Research Article

Neutrosophic β-Baire Spaces

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Abstract— in this paper the concept of neutrosophic β-Baire spaces are introduced and characterization of neutrosophic β -Baire spaces are studied. Examples are given to illustrate the concepts introduced in this paper.

Keywords: Neutrosophic β-open set, Neutrosophic β -dense set, Neutrosophic β-nowhere dense set, Neutrosophic β-first category, Neutrosophic β -Baire spaces**.**

I. INTRODUCTION

The fuzzy set was introduced by L.A.Zadeh in 1965, where each element had a degree of membership. The concept of fuzzy topological space was introduced by C.L.Chang in 1968. The notion of intuitionistic fuzzy set introduced by K.Atanassov is one of the generalisation of the notion of fuzzy set. The concept of Neutrosophic set was introduced by Smarandache. Neutrosophic operations were introduced by A.A.Salama. The concept of Neutrosophic β-open set was given by I.Arokiarani and R.Dhavaseelan[4].The concept of Baire space in fuzzy setting was introduced by G.Thangaraj and S.Anjalmose[10].The idea of Fuzzy β Baire spaces was given by G.Thangaraj and R.Palani[11]. The idea of Neutrosophic Baire space was introduced by R.Dhavaseelan, S.Jafari, R.Narmada Devi[6].

II. PRELIMINARIES

In this work by a Neutrosophic Topological space we mean that a non-empty set X together with a Neutrosophic Topology N_τ and denote it by (X, N_τ). The interior, closure and the complement of a Neutrosophic set P will be denoted by int(P), cl(P) and 1-P (or)$\overline{\text{P}}$ respectively.

Definition 2.1. [7,8] Let T,I,F be real standard or non standard subsets of $]0^-,1^+[$, with $sup_T = t_{sup}$, $inf_T = t_{inf}$
$sup_I = i_{sup}$, $inf_I = i_{inf}$
$sup_F = F_{sup}$, $inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. T,I,F are neutrosophic components.

Definition 2.2. [7,8] Let X be a nonempty fixed set. A neutrosophic set [briefly Neu.Set] P is an object having the form P= $\{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$ where $\mu_P(x), \sigma_P(x) and\ \gamma_{P(x)}$ represents the degree of membership function, the degree of indeterminacy and the degree of nonmembership respectively of each element $x \in X$ to the set P.

Remark 2.1. [7,8]
(1) A Neu.Set $P = \{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_P, \sigma_P, \gamma_P\rangle$ in $]0^-,1^+[$ on X.
(2) For the sake of simplicity we shall use the symbol $P = \langle \mu_P, \sigma_P, \gamma_P\rangle$ for the Neu.Set $P = \{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$.

Definition 2.3 . [7,8] Let X be a nonempty set and the Neu.Sets P and Q in the form P= $\{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$,
$Q = \{\langle x, \mu_Q(x), \sigma_Q(x), \gamma_Q(x)\rangle : x \in X\}$. Then
(a)$P \subseteq Q$ iff $\mu_P(x) \le \mu_Q(x), \sigma_P(x) \le \sigma_Q(x)$, and $\gamma_P(x) \ge \gamma_Q(x)$ for all $x \in X$;
(b) $P = Q\ iff\ P \subseteq Q\ and\ Q \subseteq P$;
(c) $\bar{P} = \{\langle x, \gamma_P(x), \sigma_P(x), \mu_P(x)\rangle : x \in X\}$
(d)$P \cap Q = \{\langle x, \mu_P(x) \wedge \mu_Q(x), \sigma_P(x) \wedge \sigma_Q(x), \gamma_P(x) \vee \gamma_Q(x)\rangle : x \in X\}$;
(e)$P \cup Q = \{\langle x, \mu_P(x) \vee \mu_Q(x), \sigma_P(x) \vee \sigma_Q(x), \gamma_P(x) \wedge \gamma_Q(x)\rangle : x \in X\}$;
(f) $[\,]P = \{\langle x, \mu_P(x), \sigma_P(x), 1 - \mu_P(x)\rangle : x \in X\}$;

(g) $\langle \quad \rangle P = \{\langle x, 1-\gamma_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$.
Definition 2.4.[7,8] Let $\{P_i : i \in J\}$ be an arbitrary family of neutrosophic sets X. Then
(a) $\bigcap P_i = \left\{\langle x, \wedge \mu_{P_i}(x), \wedge \sigma_{P_i}(x), \vee \gamma_{P_i}(x)\rangle : x \in X\right\}$;
$(b) \bigcup P_i = \left\{\langle x, \vee \mu_{P_i}(x), \vee \sigma_{P_i}(x), \wedge \gamma_{P_i}(x)\rangle : x \in X\right\}$.
Since our main purpose is to construct the tools for developing Neutrosophic topological spaces (Neu.T.S), we introduce the Neu. sets 0_N and 1_N in X as follows:
Definition.2.5.[7,8] $0_N = \{\langle x, 0,0,1\rangle : x \in X\}\ and\ 1_N = \{\langle x, 1,1,0\rangle : x \in X\}$.
Definition 2.6.[13] A Neu. topology(N_τ) on a nonempty set X is a family τ of Neu.Sets in X satisfying the following axioms:
(i) $0_N, 1_N \in \tau$
(ii)$G_1 \cap G_2 \in \tau$.
(iii)$\cup G_i \in \tau\ for\ arbitrary\ family\ \{G_i/i \in \wedge\} \subseteq \tau$.
In this case the ordered pair (X,N_τ) or simply X is called a neutrosophic topological space (briefly Neu.T.S) and each Neu. Set in $\boldsymbol{\tau}$ is called a neutrosophic open set (briefly Neu.O.S) . The complement $\overline{\mathrm{P}}$ of a Neu.O.S P in X is called a neutrosophic closed set (briefly Neu.C.S) in X.
Definition 2.7.[13] Let P be a Neu. Set in a Neu.T.S (X, N_τ) . Then Neu.int(P) = ∪{G/ G is a Neu.O.Set in X and G $\subseteq P$ } is called the neutrosophic interior of P.
Neu.cl(P) = ∩{G/G is a Neu.C.Set in X and G $\supseteq P$} is called the neutrosophic closure of P.
It can also be shown that Neu.int(P) is Neu.O.Set and Neu.cl(P) is Neu.C.Set in X.
a) P is Neu.O.Set if and only if P = Neu.int(P).
b) P is Neu.C.Set if and only if P = Neu.cl(P)
Proposition 2.1[13] For any Neu.Set P in (X,N_τ) we have
a) Neu.int(C(P))=C(Neu.cl(P)).
b) Neu.cl(C(P))=C(Neu.int(P)).
Definition 2.8. [6] Let X be a nonempty set. If r, t, s be a real standard or non standard subsets of $]0^-, 1^+[$ then the Neu. set $x_{r,t,s}$ is called a Neu. point (in short Neu.P) in X given by

$$x_{r,t,s}(x_p) = \begin{Bmatrix} (r,t,s) & if\ x = x_p \\ (0,0,1), & if\ x \neq x_p \end{Bmatrix}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$. where r denotes the degree of membership value, t denotes the degree of membership value, t denotes the degree of indeterminacy and s denotes the degree of non-membership value.
Proposition 2.2[16]. Let (X,N_τ) be a Neu.T.S and P, Q be the two Neu.Sets in X. Then the following properties hold:
a) Neu.int(P)⊆P.
b) P⊆Neu.cl(P).
c) P⊆Q⇒ Neu.int(P) ⊆Neu.int(Q).
d) P⊆Q⇒ Neu.cl(P) ⊆Neu.cl(Q).
e) Neu.int(Neu.int(P))=Neu.int(P).
f) Neu.cl(Neu.cl(P))=Neu.cl(P).
g) Neu.int(P∪Q) ⊇ Neu.int(P) ∪ Neu.int(Q).
h)Neu.int(P∩Q) = Neu.int(P) ∩ Neu.int(Q).
i) Neu.cl(P∪Q)= Neu.cl(P) ∪ Neu.cl(Q).
j) Neu.cl(P∩Q)⊆ Neu.cl(P) ∩ Neu.cl(Q).
k) Neu.int(0_N) =0_N.
l) Neu.int(1_N) =1_N.
m) Neu.cl(0_N) =0_N.
n) Neu.cl(1_N) =1_N.
o) P⊆Q⇒ C(Q) ⊆C(P).
Definition 2.9[7]. A Neu.Set P in Neu.T.S (X,N_τ) is called neutrosophic dense(Neu.D) if there exists no neutrosophic closed set Q in (X,N_τ) such that P⊂Q⊂ 1_N.
Definition 2.10[7]. A Neu. set P in Neu.T.S (X,N_τ) is called neutrosophic nowhere dense set (Neu. N.D.Set) if there exists no Neu.O.Set Q in (X,N_τ) such that Q ⊂Neu.cl(P) that is Neu.int(Neu.cl(P))= 0_N.
Proposition 2.3. If P is a Neu.N.D.Set in (X,N_τ),then $\bar{P}$ is a Neu.D.set in (X,N_τ).
Definition 2.11[4] A Neu.Set P in Neu.T.S X is said to be a neutrosophic β-open set (Neu.β OS) if $P \subseteq Ncl(Nint(Ncl(P)))$ and neutrosophic β-closed set(Neu. βCS) if. $Nint\left(Ncl\left(Nint(P)\right)\right) \subseteq P$.
Definition 2.12. Let P be a Neu.Set in a Neu.T.S in (X,N_τ). Then
Neu.βint(P) = ∪{G/ G is a Neu.βO.S in X and G $\subseteq P$ } is called the Neutrosophic β interior of P.
Neu.βcl(P) = ∩{G/G is a Neu.βC.S in X and G $\supseteq P$} is called the Neutrosophic βclosure of P.
Theorem 2.13. In a Neu.T.S (X,N_τ) the following are valid.

a) P is Neu. β −open if and only if Neu.βint(P)=P.
b) P is Neu. β −closed if and only if Neu.βcl(P)=P.
Result 2.14. Let P be a Neu.Set in a Neu.T.S (X,N_τ). Then
Neu.βcl(P) =P∪ Nint(Ncl(Nint(P))).
Neu.βint(P) = P∩ Ncl(Nint(Ncl(P))).

III. NEUTROSOPHIC β-NOWHERE DENSE SETS

Definition 3.1. A Neu.Set P in a Neu.T.S (X,N_τ) is called neutrosophic β-dense(Neu.$\beta.D$) if there exists no Neu. β.C.Set Q in (X, N_τ) such that P ⊂ Q ⊂ 1_N That is Neu.βcl(P)= 1_N.
Definition 3.2. Let (X,N_τ) be a Neu.T.S. A Neu.Set P in (X,N_τ) is called a neutrosophic β-nowhere dense set(Neu.$\beta.N.D$) if there exists no non-zero neutrosophic β-open set Q in (X,N_τ) such that Q⊂ Neu. βcl(P). That is $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = 0_N$.
Example 3.1 Let X={p,q}.Define the Neu. sets P,Q as follows:
P=$\langle x, \left(\frac{p}{0.6},\frac{q}{0.6}\right), \left(\frac{p}{0.6},\frac{q}{0.6}\right), \left(\frac{p}{0.3},\frac{q}{0.4}\right)\rangle$
Q=$\langle x, \left(\frac{p}{0.6},\frac{q}{0.5}\right), \left(\frac{p}{0.6},\frac{q}{0.5}\right), \left(\frac{p}{0.4},\frac{q}{0.5}\right)\rangle$
Then $N_\tau = \{0_N, 1_N, P, Q\}$ is a Neu. topology on X. Thus (X, N_τ) is a Neu. Topological space (Neu.T.S). $\overline{P}, \overline{Q}$ are Neu. β-nowhere dense sets.
Proposition 3.1: If P is a Neu. β. N. D set in (X,N_τ), then $\bar{P}$ is a Neu. β.D set in (X,N_τ).
Proof: Let P be a Neu. β.N.D set in (X, N_τ). Then $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = 0_N$.
Now 1- $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = 1 - 0_N = 1_N$ and hence $\text{Neu.}\,\beta\text{cl}(\text{Neu.}\,\beta\text{int}(1 - P)) = 1_N$.
But $\text{Neu.}\,\beta\text{cl}(\text{Neu.}\,\beta\text{int}(1 - P)) \leq \text{Neu.}\,\beta\text{cl}(1 - P)$ implies that $1_N \leq \text{Neu.}\,\beta\text{cl}(1 - P)$.
That is $\text{Neu.}\,\beta\text{cl}(1 - P) = 1_N$ in (X, N_τ). Therefore,(1-P) is a Neu.β.D set in (X, N_τ).
Proposition 3.2: If P is a Neu. β.C.Set in (X, N_τ), then P is a Neu.β.N.D set in (X, N_τ) if and only if $\text{Neu.}\,\beta\text{int}(P) = 0_N$.
Proof: Let P be a Neu.β.C.Set in (X, N_τ), then $\text{Neu.}\,\beta\text{cl}(P) = P$. If $\text{Neu.}\,\beta\text{int}(P) = 0_N$, Then $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = \text{Neu.}\,\beta\text{int}(P) = 0_N$. So P is a Neu. β.N.D set in (X, N_τ). Conversely, let P be a Neu.β.N.D set in(X, N_τ), then $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = 0_N$ which implies that $\text{Neu.}\,\beta\text{int}(P) = \text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = 0_N$,since P is a Neu.$\beta$CS, $\text{Neu.}\,\beta\text{cl}(P) = P$.
Proposition 3.3: If P is a Neu.β.N.D set in a Neu.T.S(X, N_τ), then $\text{Neu.}\,\beta\text{int}(P) = 0_N$.
Proof: Let P be a Neu.β.N.D set in (X, N_τ). Then $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = 0_N$ in (X, N_τ). Now $\text{Neu.}\,\beta\text{int}(P) \leq \text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P))$ implies that $\text{Neu.}\,\beta\text{int}(P) \leq 0_N$ in (X, N_τ). (i.e) $\text{Neu.}\,\beta\text{int}(P) = 0_N$ in (X, N_τ).
Proposition 3.4: If P is a Neu.β.N.D set in a Neu.T.S (X, N_τ), then Neu. βcl(P) is a Neu.$\beta.N.D$ set in (X, N_τ).
Proof: Let P be a Neu.β.N.D set in (X, N_τ). Then $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = 0_N$ in (X, N_τ). Now, $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(\text{Neu.}\,\beta\text{cl}(P))) = \text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P))$ and hence $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(\text{Neu.}\,\beta\text{cl}(P))) = 0_N$ in (X, N_τ).Therefore Neu. βcl(P) is a Neu.β.N.D set in (X, N_τ).
Proposition 3.5: If P is a Neu.β.N.D Set in a Neu.T.S (X, N_τ),then 1-Neu. βcl(P) is a Neu. β.D.Set in (X, N_τ).
Proof: Let P be a Neu. β.N.D.Set in (X, N_τ). Then by proposition 3.4, Neu. βcl(P) is a Neu.$\beta.N.D$ set in (X, N_τ). By proposition 2.1 1- Neu. βcl(P) is a Neu.$\beta.D$ set in (X, N_τ).
Proposition 3.6: If P is a Neu.β.N.D Set in a Neu.T.S (X, N_τ), then $\text{Neu.}\,\beta\text{int}(1 - P)$ is a Neu. β.D.Set in (X, N_τ).
Proof: Let P be a Neu. β.N.D.Set in (X, N_τ). Then by proposition 3.5, 1-Neu. βcl(P) is a Neu. β.D.Set in (X, N_τ). Now 1-Neu. βcl(P)=$\text{Neu.}\,\beta\text{int}(1 - P)$ in (X, N_τ) and hence $\text{Neu.}\,\beta\text{int}(1 - P)$ is a Neu. β. D. Set (X, N_τ).
Proposition 3.7: If P is a Neu.β.N.D and Neu.C.Set in a Neu.T.S (X, N_τ), then P is a Neu.N.D set in (X, N_τ).
Proof: Let P be a Neu. β.N.D and Neu.CS in (X, N_τ). Then, $\text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P)) = 0_N$ and $\text{Neu.}\,\beta\text{cl}(P) = P$ in (X, N_τ). But $\text{Neu.}\,\beta\text{int}(P) \leq \text{Neu.}\,\beta\text{int}(\text{Neu.}\,\beta\text{cl}(P))$, implies that $\text{Neu.}\,\beta\text{int}(P) \leq 0_N$ (i.e) $\text{Neu.}\,\beta\text{int}(P) = 0_N$ in (X,N_τ).We have $\text{Neu.}\,\text{int}(P) \leq \text{Neu.}\,\beta\text{int}(P)$, and hence $\text{Neu.}\,\text{int}(P) = 0_N$.Then $\text{Neu.}\,\text{int}(\text{cl}(P)) = \text{Neu.}\,\text{int}(P) = 0_N$ in (X, N_τ).Therefore, P is a Neu.N.D set in (X, N_τ).

IV. NEUTROSOPHIC β − BAIRE SPACE

Definition 4.1: Let (X, N_τ) be a neutrosophic topological space. A Neu. set P in (X, N_τ)is called neutrosophic β − first category(Neu.$\beta.F.C$) if $P = \bigcup_{i=1}^{\infty} P_i$, where P_i's are Neu. β.N.D set in (X, N_τ). Anyother Neu.set in (X, N_τ) is said to be of neutrosophic β − second category(Neu.$\beta.S.C.$).
Definition 4.2: Let P be a Neu.$\beta.F.C$ set in a Neu.T.S (X, N_τ).Then 1-P is called a neutrosophic β − residual set in (X, N_τ).
Example 4.1: Let X={p,q}.Define the Neu. sets P,Q as follows:

P=⟨x, $\left(\frac{p}{0.6},\frac{q}{0.5}\right),\left(\frac{p}{0.6},\frac{q}{0.5}\right),\left(\frac{p}{0.3},\frac{q}{0.4}\right)$⟩
Q=⟨x, $\left(\frac{p}{0.6},\frac{q}{0.6}\right),\left(\frac{p}{0.6},\frac{q}{0.6}\right),\left(\frac{p}{0.3},\frac{q}{0.5}\right)$⟩
Then $N_\tau = \{0_N, 1_N, P, Q, P\cup Q, P\cap Q\}$ is a Neutrosophic topology on X. Thus (X, N_τ) is a neutrosophic topological space (Neu.T.S). $\overline{P}, \overline{Q}, \overline{P\cup Q}, \overline{P\cap Q}$ are neutrosophic β-nowhere dense sets and $[\overline{P}\cup\overline{Q}\cup\overline{P\cup Q}\cup\overline{P\cap Q}] = \overline{P\cap Q}$ is a Neu.β.F.C Set.
Definition 4.3: Let(X, N_τ) be a Neu.T.S. Then (X, N_τ) is called a neutrosophic $\beta-$ Baire space if $\mathrm{N\beta int}(\bigcup_{i=1}^{\infty} P_i) = 0_N$, where P_i's are neutrosophic β-nowhere dense set in (X, N_τ).
In Example 4.1, The sets $\overline{P}, \overline{Q}, \overline{P\cup Q}, \overline{P\cap Q}$ are neutrosophic β-nowhere dense sets and Neu.βint$[\overline{P}\cup\overline{Q}\cup\overline{P\cup Q}\cup\overline{P\cap Q}] = \mathrm{N\beta int}(\overline{P\cap Q})$ =0_N is a Neu.β.B.Space.
Proposition 4.1: If $\mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty} P_i) = 0_N$, where $\mathrm{Neu.\beta int}(P_i) = 0_N$ where P_i's are Neu. β.C.set in a Neu.T.S (X, N_τ). Then (X, N_τ) is a Neu.β. B. space.
Proof: Let P_i's be the Neu.β.C.Sets in a Neu.T.S(X, N_τ).Since $\mathrm{Neu.\beta int}(P_i) = 0_N$ by proposition 3.3, P_i's are Neu.β.N.D sets in (X, N_τ), implies that (X, N_τ) is a Neu. β.B space.
Proposition 4.2: If $\mathrm{Neu.\beta cl}(\bigcap_{i=1}^{\infty} P_i) = 1_N$, where P_i's are Neu. β.D and Neu.β.O.Sets in a Neu.T.S (X, N_τ). Then (X, N_τ) is a Neu. β.B space.
Proof: Now $\mathrm{Neu.\beta cl}(\bigcap_{i=1}^{\infty} P_i) = 1_N$, implies that $1-\mathrm{Neu.\beta cl}(\bigcap_{i=1}^{\infty} P_i) = 1-1 = 0_N$. Then $\mathrm{Neu.\beta int}(1-\bigcap_{i=1}^{\infty} P_i) = 0_N$ in (X, N_τ). This implies that $\mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty}(1-P_i)) = 0_N$.Since P_i's are Neu.β.D in (X, N_τ) , $\mathrm{Neu.\beta cl}(P_i) = 1_N$ and $\mathrm{Neu.\beta int}(1-P_i) = 1-\mathrm{Neu.\beta cl}(P_i) = 1-1 = 0_N$ and $(1-P_i)'$s are Neu.β.C sets in (X, N_τ).Then by proposition 4.1 , the Neu.T.S (X, N_τ) is a Neu. $\beta.B$ space.
Proposition 4.3: Let (X, N_τ) be a Neu.T.S.Then the following results are equivalent.
(1) (X, N_τ) is a Neu. β.B.space.
(2) $\mathrm{Neu.\beta int}(P) = 0_N$, for every Neu. β.F.C set P in (X, N_τ).
(3) $\mathrm{Neu.\beta cl}(Q) = 1_N$, for every Neu.$\beta-$ residual set Q in (X, N_τ).
Proof: (1)⇒(2),Let P be a Neu. β.F.C set in (X, N_τ). Then, $P = \bigcup_{i=1}^{\infty} P_i$ where P_i's are Neu. β.N.D set in (X, N_τ). Now $\mathrm{N}eu.\mathrm{\beta int}(P) = \mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty} P_i) = 0_N$ (since (X, N_τ) is a Neu.β.B. space).Therefore, $\mathrm{Neu.\beta int}(P_i) = 0_N$ in (X, N_τ).
(2) ⇒(3), Let Q be a Neu.$\beta-$ residual set in (X,N_τ). Then 1-Q is a Neu. β.F.C set in (X, N_τ). By hypothesis, $\mathrm{Neu.\beta int}(1-Q) = 0_N$ in (X, N_τ). This implies that $1-\mathrm{Neu.\beta cl}(Q) = 0_N$ and hence $\mathrm{Neu.\beta cl}(Q) = 1_N$ in (X, N_τ).
(3)⇒(1),Let P be a Neu.β.F.C set in (X, N_τ). Then, $P = \bigcup_{i=1}^{\infty} P_i$ where P_i's are Neu.β.N.D set in (X, N_τ). Since P is a Neu.β.F.C set in (X, N_τ), 1-P is a Neu.$\beta-$ residual set in (X, N_τ). By hypothesis, $\mathrm{Neu.\beta cl}(1-P) = 1_N$. Then, $1-\mathrm{Neu.\beta int}(P) = 1_N$ in (X, N_τ). This implies that $\mathrm{Neu.\beta int}(P) = 0_N$ in (X, N_τ). Hence $\mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty} P_i) = 0_N$, where P_i's are Neu.β.N.D set in (X, N_τ). This implies that (X, N_τ) is a Neu.β.B. space.
Proposition 4.4: If a Neu.T.S(X, N_τ) is a Neu.β.B space and if every Neu.β.N.D set in (X,N_τ) is a Neu.C.Set in (X, N_τ),Then the Neu.T.S(X, N_τ) is a Neu.B.space.
Proof: Let (X, N_τ) be a Neu.β.B. space such that Neu.β.N.D set in (X, N_τ) is a Neu.C.Set in (X, N_τ). Since, (X, N_τ) is a Neu.$\beta.B$ space then $\mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty} P_i) = 0_N$, where P_i's are Neu.β.N.D set in (X, N_τ). Since the Neu.β.N.D set P_i's in (X, N_τ) are Neu.C.Sets in (X, N_τ) by proposition 3.6, P_i's are Neu.N.D sets in(X, N_τ) in (X, N_τ). Now $\mathrm{Neu.int}(\bigcup_{i=1}^{\infty} P_i) \le \mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty} P_i)$, and $\mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty} P_i)= 0_N$, implies that $\mathrm{Neu.int}(\bigcup_{i=1}^{\infty} P_i) = 0_N$ in (X, N_τ).Thus $\mathrm{Neu.int}(\bigcup_{i=1}^{\infty} P_i) = 0_N$ where P_i's are Neu.N.D set in (X, N_τ), implies that (X, N_τ) is Neu.B.space.
Proposition 4.5: If a Neu.T.S (X, N_τ) is a Neu.B.space and every Neu.N.D.Set P in (X, N_τ) is a Neu.C.Set, then (X, N_τ) is not a Neu.β. B.space.
Proof: Let (X, N_τ) be a Neu.B.space such that every Neu.N.D set in (X, N_τ) is a Neu.C.Set in (X, N_τ). Since, (X, N_τ) is a Neu.B.space, $\mathrm{Neu.int}(\bigcup_{i=1}^{\infty} P_i) = 0_N$,where P_i's are Neu.N.D set in (X, N_τ).Since, the Neu.N.D set (P_i)'s in (X, N_τ) are Neu.C.Set in (X, N_τ) by proposition 3.6 P_i's are Neu.β.N.D set in (X, N_τ). Now $\mathrm{Neu.int}(\bigcup_{i=1}^{\infty} P_i) \le \mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty} P_i)$, and $\mathrm{Neu.int}(\bigcup_{i=1}^{\infty} P_i)= 0_N$, implies that $\mathrm{Neu.\beta int}(\bigcup_{i=1}^{\infty} P_i) \neq 0_N$ implies that (X, N_τ) is not a Neu. β.B. space.

References

1. S.A.Alblowi, A.a.Salama and Mohamed Eisa, New Concepts of Neutrosophic Sets, Interrnational Journal of Mathematics and Computer Applications research(IJMCAR),Vol 3, Issue 3, Oct(2013)95-102.
2. K.Atanassov, Intuitionistic fuzzy sets in V.Sgurev,ed., Vii ITKRS Session,Sofia(June 1983 central Sci. and Techn.Library,Bulg.Academy of Sciences(1984)).
3. K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems20(1986)87-96.

4. Arokiarani, R.Dhavaseelan, S.Jafari, M.Parimala "On Some New Notions and Functions in Neutrosophic Topological Spaces" Neutrosophic Sets and Systems,Vol.16,2017.
5. C.L.Chang, Fuzzy Topological Spaces,J.Math. Anal.Appl.24(1968)182-1 90.
6. R.Dhavaseelan, S.Jafari, R.Nirmala Devi,4Md.Hanif Page, Neutrosophic Baire Spaces,Neutrosophic Sets and Systems,Vol.16,2017.
7. R.Dhavaseelan, R.Narmada devi, S.Jafari,Characterization of Neutrosophic Nowhere Dense Sets,International Journal of Mathematical Archive,Vol.9.No:3,March-2018.
8. F.Smarandache , Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, eutrosophic Logic ,Set, Probability, and Statistics University of New Mexico,Gallup,NM 87301,USA(2002).
9. F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic set, Neutrosophic Probability, American Research Press.
10. G.Thangaraj and S.Anjalmose, On Fuzzy Baire Space,J. Fuzzy Math.Vol.21(3),(2013)667-676.
11. G.Thangaraj and R.Palani , On Fuzzy β− Baire spaces, The Journal of Fuzzy Mathematics Vol.26, No.1,2018 Los Angeles.
12. S.S.Thakur and S.Singh, On Fuzzy semi-preopen sets and fuzzy semi-pre continuity,Fuzzy Sets and Systems,98(1998),383-391.
13. Tugba Han Simsekler,Naime Tozlu and Saziye Yuksel, β- Baire spaces and β− Baire property, Inter.J.Contem Math. Sci.Vol.11, No. 5,(2016),211-216.
14. A.A.Salama and S.A.Albowi, Neutrosophic Set and Neutrosophic Topological Spaces, ISORJ. Marhematics.Vol.(3) ,Issue(3),(2012) pp-31-35.
15. A.A.Salama and S.A.Albowi," Generalised Neutrosophic Set And Generalised Neutrosophic Topological Spaces," Journal Computer Sci. Engineering, Vol.(2) No.(7)(2012) pp129-132.
16. L.A.zadeh, Fuzzy Sets, Inform and Control 8(1965) 338-353.