutrosophic sets and its fundamental results.

Definition 2.1 ([9])**.** *Let N^X be a non empty fixed set. A Neutrosophic set $V_{A_1^*}$ in N^X is an object having the form*

$$V_{A_1^*} = \left\{\left\langle x, \mu_{V_{A_1^*}}(x), \sigma_{V_{A_1^*}}(x), \nu_{V_{A_1^*}}(x)\right\rangle \mid x \in N^X\right\},$$

where $\mu_{V_{A_1^*}}(x)$ represents the degree of membership function, $\sigma_{V_{A_1^*}}(x)$ represents the degree of indeterminacy and $\nu_{V_{A_1^*}}(x)$represents the degree of non-membership function. $NS(N^X)$ denote the set of all Neutrosophic sets N^X.

Definition 2.2 ([9])**.** *If the Neutrosophic set*

$$V_{A_1^*} = \left\{\left\langle x, \mu_{V_{A_1^*}}(x), \sigma_{V_{A_1^*}}(x), \nu_{V_{A_1^*}}(x)\right\rangle \mid x \in N^X\right\}$$

on N^X then its compliment is

$$V^c_{A_1^*} = \left\{\left\langle, \nu_{V_{A_1^*}}(x), 1-\sigma_{V_{A_1^*}}(x), \mu_{V_{A_1^*}}(x)\right\rangle \mid x \in N^X\right\}.$$

Definition 2.3 ([9])**.** *Let $V_{A_1^*}$ and $V_{B_1^*}$ be two Neutrosophic sets, $\forall x \in N^X$,*

$$V_{A_1^*} = \left\{\left\langle x, \mu_{V_{A_1^*}}(x), \sigma_{V_{A_1^*}}(x), \nu_{V_{A_1^*}}(x)\right\rangle \mid x \in N^X\right\}$$

and

$$V_{B_1^*} = \left\{\left\langle x, \mu_{V_{B_1^*}}(x), \sigma_{V_{B_1^*}}(x), \nu_{V_{B_1^*}}(x)\right\rangle \mid x \in N^X\right\}$$

then $V_{A_1^*} \subseteq V_{B_1^*}$ if and only if $\mu_{V_{A_1^*}}(x) \leq \mu_{V_{B_1^*}}(x), \sigma_{V_{A_1^*}}(x) \leq \sigma_{V_{B_1^*}}(x)$ and $\nu_{V_{A_1^*}}(x) \geq \nu_{V_{B_1^*}}(x)$.

Definition 2.4. *Let $V_{A_1^*}$ and $V_{B_1^*}$ be two Neutrosophic sets, $\forall x \in N^X$, $V_{A_1^*} = \left\{ \left\langle x, \mu_{V_{A_1^*}}(x), \sigma_{V_{A_1^*}}(x), \nu_{V_{A_1^*}}(x) \right\rangle \mid x \in N^X \right\}$ and $V_{B_1^*} = \left\{ \left\langle x, \mu_{V_{B_1^*}}(x), \sigma_{V_{B_1^*}}(x), \nu_{V_{B_1^*}}(x) \right\rangle \mid x \in N^X \right\}$ then*

1. $V_{A_1^*} \cap V_{B_1^*} = \left\{ \left\langle x, \mu_{V_{A_1^*}}(x) \wedge \mu_{V_{B_1^*}}(x), \sigma_{V_{A_1^*}}(x) \wedge \sigma_{V_{B_1^*}}(x), \nu_{V_{A_1^*}}(x) \vee \nu_{V_{B_1^*}}(x) \right\rangle \mid x \in N^X \right\}$.
2. $V_{A_1^*} \cup V_{B_1^*} = \left\{ \left\langle x, \mu_{V_{A_1^*}}(x) \vee \mu_{V_{B_1^*}}(x), \sigma_{V_{A_1^*}}(x) \vee \sigma_{V_{B_1^*}}(x), \nu_{V_{A_1^*}}(x) \wedge \nu_{V_{B_1^*}}(x) \right\rangle \mid x \in N^X \right\}$.

Definition 2.5 ([20, 21])**.** *Let N^X be a non-empty set and N^τ be the collection of Neutrosophic subsets of N^X satisfying the following properties:*

1. $0_N, 1_N \in N^\tau$.
2. *$\lambda_1 \cap \lambda_2 \in N^\tau$ for any $\lambda_1, \lambda_2 \in N^\tau$.*
3. *$\cup \lambda_i \in N^\tau$ for every $\{\lambda_i \mid i \in I\} \subseteq N^\tau$.*

The space (N^X, N^τ) is called a Neutrosophic topological space (NTS). The elements of N^τ are called Neutrosophic open set (NOS) and its complement is called Neutrosophic closed set (NCS).

Definition 2.6 ([2, 5, 7, 11, 12, 22])**.** *Let (N^X, N^τ) be a Neutrosophic topological space. Neutrosophic set $V_{A_1^*}$ is said to be*

1. *Neutrosophic α-closed set (N.αCS) if $N.cl(N.int(N.cl(V_{A_1^*}))) \subseteq V_{A_1^*}$*
2. *Neutrosophic semi closed set (N.SCS) if $N.int(N.cl(V_{A_1^*})) \subseteq V_{A_1^*}$*
3. *Neutrosophic generalized closed set (N.GCS) if $N.cl(V_{A_1^*}) \subseteq H$ whenever $V_{A_1^*} \subseteq H$ and H is a N.OS*
4. *Neutrosophic α generalized closed set (N.αGCS) if $N.\alpha cl(V_{A_1^*}) \subseteq H$ whenever $V_{A_1^*} \subseteq H$ and H is a N.OS*
5. *Neutrosophic generalized semi closed set (N.GSCS) if $N.Scl(V_{A_1^*}) \subseteq H$ whenever $V_{A_1^*} \subseteq H$ and H is a N.OS*
6. *Neutrosophic α generalized semi closed set (N.αGSCS) if $N.\alpha cl(V_{A_1^*}) \subseteq H$ whenever $V_{A_1^*} \subseteq H$ and H is a N.SOS.*

3. Main Results

Definition 3.1. *Let N^{f_*} be a bijection from a NTS (N^X, N^τ) into a NTS (N^Y, N^σ). Then N^{f_*} is said to be*

1. *Neutrosophic homeomorphism if N^{f_*} and $N^{f_*^{-1}}$ are Neutrosophic continuous (N-CTS) maps*
2. *Neutrosophic α homeomorphism if N^{f_*} and $N^{f_*^{-1}}$ are Neutrosophic α CTS maps*
3. *Neutrosophic α generalized homeomorphism (briefly NαG homeomorphism) if N^{f_*} and $N^{f_*^{-1}}$ are NG CTS maps.*

Definition 3.2. *A bijective map $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ is called a Neutrosophic α generalized semi homeomorphism (briefly NαGS homeomorphism) if N^{f_*} and $N^{f_*^{-1}}$ are NαGS CTS maps.*

Example 3.3. *Let $N^X = \{a, b\}$, $N^Y = \{u, v\}$,*

$$G_1^* = \left\langle x, \left(\frac{1}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right) \right\rangle$$

and

$$G_2^* = \left\langle y, \left(\frac{3}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right) \right\rangle.$$

Then $N^\tau = \{0_N, G_1^*, 1_N\}$ and $N^\sigma = \{0_N, G_2^*, 1_N\}$ are NTs on N^X and N^Y respectively. Define a bijective map $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ by $N^{f_*}(a) = u$ and $N^{f_*}(b) = v$. Then N^{f_*} is a NαGS CTS and $N^{f_*^{-1}}$ is also a NαGS CTS map. Therefore, the bijective map N^{f_*} is a NαGS homeomorphism.

Theorem 3.4. *Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a bijective map from a NTS N^X into a NTS N^Y. Then the following conditions are equivalent:*

1. *N^{f_*} is a Neutrosophic homeomorphism*
2. *N^{f_*} is a N-CTS map and N^{f_*} is a Neutrosophic open map*
3. *N^{f_*} and $N^{f_*^{-1}}$ are N-CTS maps.*

Proof. (1) $\Rightarrow$ (2): It is obviously true.
(2) $\Rightarrow$ (3): Let N^{f_*} is a Neutrosophic open map. That is $N^{f_*}(V_{A_1^*})$ is NOS in N^Y for each NOS $V_{A_1^*}$ in N^X. Now define a map $N^{f_*^{-1}} : (N^Y, N^\sigma) \to (N^X, N^\tau)$. By hypothesis, for every NOS $V_{A_1^*}$ in N^X, we have $N^{f_*^{-1}}(V_{A_1^*})$ is a NOS in N^Y. Hence $N^{f_*^{-1}}$ is a N-CTS map. That is N^{f_*} and $N^{f_*^{-1}}$ are N-CTS maps.
(3) $\Rightarrow$ (1): Let N^{f_*} and $N^{f_*^{-1}}$ be N-CTS map. Since $N^{f_*^{-1}} : (N^Y, N^\sigma) \to (N^X, N^\tau)$ is a N-CTS map, $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ is a Neutrosophic open map. Hence N^{f_*} is a Neutrosophic homeomorphism. □

Theorem 3.5. *Every Neutrosophic homeomorphism is a NαGS homeomorphism but not conversely.*

Proof. Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a Neutrosophic homeomorphism. Then N^{f_*} and $N^{f_*^{-1}}$ are N-CTS maps. Since every N-CTS map is a $N\alpha GS$ CTS map, N^{f_*} and $N^{f_*^{-1}}$ are $N\alpha GS$ CTS maps. Therefore N^{f_*} is a $N\alpha GS$ homeomorphism. □

Example 3.6. *Let $N^X = \{a, b\}$, $N^Y = \{u, v\}$,*

$$G_1^* = \left\langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{1}{10}, \frac{5}{10}, \frac{2}{10}\right) \right\rangle$$

and

$$G_2^* = \left\langle y, \left(\frac{1}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{4}{10}\right)\right\rangle.$$

Then $N^\tau = \{0_N, G_1^*, 1_N\}$ and $N^\sigma = \{0_N, G_2^*, 1_N\}$ are NTs on N^X and N^Y respectively. Define a bijective $N^{f*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ by $N^{f_*}(a) = u$ and $N^{f_*}(b) = v$. Since the inverse image of every NCS in (N^Y, N^σ) is a $N\alpha GSCS$ in (N^X, N^τ), N^{f_*} is a $N\alpha GS$ CTS map and the inverse image of every NCS in (N^X, N^τ) is a $N\alpha GSCS$ in (N^Y, N^σ), $N^{f_*^{-1}}$ is a $N\alpha GS$ CTS map. Hence N^{f_*} is a $N\alpha GS$ homeomorphism. But N^{f_*} is not a Neutrosophic homeomorphism since N^{f_*} and $N^{f_*^{-1}}$ are not N-CTS maps.

Theorem 3.7. *Every Neutrosophic α homeomorphism is a $N\alpha GS$ homeomorphism but not conversely.*

Proof. Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a Neutrosophic α homeomorphism. Then N^{f_*} and $N^{f_*^{-1}}$ are Neutrosophic α CTS maps. Since every Neutrosophic α CTS is a $N\alpha GS$ CTS map, N^{f_*} and $N^{f_*^{-1}}$ are $N\alpha GS$ CTS maps. Therefore N^{f_*} is a $N\alpha GS$ homeomorphism. □

Example 3.8. *Let $N^X = \{a, b\}$, $N^Y = \{u, v\}$,*

$$G_1^* = \left\langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right)\right\rangle$$

and

$$G_2^* = \left\langle y, \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right)\right\rangle.$$

Then $N^\tau = \{0_N, G_1^*, 1_N\}$ and $N^\sigma = \{0_N, G_2^*, 1_N\}$ are NTs on N^X and N^Y respectively. Define a bijective $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ by $N^{f_*}(a) = u$ and $N^{f_*}(b) = v$. Consider, NCS

$$G_2^{*'} = \left\langle y, \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{2}{10}\right)\right\rangle$$

in N^Y. Then

$$N^{f_*^{-1}}(G_2^{*'}) = \left\langle x, \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{2}{10}\right)\right\rangle$$

is not a $N\alpha CS$ in N^X. This implies N^{f_*} is not a Neutrosophic α CTS map. Hence N^{f_*} is not a Neutrosophic α homeomorphism.

Theorem 3.9. *Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a $N\alpha GS$ homeomorphism. Then N^{f_*} is a Neutrosophic homeomorphism if N^X and N^Y are $N\alpha gaT_{\frac{1}{2}}$ spaces.*

Proof. Let $V_{B_1^*}$ be a NCS in N^Y. By hypothesis, $N^{f_*^{-1}}(V_{B_1^*})$ is a $N\alpha GSCS$ in N^X. Since N^X is a $N\alpha gaT_{\frac{1}{2}}$ space, $N^{f_*^{-1}}(V_{B_1^*})$ is a NCS in N^X. Hence N^{f_*} is a N-CTS map. By hypothesis $N^{f_*^{-1}} : (N^Y, N^\sigma) \to (N^X, N^\tau)$ is a $N\alpha GS$ CTS map. Let $V_{A_1^*}$ be a NCS in N^X. Then $(N^{f_*^{-1}})^{-1}(V_{A_1^*}) = N^{f_*}(V_{A_1^*})$ is a $N\alpha GSCS$ in N^Y. Since N^Y is a $N\alpha gaT_{\frac{1}{2}}$ space, $N^{f_*}(V_{A_1^*})$ is a NCS in N^Y. Hence $N^{f_*^{-1}}$ is a N-CTS map. Therefore N^{f_*} is a Neutrosophic homeomorphism. □

Theorem 3.10. *Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a $N\alpha GS$ homeomorphism. Then N^{f_*} is a Neutrosophic Generalised homeomorphism if N^X and N^Y are $N\alpha gaT_{\frac{1}{2}}$ spaces.*

Proof. Let $V_{B_1^*}$ be a NCS in N^Y. By hypothesis, $N^{f_*^{-1}}(V_{B_1^*})$ is a $N\alpha GSCS$ in N^X. Since N^X is a $N\alpha gaT_{\frac{1}{2}}$ space, $N^{f_*^{-1}}(V_{B_1^*})$ is a $NGCS$ in N^X. Hence N^{f_*} is a Neutrosophic Generalised CTS map. By hypothesis $N^{f_*^{-1}} : (N^Y, N^\sigma) \to (N^X, N^\tau)$ is a $N\alpha GS$ CTS map. Let $V_{A_1^*}$ be a NCS in N^X. Then $(N^{f_*^{-1}})^{-1}(V_{A_1^*}) = N^{f_*}(V_{A_1^*})$ is a $N\alpha GSCS$ in N^Y. Since N^Y is a $N\alpha gaT_{\frac{1}{2}}$ space, $N^{f_*}(V_{A_1^*})$ is a $NGCS$ in N^X. Hence $N^{f_*^{-1}}$ is a Neutrosophic Generalised CTS map. Therefore N^{f_*} is a Neutrosophic Generalised homeomorphism.

□

Theorem 3.11. *Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a bijective map. Neutrosophic N^{f_*} is a $N\alpha GS$ CTS map, then the following are equivalent:*

1. *N^{f_*} is a $N\alpha GS$ closed map*
2. *N^{f_*} is a $N\alpha GS$ open map*
3. *N^{f_*} is a $N\alpha GS$ homeomorphism.*

Proof. $(1) \Rightarrow (2)$: Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a bijective map and let N^{f_*} be a $N\alpha GS$ closed map. This implies $N^{f_*^{-1}} : (N^Y, N^\sigma) \to (N^X, N^\tau)$ is a $N\alpha GS$ CTS map. Assume that $V_{A_1^*}$ is a NOS in N^X. Then by hypothesis, $(N^{f_*^{-1}})^{-1}(V_{A_1^*})$ is a $N\alpha GSOS$ in N^Y. Hence N^{f_*} is a $N\alpha GS$ open map.
$(2) \Rightarrow (3)$: Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a bijective map and let N^{f_*} is a $N\alpha GS$ open map. This implies $N^{f_*^{-1}} : (N^Y, N^\sigma) \to (N^X, N^\tau)$ is a $N\alpha GS$ CTS map. Hence N^{f_*} and $N^{f_*^{-1}}$ are $N\alpha GS$ CTS maps. Therefore, N^{f_*} is a $N\alpha GS$ homeomorphism.
$(3) \Rightarrow (1)$: Let N^{f_*} be a $N\alpha GS$ homeomorphism. That is N^{f_*} and $N^{f_*^{-1}}$ are $N\alpha GS$ CTS maps. Assume that $V_{A_1^*}$ is a NCS in N^X. Then by hypothesis, $V_{A_1^*}$ is a $N\alpha GSCS$ in N^Y. Hence N^{f_*} is a $N\alpha GS$ closed map.

□

Remark 3.12. *The composition of two $N\alpha GS$ homeomorphisms need not be a $N\alpha GS$ homeomorphism in general.*

Example 3.13. *Let $N^X = \{a, b\}$, $N^Y = \{c, d\}$ and $N^Z =$*

$\{u,v\}$. *Let*

$$G_1^* = \left\langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{3}{10}\right)\right\rangle$$
$$G_2^* = \left\langle y, \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\right\rangle$$
$$G_3^* = \left\langle z, \left(\frac{1}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{4}{10}\right)\right\rangle.$$

Then $N^\tau = \{0_N, G_1^*, 1_N\}$, $N^\sigma = \{0_N, G_2^*, 1_N\}$ *and* $N^\nu = \{0_N, G_3^*, 1_N\}$ *are* NTs *on* N^X, N^Y *and* N^Z *respectively. Define a bijective map* $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ *by* $N^{f_*}(a) = c$ *and* $N^{f_*}(b) = d$ *and* $N^{g_*} : (N^Y, N^\sigma) \to (N^Z, N^\nu)$ *by* $N^{g_*}(c) = u$ *and* $N^{g_*}(d) = v$. *Then* N^{f_*} *and* $N^{f_*^{-1}}$ *are* $N\alpha GS$ *CTS maps. Also* N^{g_*} *and* $N^{g_*^{-1}}$ *are* $N\alpha GS$ *CTS maps. Hence* N^{f_*} *and* N^{g_*} *are* $N\alpha GS$ *homeomorphisms. But the composition* $N^{g_*} \circ N^{f_*} : N^X \to N^Z$ *is not a* $N\alpha GS$ *homeomorphism since* $N^{g_*} \circ N^{f_*}$ *is not a* $N\alpha GS$ *CTS map.*

Definition 3.14. *A bijective map* $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ *is called a Neutrosophic* $i\alpha$ *-generalized semi homeomorphism (briefly* $Ni\alpha GS$ *homeomorphism) if* N^{f_*} *and* $N^{f_*^{-1}}$ *are* $N\alpha GS$ *irresolute maps.*

Theorem 3.15. *Every* $Ni\alpha GS$ *homeomorphism is a* $N\alpha GS$ *homeomorphism but not conversely.*

Proof. Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ be a $Ni\alpha GS$ homeomorphism. Let $V_{B_1^*}$ be NCS in N^Y. Since every NCS is a $N\alpha GSCS$, $V_{B_1^*}$ is a $N\alpha GSCS$ in N^Y. By hypothesis, $N^{f_*^{-1}}(V_{B_1^*})$ is a $N\alpha GSCS$ in N^X. Hence N^{f_*} is a $N\alpha GS$ CTS map. Similarly we can prove $N^{f_*^{-1}}$ is a $N\alpha GS$ CTS map. Hence N^{f_*} and $N^{f_*^{-1}}$ are $N\alpha GS$ CTS maps. Therefore, the map N^{f_*} is a $N\alpha GS$ homeomorphism. □

Example 3.16. *Let* $N^X = \{a,b\}$, $N^Y = \{u,v\}$,

$$G_1^* = \left\langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{1}{10}, \frac{5}{10}, \frac{2}{10}\right)\right\rangle$$
$$G_2^* = \left\langle y, \left(\frac{1}{10}, \frac{5}{10}\frac{2}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{4}{10}\right)\right\rangle.$$

Then $N^\tau = \{0_N, G_1^*, 1_N\}$ *and* $N^\sigma = \{0_N, G_2^*, 1_N\}$ *are* NTs *on* N^X *and* N^Y *respectively. Define a bijective map* $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ *by* $N^{f_*}(a) = u$ *and* $N^{f_*}(b) = v$. *Then* N^{f_*} *is* $N\alpha GS$ *homeomorphism. Let us consider a NS*

$$V_{A_1^*} = \left\langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{3}{10}\right)\right\rangle$$

in N^X. *Clearly* $V_{A_1^*}$ *is a* $N\alpha GSCS$ *in* N^X. *But* $f(V_{A_1^*})$ *is not a* $N\alpha GSCS$ *in* N^Y. *That is* $N^{f_*^{-1}}$ *is not a* $N\alpha GS$ *irresolute map. Hence* N^{f_*} *is not a* $Ni\alpha GS$ *homeomorphism.*

Theorem 3.17. *Let* $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ *be a* $Ni\alpha GS$ *homeomorphism. Then* N^{f_*} *is a Neutrosophic homeomorphism if* N^X *and* N^Y *are* $N\alpha gaT_{\frac{1}{2}}$ *spaces.*

Proof. Let $V_{B_1^*}$ be a NCS in N^Y. Since every NCS is a $N\alpha GSCS$, $V_{B_1^*}$ is a $N\alpha GSCS$ in N^Y. Since N^{f_*} is a $N\alpha GS$ irresolute map, $N^{f_*^{-1}}(V_{B_1^*})$ is a $N\alpha GSCS$ in N^X. Since N^X is a $N\alpha gaT_{\frac{1}{2}}$ space, $N^{f_*^{-1}}(V_{B_1^*})$ is a NCS in N^X. Hence N^{f_*} is a N-CTS map. By hypothesis, $N^{f_*^{-1}} : (N^Y, N^\sigma) \to (N^X, N^\tau)$ is a $N\alpha GS$ irresolute map. Let $V_{A_1^*}$ be a NCS in N^X. Since every NCS is a $N\alpha GSCS$, $V_{A_1^*}$ is a $N\alpha GSCS$ in N^X. Then $(N^{f_*^{-1}})^{-1}(V_{A_1^*}) = f(V_{A_1^*})$ is a $N\alpha GSCS$ in N^Y. Since N^Y is a $N\alpha gaT_{\frac{1}{2}}$ space, $N^{f_*}(V_{A_1^*})$ is a NCS in N^Y. Hence $N^{f_*^{-1}}$ is a N-CTS map. Therefore N^{f_*} is a Neutrosophic homeomorphism. □

Theorem 3.18. *If* $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ *is a* $Ni\alpha GS$ *homeomorphism, then* $N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})) \subseteq N^{f_*^{-1}}(N\alpha cl(V_{B_1^*}))$ *for every NS* $V_{B_1^*}$ *in* N^Y.

Proof. Let $V_{B_1^*}$ be a NS in N^Y. Then $N\alpha cl(V_{B_1^*})$ is a $N\alpha CS$ in N^Y. This implies $N\alpha cl(V_{B_1^*})$ is a $N\alpha GSCS$ in N^Y. Since N^{f_*} is a $N\alpha GS$ irresolute map, $N^{f_*^{-1}}(N\alpha cl(V_{B_1^*}))$ is a $N\alpha GSCS$ in N^X. This implies $N\alpha gscl(N^{f_*^{-1}}(N\alpha cl(V_{B_1^*}))) = N^{f_*^{-1}}(N\alpha cl(V_{B_1^*}))$. i.e. $N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})) \subseteq N\alpha gscl(N\alpha gscl(N^{f_*^{-1}}(N\alpha cl(V_{B_1^*}))) = N^{f_*^{-1}}(N\alpha cl(V_{B_1^*}))$. Hence, $N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})) \subseteq N^{f_*^{-1}}(N\alpha cl(V_{B_1^*}))$ for every NS $V_{B_1^*}$ in N^Y. □

Theorem 3.19. *If* $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ *is a* $Ni\alpha GS$ *homeomorphism, then* $N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})) = N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*}))$ *for every NS* $V_{B_1^*}$ *in* N^Y.

Proof. Since N^{f_*} is a $Ni\alpha GS$ homeomorphism, N^{f_*} is a $N\alpha GS$ irresolute map. Consider a NS $V_{B_1^*}$ in N^Y. Clearly $N\alpha gscl(V_{B_1^*})$ is a $N\alpha GSCS$ in N^Y. By hypothesis, $N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*}))$ is a $N\alpha GSCS$ in N^X. Since

$$N^{f_*^{-1}}(V_{B_1^*}) \subseteq N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*})),$$
$$N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})) \subseteq N\alpha gscl(N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*})))$$
$$= N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*})).$$

This implies $N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})) \subset N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*}))$. Since N^{f_*} is a $Ni\alpha GS$ homeomorphism, $N^{f_*^{-1}} : N^Y \to N^X$ is a $N\alpha GS$ irresolute map. Consider a NS $N^{f_*^{-1}}(V_{B_1^*})$ in N^X. Clearly $N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*}))$ is a $N\alpha GSCS$ in N^X.
This implies $(N^{f_*^{-1}})^{-1}(N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*}))) = N^{f_*}(N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})))$ is a $N\alpha GSCS$ in N^Y. Hence

$$V_{B_1^*} = (N^{f_*^{-1}})^{-1}(N^{f_*^{-1}}(V_{B_1^*})) \subseteq (N^{f_*^{-1}})^{-1}(N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})))$$
$$= N^{f_*}(N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*}))).$$

Therefore,

$$N\alpha gscl(V_{B_1^*}) \subseteq N\alpha gscl(N^{f_*}(N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})))) = N^{f_*}(N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*}))).$$

Since $N^{f_*^{-1}}$ is a $N\alpha GS$ irresolute map. Hence,

$$N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*})) \subseteq N^{f_*^{-1}}(N^{f_*}(N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*}))) = N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})).$$

That is $N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*})) \subseteq N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*}))$. Hence, $N\alpha gscl(N^{f_*^{-1}}(V_{B_1^*})) = N^{f_*^{-1}}(N\alpha gscl(V_{B_1^*}))$. □

Theorem 3.20. *If $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ is a $Ni\alpha GS$ homeomorphism, then $N\alpha gscl(N^{f_*}(V_{B_1^*})) = N^{f_*}(N\alpha gscl(V_{B_1^*}))$ for every NS $V_{B_1^*}$ in N^X.*

Proof. Since N^{f_*} is a $Ni\alpha GS$ homeomorphism, $N^{f_*^{-1}}$ is a $Ni\alpha GS$ homeomorphism. Let us consider a NS $V_{B_1^*}$ in N^X. By the Theorem (3.18), $N\alpha gscl((N^{f_*^{-1}})^{-1}(V_{B_1^*})) = (N^{f_*^{-1}})^{-1}(N\alpha gscl(V_{B_1^*}))$. Hence $N\alpha gscl(N^{f_*}(V_{B_1^*})) = N^{f_*}(N\alpha gscl(V_{B_1^*}))$ for every NS $V_{B_1^*}$ in N^X. □

Proposition 3.21. *The composition of two $Ni\alpha GS$ homeomorphisms is a $Ni\alpha GS$ homeomorphism in general.*

Proof. Let $N^{f_*} : (N^X, N^\tau) \to (N^Y, N^\sigma)$ and $N^{g_*} : (N^Y, N^\sigma) \to (N^Z, N^\eta)$ be two $Ni\alpha GS$ homeomorphisms. Let $V_{A_1^*}$ be a $N\alpha GSCS$ in N^Z. Then by hypothesis, $N^{g_*^{-1}}(V_{A_1^*})$ is a $N\alpha GSCS$ in N^Y. Hence, $N^{f_*^{-1}}(N^{g_*^{-1}}(V_{A_1^*}))$ is a $N\alpha GSCS$ in N^X. Hence $(N^{g_*} \circ N^{f_*})^{-1}$ is a $N\alpha GS$ irresolute map. Let $V_{B_1^*}$ be a $N\alpha GSCS$ in N^X. Then by hypothesis, $N^{f_*}(V_{B_1^*})$ is a $N\alpha GSCS$ in N^Y. Then by hypothesis $N^{g_*}(N^{f_*}(V_{B_1^*}))$ is a $N\alpha GSCS$ in N^Z. This implies $N^{g_*} \circ N^{f_*}$ is a $N\alpha GS$ irresolute map. Hence $N^{g_*} \circ N^{f_*}$ is a $Ni\alpha GS$ homeomorphism. Therefore the composition of two $Ni\alpha GS$ homeomorphisms is a $Ni\alpha GS$ homeomorphism in general. We denote the family of all $Ni\alpha GS$ homeomorphisms of a NTS (N^X, N^τ) onto itself by $Ni\alpha GS\text{-}h(N^X, N^\tau)$. □

Theorem 3.22. *The set $Ni\alpha GS\text{-}h(N^X, N^\tau)$ is a group under the composition of maps.*

Proof. Define a binary operation $* : Ni\alpha GS\text{-}h(N^X, N^\tau) \times Ni\alpha GS\text{-}h(N^X, N^\tau) \to Ni\alpha GS\text{-}h(N^X, N^\tau)$ by $N^{f_*} * N^{g_*} = N^{g_*} \circ N^{f_*}$ for all $N^{f_*}, N^{g_*} \in Ni\alpha GS\text{-}h(N^X, N^\tau)$ and $\circ$ is the usual operation of composition of maps. Then by Theorem (3.20), $N^{g_*} \circ N^{f_*} \in Ni\alpha GS\text{-}h(N^X, N^\tau)$. We know that, the composition of maps is associative and the identity map $I : (N^X, N^\tau) \to (N^X, N^\tau)$ belonging to $Ni\alpha GS\text{-}h(N^X, N^\tau)$ serves as the identity element. If $N^{f_*} \in Ni\alpha GS\text{-}h(N^X, N^\tau)$, then $N^{f_*^{-1}} \in Ni\alpha GS\text{-}h(N^X, N^\tau)$ such that $N^{f_*} \circ N^{f_*^{-1}} = N^{f_*^{-1}} \circ N^{f_*} = I$ and so inverse exists for each element of $Ni\alpha GS\text{-}h(N^X, N^\tau)$. Therefore, $(Ni\alpha GS\text{-}h(N^X, N^\tau), \circ)$ is a group under the operation of composition of maps. □

4. Conclusion

In this paper, we discussed Neutrosophic α-generalized semi homeomorphism and Neutrosophic i α-generalized semi homeomorphism. Also we have studied some of its basic properties.The results are illustrated with well-analyzed examples.

References

[1] K.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems,* 20(1986), 87–95.

[2] Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala, On Some New Notions and Functions in Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems,* 16(2017), 16–19.

[3] A. Atkinswestley and S. Chandrasekar, Neutrosophic g^*-Closed Sets and its Maps, *Neutrosophic Sets and Systems*, 36(2020), 96–107.

[4] A. Atkinswestley S. Chandrasekar, Neutrosophic Weakly G^* closed sets, *Advances in Mathematics: Scientific Journal*, 09(2020), 2853–2861.

[5] V. Banu Priya S. Chandrasekar, Neutrosophic αgs Continuity and Neutrosophic αgs Irresolute Maps, *Neutrosophic Sets and Systems*, 28(2019), 162–170.

[6] V. Banu Priya, S. Chandrasekar and M. Suresh, Neutrosophic Strongly α-Generalized Semi Closed Sets, *Advances in Mathematics: Scientific Journal*, 09(2020), 8605–8613.

[7] R. Dhavaseelan and S. Jafari, Generalized Neutrosophic closed sets, *New trends in Neutrosophic Theory and Applications*, 02(2018), 261–273.

[8] R. Dhavaseelan, S.Jafari, and Md. Hanif page, Neutrosophic generalized α-contra-continuity, *Creat. Math. Inform,* 27(2), (2018), 133–139.

[9] Florentin Smarandache, Neutrosophic and Neutrosophic Logic, *First International conference On Neutrosophic, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico*, 2008.

[10] Md. Hanif Page and Qays Hatem Imran, Neutrosophic Generalized Homeomorphism, *Neutrosophic Sets and Systems*, 35(2020), 340–346.

[11] P. Ishwarya, and K. Bageerathi, On Neutrosophic semiopen sets in NTSs, *International Jour. of Math.Trends and Tech.,* 37(3)(2016), 214–223.

[12] D. Jayanthi, α Generalized Closed Sets in Neutrosophic Topological Spaces, *International Journal of Mathematics Trends and Technology, Special Issue-ICRMIT*, (2018), 88–91.

[13] M. Parimala, R. Jeevitha, F. Smarandache, S. Jafari and R. Udhayakumar, Neutrosophic $\alpha\psi$ Homeomorphism in Neutrosophic Topological Spaces, *Information*, 09(187)(2018), 01–10.

[14] C. Maheswari, M. Sathyabama, S. Chandrasekar, Neutrosophic generalized b-closed Sets in Neutrosophic Topo-

logical Spaces, *Journal of Physics: Conference Series*, 1139(2018).

[15] C. Maheswari, S. Chandrasekar, Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity, *Neutrosophic Sets and Systems*, 29(2019), 89–100.

[16] C. Maheswari and S. Chandrasekar, Neutrosophic bg-closed Sets and its Continuity, *Neutrosophic Sets and Systems*, 36(2020), 108–120.

[17] T. Rajesh kannan, S. Chandrasekar, Neutrosophic $\omega\alpha$closed sets in Neutrosophic topological space, *Journal Of Computer And Mathematical Sciences*, 9(10)(2018), 1400–1408.

[18] T. Rajesh kannan, S. Chandrasekar, Neutosophic α-continuous multifunction in Neutrosophic topological space, *The International Journal of Analytical and Experimental Modal Analysis*, 11(2019), 1360–1367.

[19] T. RajeshKannan and S. Chandrasekar, Neutrosophic α-Irresolute Multifunction In Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems*, 32(1)(2020), 390–400.

[20] A.A. Salama, S.A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, *IOSR Journal of Mathematics*, 3(4)(2012), 31–35.

[21] A.A. Salama, S.A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, *Computer Science and Engineering*, 2(7)(2012), 129–132.

[22] V.K. Shanthi, S.Chandrasekar, K.Safina Begam, Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces, *International Journal of Research in Advent Technology*, 6(7)(2018), 1739–1743.

[23] L. A. Zadeh, Fuzzy sets, *Information and Control*, 8(3)(1965), 338–353.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆