Neutrosophic α-Continuous Multifunction In Neutrosophic Topological Spaces

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Abstract— **Aim of this present paper is, we introduce and investigate a new class of continuous multivalued function is called Neutrosophic α- continuous multi valued function in Neutrosophic topological spaces and its properties and characterization are discussed details.**

Keywords-Neutrosophic α-closed sets, Neutrosophic α- continuous , Neutrosophic α-continuous multi valued function- Neutrosophic topological spaces

I. Introduction

C.L. Chang [3] was introduced and developed fuzzy topological space by using L.A. Zadeh's[18] fuzzy sets. Coker [4] introduced the notion of Intuitionistic fuzzy topological spaces by using Atanassov's[1] Intuitionistic fuzzy set. Neutrality the degree of indeterminacy, as an independent concept was introduced by Smarandache [7] in 1998. He also defined the Neutrosophic set on three component (t,f,i) =(Truth, Falsehood, Indeterminacy),The Neutrosophic crisp set concept was converted to Neutrosophic topological spaces by A.A.Salama [12]. I.Arokiarani.[2] et al, introduced Neutrosophic α-closed sets. Wadei and saeid[17] are introduced Neutrosophic upper and lower pre continuous multivalued function and R.Dhavaseelan and e.tal.[6] are investigated Neutrosophic semi continuous function. Aim of this present paper is, we introduce and investigate a new class of continuous multivalued function is called Neutrosophic α-continuous multivalued function in Neutrosophic topological spaces and its properties and characterization are discussed details

II. PRELIMINARIES

In this section, we introduce the basic Definition for Neutrosophic sets and its operations.
Throughout this paper, (X,τ) is called classical topological spaces on X (represent as CTSX),(Y,τ_{N_Y}) is called Neutrosophic topological spaces on Y(represent as NUTSY),The family of all open set in X (α-Open inX, semi-open in X and pre-open in X respectively) is denoted by O(CTSX).(αO(CTSX) , SO(CTSX) and PO(CTSX) respectively). The family of all Neutrosophic open set in Y (α-Open in Y, semi-open in Y and pre-open in Y respectively) is denoted by O(NUTSY).(αO(NUTSY) , SO(NUTSY) and PO(NUTSY) respectively). The family of all closed set in X (α-closed inX , semi-closed in X and pre-Closed in X respectively) is denoted by C(CTSX).(α-C(CTSX) , SC(CTSX) and PS(CTSX) respectively). The family of all Neutrosophic Closed in Y (α-closed in Y , semi-closed in Y and pre-closed in Y respectively) is denoted by C(NUTSY).(αC(NUTSY) , SC(NUTSY) and PC(NUTSY) respectively)

Definition 2.1 [7]
Let X be a non-empty fixed set. A Neutrosophic set A is an object having the form A= {<x, μ_A(x),σ_A(x),γ_A(x)> : x ∈ X}.Where μ_A(x), σ_A(x) and γ_A(x) which represent Neutrosophic of the degree of membership function, the degree indeterminacy and the degree of non membership function respectively of each element x ∈ X to the set Awith 0 ≤μ_A(x)+σ_A(x)+γ_A(x) ≤ 1.
Remark 2.2[7]
we shall use the symbol
A =<x, μ_A, σ_A, γ_A> for the Neutrosophic set A = {<x, μ_A(x), σ_A(x), γ_A(x) >:x∈X}.
Example 2.3 [7]
Every Intuitionistic fuzzy set A is a non-empty set in X is obviously on Neutrosophic set having the form A = { <x, μ_A(x), 1-((μ_A(x) + γ_A(x)), γ_A(x) > : x ∈ X }.
Definition 2.4 [7]
we must introduce the Neutrosophic set0_N and 1_N in X as follows:
0_N be defined as:
0_N= {<x, 0, 0, 1>: x ∈X}
1_N be defined as :
1_N = {<x, 1, 0, 0>: x ∈ X}

Definition 2.5 [7]
Let A= {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$>: x∈X} be a Neutrosophic set on X, Then the complement of the set A (A^C) defined as A^C = {<x , $\gamma_A(x)$, $\sigma_A(x)$, $\mu_A(x)$>: x ∈X}

Definition 2.6 [7]
Let X be a non-empty set and Neutrosophic sets A and B in the form
A = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$>: x∈X} and
B = {<x, $\mu_B(x)$, $\sigma_B(x)$, $\gamma_B(x)$)>: x∈X }.
Then we consider A subsets of B (A⊆B).
defined as: A⊆B ⟺ $\mu_A(x) \le \mu_B(x)$, $\sigma_A(x) \le \sigma_B(x)$, and $\gamma_A(x) \ge \gamma_B(x)$ for all x∈X

Definition 2.7 [7]
Let X be a non-empty set, and Take A= {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$>: x∈X} andB = {<x, $\mu_B(x)$, $\sigma_B(x)$, $\gamma_B(x)$)>: x∈X }.are Neutrosophic sets. Then
(i) A∩B defined as :A∩B ={<x, $\mu_A(x) \wedge \mu_B(x)$, $\sigma_A(x) \wedge \sigma_B(x)$,$\gamma_A(x \vee \gamma_B(x)$>: x∈X}
(ii) A∪B defined as :A∪B ={<x, $\mu_A(x) \vee \mu_B(x)$, $\sigma_A(x) \vee \sigma_B(x)$, $\gamma_A(x) \wedge \gamma_B(x)$> : x∈X}

Definition 2.8 [7]
We can easily generalize the operation of intersection and union in Definition 2.7 to arbitrary family of Neutrosophic sets as follows:
Let { A_j: j ∈J } be an arbitrary family of Neutrosophic sets in X, then
(i) ∩Ajdefined as : ∩A_j={<x, $\wedge_{j\in J}\mu_{Aj}(x)$,$\wedge_{j\in J}\sigma_{Aj}(x)$,$\vee_{j\in J}\gamma_{Aj}(x)$ > :x ∈X }
(ii) ∪Ajdefined as :∪Aj={$\vee_{j\in J}\mu_{Aj}(x)$, $\vee_{\in J}\sigma_{Aj}(x)$,$\wedge_{j\in J}\gamma_{Aj}(x)$ > :x ∈X}

Proposition 2.9 [9]
For all A and B are two Neutrosophic sets then the following condition are true:
(1) $(A\cap B)^C = A^C \cup B^C$
(2) $(A\cup B)^C = A^C \cap B^C$.

Definition 2.10 [10]
A Neutrosophic topology is a non -empty set X is a family τ_N of Neutrosophic subsets in X satisfying the following axioms:
(i) 0_N, $1_N \in \tau_N$,
(ii) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
(iii) ∪Gi∈τ_N for every Gi∈τ_N,i∈J
the pair (X, τ_N) is called a Neutrosophic topological space.
The element Neutrosophic topological spaces of τ_N are called Neutrosophic open sets.
A Neutrosophic set A is closed if and only if A^C is Neutrosophic open.

Definition 2.11[10]
Let (X, $\tau_{N)}$be Neutrosophic topological spaces andA = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$>: x∈X} be a Neutrosophic set in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by
Neu-cl(A) =∩{ K :K is a Neutrosophic closed set in X and A⊆K}
Neu-int(A) = ∪{G :G is a Neutrosophic open set in X and G⊆A}.

Definition: 2.12[8]
Let (X, τ_N) be Neutrosophic topological spaces andA = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$>: x∈X} be a Neutrosophic set in X Then A is called if Neutrosophic semi-open if A⊆Neu-cl(Neu-int(A)).
The complement of Neutrosophic semi-open set is called Neutrosophic semi-closed.

Definition: 2.13[10]
Let (X, τ_N) be Neutrosophic topological spaces and A = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$>: x∈X} be a Neutrosophic set in X Then A is called if Neutrosophic α-open set if A⊆Neu-int(Neu-cl(Neu-int(A))).
The complement of Neutrosophic α-open set is called Neutrosophic α-closed.

Definition: 2.14[10]
Let (X, τ_N) be Neutrosophic topological spaces andA = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$>: x∈X} be a Neutrosophic set in X then A is called if Neutrosophic pre open set if A⊆Neu-int(Neu-clA)).
The complement of Neutrosophic pre-open set is called Neutrosophic pre-closed

Remark:2.15[11]
Let A be an Neutrosophic topological space (X, τ_N) .Then
(i) Neu α-cl(A) = A∪Neu-cl(Neu-int(Neu-cl(A))).
(ii) Neu α-int(A) = A⋂Neu-int(Neu-cl(Neu-int(A))).

Definition 2.16[9]
Take r,s,t are belongs to real numbers 0 to 1 such that 0≤r+s+t≤1 .An Neutrosophic point $\beta_{(r,s,t)}$ is Neutrosophic set defined by

$$\beta_{(r,s,t)} = \begin{cases} (r,s,t) \text{ if} x = p \\ (0,0,1) \text{ if} x \neq p \end{cases}$$

Take β(r,s,t) =<,$β_r$,$β_s$.$β_t$>Where $β_r$, $β_s$.$β_t$are represent Neutrosophic topological spaces the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element x ∈ X to the set A

Definition:2.17

A Neutrosophic set A in Y is said to be quasi-coincident (q-coincident) with a Neutrosophic set B denoted by AqB, if and only if there exists y∈Y such that A(y) + B(y) >1.

A Neutrosophic set Γ of Y is called a Neutrosophic neighborhood of a fuzzy point $y_α$ in Y if there exists a Neutrosophic open set μ in Y such that $y_α$ ∈ μ ≤ Γ

Remark: **2.18**

AqB⟺ A⊈ B^C

Definition 2.19[9]

let X and Y be two finite sets. Define f:X→Y.If

A={<y, μ_A(y), $σ_A$(y), $γ_A$(y)>y∈y} is an NS in Y, then the inverse image(pre image) A under f is an NS defined by f^{-1}(A)=<x, $f^{-1}\mu_A$(x), $f^{-1}σ_A$(x), $f^{-1}γ_A$(x) :x∈X>. Also define image NS U=<x, μ_U(x), σ_U(x), γ_U(x) :x∈X:> under f is an NS defined by

f (U)=<y, f μ_A(y), f $σ_A$ (y), f $γ_A$ (y):y∈y>.where

fμ_A(y)= {supμ_A(x) if f^{-1} (y) ≠ϕ, x∈f^{-1}(y)
0 otherwise

f$σ_A$ (y)= {sup$σ_A$(x) if f^{-1} (y) ≠ϕ, x∈f^{-1}(y)
0 Otherwise

f$γ_A$ (y)= {inf$γ_A$ (x) if f^{-1} (y) ≠ϕ, x∈f^{-1}(y)
0 Otherwise

Definition 2.20[2]

A mapping f:(X, τ_{N_X})→(Y, σ_{N_Y}) is called a

(i)Neutrosophiccontinuous(Neu-continuous in short) if the pre image (inverse image) of Neutrosophic closed (Neu- closed in short) set in σ_{N_Y} is Neutrosophic closed sets (Neu- closed in short) in τ_{N_X}

(ii)Neutrosophic α-continuous(Neu α - continuous in short) if the pre image (inverse image) of Neutrosophic closed (Neu- closed in short) set in σ_{N_Y} is Neutrosophic α -closed sets (Neu α - closed in short) in τ_{N_X}

(iii)Neutrosophic semi-continuous(Neu semi - continuous in short) if the pre image (inverse image) of Neutrosophic closed (Neu- closed in short) set in σ_{N_Y} is Neutrosophic semi -closed sets (Neu semi- closed in short) in τ_{N_X}

Definition 2.21.

Let (X,τ) be a topological space in the classical sense and (Y,τ_{N_Y}) be an Neutrosophic topological space. F : (X,τ) → (Y,τ_{N_Y}) is called a Neutrosophic multifunction if and only if for each x ∈X, F (x) is a Neutrosophic set in Y

Definition 2.22.

For a Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}), the upper inverse

F^+(Γ) and lower inverse F^- (Γ) of a Neutrosophic set Γ in Y are defined as follows:

F^+(Γ) ={x ∈ X\F (x) ≤ Γ}and

F^- (Γ) = {x ∈X\F(x)qΓ}.

Lemma 2.23.

For a Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}),

we have F^- (1- Γ) =X - F^+(Γ), for any Neutrosophic set Γ in Y

Definition 2.24[6]

A Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}) is said to be

1.Neutrosophic upper semi continuous at a point x∈X if for any Γ ∈ O(NUTSY), Γ containing F(x) (that is , F (x) ≤ Γ), there exist x∈ U ∈ O(CTSX) such that F (U) ≤ Γ. (that is U ⊂ F^+(Γ)).

2.Neutrosophic lower semi continuous at a point x ∈X if for any Γ ∈ O(NUTSY), with F (x)qΓ , there exist x ∈ U ∈ O(CTSX) such that U ⊆F^- (Γ).

3.Neutrosophic upper semi continuous (Neutrosophic lower semi continuous) if it is Neutrosophic upper semi continuous (Neutrosophic lower semi continuous) at each point x ∈X.

Definition 2.25[17]

A Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}) is said to be

1.Neutrosophic upper pre -continuous at a point x∈X if for any Γ ∈ O(NUTSY), Γ containing F(x) (that is , F (x) ≤ Γ), there exist x∈ U ∈ PO(CTSX) such that F (U) ≤ Γ. (that is U ⊂ F^+(Γ)).

2.Neutrosophic lower pre- continuous at a point x ∈X if for any Γ ∈ O(NUTSY), with F (x)qΓ , there exist x ∈ U ∈ PO(CTSX) such that U ⊆F^- (Γ).

3.Neutrosophic upper pre-continuous (Neutrosophic lower pre-continuous) if it is Neutrosophic upper pre-continuous (Neutrosophic lower pre-continuous) at each point x ∈X.

Definition 2.26.

A Neutrosophic multifunction F :(X,τ) → (Y,τ_{N_Y}) is said to be

(i).Neutrosophic upper quasi -continuous at a point x∈ for any Γ ∈ O(NUTSY), Γ containing F(x) (that is , F (x) ≤ Γ), there exist x∈ U ∈ SO(CTSX) such that F (U) ≤ Γ. (that is U ⊂ F^+(Γ))

(ii).Neutrosophic lower quasi- continuous at a point x ∈X if for any Γ ∈ O(NUTSY), with F (x)qΓ , there exist x ∈ U ∈ SO(CTSX) such that U ⊆F^- (Γ).

(iii).Neutrosophic upper quasi-continuous (Neutrosophic lower quasi-continuous) if it is Neutrosophic upper quasi-continuous (Neutrosophic lower quasi-continuous) at each point x∈X.

Definition:2.27

Let A be an Neutrosophic set in Neutrosophic fuzzy topology space(Y,τ_{N_Y}).Then V is said to be a neighburood of A in Y if there exist an Neutrosophic open set U of Y such that A⊆U⊆V.

III.NEUTROSOPHIC LOWER Α- CONTINUOUS MULTIFUNCTION

In this section, we introduce the Definition for Neutrosophic Lower α- continuous multifunction and its properties

Definition 3.1.

A Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}) is said to be

(i).Neutrosophic lower α- continuous at a point x ∈X if if for any Γ ∈O(NUTSY), with F (x)qΓ , there exist x ∈ U ∈ αO(CTSX) such that U ⊆F^- (Γ).

(ii).Neutrosophic lower α-continuous) if it is Neutrosophic lower α-continuous at each point x∈X.

Theorem:3.2

Every Neutrosophic lower semi continuous multifunction is Neutrosophic lower α continuous multifunction.

Proof:

Take for any Γ ∈ O(NUTSY), with F (x)qΓ , By our assumption ,there exist x ∈ U ∈ O(CTSX) such that U ⊆F^- (Γ).This implies ,there exist x ∈ U ∈ αO(CTSX) such that U ⊆F^- (Γ).since open sets are α- open set in X.

Remark:3.3

Converse of the above theorem need not be true.

Example:3.4 Consider X={a , b, c} , Y=[0,1] and take τ ={∅, {a } , X } and τ_{N_Y}={0_N, 1_N , þ(0.25,0.25,0.5), þ(0.3,0.3,0.4)} are topology and Neutrosophic topology on X and Y respectively. We using the notion Neutrosophic point (constant) þ(r,s,t) =<(y,$þ_r þ_s . þ_t$), ∀y>.Define the Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}) by F(a)=þ(0.7,0.2,0.1) ,F(b)= þ(0.4,0.3,0.3) , F(c)= þ(0.75,0.10,0.15) ,F is Neutrosophic lower α continuous multifunction but not Neutrosophic lower semi continuous multifunction. Since F^-(þ(0.25,0.25,0.5)) = {a , c} and F^-(þ((0.3,0.3,0.4))) = {a , c}are α- open set in X but not open set in X.

Theorem:3.5

Every Neutrosophic lower αcontinuous multifunction is Neutrosophic lower quasi semi continuous multifunction

Proof:

For any Γ ∈O(NUTSY), with F (x)qΓ . By our assumption , there exist x ∈ U ∈ αO(CTSX) such that U ⊆F^- (Γ).This implies , there exist x ∈ U ∈ SO(CTSX) such that U ⊆F^- (Γ)..since α- open sets are semi open set in X.

Remark:3.6

Converse of the above theorem need not be true.

Example:3.7

Consider X={a , b, c} , Y=[0,1] and take τ={∅, {a }{c}{a , c} , X } and τ_{N_Y}={0_N, 1_N , þ(0.25,0.25,0.5), þ(0.3,0.3,0.4)} are topology and Neutrosophic topology on X and Y respectively. We using the notion Neutrosophic point (constant) þ(r,s,t) =<(y,$þ_r þ_s . þ_t$), ∀y>.Define the Neutrosophic multifunction F : (X,τ)→(Y,τ_{N_Y}) by F(a)=þ(0.7,0.2,0.1) ,F(b)= þ(0.4,0.3,0.3) , F(c)= þ(0.75,0.10,0.15) ,F is Neutrosophic lower quasi semi continuous multifunction but not Neutrosophic lower α semi continuous multifunction. Since F^-(þ((0.25,0.25,0.5))) = {a , c} and F^-þ(((0.3,0.3,0.4))) = {a , c}are semi-open set in X but not α- open set in X.

Theorem:3.8

Every Neutrosophic lower α continuous multifunction is Neutrosophic lower pre continuous multifunction.

Proof: :

For any Γ ∈ O(NUTSY), with F (x)qΓ ,By our assumption , there exist x ∈ U ∈ αO(CTSX) such that U ⊆F^- (Γ).This implies , there exist x ∈ U ∈ PO(CTSX) such that U ⊆F^- (Γ).since α- open sets are an pre open set in X.

Remark:3.9

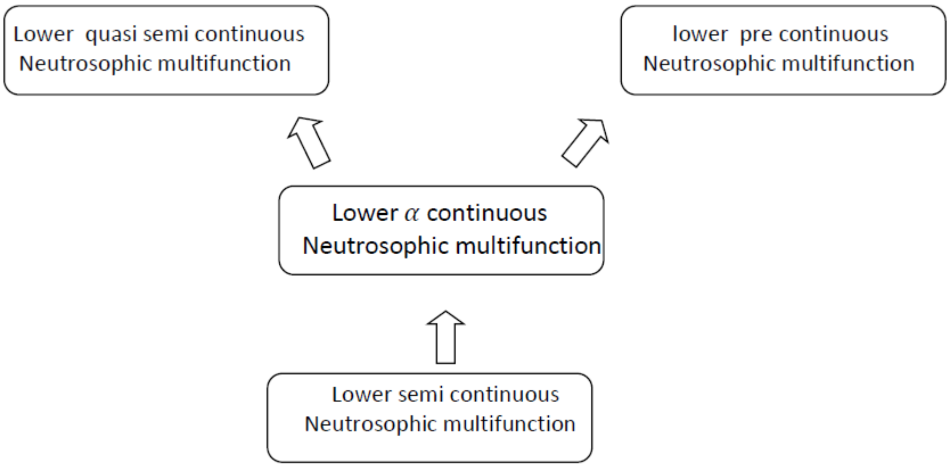
Converse of the above theorem need not be true.

Example:3.10

Consider X=$\{a, b, c, d\}$, , Y=[0,1] and take τ=$\{\emptyset, \{a\}\{c\}\{a, c\}, X\}$ and $\tau_{N_Y}=\left\{0_N, 1_N, \text{ƥ}\left(\frac{1}{4},\frac{1}{4},\frac{1}{2}\right), \text{ƥ}\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$ are topology and Neutrosophic topology on X and Y respectively. We using the notion Neutrosophic point (constant) ƥ(r,s,t) =<(y, $\text{ƥ}_r\text{ƥ}_s.\text{ƥ}_t), \forall y$ >.Define the Neutrosophic multifunction F:(X,τ)→(Y, τ_{N_Y}) by F(a)= ƥ $\left(\frac{5}{6},\frac{1}{12},\frac{1}{12}\right)$,F(b)= ƥ $\left(\frac{1}{4},\frac{1}{4},\frac{1}{2}\right)$, F(c)= ƥ $\left(\frac{3}{4},\frac{1}{8},\frac{1}{8}\right)$ and F(d)= ƥ $\left(\frac{1}{5},\frac{1}{5},\frac{3}{5}\right)$.$F$ is Neutrosophic lower pre continuous multifunction but not Neutrosophic lower α continuous multifunction. Since F^{-}ƥ$\left(\frac{1}{2},\frac{1}{4},\frac{1}{4}\right)$) = $\{a, b, c\}$ and F^{-}(ƥ $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$) = $\{a, b, c\}$are pre-open set in X but not α- vopen set in X.

Remark 3.11. We obtain the following diagram from the results we discussed above.

Diagram-I



Where A ⟶ B represents A implies B

IV. NEUTROSOPHIC UPPER Α- CONTINUOUS MULTIFUNCTION.

In this section, we introduce the definition for Neutrosophic upper **α**- continuous multifunction and its properties

Definition 4.1.

A Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}) is said to be

(i). Neutrosophic upper α -continuous at a point x∈X if for any Γ∈ O(NUTSY), Γ containing F(x) (that is , F (x) ≤ Γ), there existsx ∈ U ∈ αO(CTSX) such that F (U) ≤ Γ (that is,U⊂F^{+}(Γ)).

(ii). Neutrosophic upper α-continuous) if it is Neutrosophic upper α-continuous at each pointx∈X.

Theorem:4.2

Every Neutrosophicupper semi continuous multifunction is Neutrosophicupperα continuous multifunction.

Proof:

For any Γ∈ O(NUTSY), Γ containing F(x) .By our assumption , there exists x ∈ U ∈O(CTSX) such that F (U) ≤ Γ. This implies there exists x ∈ U ∈ αO(CTSX) such that F (U) ≤ Γ .since open sets are an α- open set in X.

Remark:4.3:

Converse of the above theorem need not be true.

Example:4.4

Consider X=$\{a, b, c\}$, Y=[0,1] and take τ=$\{\emptyset, \{b\}, X\}$ and τ_{N_Y}=$\{0_N, 1_N$,ƥ(0.7,0.1,0.2),ƥ(0.3,0.4,0.3)} are topology and Neutrosophic topology on X and Y respectively. We using the notion Neutrosophic point (constant) ƥ(r,s,t) =<(y,$\text{ƥ}_r\text{ƥ}_s.\text{ƥ}_t), \forall y$>.Define the Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}) by F(a)=ƥ(0.3,0.1,0.6) ,F(b)= ƥ(0.5,0.2,0.3) , F(c)= ƥ(0.8,0.1,0.1) ,F is Neutrosophic upper α continuous Neutrosophic multifunction but not Neutrosophic upper semi continuous Neutrosophic multifunction. Since F^{+}(ƥ(0.7,0.1,0.2)) = $\{a, b\}$ and F^{+}(ƥ(0.3,0.4,0.3)) = $\{a, b\}$ are α- open set in X but not open set in X.

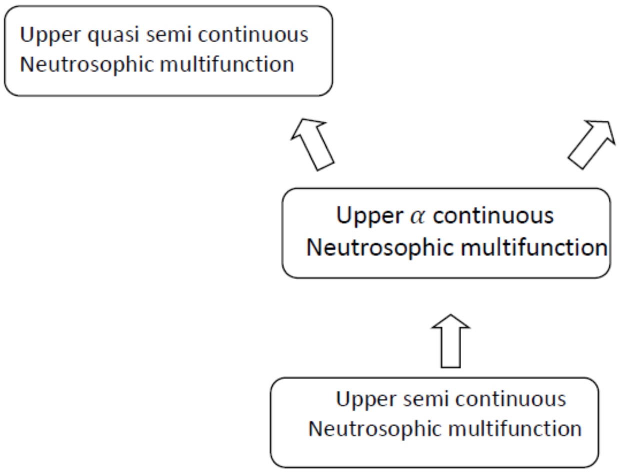
Theorem:4.5

Every Neutrosophic upper α- continuous multifunction is Neutrosophic upper quasi semi continuous multifunction.

Proof:

For any Γ∈ O(NUTSY), Γ containing F(x) .By our assumption , there exists x ∈ U ∈ αO(CTSX) such that F (U) ≤ Γ. This implies there exists x ∈ U ∈ SO(CTSX) such that F (U) ≤ Γ.sinceα- open sets are semi open set in X.

Remark:4.6
Converse of the above theorem need not be true.
Example:4.7
let X= $\{a, b, c\}$, Y= [0,1] and take τ= $\{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ and $\tau_{N_Y} = \left\{0_N, 1_N, ƥ\left(\frac{4}{6}, \frac{1}{6}, \frac{1}{6}\right), ƥ\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\right\}$ are topology and Neutrosophic topology on X and Y respectively. We using the notion Neutrosophic point (constant) ƥ(r,s,t) =<(y, $ƥ_r ƥ_s . ƥ_t), \forall y$ >.Define the Neutrosophic multifunction F:(X,τ)→(Y, τ_{N_Y}) by F(a)= $ƥ\left(\frac{4}{6}, \frac{1}{6}, \frac{1}{6}\right)$,F(b)= $ƥ\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$, F(c)= $ƥ\left(\frac{6}{7}, \frac{1}{14}, \frac{1}{14}\right)$ is Neutrosophic upper α continuous Neutrosophic multifunction but not Neutrosophic upper semi continuous Neutrosophic multifunction. Since $F^+(ƥ\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right))$ = $\{a, b\}$ and $F^+(ƥ\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right))$ = $\{a, b\}$ are semi-open set in X but not α- open set in X.
Theorem: 4.8
Every Neutrosophic upper α continuous multifunction is Neutrosophic upper pre continuous multifunction.
Proof:
For any Γ∈ O(NUTSY), Γ containing F(x) .By our assumption , there exists x ∈ U ∈ αO(CTSX) such that F (U) ≤ Γ. This implies there exists x ∈ U ∈ PO(CTSX) such that F (U) ≤ Γ since α- open sets are pre open set in X.
Remark:4.9 Converse of the above theorem need not be true.
Example:4.91 Consider X= $\{a, b, c, d\}$, , Y= [0,1] and take τ= $\{\emptyset, \{a\}\{c\}\{a, c\}, X\}$ and τ_{N_Y}=$\{0_N, 1_N$, ƥ(0.5,0.25,0.25), ƥ(0.3,0.3,0.4)$\}$ are topology and Neutrosophic topology on X and Y respectively. We using the notion Neutrosophic point (constant) ƥ(r,s,t) =<(y,$ƥ_r ƥ_s . ƥ_t), \forall y$>.Define the Neutrosophic multifunction F : (X,τ) → (Y,τ_{N_Y}) by F(a)= ƥ(0.4,0.2,0.4) ,F(b)= ƥ(0.4,0.35,0.15) , F(c)= ƥ(0.5,0.25,0.25) and F(d)= ƥ(0.4,0.3,0.3) ,F is Neutrosophic upper pre continuous Neutrosophic multifunction but not Neutrosophic upper α continuous Neutrosophic multifunction. Since F^+(ƥ(0.5,0.25,0.25)) = $\{a, b, c\}$ and F^+ƥ(0.3,0.3,0.4)) = $\{a, b, c\}$ are pre-open set in X but not α- vopen set in X.
Remark 4.92
We obtain the following diagram from the results we discussed above.
Diagram-II



Where A⟶B represents A implies B
V. PROPERTIES ABOUT LOWER AND UPPER α CONTINUOUS MULTIFUNCTION

In this section, we derive some application about Lower and Upper α Continuous Multifunction.
Theorem 5.1.
Let $F: (X, \tau) \to (Y, \tau_{N_Y})$ be an Neutrosophic multifunction and let $x \in X$. Then the following statements are equivalent:
(a) F is Neutrosophic lower α-continuous at x.
(b) Every $\ddot{\Omega}$ ∈ O(NUTSY) with $F(x)q\ddot{\Omega}$, implies $x \in sCl\left(IntF^-(\ddot{\Omega})\right)$.
(c) For any x ∈ U ∈ SO(CTSX)and for any $\ddot{\Omega}$ ∈ O(NUTSY) with $F(x)q\ddot{\Omega}$, there exists a non empty open set $B \subset W$ such that $F(v)q\ddot{\Omega}$, for all $v \in B$.

Proof.

(a)$\Longrightarrow$ (b). Assume that F is Neutrosophic lower α-continuous at x and $x \epsilon X$ and $\ddot{\Omega} \in$ O(NUTSY) with $F(x)q\ddot{\Omega}$.By our assumption . there exist $W \in \alpha O(CTSX)$ such that $x \in W$ and $F(W)q\ddot{\Omega}$.Thus $x \in W \subset F^{-}(\ddot{\Omega})$. Then, $W \in \alpha O(CTSX)$ implies $W \subset sCl(Int(W))$.we get $x \in sCl(IntF^{-}\ddot{\Omega})$.(b)$\Longrightarrow$(c).Take,$\ddot{\Omega} \in$ O(NUTSY) with $F(x)q\ddot{\Omega}$ then $x \in sCl(IntF^{-}(\ddot{\Omega}))$. Let x $\in$U$\in$ SO(CTSX).Then $W \cap Int(F^{-}(\ddot{\Omega})) \neq \emptyset$ .Let us consider, $B = W \cap Int(F^{-}(\ddot{\Omega})) \neq \emptyset$. Then V $\in$ SO(CTSX). $B \subset W, B \neq \emptyset$and$F(v)q\ddot{\Omega}$for all $v \in B$ (c) $\Longrightarrow$(a). Let $\{W_x\}$ $\in$SO(CTSX) be the system of the semi-open sets in X containing x. For any semi-open set $W \subset X$such that $x \in W$ and any Neutrosophic open set $\ddot{\Omega}$ of Ysuch that $F(x)q\ddot{\Omega}$, there exists a non empty open set $B_W \subset W$.such that $F(v)q\ddot{\Omega}$,. For all $v \in B_W$ .Let $V = \cup_{W \in W_x} B_W$, Then W is open in X , $x \in sCl(W)$ and $F(v)q\ddot{\Omega}$, for all $v \in W$ Put $A = V \cup \{x\}$, then $A \subset S \subset sCl(A)$.Thus $S \in \alpha O(X), x \in S$and $F(v)q\ddot{\Omega}, for\ all\ v \in S$. Hence F is Neutrosophic lower α-continuous at x .Hence F is Neutrosophic lower α-continuous at x.

Theorem 5.2.

For an Neutrosophic multifunction $F: (X, \tau) \to (Y, \tau_{N_Y})$ and let $x \in X$, the following statements are equivalent:

(1) F is Neutrosophic lower α-continuous.

(2) $F^{-}(\bar{G}) \in \alpha O(CTSX)$,for every $\bar{G} \in O(NUTSY)$.

(3)$F^{+}(\bar{V}) \in \alpha C(CTSX)$for each $\bar{V} \in C(NUTSY)$.

(4) $sInt(Cl(F^{+}(\bar{B}))) \subset F^{+}(Cl(\bar{B}))$for any Neutrosophic set $\bar{B}$of Y.

(5) .F(sInt(Cl(A) $\subset$Cl(F(A) for each subset A of X.

(6)F($\alpha Cl(A)$)$\subset$Cl(F(A)) for each subset A of X.

(7) $\alpha Cl(F^{+}(\bar{B})) \subset F^{+}(Cl(\bar{B}))$ for each Neutrosophic set $\bar{B}$of Y.

(8)F(Cl(Int(Cl(A)))) $\subset Cl(F(A)$ for any A of X.

Proof.

(1)$\Longrightarrow$(2). Let $\bar{G}$ be any Neutrosophic open set of Y and $x \in F^{-}(\bar{G})$. So F(x) q $\bar{G}$, since F is Neutrosophic lower α - continuous multi function, it follows that $x \in sCl\big(IntF^{-}(\bar{G})\big)$. we obtain $F^{-}(\bar{G}) \subset sCl(IntF^{+}(\bar{G})))$.Hence ,$F^{-}(\bar{G}) \in \alpha O(X)$.(2)$\Longrightarrow$ (1). Let x be arbitrarily chosen in X and $\bar{G}$ be any Neutrosophic open set of Y such that $F(x)q\bar{G}$, so $x \in F^{-}(\bar{G})$. By hypothesis $F^{-}(\bar{G}) \in \alpha O(CTSX)$, we have $x \in F^{-}(\bar{G}) \subset sCl\left(Int\big(F^{-}(\bar{G})\big)\right)$and thus F is Neutrosophic lower$\alpha -$ continuous at x ,As x is arbitrarily chosen, F Neutrosophic lower α-continuous.(2) $\Longrightarrow$ (3). It follows from the fact that $[F^{-}(\bar{A})]^{c} = F^{+}(\bar{A}^{c})$ for every Neutrosophic set $\bar{A}$ of Y and compliment of every open set is always closed.(3) $\Longrightarrow$ (4). Let$\bar{B}$be any Neutrosophic open set of Y .since Cl$\overline{(B)}$ is Neutrosophic closed set in Y .Then by $(c), F^{+}\big(Cl(\bar{B})\big)$ is an $\alpha -$ closed set in X. Thus we have $F^{+}\big(Cl(\bar{B})\big) \supset sInt\left(Cl\left(F^{+}\big(Cl(\bar{B})\big)\right)\right) \supset sInt\left(Cl\big(F^{+}(\bar{B})\big)\right)$

(4) $\Longrightarrow$ (5). Let A be an arbitrary subset of X.Let us put F(A)= $\bar{B}$Then A $\subset F^{+}\big((\bar{B})\big)$.Therefore , sIntCL(A) $\subset$sIntCl($F^{+}(\bar{B})$))$\subset$ $F^{+}\big(cl(\bar{B})\big)$. Therefore F(sIntCl(A)) $\subset$ F(sIntCl($F^{+}(\bar{B})$)) $\subset F(F^{+}\big(cl(\bar{B})\big) \subset cl(\bar{B}) = cl\big($F(A)$\big)$. (5) $\Longrightarrow$ (3) . Let $\bar{B}$ be any Neutrosophic closed set of Y .

Put A= $F^{+}(\bar{B})$. Then F(A) $\subset$ $(\bar{B})$. Therefore , we have F(sInt(Cl(A)) $\subset$ $cl(\bar{B}) = (\bar{B})$and F^{+}(F(sInt(Cl(A)) $\subset$ $F^{+}cl(\bar{B}) = F^{+}(\bar{B})$ but we know that F^{+}(F(sInt(Cl(A))$\supset$sIntCl(A).Hence sIntCl(A) $\subset$ $F^{+}(\bar{B})$.$F^{+}(\bar{B})$is closed set inX. (3) $\Longrightarrow$ (6) Since A$\subset$ $F^{+}\big(F(A)\big)$, We have A$\subset$ $F^{+}\big(ClF(A)\big)$. Now $ClF(A)$ is Neutrosophic closed set in Y and by hypothesis $F^{+}\big(ClF(A)\big)$ is $\alpha - c$ losed set in X.Thus $\alpha Cl(A) \subset F^{+}\big(ClF(A)\big)$.We get $F(\alpha Cl(A)) \subset F(F^{+}(ClF(A)) \subset (ClF(A)$. (6) $\Longrightarrow$ (3) Let $\bar{B}$ be any Neutrosophic closed set of Y.Put A= $F^{+}(\bar{B})$.We get $F(\alpha Cl(F^{+}(\bar{B})) \subset F(F^{+}(ClF(F^{+}(\bar{B})) \subset (ClF(F^{+}(\bar{B})$=$\bar{B}$.

Consequently $\alpha Cl(F^{+}(\bar{B}) \subset F^{+}(\bar{B}\ but\ \alpha Cl(F^{+}(\bar{B}) \supset F^{+}(\bar{B}$).Thus $F^{+}(\bar{B}$) is α -closed set in X. (6) $\Longrightarrow$ (7) Let $\bar{B}$ be any Neutrosophic closed set of Y $and\ F\big(\alpha Cl(F^{+}(\bar{B})\big) \subset ClF(F^{+}(\bar{B}) \subset Cl(\bar{B})$. Thus $\alpha Cl(F^{+}(\bar{B}) \subset F^{+}Cl(\bar{B})$. (7) $\Longrightarrow$ (8) replacing $(\bar{B})by$ F(A).where A is a sub set of X. $\alpha Cl(A) \subset \alpha Cl(F^{+}(F(A)) \subset F^{+}Cl\big(F(A)\big)$. This implies $F\big(\alpha Cl(A)\big) \subset F(\alpha Cl(F^{+}(F(A))$ $\subset F^{+}Cl\big(F(A)\big) = Cl\big(F(A)\big)$. (5) $\Longrightarrow$ (8). It is clearly true. (8) $\Longrightarrow$ (1). Let $x \in X$ and $\bar{V}$ be an Neutrosophic set in such that $F(x)q\bar{V}$. Thus $x \in F^{-}(\bar{V})$. We have to prove that $F^{-}(\bar{V})is\ \alpha -$ open set in X. We have F(Cl(Int(Cl($F^{+}Cl(\bar{V}^{c})$)) $\subset Cl\big(F(F^{+}(\bar{V}^{c})\big) \subset \bar{V}^{c}$ Which implies Cl(Int(Cl($F^{+}Cl(\bar{V}^{c})$)) $\subset F^{+}(\bar{V}^{c})$=$F^{-}(V)^{C}$. Therefore $F^{-}(V)$ $\subset$ Int (Cl(Int $F^{-}(V)$)). Hence $F^{-}(V)$ is an α-open set in X.we get $U \in \alpha O(CTSX)$ such that $x \in U$and $F(u)q\bar{V}$, for all $u \in U$.Hence F is Neutrosophic lower α-continuous

Theorem 5.3.
For an Neutrosophic multifunction $F : (X,\tau) \to (Y,\tau_{N_Y})$ and let $x \in X$, the following statements are equivalent:
(a) F is Neutrosophic upper α-continuous at x.
(b) For each Neutrosophic open set $\bar{G}$ of Y with $F(x) \subset \bar{G}$, there results the relation $x \in sCl(Int(F^+(\bar{G}))$.
(c) For any semi-open set $U \subset X$ containing x and for any Neutrosophic open set $\bar{G}$of Y, $F(x) \subset \bar{G}$, there exists a non empty open set $V \in U$. such that $F(V) \subset \bar{G}$.
Proof.
(a)⟹(b). Let $x \in X$ and $\bar{G}$ be any Neutrosophic open set of Y such that $F(x) \subset \bar{G}$, there is a $U \in \alpha O(CTSX)$.such that $x \in U$ and $F(u) \subset \bar{G}, for\ all\ u \in U$ .Thus $x \in U \subset F^+(\bar{G})$. Since $U \in \alpha O(X), U \subset sCl(Int(U)) \subset sCl(Int(F^+(\bar{G})$).Hence ,$x \in sCl\left(Int\left(F^+(\bar{G})\right)\right)$.(b)⟹(c). Let$\bar{G}$be any Neutrosophic open set of Y such that $F(x) \subset \bar{G}$, then $x \in sCl\left(Int\left(F^+(\bar{G})\right)\right)$.Let $U \subset X$ be any semi-open set such that $x \in U$.Then $U \cap Int(F^+(\bar{G}))) \neq \emptyset$. Put $V = U \cap Int\left(F^+(\bar{G})\right)$.Then V is an semi-open set in X, $V \subset U, V \neq \emptyset$ and $F(V) \subset \bar{G}$.(c)⟹(a). Let $\{U_x\}$be the system of the semi-open sets in X containing x. For any semi-open set $U \subset X$ such that $x \in U$ and $\bar{G}$be any Neutrosophic open set of Y such that $F(x) \subset \bar{G}$, there exists a non empty open set $G_U \subset U$.such that $F(G_U) \subset \bar{G}$
Let $W = U_{U \in U_x} G_u$, Then W is open , $x \in sCl(W)$ and $F(w) \subset \bar{G}$. for all $v \in W$ .Put $S = W \cup \{x\}$, then $W \subset S \subset sCl(W)$.Thus $S \in \alpha O(CTSX), x \in S$ and $F(w) \subset \bar{G}$,.for all $w \in S$Hence F is Neutrosophic upper α −continuous at x.
Theorem 5.4.
For an Neutrosophic multifunction $F : (X,\tau) \to (Y,\Gamma)$ and let $x \in X$, the following statements are equivalent:
(a) F is Neutrosophic upper α-continuous.
(b) $F^+(\bar{G}) \in \alpha O(CTSX)$,for every Neutrosophic open set$\bar{G}$ of Y .
(c)$F^-(\bar{B}) \in \alpha C(CTSX)$for each Neutrosophic closed set $\bar{B}$of Y.
(d) For each point $x \in X$ and for each neighborhood $\bar{V}$of $F(x)$ in Y, $F^+(\bar{V})$ is a α- neighborhood of x.
(e) For each point $x \in X$ and for each neighborhood $\bar{V}$ of F(x) in Y , there is an α - neighborhood U of x such that $F(U) \subset \bar{V}$.
(f) $\alpha Cl(F^-(\bar{B})) \subset F^-(Cl(\bar{B}))$ for each Neutrosophic set $\bar{B}$of Y.
(g) $sInt(Cl(F^-(\bar{B}))) \subset F^-(Cl(\bar{B}))$for any Neutrosophic set $\bar{B}$of Y.
Proof.
(a)⟹(b). Let $\bar{V}$ be any Neutrosophic open set of Y and $x \in F^+(\bar{V})$. We get $x \in sCl\left(IntF^+(\bar{V})\right)$. Therefore, we obtain $F^+(\bar{V}) \subset sCl(IntF^+(\bar{V})))$.Hence ,$F^+(\bar{V}) \in \alpha O(CTSX)$.(b) ⟹ (a). Let x be in X and $\bar{G}$ be any Neutrosophic open set of Y such that $F(x) \subset \bar{G}$, so $x \in F^+(\bar{G})$. By assumption $F^+(\bar{G}) \in \alpha O(CTSX)$, we have $x \in F^+(\bar{G}) \subset sCl\left(Int\left(F^+(\bar{G})\right)\right)$ and Thus F is Neutrosophic upper α − continuous at x .Hence F is Neutrosophic upper α-continuous.(b) ⟹ (c). It follows from the fact that $[F^-(\bar{A})]^c = F^+(\bar{A}^c)$ for every Neutrosophic set $\bar{A}$ of Y and compliment of every open set is always closed.(c) ⟹ (f). Let$\bar{B}$be any Neutrosophic open set of Y .Then by (c), $F^-\left(Cl(\bar{B})\right)$is an α-closed set in X.
Thus we have $F^-\left(Cl(\bar{B})\right) \supset sInt\left(Cl\left(F^-\left(Cl(\bar{B})\right)\right)\right) \supset sInt(Cl(F^-(\bar{B}))) \supset. F^-(\bar{B})[\ sInt(Cl(F^-(\bar{B}))) \supset \alpha Cl\left(F^-(\bar{B})\right)$. (f) ⟹ (g). Let $\bar{B}$be any Neutrosophic open set of Y .we have $\alpha Cl(F^-(\bar{B})) = F^-(\bar{B})\ (\ sInt(Cl(F^-(\bar{B}))) \subset F^-(Cl(\bar{B}))$(g) ⟹ (c). Let $\bar{B}$be any Neutrosophic closed set of Y . Then we have, $sInt(Cl(F^-(\bar{B}))) \subset F^-(\bar{B})(\ sInt(Cl(F^-(\bar{B}))) \subset F^-(Cl(\bar{B})$: Hence $F^-(\bar{B}) \in \alpha C(CTSX)$.(b) ⟹ (d). Let $x \in X$ and $\bar{V}$ be a neighborhood of F(x) in Y . Then there is an Neutrosophic open set $\bar{G}$of Y.such that $(x) \subset \bar{G} \subset \bar{V}$. Hence, $x \in F^+(\bar{G}) \subset F^+(\bar{V})$.Now by hypothesis $F^+(\bar{G}) \in \alpha O(CTSX)$, and Thus $F^+(\bar{V})$ is an α-neighborhood of x.(d) ⟹(e). Let $x \in X$ and $\bar{V}$ be a neighborhood of F(x) in Y . Put $U = F^+(\bar{V})$.Then U is an α-neighborhood of x and $F(U) \subset \bar{V}$. (e) ⟹ (a). Let $x \in X$ and $\bar{V}$ be an Neutrosophic set in Y.such that $F(x) \subset \bar{V}$., Being an Neutrosophic open set in Y , is a neighborhood of F(x) and according to the hypothesis there is a α-neighborhood U of x such that $F(U) \subset \bar{V}$.Therefore there is $A \in \alpha O(CTSX)$ such that $x \in A \subset U$ and hence $F(A) \subset F(U) \subset \bar{V}$.
Corollary 3.10
For a multifunction $F: X \longrightarrow Y$ and point $x \epsilon X$ the following statements are equivalent :
(a) F is lower α-continuous at x.
(b) For each non-empty open set B of Y with $F(x) \cap B \neq \emptyset$, implies$x \in sCl(Int(F^-(B))$.
(c) For any semi-open set U of X containing x and for any non-empty open set B of Ywith$F(x) \cap B \neq \emptyset$,, there exists a non empty open set $V \subset U$such that $F(x) \cap B \neq \emptyset$, for all $v \in V$

REFERENCES

[1] K. Atanassov, Intuitionistic Fuzzy sets, Fuzzy sets and Systems 20(1986),87-94.

[2] I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala,On Some New Notions And Function In Neutrosophic Topological Spaces,Neutrosophic Sets and systems, vol. 16, 2017,(16-19)

[3] C.l. Chang, Fuzzy topological spaces, j. Math. Anal.appl.24 (1968), 182-190.

[4] Dogan Coker, An Introduction To Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets And Systems,88(1997), 81-89

[5] R .Dhavaseelan and S.Jafari, Generalized Neutrosophic Closed sets, New Trends in Neutrosophic Theory and Applications volume ii- 261-273,(2018)

[6] R .Dhavaseelan and S.Jafari, Neutrosophic Semi Continuous Multifunction , New Trends in Neutrosophic Theory and Applications volume ii - 345-354,(2017)

[7] Florentinsmarandache , Neutrosophic and NeutrosophicLogic,First International Conference On Neutrosophic ,Neutrosophic Logic, Set, Probability, And Statistics University Of New Mexico, Gallup, Nm 87301, usa (2002), smarand@unm.edu

[8] Floretinsmaradache, Neutrosophic Set: - A Generalization Of Intuitionistic Fuzzy Set, Journal Of Defense Resourses Management. 1(2010), 107-114.

[9] P.Iswaryaand.K.Bageerathi, On Neutrosophic Semi-Open Sets In NeutrosophicTopological Spaces, International Journal Of Mathematics Trends And Technology (Ijmtt), Vol37, No.3, (2016), 24-33.

[10] Mani Parimala,FlorentinSmarandache,SaeidJafari and RamalingamUdhayakumar, Article In Information (Switzerland) October 2018.

[11] T Rajesh kannanand .S.Chandrasekar,Neutosophicωα-Closed Sets In Neutrosophic Topological Space, Journal Of Computer And Mathematical sciences,vol.9(10),1400-1408 october 2018.

[12] A.A. Salama and S.A. Alblowi, Generalized Neutrosophic Set And Generalized Neutrosophic Topological spaces, journal computer sci. Engineering, vol.(2) no.(7)(2012).

[13] A.A. Salama and S.A. Alblowi, Neutrosophic Set And Neutrosophic Topological Space, IsorJ.Mathematics,vol.(3),issue(4),(2012).pp-31-35

[14] Santhi R. And Udhayaranin ω -Closed Sets In Neutrosophic Topological Spaces,Neutrosophic Sets And Systems, vol. 12, 2016, 114-117

[15] V.K.Shanthi ,S.Chandrasekar, k.Safinabegam, Neutrosophic Generalized Semi closed Sets In Neutrosophic Topological Spaces, International Journal Of Research In Advent Technology, vol.6, no.7, july 2018, 1739-1743

[16] S S Thakur and Kush bohre , on Lower and Upper α −continous Intuitionistic Fuzzy Multifunction, Annalas Of Fuzzy Mathematics And Informatics volume 9, no 5, may 2015, page 801-815

[17] Wadeif.Al-Omeri and saeidjafari , Neutrosophic Pre Continuous Multifunction And Almost Pre ContinuousMultifunctions,Netrosophic Sets and system, vol 27 2019,53-69

[18] L.a.Zadeh, Fuzzy Sets, Inform And Control 8(1965), 338- 353.