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Neutrosophic Cubic Einstein Hybrid Geometric Aggregation Operators with Application in Prioritization Using Multiple Attribute Decision-Making Method

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Abstract: Viable collection is one of the imperative instruments of decision-making hypothesis. Collection operators are not simply the operators that normalize the value; theyrepresent progressively broad values that can underline the entire information. Geometric weighted operators weight the values only, andthe ordered weighted geometric operators weight the ordering position only.Both of these operators tend to the value that relates to the biggest weight segment. Hybrid collection operators beat these impediments of weighted total and request total operators. Hybrid collection operators weight the incentive as well as the requesting position. Neutrosophic cubic sets (NCs) are a classification of interim neutrosophic set and neutrosophic set. This distinguishing of neutrosophic cubic set empowers the decision-maker to manage ambiguous and conflicting data even more productively. In this paper, we characterized neutrosophic cubic hybrid geometric accumulation operator (NCHG) and neutrosophic cubic Einstein hybrid geometric collection operator (NCEHG). At that point, we outfitted these operators upon an everyday life issue which empoweredus to organize the key objective to develop the industry.

Keywords:neutrosophic cubic set, neutrosophic cubic hybrid geometric operator; neutrosophic cubic Einstein hybrid geometric operator; multiattributedecision-making (MADM)

1. Introduction

Life is loaded with indeterminacy and vagueness, which makes it hard to get adequate and exact information. This uncertain and obscure information can be tended to by fuzzy set [1], interim-valued fuzzy set(IVFS) [2,3], intuitionistic fuzzy set(IFS) [4], interim-valued intuitionistic fuzzy set(IVIFS) [5], cubic sets [6], neutrosophic set(Ns) [7], single-valued neutrosophic set(SVNs) [8], interim neutrosophic set(INs) [9], and neutrosophic cubic set[10]. Smarandache first investigated the hypothesis of neutrosophic sets [7].

Not long after thisinvestigation, it became a vital tool to manage obscure and conflicting information. The neutrosophic set comprises of three segments: truth enrollment, indeterminant participation, and deception enrollment. These segments can, likewise,be alluded to as participation, aversion, andnon-membership, and these segments range from $]0^{-},1^{+}[$. For science and designing issues, Wang et al. [8] proposed the idea of a single-valued neutrosophic set, which is a class of neutrosophic set, where the parts of single-valued neutrosophic set are in [0,1]. Wang et al. stretched it outto the interim neutrosophic set [9]. Jun et al. [10] consolidated both of these structures to frame the neutrosophic cubic set, which is the speculation of single-valued neutrosophic set and interim neutrosophic set. These structures drew scientistsinto apply it to various fields of sciences, building day-by-day life issues.

Decision-making is a basic instrument of everyday life issues. Analysts connected distinctive collection operators to neutrosophic sets and its augmentations. Zhan et. al. [11] took a shot at multicriteria decision-making on neutrosophic cubic sets. Banerjee et al. [12] utilized GRA(Grey Rational Analysis) for multicriteria decision-making on neutrosophic cubic sets. Lu and Ye [13] characterized cosine measure inneutrosophic cubic sets. Pramanik et al. [14] utilized a likeness measure to neutrosophic cubic sets. Shi and Ye [15] characterized Dombi total operators on neutrosophic cubic sets. Baolin et al. [16] connected Einstein accumulations to neutrosophic sets. Majid et al. [17] proposed neutrosophic cubic geometric and Einstein geometric collection operators. Different applied aspects of different types of fuzzy sets can be seen in [18–27].

A compelling accumulation is one of the imperative instruments of decision-making. Collection operators are not simply the operators that normalize the value, theyrepresent progressively broad values that can underline the entire data. The geometric weighted operator weights the values just where the requested weighted geometric collection operators weight the requesting position of values. In any case, the issue emerges when the load segments of weight vectors are so that one segment is a lot bigger than the other in parts of the weight vector. Motivated by such a circumstance, the thought of neutrosophic cubic crossbreed geometric and neutrosophic cubic Einstein hybrid geometric total operators are proposed. That is the reason we present the idea of neutrosophic cubic hybrid geometric and neutrosophic cubic Einstein hybrid geometric (NCEHG) collection operators. More often than not, the decision-making strategies are produced to pick one fitting option among the given. Be that as it may, frequently, in certain circumstances,we instead organize the option to pick a suitable one. Roused by such a circumstance, a technique is being created toprioritize the options. A numerical model is outfitted upon these operators to organize the vital objective to develop the industry.

2. Preliminaries

This section consists of some predefined definitions and results. We recommend the reader to see [1–3,6–10,16].

Definition 1. [1] *Mapping* $\psi : U \to [0,1]$ *is called fuzzy set,* $\psi(u)$ *is called membership function. Simply denoted by* ψ *.*

Definition 2. [2,3] *Mapping* $\tilde{\Psi} : U \to D[0,1]$, $D[0,1]$ *has interval value of* $[0,1]$, *and is called interval-valued fuzzy set(IVF). For all* $u \in U$ $\tilde{\Psi}(u) = \left\{ \left[\psi^{L}(u), \psi^{U}(u) \right] \mid \psi^{L}(u), \psi^{U}(u) \in [0,1] and\, \psi^{L}(u) \le \psi^{U}(u) \right\}$ *is membership degree of* u *in* $\tilde{\Psi}$ *. Simply denoted by* $\tilde{\Psi} = \left[\Psi^{L}, \Psi^{U} \right]$.

Definition 3. [6] *A structure* $C = \left\{ \left(u, \tilde{\Psi}(u), \Psi(u) \right) \mid u \in U \right\}$ *is cubic set in* U *, in which* $\tilde{\Psi}(u)$ *is IVF in* U *,i.e,.* $\tilde{\Psi} = \left[\Psi^{L}, \Psi^{U} \right]$, *and* Ψ *is fuzzy set in* U *. Simply denoted by* $C = \left(\tilde{\Psi}, \Psi \right)$. C^{U} *denotes collection of cubic sets in* U *.*

Definition 4. [7] A structure $N=\{(T_N(u),I_N(u),F_N(u))\,|\,u\in U\}$ is neutrosophic set(Ns), where $\{T_N(u),I_N(u),F_N(u)\in\,]0^-,1^+[\}$ *and* $T_N(u),I_N(u),F_N(u)$ are truth,indeterminacy, andfalsity function.

Definition 5. [8] *A structure* $N=\{(T_N(u),I_N(u),F_N(u))\,|\,u\in U\}$ *is single value neutrosophic set(SVNs), where* $\{T_N(u),I_N(u),F_N(u)\in[0,1]\}$ *are called truth, indeterminacy, and falsity functions respectively. Simply denoted by* $N=(T_N,I_N,F_N)$.

Definition 6. [9] *An interval neutrosophic set (INs) in* U *is a structure* $N=\{(\tilde{T}_N(u),\tilde{I}_N(u),\tilde{F}_N(u))\,|\,u\in U\}$ *where* $\{\tilde{T}_N(u),\tilde{I}_N(u),\tilde{F}_N(u)\in D[0,1]\}$ *respectively called truth, indeterminacy, and falsity function in* U. *Simply denoted by* $N=(\tilde{T}_N,\tilde{I}_N,\tilde{F}_N)$. *For convenience, we denote* $N=(\tilde{T}_N,\tilde{I}_N,\tilde{F}_N)$ *by* $N=(\tilde{T}_N=[T_N^L,T_N^U],\tilde{I}_N=[I_N^L,I_N^U],\tilde{F}_N=[F_N^L,F_N^U])$.

Definition 7. [10] *A structure* N= $\{(u,\tilde{T}_N(u),\tilde{I}_N(u),\tilde{F}_N(u),T_N(u),I_N(u),F_N(u))\,|\,u\in U\}$ *is neutrosophic cubic setin* U, *in which* $(\tilde{T}_N=[T_N^L,T_N^U],\tilde{I}_N=[I_N^L,I_N^U],\tilde{F}_N=[F_N^L,F_N^U])$ *is an interval neutrosophic set and* (T_N,I_N,F_N) *is neutrosophic set in* U. *Simply denoted by* $N=(\tilde{T}_N,\tilde{I}_N,\tilde{F}_N,T_N,I_N,F_N)$, $[0,0]\le\tilde{T}_N+\tilde{I}_N+\tilde{F}_N\le[3,3]$, *and* $0\le T_N+I_N+F_N\le 3$. N^U *denotes the collection of neutrosophic cubic sets in* U. *Simply denoted by* $N=(\tilde{T}_N,\tilde{I}_N,\tilde{F}_N,T_N,I_N,F_N)$.

Definition 8. [16] *The t-operators are basically Union and Intersection operators in the theory of fuzzy sets which are denoted by t-conorm* (Γ^*) *and t-norm* (Γ), *respectively. The role of t-operators is very important in fuzzy theory and its applications.*

Definition 9. [16] $\Gamma^*:[0,1]\times[0,1]\to[0,1]$ *is called t-conorm if it satisfies the following axioms.*

Axiom 1 $\Gamma^*(1,u)=1$ *and* $\Gamma^*(0,u)=0$
Axiom 2 $\Gamma^*(u,v)=\Gamma^*(v,u)$ *for all a and b.*
Axiom 3 $\Gamma^*(u,\Gamma^*(v,w))=\Gamma^*(\Gamma^*(u,v),w)$ *for all a,b, and c.*
Axiom 4 *If* $u\le u'$ *and* $v\le v'$, *then* $\Gamma^*(u,v)\le\Gamma^*(u',v')$

Definition 10. [16] $\Gamma:[0,1]\times[0,1]\to[0,1]$ *is called t-norm if it satisfies the following axioms.*

Axiom 1 $\Gamma(1,u)=u$ *and* $\Gamma(0,u)=0$
Axiom 2 $\Gamma(u,v)=\Gamma(v,u)$ *for all a and b.*
Axiom 3 $\Gamma(u,\Gamma(v,w))=\Gamma(\Gamma(u,v),w)$ *for all a,b, and c.*
Axiom 4 *If* $u\le u'$ *and* $v\le v'$, *then* $\Gamma(u,v)\le\Gamma(u',v')$

The t-conorms and t-norms families have a vast range, which correspond to unions and intersections, among these, Einstein sum and Einstein product are good choices since they give smooth approximations, like algebraic sum and algebraic product, respectively. Einstein sums $\oplus_E$ and Einstein products $\otimes_E$ are respectively the examples of t-conorm and t-norm:

$$\Gamma^*_E(u,v)=\frac{u+v}{1+uv},$$

$$\Gamma_E(u,v)=\frac{uv}{1+(1-u)(1-v)}.$$

Definition 11. [17] *The sum of two neutrosophic cubic sets,* $A=\left(\tilde{T}_A,\tilde{I}_A,\tilde{F}_A,T_A,I_A,F_A\right)$, where $\tilde{T}_A=\left[T_A^L,T_A^U\right],\tilde{I}_A=\left[I_A^L,I_A^U\right],\tilde{F}_A=\left[F_A^L,F_A^U\right]$, and $B=\left(\tilde{T}_B,\tilde{I}_B,\tilde{F}_B,T_B,I_B,F_B\right)$, where $\tilde{T}_B=\left[T_B^L,T_B^U\right],\tilde{I}_B=\left[I_B^L,I_B^U\right],\tilde{F}_B=\left[F_B^L,F_B^U\right]$ is defined as

$$A\oplus B=\begin{pmatrix}\left[T_A^L+T_B^L-T_A^LT_B^L,T_A^U+T_B^U-T_A^UT_B^U\right],\\ \left[I_A^L+I_B^L-I_A^LI_B^L,I_A^U+I_B^U-I_A^UI_B^U\right],\\ \left[F_A^LF_B^L,F_A^UF_B^U\right],\\ T_AT_B,I_AI_B,F_A+F_B-F_AF_B\end{pmatrix}.$$

Definition 12. [17] *The product between two neutrosophic cubic sets,* $A=\left(\tilde{T}_A,\tilde{I}_A,\tilde{F}_A,T_A,I_A,F_A\right)$, where $\tilde{T}_A=\left[T_A^L,T_A^U\right],\tilde{I}_A=\left[I_A^L,I_A^U\right],\tilde{F}_A=\left[F_A^L,F_A^U\right]$, and $B=\left(\tilde{T}_B,\tilde{I}_B,\tilde{F}_B,T_B,I_B,F_B\right)$, where $\tilde{T}_B=\left[T_B^L,T_B^U\right],\tilde{I}_B=\left[I_B^L,I_B^U\right],\tilde{F}_B=\left[F_B^L,F_B^U\right]$ is defined as

$$A\otimes B=\begin{pmatrix}\left[T_A^LT_B^L,T_A^UT_B^U\right],\\ \left[I_A^LI_B^L,I_A^UI_B^U\right],\\ \left[F_A^L+F_B^L-F_A^LF_B^L,F_A^U+F_B^U-F_A^UF_B^U\right],\\ T_A+T_B-T_AT_B,I_A+I_B-I_AI_B,F_AF_B\end{pmatrix}.$$

Definition 13. [17] *The scalar multiplication on a neutrosophic cubic set* $A=\left(\tilde{T}_A,\tilde{I}_A,\tilde{F}_A,T_A,I_A,F_A\right)$, where $\tilde{T}_A=\left[T_A^L,T_A^U\right],\tilde{I}_A=\left[I_A^L,I_A^U\right],\tilde{F}_A=\left[F_A^L,F_A^U\right]$, and a scalar k is defined.

$$kA=\begin{pmatrix}\left[1-(1-T_A^L)^k,1-(1-T_A^U)^k\right],\\ \left[1-(1-I_A^L)^k,1-(1-I_A^U)^k\right],\\ \left[\left(F_A^L\right)^k,\left(F_A^U\right)^k\right],\\ \left(T_A\right)^k,\left(I_A\right)^k,1-\left(1-F_A\right)^k\end{pmatrix}$$

The exponential multiplication is followed by the following result.

Theorem 1. [17] *Let* $A=\left(\tilde{T}_A,\tilde{I}_A,\tilde{F}_A,T_A,I_A,F_A\right)$, *where* $\tilde{T}_A=\left[T_A^L,T_A^U\right],\tilde{I}_A=\left[I_A^L,I_A^U\right],\tilde{F}_A=\left[F_A^L,F_A^U\right]$, *is a neutrosophic cubic value, then, the exponential operation defined by*

$$A^k=\begin{pmatrix}\left[(T_A^L)^k,(T_A^U)^k\right],\\ \left[(I_A^L)^k,(I_A^U)^k\right],\\ \left[1-\left(1-F_A^L\right)^k,1-\left(1-F_A^U\right)^k\right],\\ 1-\left(1-T_A\right)^k,1-\left(1-I_A\right)^k,\left(F_A\right)^k\end{pmatrix},$$

where $A^k=A\otimes A\otimes,\ldots.\otimes A(k-times)$, moreover, A^k is a neutrosophic cubic value for every positive value of k.

Definition 14. [17] *The Einstein sum between two neutrosophic cubic sets* $A=\left(\tilde{T}_A,\tilde{I}_A,\tilde{F}_A,T_A,I_A,F_A\right)$, where $\tilde{T}_A=\left[T_A^L,T_A^U\right],\tilde{I}_A=\left[I_A^L,I_A^U\right],\tilde{F}_A=\left[F_A^L,F_A^U\right]$, and $B=\left(\tilde{T}_B,\tilde{I}_B,\tilde{F}_B,T_B,I_B,F_B\right)$, where $\tilde{T}_B=\left[T_B^L,T_B^U\right],\tilde{I}_B=\left[I_B^L,I_B^U\right],\tilde{F}_B=\left[F_B^L,F_B^U\right]$ is defined as

$$A\oplus_E B=\begin{pmatrix}\left[\frac{T_A^L+T_B^L}{1+T_A^LT_B^L},\frac{T_A^U+T_B^U}{1+T_A^UT_B^U}\right],\\ \left[\frac{I_A^L+I_B^L}{1+I_A^LI_B^L},\frac{I_A^U+I_B^U}{1+I_A^UI_B^U}\right],\\ \left[\frac{F_A^LF_B^L}{1+(1-F_A^L)\left(1-F_B^L\right)},\frac{F_A^UF_B^U}{1+(1-F_A^U)\left(1-F_B^U\right)}\right]\\ \frac{T_AT_B}{1+(1-T_A)\left(1-T_B\right)},\frac{I_AI_B}{1+(1-I_A)\left(1-I_B\right)},\frac{F_A+F_B}{1+F_AF_B}\end{pmatrix}.$$

Definition 15. [17] *The Einstein product between two neutrosophic cubic sets,* $A=\left(\tilde{T}_A,\tilde{I}_A,\tilde{F}_A,T_A,I_A,F_A\right)$, where $\tilde{T}_A=\left[T_A^L,T_A^U\right],\tilde{I}_A=\left[I_A^L,I_A^U\right],\tilde{F}_A=\left[F_A^L,F_A^U\right]$, and $B=\left(\tilde{T}_B,\tilde{I}_B,\tilde{F}_B,T_B,I_B,F_B\right)$, where $\tilde{T}_B=\left[T_B^L,T_B^U\right],\tilde{I}_B=\left[I_B^L,I_B^U\right],\tilde{F}_B=\left[F_B^L,F_B^U\right]$ is defined as

$$A\otimes_E B=\begin{pmatrix}\left[\frac{T_A^LT_B^L}{1+(1-T_A^L)\left(1-T_B^L\right)},\frac{T_A^UT_B^U}{1+(1-T_A^U)\left(1-T_B^U\right)}\right],\\ \left[\frac{I_A^LI_B^L}{1+(1-I_A^L)\left(1-I_B^L\right)},\frac{I_A^UI_B^U}{1+(1-I_A^U)\left(1-I_B^U\right)}\right],\\ \left[\frac{F_A^L+F_B^L}{1+F_A^LF_B^L},\frac{F_A^U+F_B^U}{1+F_A^UF_B^U}\right]\\ \frac{T_A+T_B}{1+T_AT_B},\frac{I_A+I_B}{1+I_AI_B},\frac{F_AF_B}{1+(1-F_A)\left(1-F_B\right)}\end{pmatrix}.$$

Definition 16. [17] *The scalar multiplication on a neutrosophic cubic set,* $A=\left(\tilde{T}_A,\tilde{I}_A,\tilde{F}_A,T_A,I_A,F_A\right)$, *where* $\tilde{T}_A=\left[T_A^L,T_A^U\right],\tilde{I}_A=\left[I_A^L,I_A^U\right],\tilde{F}_A=\left[F_A^L,F_A^U\right]$, and a scalar k is defined

$$k_E A = \begin{pmatrix} \left[\dfrac{(1+T_A^L)^k-(1-T_A^L)^k}{(1+T_A^L)^k+(1-T_A^L)^k}, \dfrac{(1+T_A^U)^k-(1-T_A^U)^k}{(1+T_A^U)^k+(1-T_A^U)^k}\right], \\ \left[\dfrac{(1+I_A^L)^k-(1-I_A^L)^k}{(1+I_A^L)^k+(1-I_A^L)^k}, \dfrac{(1+I_A^U)^k-(1-I_A^U)^k}{(1+I_A^U)^k+(1-I_A^U)^k}\right], \\ \left[\dfrac{2\left(F_A^L\right)^k}{\left(2-F_A^L\right)^k+\left(F_A^L\right)^k}, \dfrac{2\left(F_A^U\right)^k}{\left(2-F_A^U\right)^k+\left(F_A^U\right)^k}\right], \\ \dfrac{2\left(T_A\right)^k}{\left(2-T_A\right)^k+\left(T_A\right)^k}, \dfrac{2\left(I_A\right)^k}{\left(2-I_A\right)^k+\left(I_A\right)^k}, \dfrac{(1+F_A)^k-(1-F_A)^k}{(1+F_A)^k+(1-F_A)^k} \end{pmatrix}.$$

The Einstein exponential multiplication is followed by the following result.

Theorem 2. [17] *Let* $A=\left(\tilde{T}_A,\tilde{I}_A,\tilde{F}_A,T_A,I_A,F_A\right)$, *where* $\tilde{T}_A=\left[T_A^L,T_A^U\right],\tilde{I}_A=\left[I_A^L,I_A^U\right],\tilde{F}_A=\left[F_A^L,F_A^U\right]$, *is a neutrosophic cubic value, then, the exponential operation defined by*

$$A^{E^k} = \begin{pmatrix} \left[\dfrac{2(T_A^L)^k}{(2-T_A^L)^k+(T_A^L)^k}, \dfrac{2(T_A^U)^k}{(2-T_A^U)^k+(T_A^U)^k}\right], \\ \left[\dfrac{2(I_A^L)^k}{(2-I_A^L)^k+(I_A^L)^k}, \dfrac{2(I_A^U)^k}{(2-I_A^U)^k+(I_A^U)^k}\right], \\ \left[\dfrac{(1+F_A^L)^k-(1-F_A^L)^k}{(1+F_A^L)^k+(1-F_A^L)^k}, \dfrac{(1+F_A^U)^k-(1-F_A^U)^k}{(1+F_A^U)^k+(1-F_A^U)^k}\right], \\ \dfrac{(1+T_A)^k-(1-T_A)^k}{(1+T_A)^k+(1-T_A)^k}, \dfrac{(1+I_A)^k-(1-I_A)^k}{(1+I_A)^k+(1-I_A)^k}, \dfrac{2\left(F_A\right)^k}{\left(2-F_A\right)^k+\left(F_A\right)^k} \end{pmatrix},$$

where $A^{E^k}=A\otimes_E A\otimes_E \ldots\otimes_E A(k-times)$, moreover, A^{E^k} is a neutrosophic cubic value for every positive value of k.

To compare two neutrosophic cubic values the score function is defined.

Definition 17. [17] *Let* $N=\left(\tilde{T}_N,\tilde{I}_N,\tilde{F}_N,T_N,I_N,F_N\right)$, *where* $\tilde{T}_N=\left[T_N^L,T_N^U\right],\tilde{I}_N=\left[I_N^L,I_N^U\right],\tilde{F}_N=\left[F_N^L,F_N^U\right]$ *is a neutrosophic cubic value, and the score function is defined as*

$$S(N)=\left[T_N^L-F_N^L+T_N^U-F_N^U+T_N-F_N\right].$$

If the score function of two values are equal, the accuracy function is used.

Definition 18. [17] *Let* $N=\left(\tilde{T}_N,\tilde{I}_N,\tilde{F}_N,T_N,I_N,F_N\right)$, *where* $\tilde{T}_N=\left[T_N^L,T_N^U\right],\tilde{I}_N=\left[I_N^L,I_N^U\right],\tilde{F}_N=\left[F_N^L,F_N^U\right]$ *isa neutrosophic cubic value, and theaccuracy function is defined as*

$$H(u)=\frac{1}{9}\left\{T_N^L+I_N^L+F_N^L+T_N^U+I_N^U+F_N^U+T_N+I_N+F_N\right\}.$$

The following definition describes the comparison relation between two neutrosophic cubic values.

Definition 19. [17] *Let* N_1, N_2 *be two neutrosophic cubic values, with core functions* S_{N_1},S_{N_2}, *and accuracy function* H_{N_1},H_{N_2}. *Then,*

1) $S_{N_1}>S_{N_2}\Rightarrow N_1>N_2$

2) If $S_{N_1} = S_{N_2}$

i) $H_{N_1} > H_{N_2} \Rightarrow N_1 > N_2$

ii) $H_{N_1} = H_{N_2} \Rightarrow N_1 = N_2$

Definition 20. [17] *The neutrosophic cubic weighted geometric operator(NCWG) is defined as*

$$NCWG : R^m \to R \ \textit{defined by}\ NCWG_w(N_1, N_2, ..., N_m) = \overset{m}{\underset{j=1}{\otimes}} N_j^{w_j},$$

where the weight $W = (w_1, w_2, ..., w_m)^T$ of $N_j (j = 1, 2, 3, ..., m)$, such that $w_j \in [0,1]$ and $\sum\limits_{j=1}^{m} = 1.$

Definition 21. [17] *The neutrosophic cubic ordered weighted geometric operator(NCOWG) is defined as*

$$NCOWG : R^m \to R \ \textit{defined by}\ NCOWG_w(N_1, N_2, ..., N_m) = \overset{m}{\underset{j=1}{\otimes}} N_{(\gamma)_j}^{w_j},$$

where $N_{(\gamma)_j}$ is the descending ordered neutrosophic cubic values, $W = (w_1, w_2, ..., w_m)^T$ of $N_j (j = 1, 2, 3, ..., m)$, such that $w_j \in [0,1]$ and $\sum\limits_{j=1}^{m} = 1.$

Definition 22. [17] *The neutrosophic cubic Einstein weighted geometric operator(NCEWG) is defined as*

$$NCEWG: R^m \to R \ \textit{defined by}\ NCEWG_w(N_1, N_2, ..., N_m) = \overset{m}{\underset{j=1}{\otimes}} \left(N_j\right)^{E^{w_j}},$$

where $W = (w_1, w_2, ..., w_m)^T$ is weight of $N_j (j = 1, 2, 3, ..., m), \textit{such that}\ w_j \in [0,1]$ and $\sum\limits_{j=1}^{m} = 1.$

Definition 23. [17] *Order neutrosophic cubic Einstein weighted geometric operator(NCEOWG) is defined as*

$$NCEOWG: R^m \to R \ \ by \ \ NCEOWG_w(N_1, N_2, ..., N_m) = \overset{m}{\underset{j=1}{\otimes}} \left(B_j\right)^{E^{w_j}},$$

where B_j is the jth largest neutrosophic cubic value, and $W = (w_1, w_2, ..., w_m)^T$ is weight of $N_j (j = 1, 2, 3, ..., m)$, such that $w_j \in [0,1]$ and $\sum\limits_{j=1}^{m} = 1.$

The neutrosophic cubic geometric aggregation operators weight only the neutrosophic cubic values, whereas neutrosophic cubic order geometric aggregation operators weight the orders of the values first then weight them. In the two cases, the amassed values that focused on the value relate to the biggest weight. The accompanying precedents represent the impediments of the NCWG and NCEWG.

Let $W = (0.7, 0.2, 0.1)$ be the weight corresponding to the neutrosophic cubic values

$$N_1 = ([0.2, 0.7], [0.2, 0.4], [0.2, 0.5], 0.8, 0.5, 0.8)$$
$$N_2 = ([0.4, 0.6], [0.4, 0.7], [0.1, 0.3], 0.2, 0.6, 0.5)$$
$$N_3 = ([0.5, 0.8], [0.3, 0.6], [0.4, 0.9], 0.5, 0.8, 0.9),$$

Then $S(N_1) = 0.2$, $S(N_2) = 0.3$, and $S(N_3) = -0.4$.

Therefore, $NCWG = ([0.251, 0.688], [0.239, 0.465], [0.204, 0.524], 0.711, 0.563, 0.737)$

and $NCOWG = ([0.356, 0.636], [0.338, 0.616], [0.155, 0.544], 0.421, 0.609, 0.737)$.

We observe that the higher the weight component, the aggregated value will tend to the corresponding neutrosophic cubic value of that vector. In NCWG, the value tendsto N_1, as the weight that corresponds to N_1 is highest, and in NCOWG, the highest component of weight corresponds to N_2. This situation often arises in aggregation problems. Motivated by such a situation, the idea of neutrosophic cubic hybrid geometric and neutrosophic cubic Einstein hybrid geometric operators are proposed.

3. Neutrosophic Cubic Hybrid Geometric and Neutrosophic Cubic Einstein Geometric Operators

This segment comprises of the following subsections. In Section 3.1 neutrosophic cubic crossbreed, the geometric operator is characterized. In Section 3.2 neutrosophic cubic Einstein crossbreed,the geometric operator is characterized. In Section 3.3, a calculation is characterized to organize the neutrosophic cubic values utilizing these tasks. In Section 3.4, a numerical model is outfitted upon Section 3.3.

3.1. Neutrosophic Cubic Hybrid Geometric Operator

NCWG operator weights only the neutrosophic cubic values, where NCOWG weights only the ordering positions. The idea of neutrosophic cubic hybrid geometric aggregation operators is developed to overcome these limitations. NCHG weights both the neutrosophic cubic values and its order positioning as well.

Definition 24. $NCHG : \Omega^m \to \Omega$ *is a mapping from* $\boldsymbol{m}$*-dimenion, which has associated weight* $W = (w_1, w_2, \ldots, w_m)^T$ *,such that* $w_j \in [0,1]$ *and* $\sum_{j=1}^{m} w_j = 1$, *such that*

$$NCHG_w(N_1, N_2, \ldots, N_m) = \left(N^{\sim}_{\sigma(1)}\right)^{w_1} \otimes \left(N^{\sim}_{\sigma(2)}\right)^{w_2} \otimes, \ldots, \otimes \left(N^{\sim}_{\sigma(m)}\right)^{w_m},$$

where $N^{\sim}_j$ jth largest of the weighted neutrosophic cubic values $\left\{N^{\sim}_{(j)}\left(N^{\sim}_{(j)} = N_j^{mw_j}\right), j = 1, 2, 3,,, m), W = (w_1, w_2, \ldots, w_m)^T\right\}$, such that $w_j \in [0,1]$ *and* $\sum_{j=1}^{m} w_j = 1$, and m is the balancing coefficient.

Theorem 3. *Let* $N_j = \left(\tilde{T}_{N_j}, \tilde{I}_{N_j}, \tilde{F}_{N_j}, T_{N_j}, I_{N_j}, F_{N_j}\right)$, where $\tilde{T}_{N_j} = \left[T^L_{N_j}, T^U_{N_j}\right], \tilde{I}_{N_j} = \left[I^L_{N_j}, I^U_{N_j}\right], \tilde{F}_{N_j} = \left[F^L_{N_j}, F^U_{N_j}\right]$, $(j = 1, 2, \ldots, m)$ be collection of neutrosophic cubic values, then the aggregated value (NCHWG) is also a cubic value and

$$NCHG(N_j) = \begin{pmatrix} \left[\bigotimes_{j=1}^{m}\left(T^{\sim L}_{\sigma(j)}\right)^{w_j}, \bigotimes_{j=1}^{m}\left(T^{\sim U}_{\sigma(j)}\right)^{w_j}\right], \left[\bigotimes_{j=1}^{m}\left(I^{\sim L}_{\sigma(j)}\right)^{w_j}, \left(I^{\sim U}_{\sigma(j)}\right)^{w_j}\right], \\ \left[1 - \bigotimes_{j=1}^{m}\left(1 - F^{\sim L}_{\sigma(j)}\right)^{w_j}, 1 - \bigotimes_{j=1}^{m}\left(1 - F^{\sim U}_{\sigma(j)}\right)^{w_j}\right], \\ 1 - \bigotimes_{j=1}^{m}\left(1 - T^{\sim}_{\sigma(j)}\right)^{w_j}, 1 - \bigotimes_{j=1}^{m}\left(1 - I^{\sim}_{\sigma(j)}\right)^{w_j}, \bigotimes_{j=1}^{m}\left(F^{\sim}_{\sigma(j)}\right)^{w_j} \end{pmatrix},$$

the weight $W = (w_1, w_2, \ldots, w_m)^T$, such that $w_j \in [0,1]$ and $\sum_{j=1}^{m} = 1$.

Proof. By mathematical induction for $m = 2$, using

$$\underset{j=1}{\overset{2}{\otimes}} N_j^{w_j} = N_1^{w_1} \otimes N_2^{w_2}$$

$$= \begin{pmatrix} \left[(T^L_{N\sigma(j)})^{w_1}, (T^U_{N_{\sigma(j)}})^{w_1} \right], \\ \left[(I^L_{N_{\sigma(j)}})^{w_1}, (I^U_{N_{\sigma(j)}})^{w_1} \right], \\ \left[1-\left(1-F^L_{N_{\sigma(j)}}\right)^{w_1}, 1-\left(1-F^U_{N_{\sigma(j)}}\right)^{w_1} \right], \\ 1-\left(1-\left(T_{N_{\sigma(j)}}\right)\right)^{w_1}, 1-\left(1-\left(I_{N\sigma(j)}\right)\right)^{w_1}, \left(F_{N_{\sigma(j)}}\right)^{w_1} \end{pmatrix} \otimes \begin{pmatrix} \left[(T^L_{N_{\sigma(j)}})^{w_2}, (T^U_{N_{\sigma(j)}})^{w_2} \right], \\ \left[(I^L_{N_{\sigma(j)}})^{w_2}, (I^U_{N_{\sigma(j)}})^{w_2} \right], \\ \left[1-\left(1-F^L_{N_{\sigma(j)}}\right)^{w_2}, 1-\left(1-F^U_{N_{\sigma(j)}}\right)^{w_2} \right], \\ 1-\left(1-\left(T_{N_{\sigma(j)}}\right)\right)^{w_2}, 1-\left(1-\left(I_{N_{\sigma(j)}}\right)\right)^{w_2}, \left(F_{N_{\sigma(j)}}\right)^{w_2} \end{pmatrix}$$

$$= \begin{pmatrix} \left[\underset{j=1}{\overset{2}{\otimes}} (T^L_{N_{\sigma(j)}})^{w_j}, \underset{j=1}{\overset{2}{\otimes}} (T^U_{N_{\sigma(j)}})^{w_j} \right], \left[\underset{j=1}{\overset{2}{\otimes}} (I^L_{N_{\sigma(j)}})^{w_j}, \underset{j=1}{\overset{2}{\otimes}} (I^U_{N_{\sigma(j)}})^{w_j} \right], \\ \left[1-\underset{j=1}{\overset{2}{\otimes}}\left(1-F^L_{N_{\sigma(j)}}\right)^{w_j}, 1-\underset{j=1}{\overset{2}{\otimes}}\left(1-F^U_{N_{\sigma(j)}}\right)^{w_j} \right], \\ 1-\underset{j=1}{\overset{2}{\otimes}}\left(1-\left(T_{N_{\sigma(j)}}\right)\right)^{w_j}, 1-\underset{j=1}{\overset{2}{\otimes}}\left(1-\left(I_{N_{\sigma(j)}}\right)\right)^{w_j}, \underset{j=1}{\overset{2}{\otimes}}\left(F_{N_{\sigma(j)}}\right)^{w_j} \end{pmatrix}.$$

Let the results hold for m.

$$\underset{j=1}{\overset{m}{\otimes}} N_j^{w_j} = \begin{pmatrix} \left[\underset{j=1}{\overset{m}{\otimes}} (T^L_{N_{\sigma(j)}})^{w_j}, \underset{j=1}{\overset{m}{\otimes}} (T^U_{N_{\sigma(j)}})^{w_j} \right], \left[\underset{j=1}{\overset{m}{\otimes}} (I^L_{N_{\sigma(j)}})^{w_j}, \underset{j=1}{\overset{m}{\otimes}} (I^U_{N_{\sigma(j)}})^{w_j} \right], \\ \left[1-\underset{j=1}{\overset{m}{\otimes}}\left(1-F^L_{N_{\sigma(j)}}\right)^{w_j}, 1-\underset{j=1}{\overset{m}{\otimes}}\left(1-F^U_{N_{\sigma(j)}}\right)^{w_j} \right], \\ 1-\underset{j=1}{\overset{m}{\otimes}}\left(1-T_{N_{\sigma(j)}}\right)^{w_j}, 1-\underset{j=1}{\overset{m}{\otimes}}\left(1-I_{N_{\sigma(j)}}\right)^{w_j}, \underset{j=1}{\overset{m}{\otimes}}\left(F_{N_{\sigma(j)}}\right)^{w_j} \end{pmatrix}$$

We prove the result for $m+1$,

$$\text{as} \left(N_{j+1}\right)^{w_{j+1}} = \begin{pmatrix} \left[(T^L_{N_{j+1}})^{w_{j+1}}, (T^U_{N_{j+1}})^{w_{j+1}} \right], \left[(I^L_{N_{j+1}})^{w_{j+1}}, (I^U_{N_{j+1}})^{w_{j+1}} \right], \\ \left[1-\left(1-F^L_{N_{j+1}}\right)^{w_{j+1}}, 1-\left(1-F^U_{N_{j+1}}\right)^{w_{j+1}} \right], \\ 1-\left(1-T_{N_{j+1}}\right)^{w_{j+1}}, 1-\left(1-I_{N_{j+1}}\right)^{w_{j+1}}, \left(F_{N_{j+1}}\right)^{w_{j+1}} \end{pmatrix}$$

$$\overset{m}{\underset{j=1}{\otimes}} N_j^{w_j} \oplus N_{j+1}^{w_{j+1}} = \begin{pmatrix} \left[\overset{m}{\underset{j=1}{\otimes}}(T^L_{N_{\sigma(j)}})^{w_j}, \overset{m}{\underset{j=1}{\otimes}}(T^U_{N_{\sigma(j)}})^{w_j}\right], \left[\overset{m}{\underset{j=1}{\otimes}}(I^L_{N_{\sigma(j)}})^{w_j}, \overset{m}{\underset{j=1}{\otimes}}(I^U_{N_{\sigma(j)}})^{w_j}\right], \\ \left[1-\overset{m}{\underset{j=1}{\otimes}}\left(1-F^L_{N_{\sigma(j)}}\right)^{w_j}, 1-\overset{m}{\underset{j=1}{\otimes}}\left(1-F^U_{N_{\sigma(j)}}\right)^{w_j}\right], \\ 1-\overset{m}{\underset{j=1}{\otimes}}\left(1-T_{N_{\sigma(j)}}\right)^{w_j}, 1-\overset{m}{\underset{j=1}{\otimes}}\left(1-I_{N_{\sigma(j)}}\right)^{w_j}, \overset{m}{\underset{j=1}{\otimes}}\left(F_{N_{\sigma(j)}}\right)^{w_j} \end{pmatrix} \oplus \begin{pmatrix} \left[(T^L_{N_{j+1}})^{w_{j+1}}, (T^U_{N_{j+1}})^{w_{j+1}}\right], \\ \left[(I^L_{N_{j+1}})^{w_{j+1}}, (I^U_{N_{j+1}})^{w_{j+1}}\right], \\ \left[1-\left(1-F^L_{N_{j+1}}\right)^{w_{j+1}}, 1-\left(1-F^U_{N_{j+1}}\right)^{w_{j+1}}\right], \\ 1-\left(1-T_{N_{j+1}}\right)^{w_{j+1}}, 1-\left(1-I_{N_{j+1}}\right)_I^{w_{j+1}}, (F_{N_{j+1}})^{w_{j+1}} \end{pmatrix}$$

$$\overset{m+1}{\underset{j=1}{\otimes}} N_j^{w_j} = \begin{pmatrix} \left[\overset{m}{\underset{j=1}{\otimes}}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}\left(T^L_{N_{m+1}}\right)^{w_{m+1}}, \overset{m}{\underset{j=1}{\otimes}}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}\left(T^U_{N_{m+1}}\right)^{w_{m+1}}\right], \left[\overset{m}{\underset{j=1}{\otimes}}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}\left(I^L_{N_{m+1}}\right)^{w_{m+1}}, \overset{m}{\underset{j=1}{\otimes}}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}\left(I^U_{N_{m+1}}\right)^{w_{m+1}}\right], \\ \begin{bmatrix} 1-\overset{m}{\underset{j=1}{\otimes}}(1-F^L_{N_{\sigma(j)}})^{w_j} + 1-(1-F^L_{N_{m+1}})^{w_{m+1}} - \left(1-\overset{m}{\underset{j=1}{\otimes}}(1-F^L_{N_{\sigma(j)}})^{w_j}\right)\left(1-(1-F^L_{N_{m+1}})^{w_{m+1}}\right), \\ 1-\overset{m}{\underset{j=1}{\otimes}}(1-F^U_{N_{\sigma(j)}})^{w_j} + 1-(1-F^U_{N_{m+1}})^{w_{m+1}} - \left(1-\overset{m}{\underset{j=1}{\otimes}}(1-F^U_{N_{\sigma(j)}})^{w_j}\right)\left(1-(1-F^U_{N_{m+1}})^{w_{m+1}}\right), \end{bmatrix}, \\ 1-\overset{m}{\underset{j=1}{\otimes}}(1-\left(T_{N_{\sigma(j)}}\right))^{w_j} + 1-(1-\left(T_{N_{m+1}}\right))^{w_{m+1}} - \left(1-\overset{m}{\underset{j=1}{\otimes}}(1-\left(T_{N_{\sigma(j)}}\right))^{w_j}\right)\left(1-(1-\left(T_{N_{m+1}}\right))^{w_{m+1}}\right) \\ 1-\overset{m}{\underset{j=1}{\otimes}}(1-\left(I_{N_{\sigma(j)}}\right))^{w_j} + 1-(1-\left(I_{N_{m+1}}\right))^{w_{m+1}} - \left(1-\overset{m}{\underset{j=1}{\otimes}}(1-\left(I_{N_{\sigma(j)}}\right))^{w_j}\right)\left(1-(1-\left(I_{N_{m+1}}\right))^{w_{m+1}}\right) \\ \overset{m}{\underset{j=1}{\otimes}}\left(F_{N_{\sigma(j)}}\right)^{w_j}\left(F_{N_{m+1}}\right)^{w_{m+1}} \end{pmatrix}$$

$$= \begin{pmatrix} \left[\overset{m+1}{\underset{j=1}{\otimes}}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}, \overset{m+1}{\underset{j=1}{\otimes}}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}\right], \left[\overset{m+1}{\underset{j=1}{\otimes}}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}, \overset{m+1}{\underset{j=1}{\otimes}}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}\right], \\ \begin{bmatrix} 2-\overset{m+1}{\underset{j=1}{\otimes}}(1-F^L_{N_{\sigma(j)}})^{w_j} - 1 + \overset{m+1}{\underset{j=1}{\otimes}}(1-F^L_{N_{\sigma(j)}})^{w_j} + (1-F^L_{N_{m+1}})^{w_{m+1}} - \left(\overset{m+1}{\underset{j=1}{\otimes}}(1-F^L_{N_{\sigma(j)}})^{w_j}\right)(1-F^L_{N_{m+1}})^{w_{m+1}}, \\ 2-\overset{m+1}{\underset{j=1}{\otimes}}(1-F^U_{N_{\sigma(j)}})^{w_j} - 1 + \overset{m+1}{\underset{j=1}{\otimes}}(1-F^U_{N_{\sigma(j)}})^{w_j} + (1-F^U_{N_{m+1}})^{w_{m+1}} - \left(\overset{m+1}{\underset{j=1}{\otimes}}(1-F^U_{N_{\sigma(j)}})^{w_j}\right)(1-F^U_{N_{m+1}})^{w_{m+1}} \end{bmatrix}, \\ 2-\overset{m+1}{\underset{j=1}{\otimes}}(1-T_{N_{\sigma(j)}})^{w_j} - 1 + \overset{m+1}{\underset{j=1}{\otimes}}(1-T_{N_{\sigma(j)}})^{w_j} + (1-T_{N_{m+1}})^{w_{m+1}} - \left(\overset{m+1}{\underset{j=1}{\otimes}}(1-T^{w_j}_{N_{\sigma(j)}}\right)(1-T_{N_{m+1}})^{w_{m+1}}, \\ 2-\overset{m+1}{\underset{j=1}{\otimes}}(1-I_{N_{\sigma(j)}})^{w_j} - 1 + \overset{m+1}{\underset{j=1}{\otimes}}(1-I_{N_{\sigma(j)}})^{w_j} + (1-I_{N_{m+1}})^{w_{m+1}} - \left(\overset{m+1}{\underset{j=1}{\otimes}}(1-I^{w_j}_{N_{\sigma(j)}}\right)(1-I_{N_{m+1}})^{w_{m+1}} \\ , \overset{m+1}{\underset{j=1}{\otimes}}\left(F_{N_j}\right)^{w_j} \end{pmatrix}$$

$$= \begin{pmatrix} \left[\overset{m+1}{\underset{j=1}{\otimes}}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}, \overset{m+1}{\underset{j=1}{\otimes}}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}\right], \left[\overset{m+1}{\underset{j=1}{\otimes}}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}, \overset{m+1}{\underset{j=1}{\otimes}}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}\right], \\ \left[1-\overset{m+1}{\underset{j=1}{\otimes}}(1-F^L_{N_{\sigma(j)}})^{w_j}, 1-\overset{m+1}{\underset{j=1}{\otimes}}(1-F^U_{N_{\sigma(j)}})^{w_j}\right], \\ 1-\overset{m+1}{\underset{j=1}{\otimes}}\left(1-T_{N_{\sigma(j)}}\right)^{w_j}, 1-\overset{m+1}{\underset{j=1}{\otimes}}\left(1-I_{N_{\sigma(j)}}\right)^{w_j}, \overset{m+1}{\underset{j=1}{\otimes}}\left(F_{N_{\sigma(j)}}\right)^{w_j} \end{pmatrix},$$

which completes the proof. □

Theorem 4. *The NCWG is a special case of NCHG operator*.

Proof. Let $W = (\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m})^T$. Then,

$$NCHG(N_1, N_2, \ldots, N_m)^w = \left(N^{\sim}_{\sigma(1)}\right)^{w_1} \otimes \left(N^{\sim}_{\sigma(2)}\right)^{w_2} \otimes, \ldots, \otimes \left(N^{\sim}_{\sigma(m)}\right)^{w_m}$$

$$= \left(N^{\sim}_{\sigma(1)}\right)^{\frac{1}{m}} \otimes \left(N^{\sim}_{\sigma(2)}\right)^{\frac{1}{m}} \otimes, \ldots, \otimes \left(N^{\sim}_{\sigma(m)}\right)^{\frac{1}{m}}$$

$$= (N_1, N_2, ..., N_m)^{\frac{1}{m}}$$
$$= (N_1)^{w_1}, (N_2)^{w_2}, ..., (N_m)^{w_m}$$
$$= NCWG(N_1, N_2, ..., N_m). \quad \square$$

Theorem 5. *The NCOWG is a special case of NCHG.*

Proof. Let $W = (\frac{1}{m}, \frac{1}{m}, ..., \frac{1}{m})^T$. Then,

$$NCHG(N_1, N_2, ..., N_m)^w = \left(N^{\sim}_{\sigma(1)}\right)^{w_1} \otimes \left(N^{\sim}_{\sigma(2)}\right)^{w_2} \otimes, ..., \otimes \left(N^{\sim}_{\sigma(m)}\right)^{w_m}$$
$$= \left(N^{\sim}_{\sigma(1)}\right)^{\frac{1}{m}} \otimes \left(N^{\sim}_{\sigma(2)}\right)^{\frac{1}{m}} \otimes, ..., \otimes \left(N^{\sim}_{\sigma(m)}\right)^{\frac{1}{m}}$$
$$= (N_1, N_2, ..., N_m)^{\frac{1}{m}}$$
$$= (N_1)^{w_1}, (N_2)^{w_2}, ..., (N_m)^{w_m}$$
$$= NCOWG(N_1, N_2, ..., N_m). \quad \square$$

3.2. Neutrosophic Cubic Einstein Hybrid Geometric Operator

NCEWG operator weights only the neutrosophic cubic values,where NCEOWGA weights only the ordering positions. The idea of neutrosophic cubic Einstein hybrid aggregation operators (NCEHG) is developed to overcome these limitations, which weights both the given neutrosophic cubic value and its order position as well.

Definition 25. $NCEHG : \Omega^m \to \Omega$ *is a map from m-dimension which has an associated vector* $W = (w_1, w_2, ..., w_m)^T$, where $w_j \in [0,1]$ and $\sum_{j=1}^{m} = 1$, such that

$$\text{NCEHG}_w(N_1, N_2, ..., N_m) = \left(N^{\sim}_{\sigma(1)}\right)^{w_1} \otimes_E \left(N^{\sim}_{\sigma(2)}\right)^{w_2} \otimes_E, ..., \otimes_E \left(N^{\sim}_{\sigma(m)}\right)^{w_m},$$

where $N^{\sim}_j$ isthe jth largest of the weighted neutrosophic cubic values $\left\{N^{\sim}_{(j)}\left(N^{\sim}_{(j)} = N_j^{mw_j}\right), j = 1, 2, 3,,, m), W = (w_1, w_2, ..., w_m)^T\right\}$,with $w_j \in [0,1]$ and $\sum_{j=1}^{m} = 1$, and m is the balancing coefficient.

Theorem 6. *Let* $N_j = \left(\tilde{T}_{N_j}, \tilde{I}_{N_j}, \tilde{F}_{N_j}, T_{N_j}, I_{N_j}, F_{N_j}\right)$, *where* $\tilde{T}_{N_j} = \left[T^L_{N_j}, T^U_{N_j}\right], \tilde{I}_{N_j} = \left[I^L_{N_j}, I^U_{N_j}\right], \tilde{F}_{N_j} = \left[F^L_{N_j}, F^U_{N_j}\right]$, $N = \left(\tilde{T}_{N_j}, \tilde{I}_{N_j}, \tilde{F}_{N_j}, T_{N_j}, I_{N_j}, F_{N_j}\right)$, where $\tilde{T}_{N_j} = \left[T^L_{N_j}, T^U_{N_j}\right], \tilde{I}_{N_j} = \left[I^L_{N_j}, I^U_{N_j}\right], \tilde{F}_{N_j} = \left[F^L_{N_j}, F^U_{N_j}\right]$ $(j = 1, 2, ..., m)$ is acollection of neutrosophic cubic values, then, their aggregated value by NCEWG operator is also a cubic value and

$$NCEHG(N_j)=\left(\begin{array}{c}\left[\dfrac{2\otimes_{j=1}^{m}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}}{\otimes_{j=1}^{m}\left(2-T^L_{N_{\sigma(j)}}\right)^{w_j}+\otimes_{j=1}^{m}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\otimes_{j=1}^{m}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}{\otimes_{j=1}^{m}\left(2-T^U_{N_{\sigma(j)}}\right)^{w_j}+\otimes_{j=1}^{m}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}\right],\\ \left[\dfrac{2\otimes_{j=1}^{m}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}}{\otimes_{j=1}^{m}\left(2-I^L_{N_{\sigma(j)}}\right)^{w_j}+\otimes_{j=1}^{m}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\otimes_{j=1}^{m}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}{\otimes_{j=1}^{m}\left(2-I^U_{N_{\sigma(j)}}\right)^{w_j}+\otimes_{j=1}^{m}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}\right],\\ \left[\dfrac{\otimes_{j=1}^{m}(1+F^L_{N_{\sigma(j)}})^{w_j}-\otimes_{j=1}^{m}(1-F^L_{N_{\sigma(j)}})^{w_j}}{\otimes_{j=1}^{m}(1+F^L_{N_{\sigma(j)}})^{w_j}+\otimes_{j=1}^{m}(1-F^L_{N_{\sigma(j)}})^{w_j}},\dfrac{\otimes_{j=1}^{m}(1+F^U_{N_{\sigma(j)}})^{w_j}-\otimes_{j=1}^{m}(1-F^U_{N_{\sigma(j)}})^{w_j}}{\otimes_{j=1}^{m}(1+F^U_{N_{\sigma(j)}})^{w_j}+\otimes_{j=1}^{m}(1-F^U_{N_{\sigma(j)}})^{w_j}}\right],\\ \dfrac{\otimes_{j=1}^{m}(1+T_{N_{\sigma(j)}})^{w_j}-\otimes_{j=1}^{m}(1-T_{N_{\sigma(j)}})^{w_j}}{\otimes_{j=1}^{m}(1+T_{N_{\sigma(j)}})^{w_j}+\otimes_{j=1}^{m}(1-T_{N_{\sigma(j)}})^{w_j}},\dfrac{\otimes_{j=1}^{m}(1+I_{N_{\sigma(j)}})^{w_j}-\otimes_{j=1}^{m}(1-I_{N_{\sigma(j)}})^{w_j}}{\otimes_{j=1}^{m}(1+I_{N_{\sigma(j)}})^{w_j}+\otimes_{j=1}^{m}(1-I_{N_{\sigma(j)}})^{w_j}},\\ \dfrac{2\otimes_{j=1}^{m}\left(F_{N_{\sigma(j)}}\right)^{w_j}}{\otimes_{j=1}^{m}\left(2-F_{N_{\sigma(j)}}\right)^{w_j}+\otimes_{j=1}^{m}\left(F_{N_{\sigma(j)}}\right)^{w_j}}\end{array}\right),$$

where $W=(w_1,w_2,...,w_m)^T$ is weight of $N_j(j=1,2,3,...,m)$, with $w_j\in[0,1]$ and $\sum_{j=1}^{m}=1$.

Proof. We use mathematical induction to prove this result for $k=2$, using definition

$$\left(N_1^E\right)^{w_1}=\left(\begin{array}{c}\left[\dfrac{2\left(T^L_{N_1}\right)^{w_1}}{\left(2-T^L_{N_1}\right)^{w_1}+T^L_{N_1}},\dfrac{2\left(T^U_{N_1}\right)^{w_1}}{\left(2-T^U_{N_1}\right)^{w_1}+T^U_{N_1}}\right],\left[\dfrac{2\left(I^L_{N_1}\right)^{w_1}}{\left(2-I^L_{N_1}\right)^{w_1}+I^L_{N_1}},\dfrac{2\left(I^U_{N_1}\right)^{w_1}}{\left(2-I^U_{N_1}\right)^{w_1}+I^U_{N_1}}\right],\\ \left[\dfrac{(1+F^L_{N_1})^{w_1}-(1-F^L_{N_1})^{w_1}}{(1+F^L_{N_1})^{w_1}+(1-F^L_{N_1})^{w_1}},\dfrac{(1+F^U_{N_1})^{w_1}-(1-F^U_{N_1})^{w_1}}{(1+F^U_{N_1})^{w_1}+(1-F^U_{N_1})^{w_1}}\right],\\ \dfrac{(1+T_{N_1})^{w_1}-(1-T_{N_1})^{w_1}}{(1+T_{N_1})^{w_1}+(1-T_{N_1})^{w_1}},\dfrac{(1+I_{N_1})^{w_1}-(1-I_{N_1})^{w_1}}{(1+I_{N_1})^{w_1}+(1-I_{N_1})^{w_1}},\dfrac{2\left(F_{N_1}\right)^{w_1}}{\left(2-F_{N_1}\right)^{w_1}+F_{N_1}}\end{array}\right)$$

$$\left(N_2^E\right)^{w_2}=\left(\begin{array}{c}\left[\dfrac{2\left(T^L_{N_2}\right)^{w_2}}{\left(2-T^L_{N_2}\right)^{w_2}+T^L_{N_2}},\dfrac{2\left(T^U_{N_2}\right)^{w_2}}{\left(2-T^U_{N_2}\right)^{w_2}+T^U_{N_2}}\right],\left[\dfrac{2\left(I^L_{N_2}\right)^{w_2}}{\left(2-I^L_{N_2}\right)^{w_2}+I^L_{N_2}},\dfrac{2\left(I^U_{N_2}\right)^{w_2}}{\left(2-I^U_{N_2}\right)^{w_2}+I^U_{N_2}}\right],\\ \left[\dfrac{(1+F^L_{N_2})^{w_2}-(1-F^L_{N_2})^{w_2}}{(1+F^L_{N_2})^{w_2}+(1-F^L_{N_2})^{w_2}},\dfrac{(1+F^U_{N_2})^{w_2}-(1-F^U_{N_2})^{w_2}}{(1+F^U_{N_2})^{w_2}+(1-F^U_{N_2})^{w_2}}\right],\\ \dfrac{(1+T_{N_2})^{w_2}-(1-T_{N_2})^{w_2}}{(1+T_{N_2})^{w_2}+(1-T_{N_2})^{w_2}},\dfrac{(1+I_{N_2})^{w_2}-(1-I_{N_2})^{w_2}}{(1+I_{N_2})^{w_2}+(1-I_{N_2})^{w_2}},\dfrac{2\left(F_{N_2}\right)^{w_2}}{\left(2-F_{N_2}\right)^{w_2}+F_{N_2}}\end{array}\right)$$

$$\bigotimes_{j=1}^{2}\left(N_j^E\right)^{w_j}=\left(\begin{array}{c}
\left[\dfrac{2\bigotimes_{j=1}^{2}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{2}\left(2-T^L_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{2}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\bigotimes_{j=1}^{2}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{2}\left(2-T^U_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{2}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}\right],\\
\left[\dfrac{2\bigotimes_{j=1}^{2}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{2}\left(2-I^L_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{2}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\bigotimes_{j=1}^{2}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{2}\left(2-I^U_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{2}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}\right],\\
\left[\dfrac{\bigotimes_{j=1}^{2}(1+F^L_{N_{\sigma(j)}})^{w_j}-\bigotimes_{j=1}^{2}(1-F^L_{N_{\sigma(j)}})^{w_j}}{\bigotimes_{j=1}^{2}(1+F^L_{N_{\sigma(j)}})^{w_j}+\bigotimes_{j=1}^{2}(1-F^L_{N_{\sigma(j)}})^{w_j}},\dfrac{\bigotimes_{j=1}^{2}(1+F^U_{N_{\sigma(j)}})^{w_j}-\bigotimes_{j=1}^{2}(1-F^U_{N_{\sigma(j)}})^{w_j}}{\bigotimes_{j=1}^{2}(1+F^U_{N_{\sigma(j)}})^{w_j}+\bigotimes_{j=1}^{2}(1-F^U_{N_{\sigma(j)}})^{w_j}}\right],\\
\dfrac{\bigotimes_{j=1}^{2}(1+T_{N_{\sigma(j)}})^{w_j}-\bigotimes_{j=1}^{2}(1-T_{N_{\sigma(j)}})^{w_j}}{\bigotimes_{j=1}^{2}(1+T_{N_{\sigma(j)}})^{w_j}+\bigotimes_{j=1}^{2}(1-T_{N_{\sigma(j)}})^{w_j}},\dfrac{\bigotimes_{j=1}^{2}(1+I_{N_{\sigma(j)}})^{w_j}-\bigotimes_{j=1}^{2}(1-I_{N_{\sigma(j)}})^{w_j}}{\bigotimes_{j=1}^{2}(1+I_{N_{\sigma(j)}})^{w_j}+\bigotimes_{j=1}^{2}(1-I_{N_{\sigma(j)}})^{w_j}},\\
\dfrac{2\bigotimes_{j=1}^{2}\left(F_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{2}\left(2-F_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{2}\left(F_{N_{\sigma(j)}}\right)^{w_j}}
\end{array}\right).$$

Let the result holds for m

$$\bigotimes_{j=1}^{m}\left(N_j^E\right)^{w_j}=\left(\begin{array}{c}
\left[\dfrac{2\bigotimes_{j=1}^{m}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{m}\left(2-T^L_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{m}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\bigotimes_{j=1}^{m}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{m}\left(2-T^U_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{m}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}\right],\\
\left[\dfrac{2\bigotimes_{j=1}^{m}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{m}\left(2-I^L_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{m}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\bigotimes_{j=1}^{m}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{m}\left(2-I^U_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{m}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}\right],\\
\left[\dfrac{\bigotimes_{j=1}^{m}(1+F^L_{N_{\sigma(j)}})^{w_j}-\bigotimes_{j=1}^{m}(1-F^L_{N_{\sigma(j)}})^{w_j}}{\bigotimes_{j=1}^{m}(1+F^L_{N_{\sigma(j)}})^{w_j}+\bigotimes_{j=1}^{m}(1-F^L_{N_{\sigma(j)}})^{w_j}},\dfrac{\bigotimes_{j=1}^{m}(1+F^U_{N_{\sigma(j)}})^{w_j}-\bigotimes_{j=1}^{m}(1-F^U_{N_{\sigma(j)}})^{w_j}}{\bigotimes_{j=1}^{m}(1+F^U_{N_{\sigma(j)}})^{w_j}+\bigotimes_{j=1}^{m}(1-F^U_{N_{\sigma(j)}})^{w_j}}\right],\\
\dfrac{\bigotimes_{j=1}^{m}(1+T_{N_{\sigma(j)}})^{w_j}-\bigotimes_{j=1}^{m}(1-T_{N_{\sigma(j)}})^{w_j}}{\bigotimes_{j=1}^{m}(1+T_{N_{\sigma(j)}})^{w_j}+\bigotimes_{j=1}^{m}(1-T_{N_{\sigma(j)}})^{w_j}},\dfrac{\bigotimes_{j=1}^{m}(1+I_{N_{\sigma(j)}})^{w_j}-\bigotimes_{j=1}^{m}(1-I_{N_{\sigma(j)}})^{w_j}}{\bigotimes_{j=1}^{m}(1+I_{N_{\sigma(j)}})^{w_j}+\bigotimes_{j=1}^{m}(1-I_{N_{\sigma(j)}})^{w_j}},\\
\dfrac{2\bigotimes_{j=1}^{m}\left(F_{N_{\sigma(j)}}\right)^{w_j}}{\bigotimes_{j=1}^{m}\left(2-F_{N_{\sigma(j)}}\right)^{w_j}+\bigotimes_{j=1}^{m}\left(F_{N_{\sigma(j)}}\right)^{w_j}}
\end{array}\right).$$

We prove the result holds for $m+1$.

$$as\left(N^E_{m+1}\right)^{w_{m+1}}=\left(\begin{array}{c}
\left[\dfrac{2\left(T^L_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-T^L_{N_{m+1}}\right)^{w_{m+1}}+\left(T^L_N\right)^{w_{m+1}}},\dfrac{2\left(T^U_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-T^U_N\right)^{w_{m+1}}+\left(T^U_N\right)^{w_{m+1}}}\right],\\
\left[\dfrac{2\left(I^L_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-I^L_{N_{m+1}}\right)^{w_{m+1}}+\left(I^L_N\right)^{w_{m+1}}},\dfrac{2\left(I^U_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-I^U_{N_{m+1}}\right)^{w_{m+1}}+\left(I^U_N\right)^{w_{m+1}}}\right],\\
\left[\dfrac{(1+F^L_{N_{m+1}})^{w_{m+1}}-(1-F^L_{N_{m+1}})^{w_{m+1}}}{(1+F^L_{N_{m+1}})^{w_{m+1}}+(1-F^L_{N_{m+1}})^{w_{m+1}}},\dfrac{(1+F^U_{N_{m+1}})^{w_{m+1}}-(1-F^U_{N_{m+1}})^{w_{m+1}}}{(1+F^U_{N_{m+1}})^{w_{m+1}}+(1-F^U_{N_{m+1}})^{w_{m+1}}}\right],\\
\dfrac{(1+T_{N_{m+1}})^{w_{m+1}}-(1-T_{N_{m+1}})^{w_{m+1}}}{(1+T_{N_{m+1}})^{w_{m+1}}+(1-T_{N_{m+1}})^{w_{m+1}}},\dfrac{(1+I_{N_{m+1}})^{w_{m+1}}-(1-I_{N_{m+1}})^{w_{m+1}}}{(1+I_{N_{m+1}})^{w_{m+1}}+(1-I_{N_{m+1}})^{w_{m+1}}},\dfrac{2\left(F_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-F_{N_{m+1}}\right)^{w_{m+1}}+\left(F_N\right)^{w_{m+1}}}
\end{array}\right)$$

$$so \mathop{\otimes}_{j=1}^{m} \left(N_j^E\right)^{w_j} \otimes_E \left(N_{m+1}^E\right)^{w_{m+1}} =$$

$$\left(\begin{array}{c}
\left[\dfrac{2\mathop{\otimes}\limits_{j=1}^{m}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}\left(2-T^L_{N_{\sigma(j)}}\right)^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\mathop{\otimes}\limits_{j=1}^{m}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}\left(2-T^U_{N_{\sigma(j)}}\right)^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}\right], \\
\left[\dfrac{2\mathop{\otimes}\limits_{j=1}^{m}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}\left(2-I^L_{N_{\sigma(j)}}\right)^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\mathop{\otimes}\limits_{j=1}^{m}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}\left(2-I^U_{N_{\sigma(j)}}\right)^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}\right], \\
\left[\dfrac{\mathop{\otimes}\limits_{j=1}^{m}(1+F^L_{N_{\sigma(j)}})^{w_j}-\mathop{\otimes}\limits_{j=1}^{m}(1-F^L_{N_{\sigma(j)}})^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}(1+F^L_{N_{\sigma(j)}})^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}(1-F^L_{N_{\sigma(j)}})^{w_j}},\dfrac{\mathop{\otimes}\limits_{j=1}^{m}(1+F^U_{N_{\sigma(j)}})^{w_j}-\mathop{\otimes}\limits_{j=1}^{m}(1-F^U_{N_{\sigma(j)}})^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}(1+F^U_{N_{\sigma(j)}})^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}(1-F^U_{N_{\sigma(j)}})^{w_j}}\right], \\
\dfrac{\mathop{\otimes}\limits_{j=1}^{m}(1+T_{N_{\sigma(j)}})^{w_j}-\mathop{\otimes}\limits_{j=1}^{m}(1-T_{N_{\sigma(j)}})^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}(1+T_{N_{\sigma(j)}})^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}(1-T_{N_{\sigma(j)}})^{w_j}},\dfrac{\mathop{\otimes}\limits_{j=1}^{m}(1+I_{N_{\sigma(j)}})^{w_j}-\mathop{\otimes}\limits_{j=1}^{m}(1-I_{N_{\sigma(j)}})^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}(1+I_{N_{\sigma(j)}})^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}(1-I_{N_{\sigma(j)}})^{w_j}}, \\
\dfrac{2\mathop{\otimes}\limits_{j=1}^{m}\left(F_{N_{\sigma(j)}}\right)^{w_j}}{\mathop{\otimes}\limits_{j=1}^{m}\left(2-F_{N_{\sigma(j)}}\right)^{w_j}+\mathop{\otimes}\limits_{j=1}^{m}\left(F_{N_{\sigma(j)}}\right)^{w_j}}
\end{array}\right) \oplus_E$$

$$\left(\begin{array}{c}
\left[\dfrac{2\left(T^L_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-T^L_{N_{m+1}}\right)^{w_{m+1}}+\left(T^L_N\right)^{w_{m+1}}},\dfrac{2\left(T^U_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-T^U_N\right)^{w_{m+1}}+\left(T^U_N\right)^{w_{m+1}}}\right], \\
\left[\dfrac{2\left(I^L_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-I^L_{N_{m+1}}\right)^{w_{m+1}}+\left(I^L_N\right)^{w_{m+1}}},\dfrac{2\left(I^U_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-I^U_{N_{m+1}}\right)^{w_{m+1}}+\left(I^U_N\right)^{w_{m+1}}}\right], \\
\left[\dfrac{(1+F^L_{N_{m+1}})^{w_{m+1}}-(1-F^L_{N_{m+1}})^{w_{m+1}}}{(1+F^L_{N_{m+1}})^{w_{m+1}}+(1-F^L_{N_{m+1}})^{w_{m+1}}},\dfrac{(1+F^U_{N_{m+1}})^{w_{m+1}}-(1-F^U_{N_{m+1}})^{w_{m+1}}}{(1+F^U_{N_{m+1}})^{w_{m+1}}+(1-F^U_{N_{m+1}})^{w_{m+1}}}\right], \\
\dfrac{(1+T_{N_{m+1}})^{w_{m+1}}-(1-T_{N_{m+1}})^{w_{m+1}}}{(1+T_{N_{m+1}})^{w_{m+1}}+(1-T_{N_{m+1}})^{w_{m+1}}},\dfrac{(1+I_{N_{m+1}})^{w_{m+1}}-(1-I_{N_{m+1}})^{w_{m+1}}}{(1+I_{N_{m+1}})^{w_{m+1}}+(1-I_{N_{m+1}})^{w_{m+1}}}, \\
\dfrac{2\left(F_{N_{m+1}}\right)^{w_{m+1}}}{\left(2-F_{N_{m+1}}\right)^{w_{m+1}}+\left(F_N\right)^{w_{m+1}}}
\end{array}\right)$$

$$\overset{m+1}{\underset{j=1}{\otimes}}\left(N_j^E\right)^{w_j}=\left(\begin{array}{c}\left[\dfrac{2\overset{m+1}{\underset{j=1}{\otimes}}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}\left(2-T^L_{N_{\sigma(j)}}\right)^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}\left(T^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\overset{m+1}{\underset{j=1}{\otimes}}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}\left(2-T^U_{N_{\sigma(j)}}\right)^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}\left(T^U_{N_{\sigma(j)}}\right)^{w_j}}\right],\\ \left[\dfrac{2\overset{m+1}{\underset{j=1}{\otimes}}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}\left(2-I^L_{N_{\sigma(j)}}\right)^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}\left(I^L_{N_{\sigma(j)}}\right)^{w_j}},\dfrac{2\overset{m+1}{\underset{j=1}{\otimes}}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}\left(2-I^U_{N_{\sigma(j)}}\right)^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}\left(I^U_{N_{\sigma(j)}}\right)^{w_j}}\right],\\ \left[\dfrac{\overset{m+1}{\underset{j=1}{\otimes}}(1+F^L_{N_{\sigma(j)}})^{w_j}-\overset{m+1}{\underset{j=1}{\otimes}}(1-F^L_{N_{\sigma(j)}})^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}(1+F^L_{N_{\sigma(j)}})^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}(1-F^L_{N_{\sigma(j)}})^{w_j}},\dfrac{\overset{m+1}{\underset{j=1}{\otimes}}(1+F^U_{N_{\sigma(j)}})^{w_j}-\overset{m+1}{\underset{j=1}{\otimes}}(1-F^U_{N_{\sigma(j)}})^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}(1+F^U_{N_{\sigma(j)}})^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}(1-F^U_{N_{\sigma(j)}})^{w_j}}\right],\\ \dfrac{\overset{m+1}{\underset{j=1}{\otimes}}(1+T_{N_{\sigma(j)}})^{w_j}-\overset{m+1}{\underset{j=1}{\otimes}}(1-T_{N_{\sigma(j)}})^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}(1+T_{N_{\sigma(j)}})^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}(1-T_{N_{\sigma(j)}})^{w_j}},\dfrac{\overset{m+1}{\underset{j=1}{\otimes}}(1+I_{N_{\sigma(j)}})^{w_j}-\overset{m+1}{\underset{j=1}{\otimes}}(1-I_{N_{\sigma(j)}})^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}(1+I_{N_{\sigma(j)}})^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}(1-I_{N_{\sigma(j)}})^{w_j}},\\ \dfrac{2\overset{m+1}{\underset{j=1}{\otimes}}\left(F_{N_{\sigma(j)}}\right)^{w_j}}{\overset{m+1}{\underset{j=1}{\otimes}}\left(2-F_{N_{\sigma(j)}}\right)^{w_j}+\overset{m+1}{\underset{j=1}{\otimes}}\left(F_{N_{\sigma(j)}}\right)^{w_j}}\end{array}\right),$$

so the result holds for all values of m. □

Theorem 7. *The NCEWG is special case of the NCEHG operator.*

Proof. Followed by Theorem 4. □

Theorem 8. *The NCOWG is a special case of NCEHG.*

Proof. Followed by Theorem 5. □

3.3. An Application of Neutrosophic Cubic Hybrid Geometric and Einstein Hybrid Geometric Aggregation Operator to Group Decision-Making Problems

In this section, we develop an algorithm for group decision-making problems using the neutrosophic cubichybrid geometric and Einstein hybrid geometric aggregation(NCHWG and NCEHWG).

Algorithm 1. Let $F=\{F_1,F_2,\ldots,F_n\}$ be the set of n alternatives, $H=\{H_1,H_2,\ldots,H_m\}$ be the m attributes subject to their corresponding weight $W=\{w_1,w_2,\ldots,w_m\}$, such that $w_j\in[0,1]$ and $\sum_{j=1}^{m}=1$. The method has the following steps.

Step 1: First of all, we construct neutrosophic cubic decision matrix $D=\left[N_{ij}\right]_{n\times m}$.

Step 2: The attributes $H=\{H_1,H_2,\ldots,H_m\}$ are weighted to their corresponding weight $W=\{w_1,w_2,\ldots,w_m\}$, and these values multipliedby the balancing coefficient m.

Step 3: The new weights are calculated using [18] so that we get new weights $V=\{v_1,v_2,\ldots,v_m\}$.

Step 4: By using aggregation operators like (NCHG, NCEHG), the decision matrix is aggregated by the new weightsassigned to the m attributes.

Step 5: The n alternatives are ranked according to their scores and arranged in descending order to select the alternative with highest score.

3.4. NumericalApplication

A steering committee is interested in prioritizingthe set of information for improvement of the project using a multiple attribute decision-making method. The committee must prioritize the development and implementation of a set of six information technology improvement projects A_j $(j=1,2,...,6)$. The three factors, B_1 productivity, to increase the effectiveness and efficiency, B_2 differentiation, from products and services of competitors, and B_3 management, to assist the management in improving their planning, are considered to assess the potential contribution of each project. The list of proposed information systems are A_1 Quality Assurance, 2) A_2 Budget Analysis, 3) A_3 Itemization, 4) A_4 Employee Skills Tracking, 5) A_5 Customer Returns and Complaints, and 6) A_6 Materials Acquisition.Suppose the weight $W=(0.5,0.3,0.2)$ corresponds to the $B_j,(j=1,2,3)$ factors and characteristics of projects A_i $(i=1,2,...,10)$ by the neutrosophic cubic value N_{ij}.

Step 1: Construction of neutrosophic cubic decision matrix $D=\left[N_{ij}\right]_{6\times 3}$

$$D=\left(\begin{array}{cccc} & B_1 & B_2 & B_3 \\ A_1 & \begin{pmatrix}[0.5,0.6],[0.2,0.5],\\ [0.4,0.8],0.7,0.8,0.4\end{pmatrix} & \begin{pmatrix}[0.3,0.7],[0.1,0.6],\\ [0.3,0.7],0.4,0.7,0.2\end{pmatrix} & \begin{pmatrix}[0.4,0.8],[0.3,0.6],\\ [0.5,0.7],0.4,0.2,0.6\end{pmatrix} \\ A_2 & \begin{pmatrix}[0.2,0.5],[0.7,0.9],\\ [0.3,0.7],0.8,0.5,0.3\end{pmatrix} & \begin{pmatrix}[0.1,0.6],[0.4,0.7],\\ [0.2,0.5],0.6,0.4,0.7\end{pmatrix} & \begin{pmatrix}[0.2,0.6],[0.1,0.7],\\ [0.3,0.7],0.6,0.4,0.8\end{pmatrix} \\ A_3 & \begin{pmatrix}[0.4,0.7],[0.2,0.5],\\ [0.5,0.7],0.3,0.6,0.2\end{pmatrix} & \begin{pmatrix}[0.3,0.8],[0.1,0.4],\\ [0.6,0.7],0.6,0.2,0.6\end{pmatrix} & \begin{pmatrix}[0.3,0.5],[0.5,0.9],\\ [0.2,0.7],0.7,0.5,0.6\end{pmatrix} \\ A_4 & \begin{pmatrix}[0.3,0.6],[0.4,0.7],\\ [0.2,0.5],0.6,0.4,0.7\end{pmatrix} & \begin{pmatrix}[0.5,0.8],[0.1,0.5],\\ [0.3,0.8],0.4,0.8,0.6\end{pmatrix} & \begin{pmatrix}[0.1,0.7],[0.2,0.6],\\ [0.4,0.7],0.5,0.6,0.8\end{pmatrix} \\ A_5 & \begin{pmatrix}[0.2,0.5],[0.3,0.7],\\ [0.2,0.6],0.5,0.3,0.8\end{pmatrix} & \begin{pmatrix}[0.4,0.8],[0.3,0.7],\\ [0.1,0.6],0.4,0.6,0.7\end{pmatrix} & \begin{pmatrix}[0.6,0.8],[0.5,0.9],\\ [0.4,0.9],0.6,0.8,0.3\end{pmatrix} \\ A_6 & \begin{pmatrix}[0.1,0.6],[0.3,0.6],\\ [0.4,0.8],0.6,0.9,0.4\end{pmatrix} & \begin{pmatrix}[0.2,0.7],[0.6,0.9],\\ [0.3,0.6],0.4,0.8,0.3\end{pmatrix} & \begin{pmatrix}[0.4,0.7],[0.3,0.5],\\ [0.1,0.6],0.4,0.6,0.7\end{pmatrix} \end{array}\right)$$

Step 2: The attributes are weighted $W=(0.5,0.3,0.2)$ and multiplied by balancing coefficient 3.

$$D = \begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ A_1 & \begin{pmatrix} [0.3535, 0.4647], \\ [0.0894, 0.3535], \\ [0.5352, 0.9105], \\ 0.8356, 0.9105, 0.2529 \end{pmatrix} & \begin{pmatrix} [0.3383, 0.7254], \\ [0.1258, 0.6314], \\ [0.27453, 0.6616], \\ 0.3685, 0.6616, 0.2349 \end{pmatrix} & \begin{pmatrix} [0.5770, 0.8746], \\ [0.4855, 0.7360], \\ [0.4229, 0.5144], \\ 0.2639, 0.1253, 0.7360 \end{pmatrix} \\ A_2 & \begin{pmatrix} [0.0894, 0.3535], \\ [0.5856, 0.8538], \\ [0.4143, 0.8356], \\ 0.9105, 0.6464, 0.1643 \end{pmatrix} & \begin{pmatrix} [0.1258, 0.6314], \\ [0.4383, 0.7254], \\ [0.1819, 0.4641], \\ 0.5616, 0.3685, 0.7254 \end{pmatrix} & \begin{pmatrix} [0.3807, 0.7360], \\ [0.2511, 0.8073], \\ [0.1926, 0.5144], \\ 0.4229, 0.2639, 0.8073 \end{pmatrix} \\ A_3 & \begin{pmatrix} [0.2529, 0.5856], \\ [0.0894, 0.3535], \\ [0.6464, 0.8356], \\ 0.4143, 0.7470, 0.0894 \end{pmatrix} & \begin{pmatrix} [0.3383, 0.8180], \\ [0.1258, 0.4383], \\ [0.5616, 0.6616], \\ 0.5616, 0.1819, 0.6314 \end{pmatrix} & \begin{pmatrix} [0.4855, 0.6597], \\ [0.6597, 0.9387], \\ [0.1253, 0.5144], \\ 0.5144, 0.3402, 0.7360 \end{pmatrix} \\ A_4 & \begin{pmatrix} [0.1643, 0.4647], \\ [0.2529, 0.5856], \\ [0.2844, 0.6464], \\ 0.7470, 0.5352, 0.5856 \end{pmatrix} & \begin{pmatrix} [0.5358, 0.8180], \\ [0.1258, 0.5358], \\ [0.2745, 0.7650], \\ 0.3685, 0.7650, 0.6314 \end{pmatrix} & \begin{pmatrix} [0.2511, 0.8073], \\ [0.3807, 0.7360], \\ [0.2639, 0.5144], \\ 0.6402, 0.4229, 0.8073 \end{pmatrix} \\ A_5 & \begin{pmatrix} [0.0894, 0.5353], \\ [0.1643, 0.5856], \\ [0.2844, 0.7470], \\ 0.6464, 0.4143, 0.7155 \end{pmatrix} & \begin{pmatrix} [0.4383, 0.8180], \\ [0.3383, 0.7254], \\ [0.0904, 0.5616], \\ 0.3685, 0.5616, 0.7254 \end{pmatrix} & \begin{pmatrix} [0.7360, 0.8073], \\ [0.6597, 0.9387], \\ [0.2639, 0.7488], \\ 0.4229, 0.6192, 0.4855 \end{pmatrix} \\ A_6 & \begin{pmatrix} [0.0316, 0.4647], \\ [0.1643, 0.4647], \\ [0.5352, 0.9105], \\ 0.7470, 0.9683, 0.2529 \end{pmatrix} & \begin{pmatrix} [0.2349, 0.7254], \\ [0.6314, 0.9035], \\ [0.2745, 0.5616], \\ 0.3685, 0.7650, 0.3383 \end{pmatrix} & \begin{pmatrix} [0.5770, 0.8073], \\ [0.4855, 0.6597], \\ [0.0612, 0.4229], \\ 0.2639, 0.4229, 0.8073 \end{pmatrix} \end{array}$$

Step 3: The new weights are calculated using the normal distribution method. Let $W = (0.2429, 0.5142, 0.2429)$ be its weighting vector derived by the normal distribution-based method [18].

Step 4: By neutrosophic cubic weighted geometric aggregation operator (NCWG), the decision matrix is aggregated by the new weights assigned to the m attributes.

$$D = \begin{pmatrix} A_1 & \begin{pmatrix} [0.3892, 0.6812], [0.1606, 0.5692], \\ [0.3840, 0.7325], 0.5273, 0.6914, 0.3270 \end{pmatrix} \\ A_2 & \begin{pmatrix} [0.1852, 0.5692], [0.4107, 0.7745], \\ [0.2480, 0.4946], 0.6813, 0.4306, 0.5190 \end{pmatrix} \\ A_3 & \begin{pmatrix} [0.3441, 0.7158], [0.1731, 0.5005], \\ [0.7078, 0.6899], 0.5178, 0.4161, 0.4076 \end{pmatrix} \\ A_4 & \begin{pmatrix} [0.3344, 0.7107], [0.1950, 0.5913], \\ [0.2743, 0.6904], 0.7010, 0.6550, 0.6580 \end{pmatrix} \\ A_5 & \begin{pmatrix} [0.3378, 0.7375], [0.3338, 0.7331], \\ [0.1848, 0.6649], 0.4633, 0.5454, 0.6557 \end{pmatrix} \\ A_6 & \begin{pmatrix} [0.1795, 0.6681], [0.4271, 0.7122], \\ [0.3068, 0.6813], 0.4751, 0.8937, 0.3893 \end{pmatrix} \end{pmatrix}$$

Step 5: The scores are

$$S(A_1) = 0.1542, S(A_2) = 0.1741, S(A_3) = -0.2276, S(A_4) = 0.1297, S(A_5) = 0.0332, S(A_6) = -0.0547,$$

$$S(A_2) > S(A_1) > S(A_4) > S(A_5) > S(A_6) > S(A_3).$$

List of priorities are as follows.

$$A_2 > A_1 > A_4 > A_5 > A_6 > A_3$$

Hence, the project A_1 has the highest potential contribution to the firm's strategic goal of gaining competitive advantage in the industry.

4. Conclusion

This paper was influenced by the impediment of neutrosophic cubic geometric and Einstein geometric collection operators as preliminarily discussed, that is, we observed that the higher the weight component, the aggregated value tended to the corresponding neutrosophic cubic value of that vector. Consequent upon such circumstances, we characterized neutrosophic cubic hybrid and neutrosophic cubic Einstein hybrid aggregation operators. At that point, these operators are outfitted upon a day-by-day life precedent structure industry to organize the potential contributions that serve to achieve the strategic objective of getting favorable circumstances in industry.

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