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Neutrosophic Extended Triplet Group Based on Neutrosophic Quadruple Numbers

Qiaoyan Li [1], Yingcang Ma [1,*], Xiaohong Zhang [2] and Juanjuan Zhang [1]

[1] School of Science, Xi'an Polytechnic University, Xi'an 710048, China; qiaoyan_li@126.con (Q.L.); zhang79jj@126.com (J.Z.)
[2] School of Arts and Sciences, Shaanxi University of Science & Technology, Xi'an 710021, China; zhangxiaohong@sust.edu.cn
* Correspondence: mayingcang@126.com

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Abstract: In this paper, we explore the algebra structure based on neutrosophic quadruple numbers. Moreover, two kinds of degradation algebra systems of neutrosophic quadruple numbers are introduced. In particular, the following results are strictly proved: (1) the set of neutrosophic quadruple numbers with a multiplication operation is a neutrosophic extended triplet group; (2) the neutral element of each neutrosophic quadruple number is unique and there are only sixteen different neutral elements in all of neutrosophic quadruple numbers; (3) the set which has same neutral element is closed with respect to the multiplication operator; (4) the union of the set which has same neutral element is a partition of four-dimensional space.

Keywords: neutrosophic extended triplet group; neutrosophic quadruple numbers; neutrosophic set

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1. Introduction

The notion of a neutrosophic set is proposed by F. Smarandache [1] in order to solve real-world problems and some in-depth analysis and research have been carried out [2–5]. Recently, Smarandache and Ali in [6] proposed a new algebraic system, neutrosophic triplet group (NTG), which different from classical groups. From the original definition of NTG, the neutral element is different from the classical algebraic unit element. By removing this restriction, the neutrosophic extended triplet group (NETG) is proposed in [7,8] and the classical group is regarded as a special case of NETG.

As a new algebraic structure, NTG (NETG) immediately attracted the attention of scholars and conducted in-depth research. These studies are mainly carried out by the following three aspects. Firstly, the structure properties of NTG (NETG) have been studied deeply. For examples, paper [8] has conducted an in-depth analysis of the nature of NTG, and the properties and structural features of NTG are studied by using theoretical analysis and software calculations. In paper [9], the notion of the neutrosophic triplet coset and its relation with the classical coset are proposed and the properties of the neutrosophic triplet cosets are given. The neutrosophic duplet sets, neutrosophic duplet semi-groups, and cancellable neutrosophic triplet groups are proposed and the characterizations of cancellable weak neutrosophic duplet semi-groups are established in paper [10]. In order to explore the structure of the algebraic system $(Z_n, \otimes)$, where $\otimes$ is the classical mod multiplication, paper [11] reveals that for each $n \in Z^+, n \geq 2, (Z_n, \otimes)$ is a commutative NETG if and only if the factorization of n is a product of single factors. Moreover, the generalized neutrosophic extended triplet group (GNETG) is proposed in [11] and verify that for each $n \in Z^+, n \geq 2, (Z_n, \otimes)$ is a commutative GNETG. Secondly, it is the application research on the algebraic system NET. For example, In paper [12], the distinguishing features between an NTG and other algebraic structures are investigated and the first isomorphism theorem was established for NTGs, furthermore, applications of the results on NTG to management and sports are discussed. In paper [13], NTGs and their applications to mathematical models, such as fuzzy cognitive maps model, neutrosophic cognitive maps model and fuzzy relational maps model, are discussed. Thirdly, extend the idea of NTG(NETG) to another algebraic system. For example, in paper [14,15], the extend to Abel–Grassmann groupoid (AG-groupoid) is studied. The neutrosophic triplet ring and a neutrosophic triplet field are discussed in paper [16,17]. A notion of neutrosophic triplet metric space is given and properties of neutrosophic triplet metric spaces are studied in [18]. The notion of neutrosophic triplet v-generalized metric space are introduced in [19]. Paper [20] applies the neutrosophic set theory to pseudo-BCI algebras. The idea of a neutrosophic triplet set to non-associative semihypergroups is given in paper [21]. The above results enrich the research content of the algebraic system NTG (NETG).

In neutrosophic logic, each proposition is approximated to represent respectively the truth (T), the falsehood (F), and the indeterminacy (I), where T, I, F are standard or non-standard subsets of the non-standard unit interval $]0^-, 1^+[= 0^- \cup [0,1] \cup 1^+$. The notion of neutrosophic quadruple number, which is represented by a known part and an unknown part to describe a neutrosophic logic proposition, was introduced by Florentin Smarandache in [22]. The algebra system $(NQ, *)$ based on neutrosophic quadruple numbers are introduced and the properties have discussed [22,23]. In this paper, we will reveal that $(NQ, *)$ is a NETG and some properties are discussed.

The paper is organized as follows. Section 2 gives the basic concepts. In Section 3, $(NQ, *)$ be a NETG is proved and some properties are discussed. In Section 4, two kinds of degradation algebra systems of $(NQ, *)$ are introduced and studied. Finally, the summary and future work are presented in Section 5.

2. Basic Concepts

In this section, we will provide the related basic definitions and properties of NETG and neutrosophic quadruple numbers, the details can be seen in [7,8,22,23].

Definition 1 ([7,8]). *Let N be a non-empty set together with a binary operation $*$. Then, N is called a neutrosophic extended triplet set if for any $a \in N$, there exists a neutral of "a" (denote by $neut(a)$), and an opposite of "a"(denote by $anti(a)$), such that $neut(a) \in N$, $anti(a) \in N$ and:*

$$a * neut(a) = neut(a) * a = a, \quad a * anti(a) = anti(a) * a = neut(a).$$

The triplet $(a, neut(a), anti(a))$ is called a neutrosophic extended triplet.

Definition 2 ([7,8]). *Let $(N, *)$ be a neutrosophic extended triplet set. Then, N is called a neutrosophic extended triplet group (NETG), if the following conditions are satisfied:*

(1) $(N, *)$ is well-defined, i.e., for any $a, b \in N$, one has $a * b \in N$.

(2) $(N, *)$ is associative, i.e., $(a * b) * c = a * (b * c)$ for all $a, b, c \in N$.

A NETG N is called a commutative NETG if for all $a, b \in N, a * b = b * a$.

Proposition 1 ([8]). *Let $(N, *)$ be a NETG. We have:*

(1) $neut(a)$ is unique for any $a \in N$.

(2) $neut(a) * neut(a) = neut(a)$ for any $a \in N$.

(3) $neut(neut(a)) = neut(a)$ for any $a \in N$.

Definition 3 ([22,23]). *A neutrosophic quadruple number is a number of the form (a, bT, cI, dF), where T, I, F have their usual neutrosophic logic meanings and $a, b, c, d \in \mathbb{R}$ or $\mathbb{C}$. The set NQ, defined by*

$$NQ = \{(a, bT, cI, dF) : a, b, c, d \in \mathbb{R} \text{ or } \mathbb{C}\}. \tag{1}$$

is called a neutrosophic set of quadruple numbers. For a neutrosophic quadruple number (a, bT, cI, dF), a is called the known part and (bT, cI, dF) is called the unknown part.

Definition 4 ([22,23])**.** *Let N be a set, endowed with a total order $a \prec b$, named "a prevailed by b" or "a less stronger than b" or "a less preferred than b". We consider $a \preceq b$ as "a prevailed by or equal to b" "a less stronger than or equal to b", or "a less preferred than or equal to b".*

For any elements $a, b \in N$, with $a \preceq b$, one has the absorbance law:

$$a \cdot b = b \cdot a = \text{ absorb}(a, b) = \max(a, b) = b, \tag{2}$$

which means that the bigger element absorbs the smaller element. Clearly,

$$a \cdot a = a^2 = \text{ absorb}(a, a) = \max(a, a) = a. \tag{3}$$

and

$$a_1 \cdot a_2 \cdots a_n = \max(a_1, a_2, \cdots, a_n). \tag{4}$$

Analogously, we say that "$a \succ b$" and we read: "a prevails to b" or "a is stronger than b" or "a is preferred to b". Also, $a \succeq b$, and we read: "a prevails or is equal to b" "a is stronger than or equal to b", or "a is preferred or equal to b".

Definition 5 ([22,23])**.** *Consider the set $\{T, I, F\}$. Suppose in an optimistic way we consider the prevalence order $T \succ I \succ F$. Then we have: $TI = IT = \max(T, I) = T$, $TF = FT = \max(T, F) = T$, $IF = FI = \max(I, F) = I$, $TT = T^2 = T$, $II = I^2 = I$, $FF = F^2 = F$.*

Analogously, suppose in a pessimistic way we consider the prevalence order $T \prec I \prec F$. Then we have: $TI = IT = \max(T, I) = I$, $TF = FT = \max(T, F) = F$, $IF = FI = \max(I, F) = F$, $TT = T^2 = T$, $II = I^2 = I$, $FF = F^2 = F$.

Definition 6 ([22,23])**.** *Let $a = (a_1, a_2T, a_3I, a_4F), b = (b_1, b_2T, b_3I, b_4F) \in NQ$, Suppose in an pessimistic way, the neutrosophic expert considers the prevalence order $T \prec I \prec F$. Then the multiplication operation is defined as following:*

$$\begin{aligned} a * b = \ & (a_1, a_2T, a_3I, a_4F) * (b_1, b_2T, b_3I, b_4F) \\ & = (a_1b_1, (a_1b_2 + a_2b_1 + a_2b_2)T, (a_1b_3 + a_2b_3 + a_3b_1 + a_3b_2 + a_3b_3)I, \\ & \quad (a_1b_4 + a_2b_4 + a_3b_4 + a_4b_1 + a_4b_2 + a_4b_3 + a_4b_4)F). \end{aligned} \tag{5}$$

Suppose in an optimistic way the neutrosophic expert considers the prevalence order $T \succ I \succ F$. Then:

$$\begin{aligned} a \star b = \ & (a_1, a_2T, a_3I, a_4F) * (b_1, b_2T, b_3I, b_4F) \\ & = (a_1b_1, (a_1b_2 + a_2b_1 + a_2b_2 + a_3b_2 + a_4b_2 + a_2b_3 + a_2b_4)T, \\ & \quad (a_1b_3 + a_3b_1 + a_3b_3 + a_3b_4 + a_4b_3)I, (a_1b_4 + a_4b_1 + a_4b_4)F). \end{aligned} \tag{6}$$

Proposition 2 ([22,23])**.** *Let $NQ = \{(a, bT, cI, dF) : a, b, c, d \in \mathbb{R} \text{ or } \mathbb{C}\}$. We have:*
(1) $(NQ, *)$ is a commutative monoid.
(2) $(NQ, \star)$ is a commutative monoid.

3. Main Results

From Proposition 2, we can see that $(NQ, *)$ (or $(NQ, \star)$) be a commutative monoid. In these section, we will show that the algebra system $(NQ, *)$(or $(NQ, \star)$) is a NETG.

Theorem 1. *For the algebra system $(NQ, *)$, for every element $a \in NQ$, there exists the neutral element $neut(a)$ and opposite element $anti(a)$.*

Proof analysis: the proof of this theorem contains two aspects. Firstly, given an element $a \in NQ, a = (a_1, a_2T, a_3I, a_4F), a_i \in \mathbb{R}, i \in \{1,2,3,4\}$. Being a_i can select every element in $\mathbb{R}$, we should discuss from different cases and in each case $netu(a)$ and $anti(a)$ should given. Secondly, we should prove that all the cases discussed above include all the elements in NQ.

Proof. Let $a = (a_1, a_2T, a_3I, a_4F)$, we consider $a_i \in \mathbb{R}, i \in \{1,2,3,4\}$ and the same results can be gotten when $a_i \in \mathbb{C}$.

Set $neut(a) = (b_1, b_2T, b_3I, b_4F), b_i \in \mathbb{R}, i \in \{1,2,3,4\}$ and $anti(a) = (c_1, c_2T, c_3I, c_4F), c_i \in \mathbb{R}, i \in \{1,2,3,4\}$. From Definition 1 we can get $a * neut(a) = a$, that is $a_1b_1 = a_1$ should hold. So we discuss from two cases, $a_1 = 0$ or $a_1 \neq 0$.

Case A: when $a_1 = 0$.

In this case, we have $a = (0, a_2T, a_3I, a_4F)$. From Definition 1, $a * anti(a) = neut(a)$, that is $0 \cdot c_1 = b_1$, so we have $b_1 = 0$, i.e., $neut(a) = (0, b_2T, b_3I, b_4F)$. Moreover, from $a * neut(a) = a$, we have $(0, a_2T, a_3I, a_4F) * (0, b_2T, b_3I, b_4F) = (0, a_2T, a_3I, a_4F)$, so we have $a_2b_2 = a_2$. So we discuss from $a_2 = 0$ or $a_2 \neq 0$.

Case A1: $a_1 = 0, a_2 = 0$. That is, $a = (0, 0, a_3I, a_4F), netu(a) = (0, b_2T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, we have $0c_1 + 0(c_1 + c_2) = b_2$, so $b_2 = 0$, i.e., $netu(a) = (0, 0, b_3I, b_4F)$. From $(0, 0, a_3I, a_4F) * (0, 0, b_3I, b_4F) = (0, 0, a_3I, a_4F)$, we have $a_3b_3 = a_3$. So we discuss from $a_3 = 0$ or $a_3 \neq 0$.

Case A11: $a_1 = a_2 = a_3 = 0$, that is, $a = (0, 0, 0, a_4F), netu(a) = (0, 0, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. In the same way, from $a * anti(a) = neut(a)$, we have $b_3 = 0$, i.e., $netu(a) = (0, 0, 0, b_4F)$. From $(0, 0, 0, a_4F) * (0, 0, 0, b_4F) = (0, 0, 0, a_4F)$, we have $a_4b_4 = a_4$. So we discuss from $a_4 = 0$ or $a_4 \neq 0$.

Case A111: $a_1 = a_2 = a_3 = a_4 = 0$, that is, $a = (0, 0, 0, 0)$, in this case, we can easily get $neut(a) = (0, 0, 0, 0)$ and $anti(a) = (c_1, c_2T, c_3I, c_4F)$, c_i can be chosen arbitrarily in $\mathbb{R}$.

Case A112: $a_1 = a_2 = a_3 = 0, a_4 \neq 0$, being that $a_4b_4 = a_4$ and $a_4 \neq 0$, we have $b_4 = 1$, that is, $a = (0, 0, 0, a_4F), netu(a) = (0, 0, 0, F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $(0, 0, 0, a_4F) * (c_1, c_2T, c_3I, c_4F) = (0, 0, 0, F)$, we have $a_4(c_1 + c_2 + c_3 + c_4) = 1$, so the opposite element of a should satisfy $c_1 + c_2 + c_3 + c_4 = \frac{1}{a_4}, c_i \in \mathbb{R}$.

Case A12: $a_1 = a_2 = 0, a_3 \neq 0$. From $a_3b_3 = a_3$ and $a_3 \neq 0$, we have $b_3 = 1$. That is $a = (0, 0, a_3I, a_4F), netu(a) = (0, 0, I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $(0, 0, a_3I, a_4F) * (0, 0, I, b_4F) = (0, 0, a_3I, a_4F)$, we have $0b_4 + 0b_4 + a_3b_4 + a_4(0 + 0 + 1 + b_4) = a_4$, so $(a_3 + a_4)b_4 = 0$. We discuss from $a_3 + a_4 = 0$ or $a_3 + a_4 \neq 0$.

Case A121: $a_1 = a_2 = 0$, $a_3 \neq 0, a_3 + a_4 = 0$, that is $a = (0, 0, a_3I, -a_3F), neut(a) = (0, 0, I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, that is $(0, 0, a_3I, -a_3F) * (c_1, c_2T, c_3I, c_4F) = (0, 0, I, b_4F)$. So we have $a_3(c_1 + c_2 + c_3) = 1$ and $a_3c_4 - a_3(c_1 + c_2 + c_3 + c_4) = b_4$ i.e., $c_1 + c_2 + c_3 = \frac{1}{a_3}$ and $b_4 = 1$. Thus $neut(a) = (0, 0, I, -F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 + c_2 + c_3 = \frac{1}{a_3}$, c_4 can be chosen arbitrarily in $\mathbb{R}$.

Case A122: $a_1 = a_2 = 0, a_3 \neq 0, a_3 + a_4 \neq 0$. From $(a_3 + a_4)b_4 = 0$, we have $b_4 = 0$. that is $a = (0, 0, a_3I, a_4F), neut(a) = (0, 0, I, 0), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, that is $(0, 0, a_3I, a_4F) * (c_1, c_2T, c_3I, c_4F) = (0, 0, I, 0)$. So we have $a_3(c_1 + c_2 + c_3) = 1$ and $a_3c_4 - a_3(c_1 + c_2 + c_3 + c_4) = 0$ i.e., $c_1 + c_2 + c_3 = \frac{1}{a_3}$ and $c_4 = -\frac{a_4}{a_3(a_3+a-4)}$. Thus $neut(a) = (0, 0, I, 0)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 + c_2 + c_3 = \frac{1}{a_3}$, $c_4 = -\frac{a_4}{a_3(a_3+a_4)}$.

Case A2: when $a_1 = 0, a_2 \neq 0$. From $a_2b_2 = a_2$, we have $b_2 = 1$, that is, $a = (0, 0, a_3I, a_4F), netu(a) = (0, T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. In the same way, from $a * neut(a) = a$, we have $(a_2 + a_3)b_3 = 0$, so we discuss from $a_2 + a_3 = 0$ or $a_2 + a_3 \neq 0$.

Case A21: when $a_1 = 0, a_2 \neq 0, a_2 + a_3 = 0$. that is, $a = (0, a_2T, -a_2I, a_4F), netu(a) = (0, T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. In the same way, from $a * neut(a) = a$, we have $a_4 + a_4(b_3 + b_4) = a_4$, that is $a_4(b_3 + b_4) = 0$, so we discuss from $a_4 = 0$ or $a_4 \neq 0$.

Case A211: when $a_1 = 0, a_2 \neq 0, a_2 + a_3 = 0, a_4 = 0$. that is, $a = (0, a_2T, -a_2I, 0), netu(a) = (0, T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $(0, a_2T, -a_2I, 0) * (c_1, c_2T, c_3I, c_4F) = (0, T, b_3I, b_4F)$, so we have $a_2(c_1 + c_2) = 1$ and $-a_2(c_1 + c_2) = b_3$, that is $b_3 = -1$. In the same way, we can get $b_4 = 0$. Thus $neut(a) = (0, T, -I, 0)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 + c_2 = \frac{1}{a_2}$, c_3, c_4 can be chosen arbitrarily in $\mathbb{R}$.

Case A212: when $a_1 = 0, a_2 \neq 0, a_2 + a_3 = 0, a_4 \neq 0$, From $a_4(b_3 + b_4) = 0$, we have $b_3 + b_4 = 0$, that is, $a = (0, a_2T, -a_2I, a_4F)$, $netu(a) = (0, T, b_3I, -b_3F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $(0, a_2T, -a_2I, a_4) * (c_1, c_2T, c_3I, c_4F) = (0, T, b_3I, -b_3F)$, so we have $a_2(c_1 + c_2) = 1$ and $-a_2(c_1 + c_2) = b_3$, i.e., $b_3 = -1$. Thus $neut(a) = (0, T, -I, F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 + c_2 = \frac{1}{a_2}$, $c_3 + c_4 = \frac{1}{a_4} - \frac{1}{a_2}$.

Case A22: when $a_1 = 0, a_2 \neq 0, a_2 + a_3 \neq 0$. From $(a_2 + a_3)b_3 = 0$, we have $b_3 = 0$. that is, $a = (0, a_2T, -a_2I, a_4F), netu(a) = (0, T, 0, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * neut(a) = a$, we have $(a_2 + a_3 + a_4)b_4 = 0$, so we discuss from $a_2 + a_3 + a_4 = 0$ or $a_2 + a_3 + a_4 \neq 0$.

Case A221: when $a_1 = 0, a_2 \neq 0, a_2 + a_3 \neq 0, a_2 + a_3 + a_4 = 0$. In this case $a = (0, a_2T, a_3I, a_4F), netu(a) = (0, T, 0, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $(0, a_2T, a_3I, a_4F) * (c_1, c_2T, c_3I, c_4F) = (0, T, 0, b_4F)$, so we have $a_2(c_1 + c_2) = 1$, $c_3 = -\frac{a_3}{a_2(2+a_3)}$, $(a_2 + a_3 + a_4)b_4 + a_4(c_1 + c_2 + c_3) = b_4$, so we have $b_4 = -1$. Thus $neut(a) = (0, T, 0, -F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 + c_2 = \frac{1}{a_2}, c_3 = -\frac{a_3}{a_2(2+a_3)}$, c_4 can be chosen arbitrarily in $\mathbb{R}$.

Case A222: when $a_1 = 0, a_2 \neq 0, a_2 + a_3 \neq 0, a_2 + a_3 + a_4 \neq 0$. From $(a_2 + a_3 + a_4)b_4 = 0$, we have $b_4 = 0$. that is, $a = (0, a_2T, a_3I, a_4F), netu(a) = (0, T, 0, 0), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $(0, a_2T, a_3I, 0) * (c_1, c_2T, c_3I, c_4F) = (0, T, 0, 0)$, so we have $a_2(c_1 + c_2) = 1$, $c_3 = -\frac{a_3}{a_2(2+a_3)}$, $(a_2 + a_3 + a_4)b_4 + a_4(c_1 + c_2 + c_3) = 0$, Thus $neut(a) = (0, T, 0, 0)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 + c_2 = \frac{1}{a_2}, c_3 = -\frac{a_3}{a_2(2+a_3)}, c_4 = -\frac{a_4}{(a_2+a_3)(a_2+a_3+a_4)}$.

Case B: when $a_1 \neq 0$.

In this case, from $a_1b_1 = a_1$ and $a_1 \neq 0$, we have $b_1 = 1$. That is $a = (a_1, a_2T, a_3I, a_4F), neut(a) = (1, b_2T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From Definition 1, $a * neut(a) = a$, that is $a_1b_2 + a_2 + a_2b_2 = a_2$, so $(a_1 + a_2)b_2 = 0$. So we discuss from $a_1 + a_2 = 0$ or $a_1 + a_2 \neq 0$.

Case B1: when $a_1 \neq 0, a_1 + a_2 = 0$. That is $a = (a_1, -a_1T, a_3I, a_4F), neut(a) = (1, b_2T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, we have $c_1 = \frac{1}{a_1}$, $a_1c_2 - a_1c_1 - a_1c_2 = b_2$, so $b_2 = -1$. From $a * neut(a) = a$, so we have $a_3 + a_3b_2 + a_3b_3 = a_3$, i.e., $a_3(b_2 + b_3) = 0$. So we discuss from $a_3 = 0$ or $a_3 \neq 0$.

Case B11: when $a_1 \neq 0, a_1 + a_2 = 0, a_3 = 0$. That is $a = (a_1, -a_1T, 0, a_4F), neut(a) = (1, -T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * neut(a) = a$, we have $a_1b_4 - a_1b_4 - 0b_4 + a_4(1 - 1 + b_3 + b_4) = a_4$, i.e., $a_4(b_3 + b_4) = a_4$. So we discuss from $a_4 = 0$ or $a_4 \neq 0$.

Case B111: when $a_1 \neq 0, a_1 + a_2 = 0, a_3 = 0, a_4 = 0$. That is $a = (a_1, -a_1T, 0, 0), neut(a) = (1, -T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, i.e., $(a_1, -a_1T, 0, 0) * (c_1, c_2T, c_3I, c_4F) = (1, -T, b_3I, b_4F)$, we have $c_1 = \frac{1}{a_1}$, $b_3 = b_4 = 0$. Thus $neut(a) = (1, -T, 0, 0)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, which satisfies $c_1 = \frac{1}{a_1}$ and c_2, c_3, c_4 can be chosen arbitrarily in $\mathbb{R}$.

Case B112: when $a_1 \neq 0, a_1 + a_2 = 0, a_3 = 0, a_4 \neq 0$. From $a_4(b_3 + b_4) = a_4$, we have $b_3 + b_4 = 1$. That is $a = (a_1, -a_1T, 0, a_4F), neut(a) = (1, -T, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, i.e., $(a_1, -a_1T, 0, a_4F) * (c_1, c_2T, c_3I, c_4F) = (1, -T, b_3I, b_4F)$, we have $c_1 = \frac{1}{a_1}$, $b_3 = 0, b_4 = 1$. Thus $neut(a) = (1, -T, 0, F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 = \frac{1}{a_1}$ and $c_2 + c_3 + c_4 = \frac{1}{a_4} - \frac{1}{a_1}$.

Case B12: when $a_1 \neq 0, a_1 + a_2 = 0, a_3 \neq 0$. From $a_3(b_2 + b_3) = 0$ and $a_3 \neq 0$, we have $b_2 + b_3 = 0$, i.e., $b_3 = 1$. That is $a = (a_1, -a_1T, a_3I, a_4F), neut(a) = (1, -T, I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * neut(a) = a$, we have $a_3b_4 + a_4 + a_4b_4 = a_4$, i.e., $(a_3 + a_4)b_4 = 0$. So we discuss from $a_3 + a_4 = 0$ or $a_3 + a_4 \neq 0$.

Case B121: when $a_1 \neq 0, a_1 + a_2 = 0, a_3 \neq 0, a_3 + a_4 = 0$. That is $a = (a_1, -a_1T, a_3I, -a_3F)$, $neut(a) = (1, -T, I, b_4F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, i.e.,

$(a_1, -a_1T, a_3I, -a_3F) * (c_1, c_2T, c_3I, c_4F) = (1, -T, I, b_4F)$, we have $c_1 = \frac{1}{a_1}$, $c_2 + c_3 = \frac{1}{a_3} - \frac{1}{a_1}$, $-a_3(c_1 + c_2 + c_3) = b_4$, i.e., $b_4 = -1$. Thus $neut(a) = (1, -T, I, -F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 = \frac{1}{a_1}$, $c_2 + c_3 = \frac{1}{a_3} - \frac{1}{a_2}$, c_4 can be chosen arbitrarily in $\mathbb{R}$.

Case B122: when $a_1 \neq 0, a_1 + a_2 = 0, a_3 \neq 0, a_3 + a_4 \neq 0$, from $(a_3 + a_4)b_4 = 0$, we have $b_4 = 0$. That is $a = (a_1, -a_1T, a_3I, a_4F), neut(a) = (1, -T, I, 0), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, i.e., $(a_1, -a_1T, a_3I, a_4F) * (c_1, c_2T, c_3I, c_4F) = (1, -T, I, 0)$, we have $c_1 = \frac{1}{a_1}, c_2 + c_3 = \frac{1}{a_3} - \frac{1}{a_1}$, $c_4 = -\frac{a_4}{a_3(a_3+a_4)}$. Thus $neut(a) = (1, -T, I, -F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 = \frac{1}{a_1}$, $c_2 + c_3 = \frac{1}{a_3} - \frac{1}{a_1}$, $c_4 = -\frac{a_4}{a_3(a_3+a_4)}$.

Case B2: when $a_1 \neq 0, a_1 + a_2 \neq 0$, from $(a_1 + a_2)b_2 = 0$, we have $b_2 = 0$. That is $a = (a_1, -a_1T, a_3I, a_4F), neut(a) = (1, 0, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * neut(a) = a$, so we have $a_1b_3 + a_2b_3 + a_3 + a_3b_3 = a_3$, i.e., $(a_1 + a_2 + a_3)b_3 = 0$. So we discuss from $a_1 + a_2 + a_3 = 0$ or $a_1 + a_2 + a_3 \neq 0$.

Case B21: when $a_1 \neq 0, a_1 + a_2 \neq 0, a_1 + a_2 + a_3 = 0$. That is $a = (a_1, a_2T, a_3I, a_4F), neut(a) = (1, 0, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * neut(a) = a$, so we have $(a_1 + a_2 + a_3)b_4 + a_4 + a_4b_3 + a_4b_4 = a_4$, i.e., $(b_3 + b_4)a_4 = 0$. So we discuss from $a_4 = 0$ or $a_4 \neq 0$.

Case B211: when $a_1 \neq 0, a_1 + a_2 \neq 0, a_1 + a_2 + a_3 = 0, a_4 = 0$. That is $a = (a_1, a_2T, a_3I, 0), neut(a) = (1, 0, b_3I, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, i.e., $(a_1, a_2T, a_3I, 0) * (c_1, c_2T, c_3I, c_4F) = (1, 0, b_3I, b_4F)$, we have $c_1 = \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_1(a_1+a_2)}$. $a_3(c_1 + c_2) = b_3$, $(a_1 + a_2 + a_3)c_4 + 0(c_1 + c_2 + c_3 + c_4) = 0$, which means $b_3 = -1, b_4 = 0$. Thus $neut(a) = (1, 0, -I, 0)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 = \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_1(a_1+a_2)}$, c_3, c_4 can be chosen arbitrarily in $\mathbb{R}$.

Case B212: when $a_1 \neq 0, a_1 + a_2 \neq 0, a_1 + a_2 + a_3 = 0, a_4 \neq 0$. From $(b_3 + b_4)a_4 = 0$, we have $b_3 + b_4 = 0$. That is $a = (a_1, a_2T, a_3I, a_4F), neut(a) = (1, 0, b_3I, -b_3F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, i.e., $(a_1, a_2T, a_3I, a_4F) * (c_1, c_2T, c_3I, c_4F) = (1, 0, b_3I, -b_3F)$, we have $c_1 = \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_1(a_1+a_2)}$. $a_3(c_1 + c_2) = b_3$, i.e., $b_3 = -1, b_4 = 1$. Thus $neut(a) = (1, 0, -I, F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 = \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_1(a_1+a_2)}$, $c_3 + c_4 = \frac{1}{a_4} - \frac{1}{a_1+a_2}$.

Case B22: when $a_1 \neq 0, a_1 + a_2 \neq 0, a_1 + a_2 + a_3 \neq 0$, from $(a_1 + a_2 + a_3)b_3 = 0$, we have $b_3 = 0$. That is $a = (a_1, a_2T, a_3I, a_4F), neut(a) = (1, 0, 0, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * neut(a) = a$, so we have $(a_1 + a_2 + a_3 + a_4)b_4 + a_4 = a_4$, i.e., $(a_1 + a_2 + a_3 + a_4)b_4 = 0$. So we discuss from $a_1 + a_2 + a_3 + a_4 = 0$ or $a_1 + a_2 + a_3 + a_4 \neq 0$.

Case B221: when $a_1 \neq 0, a_1 + a_2 \neq 0, a_1 + a_2 + a_3 \neq 0, a_1 + a_2 + a_3 + a_4 = 0$, That is $a = (a_1, a_2T, a_3I, a_4F), neut(a) = (1, 0, 0, b_4F), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, i.e., $(a_1, a_2T, a_3I, a_4F) * (c_1, c_2T, c_3I, c_4F) = (1, 0, 0, b_4F)$, we have $c_1 = \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_1(a_1+a_2)}$. $c_3 = -\frac{a_3}{(a_1+a_2)(a_1+a_2+a_3)}$, $a_4(c_1 + c_2 + c_3) = b_4$, so $b_4 = -1$. Thus $neut(a) = (1, 0, 0, -F)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 = \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_1(a_1+a_2)}$, $c_3 = -\frac{a_3}{(a_1+a_2)(a_1+a_2+a_3)}$, c_4 can be chosen arbitrarily in $\mathbb{R}$.

Case B222: when $a_1 \neq 0, a_1 + a_2 \neq 0, a_1 + a_2 + a_3 \neq 0, a_1 + a_2 + a_3 + a_4 \neq 0$. From $(a_1 + a_2 + a_3 + a_4)b_4 + a_4 = a_4$, we have $b_4 = 0$. That is $a = (a_1, a_2T, a_3I, a_4F), neut(a) = (1, 0, 0, 0), anti(a) = (c_1, c_2T, c_3I, c_4F)$. From $a * anti(a) = neut(a)$, i.e., $(a_1, a_2T, a_3I, a_4F) * (c_1, c_2T, c_3I, c_4F) = (1, 0, 0, 0)$, we have $c_1 = \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_1(a_1+a_2)}$, $c_3 = -\frac{a_3}{(a_1+a_2)(a_1+a_2+a_3)}$. $(a_1 + a_2 + a_3 + a_4)c_4 + a_4(c_1 + c_2 + c_3) = 0$, i.e., $c_4 = -\frac{a_4}{(a_1+a_2+a_3)(a_1+a_2+a_3+a_4)}$. Thus $neut(a) = (1, 0, 0, 0)$, $anti(a) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 = \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_1(a_1+a_2)}$, $c_3 = -\frac{a_3}{(a_1+a_2)(a_1+a_2+a_3)}$, $c_4 = -\frac{a_4}{(a_1+a_2+a_3)(a_1+a_2+a_3+a_4)}$.

Finally, we should show that all the above cases include each element $a \in NQ$, i.e., $a_i, i = 1, 2, 3, 4$ can take all the values on $\mathbb{R}$. It is obvious that a_1 can take all the values on $\mathbb{R}$ because $a_1 = 0$ according to case A and that $a_1 \neq 0$ according to case B. Moreover, for case A, a_2 can take all the values on $\mathbb{R}$ because case A1 according to $a_2 = 0$ and case A2 according to $a_2 \neq 0$. For case B, a_2 can take all the values on $\mathbb{R}$ because case B1 according to $a_1 + a_2 = 0$ and case B2 according to $a_1 + a_2 \neq 0$. That is for each element $a = (a_1, a_2, a_3, a_4) \in NQ$, a_1, a_2 can select all of value in $\mathbb{R}$. We will verify that a_3 and a_4 can take all the values on $\mathbb{R}$ when case A1 or A2 or B1 or B2 respectively.

For case A1, a_3 can take all the value in $\mathbb{R}$ because case A11 according to $a_3 = 0$ and case A12 according to $a_3 \neq 0$. Similarly, for case A11, a_4 can take all the value in $\mathbb{R}$ because case A111 according to $a_4 = 0$ and case A112 according to $a_4 \neq 0$. For case A12, a_4 can take all the value in $\mathbb{R}$ because case A121 according to $a_3 + a_4 = 0$ and case A122 according to $a_3 + a_4 = 0$. The top left subgraph of Figure 1 shows that the four cases A111, A112, A211 and A222. The unique $\square$ point represents the case A111, the $+$ points represent the case A112, the $*$ points represent the case A121 and the $\bullet$ points represent the case A122. This explain the that for case A1, a_3 and a_4 can take all the points on the plane. For case A2, B1 or B2, we can get that a_3 and a_4 can take all the points on the plane respectively. The top right subgraph of Figure 1 represents the case A2 if we select $a_1 = 0, a_2 = 1$, the bottom left subgraph of Figure 1 represents the case B1 if we select $a_1 = 1, a_2 = -1$ and bottom right subgraph of Figure 1 represents the case B2 if we select $a_1 = 1, a_2 = 0$. The figure intuitively illustrates that all the points $(a_1, a_2, a_3, a_4), a_i \in \mathbb{R}$ are included.

Through the above analysis, we can get that for each element $a \in NQ$, there exists the neutral element $neut(a)$ and opposite element $anti(a)$. $\square$

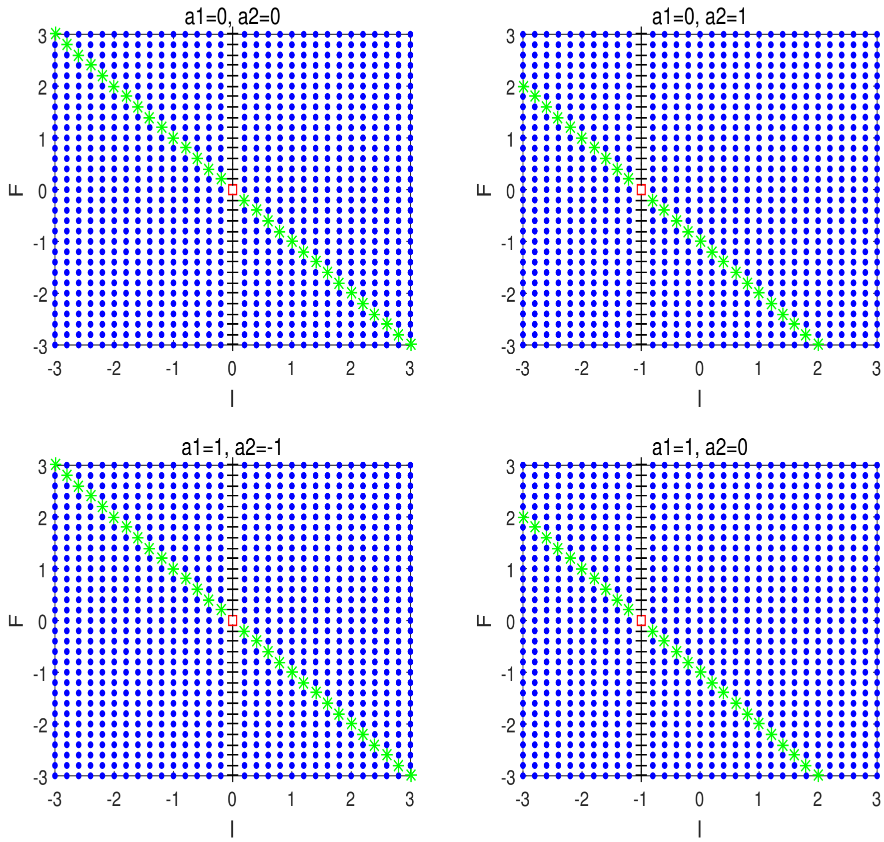

Figure 1. The demonstration figure shows that case A1 ($a_1 = a_2 = 0$, the top left subgraph) or A2 (select $a_1 = 0, 0 \neq a_2 = 1$, the top right subgraph) or B1 (Select $a_1 \neq 0, a_2 = -1$ which means $a_1 + a_2 = 0$, the bottom left subgraph) or B2 (select $a_1 = 1, a_2 = 0$, which means $a_1 + a_2 \neq 0$, the bottom right subgraph) can take all the values on the plane.

For algebra system $(NQ, *)$, Table 1 gives all the subset, which has the same neutral element, and the corresponding neutral element and opposite elements.

Table 1. The corresponding neutral element and opposite elements for $(NQ, *)$.

The Subset of NQ	Neutral Elements	Opposite Element (c_1, c_2T, c_3I, c_4F)
$\{(0,0,0,0)\}$	$(0,0,0,0)$	$c_i \in \mathbb{R}$
$\{(0,0,0,a_4F)\|a_4 \neq 0\}$	$(0,0,0,F)$	$c_1+c_2+c_3+c_4 = \frac{1}{a_4}$
$\{(0,0,a_3I,-a_3F)\|a_3 \neq 0\}$	$(0,0,I,-F)$	$c_1+c_2+c_3 = \frac{1}{a_3}, c_4 \in \mathbb{R}$
$\{(0,0,a_3I,a_4F)\|a_3 \neq 0, a_3+a_4 \neq 0\}$	$(0,0,I,0)$	$c_1+c_2+c_3 = \frac{1}{a_3}, c_4 = -\frac{a_4}{a_3(a_3+a_4)}$
$\{(0,a_2T,-a_2I,0)\|a_2 \neq 0\}$	$(0,T,-I,0)\}$	$c_1+c_2 = \frac{1}{a_2}, c_3, c_4 \in \mathbb{R}$
$\{(0,a_2T,-a_2I,a_4F)\|a_2 \neq 0, a_4 \neq 0\}$	$(0,T,-I,F)$	$c_1+c_2 = \frac{1}{a_2}, c_3+c_4 = \frac{1}{a_4} - \frac{1}{a_2}$
$\{(0,a_2T,a_3I,a_4F)\|a_2 \neq 0, a_2+a_3 \neq 0, a_2+a_3+a_4 = 0\}$	$(0,T,0,-F)$	$c_1+c_2 = \frac{1}{a_2}, c_3 = -\frac{a_3}{a_2(a_2+a_3)}, c_4 \in \mathbb{R}$
$\{(0,a_2T,a_3I,a_4F)\|a_2 \neq 0, a_2+a_3 \neq 0, a_2+a_3+a_4 \neq 0\}$	$(0,T,0,0)$	$c_1+c_2 = \frac{1}{a_2}, c_3 = -\frac{a_3}{a_2(a_2+a_3)},$ $c_4 = -\frac{a_4}{(a_2+a_3)(a_2+a_3+a_4)}$
$\{(a_1,-a_1T,0,0)\|a_1 \neq 0\}$	$(1,-T,0,0)\}$	$c_1 = \frac{1}{a_1}, c_2, c_3, c_4 \in \mathbb{R}$
$\{(a_1,-a_1T,0,a_4F)\|a_1 \neq 0, a_4 \neq 0\}$	$(1,-T,0,F)$	$c_1 = \frac{1}{a_1}, c_2+c_3+c_4 = \frac{1}{a_4} - \frac{1}{a_1}$
$\{(a_1,-a_1T,a_3I,-a_3F)\|a_1 \neq 0, a_3 \neq 0\}$	$(1,-T,I,-F)$	$c_1 = \frac{1}{a_1}, c_2+c_3 = \frac{1}{a_3} - \frac{1}{a_1}, c_4 \in \mathbb{R}$
$\{(a_1,-a_1T,a_3I,a_4F)\|a_1 \neq 0, a_3 \neq 0, a_3+a_4 \neq 0\}$	$(1,-T,I,0)$	$c_1 = \frac{1}{a_1}, c_2+c_3 = \frac{1}{a_3} - \frac{1}{a_1},$ $c_4 = -\frac{a_4}{a_3(a_3+a_4)}$
$\{(a_1,a_2T,a_3I,0)\|a_1 \neq 0, a_1+a_2 \neq 0, a_1+a_2+a_3 = 0\}$	$(1,0,-I,0)$	$c_1 = \frac{1}{a_1}, c_2 = -\frac{a_2}{a_1(a_1+a_2)}, c_3, c_4 \in \mathbb{R}$
$\{(a_1,a_2T,a_3I,a_4F)\|a_1 \neq 0, a_1+a_2 \neq 0, a_1+a_2+a_3 = 0, a_4 \neq 0\}$	$(1,0,-I,F)$	$c_1 = \frac{1}{a_1}, c_2 = -\frac{a_2}{a_1(a_1+a_2)},$ $c_3+c_4 = \frac{1}{a_4} - \frac{1}{a_1+a_2}$
$\{(a_1,a_2T,a_3I,a_4F)\|a_1 \neq 0, a_1+a_2 \neq 0, a_1+a_2+a_3 \neq 0, a_1+a_2+a_3+a_4 = 0\}$	$(1,0,0,-F)$	$c_1 = \frac{1}{a_1}, c_2 = -\frac{a_2}{a_1(a_1+a_2)},$ $c_3 = -\frac{a_3}{(a_1+a_2)(a_1+a_2+a_3)}, c_4 \in \mathbb{R}$
$\{(a_1,a_2T,a_3I,a_4F)\|a_1 \neq 0, a_1+a_2 \neq 0, a_1+a_2+a_3 \neq 0, a_1+a_2+a_3+a_4 \neq 0\}$	$(1,0,0,0)$	$c_1 = \frac{1}{a_1}, c_2 = -\frac{a_2}{a_1(a_1+a_2)},$ $c_3 = -\frac{a_3}{(a_1+a_2)(a_1+a_2+a_3)},$ $c_4 = -\frac{a_4}{(a_1+a_2+a_3)(a_1+a_2+a_3+a_4)}$

Example 1. *For the algebra system $(NQ, *)$, if $a = (a_1, a_2, a_3, a_4) = (1, -T, 3I, -F)$, i.e., $a_1 \neq 0, a_1 + a_2 = 0, a_3 \neq 0, a_3 + a_4 \neq 0$, then from Table 1, we can get $neut(a) = (1, -T, I, 0)$. Let $anti(a) = (c_1, c_2T, c_3I, c_4F)$, so $c_1 = \frac{1}{a_1} = 1, c_2 + c_3 = \frac{1}{a_3} - \frac{1}{a_1} = -\frac{2}{3}, c_4 = -\frac{a_4}{a_3(a_3+a_4)} = \frac{1}{6}$, so $anti(a) = (1, c_2T, c_3I, \frac{1}{6}F)$, where $c_2 + c_3 = -\frac{2}{3}$. Thus we can easily get the neutral element and opposite elements of each neutrosophic quadruple number. For more examples, see the following:*

1. *Let $b = (0, 0, I, -F)$, then $neut(b) = (0, 0, I, -F)$ and $anti(b) = (c_1, c_2T, c_3I, c_4F)$, where $c_1 + c_2 + c_3 = 1$, c_4 can be can be chosen arbitrarily in $\mathbb{R}$.*
2. *Let $c = (1, T, I, -F)$, then $neut(c) = (1, 0, 0, 0)$ and $anti(c) = (1, -\frac{1}{2}T, -\frac{1}{6}I, \frac{1}{6}F)$.*
3. *Let $d = (0, T, I, F)$, then $neut(d) = (0, T, 0, 0)$ and $anti(d) = (c_1, c_2T, -\frac{1}{2}I, -\frac{1}{6}F)$, where $c_1 + c_2 = 1$.*

In the following, we will discuss the algebra structure properties of $(NQ, *)$.

Proposition 3. *For algebra system $(NQ, *)$, let $NS = \{neut(a) | a \in NQ\}$, we have:*

(1) $NS = \{(1,0,0,0), (0,0,0,F), (0,0,I,-F), (0,0,I,0), (0,T,-I,0), (0,T,-I,F), (0,T,0,-F), (0,T,0,0), (1,-T,0,0), (1,-T,0,F), (1,-T,I,-F), (1,-T,I,0), (1,0,-I,0), (1,0,-I,F), (1,0,0,-F), (1,0,0,0)\}$.

(2) NS is closed with respect to operation $*$.

(3) Set $IS = \{a|a^2 = a, a \in NQ\}$, which is all the set of idempotent elements of $(NQ, *)$, then $NS = IS$.

Proof. (1) Obviously.

(2) If $c, d \in NS$, that is $neut(a) = c, neut(b) = d, a, b \in NQ$. From Proposition 1, $neut(a) * neut(b) = neut(a * b)$, i.e., $c * d = neut(a * b)$, then form Theorem 1, every element in NQ has neutral element, so $a * b$ also has neutral element, that is $neut(a * b) \in NS$, i.e., $c * d \in NS$, thus NS is closed with respect to operation $*$.

(3) From Proposition 1, $neut(a) * neut(a) = neut(a)$, so $neut(a)$ is a idempotent element and $NS \subseteq IS$. On the other hand if a is a idempotent element, so $a * a = a$, that is a exists the neutral element a and the opposite element a, so a is a neutral element, that is $IS \subseteq NS$. Thus $NS = IS$. □

Proposition 4. *For algebra system $(NQ, *)$, let $V_c = \{a|a \in NQ \wedge neut(a) = c\}$, $V_{c*d} = \{a * b|a, b \in NQ \wedge neut(a) = c \wedge neut(b) = d\}$, we have:*

(1) V_c is closed with respect to operation $*$.

(2) V_{c*d} is closed with respect to operation $*$.

Proof. (1) If $a, b \in V_c$, that is $neut(a) = neut(b) = c$. From Proposition 1, $neut(a) * neut(b) = neut(a * b)$, we can see that $neut(a * b) = neut(a) = c$, i.e., the neutral element of $a * b$ is the neutral element of a, so $a * b \in V_c$, that is V_c is closed with respect to operation $*$.

(2) If $a_1 * b_1, a_2 * b_2 \in V_{c*d}$, i.e., $neut(a_1) = neut(a_2) = c, neut(b_1) = neut(b_2) = d$. From Proposition 3(2), $a_1 * a_2 = a_3 \in V_c$, $b_1 * b_2 = b_3 \in V_d$, so $neut(a_3) = c, neut(b_3) = d$, from $(a_1 * b_1) * (a_2 * b_2) = a_3 * b_3$, so $neut(a_1 * a_2 * b_1 * b_2) = neut(a_3 * a_4)$, that is $a_3 * a_4 \in V_{c*d}$, that means $a_1 * a_2 * b_1 * b_2 \in V_{c*d}$. Thus V_{c*d} is closed with respect to operation $*$. □

Definition 7. *Assume that $(N, *)$ is a neutrosophic triplet group and H be a nonempty subset of N. Then H is called a neutrosophic triplet subgroup of N if;*

(1) $a * b \in H$ for all $a, b \in H$;

(2) there exists $anti(a) \in \{anti(a)\}$ such that $anti(a) \in H$ for all $a \in H$, where $\{anti(a)\}$ is the set of opposite element of a in $(N, *)$.

Theorem 2. *For algebra system $(NQ, *)$, let $V_c = \{a|a \in NQ \wedge neut(a) = c\}$, $V_{c*d} = \{a * b|a, b \in NQ \wedge neut(a) = c, neut(b) = d\}$, we have:*

(1) V_c is a neutrosophic triplet subgroup of NQ.

(2) V_{c*d} is a neutrosophic triplet subgroup of NQ.

Proof. (1) From Proposition 3, we can see that V_c is closed with respect to operation $*$. In the following, we will prove there exists $anti(a) \in \{anti(a)\}$ such that $anti(a) \in V_c$ for all $a \in V_c$.

Proof by contradiction.

Assume that $\{anti(a)\} \cap V_c = \emptyset$. From Proposition 1 we can see that $a * anti(a) = c$. On the other hand, $anti(a) \in NQ$, so $anti(a)$ exists neutral element, denoted by $neut(anti(a))$. Being $anti(a) \notin V_c$, so $neut(anti(a)) \neq c$.

From $a * anti(a) = c$, we have $a * anti(a) * neut(anti(a)) = c * neut(anti(a))$, being $anti(a) * neut(anti(a)) = anti(a)$ and $a * anti(a) = c$, we have $c * neut(anti(a)) = c$, and then we have $a * c * neut(anti(a)) = a * c = a$, that means $a * neut(anti(a)) = a$, so $neut(anti(a))$ is also a neutral element of a. This leads to the contradiction being the uniqueness of neutral element for each element. Therefore $\{anti(a)\} \cap V_c \neq \emptyset$. Thus from Definition 7, V_c is a neutrosophic triplet subgroup of NQ.

(2) The same way we can get V_{c*d} is a neutrosophic triplet subgroup of NQ. □

Theorem 3. *For algebra system $(NQ, *)$, let $V_c = \{a|a \in NQ \wedge neut(a) = c\}$, we have:*

(1) $V_c \cap V_d = \emptyset$ if $c \neq d$.

(2) $NQ = \cup_{c \in NS} V_c$. So $\cup_{c \in NS} V_c$ is a partition of NQ, where NS is a set, which contains all the neutral elements of $(NQ, *)$.

Proof. (1) Proof by contradiction.

Assume $V_c \cap V_d \neq \emptyset$ when $c \neq d$, so exist $a \in V_c \cap V_d$, such that a has two neutral elements c and d. This leads to the contradiction being the uniqueness of neutral element. So $V_c \cap V_d = \emptyset$ if $c \neq d$.

(2) From the proof of Theorem 1, we can get $NQ = \cup_{c \in NS} V_c$. So $\cup_{c \in NS} V_c$ is a partition of NQ. □

For the algebra system $(NQ, \star)$, we have the similar results. We describe as following and omit the proof.

Theorem 4. *For the algebra system $(NQ, \star)$, for every element $a \in NQ$, there exists the neutral element $neut(a)$ and opposite element $anti(a)$.*

For algebra system $(NQ, \star)$, Table 2 gives all the subset, which has the same neutral element, and the corresponding neutral element and opposite elements.

Table 2. The corresponding neutral element and opposite elements for $(NQ, \star)$.

The Subset of NQ	Neutral Elements	Opposite Element (c_1, c_2T, c_3I, c_4F)
$\{(0,0,0,0)\}$	$(0,0,0,0)$	$c_i \in \mathbb{R}$
$\{(0, a_2T, 0, 0) \mid a_2 \neq 0\}$	$(0, T, 0, 0)$	$c_1 + c_2 + c_3 + c_4 = \frac{1}{a_2}$
$\{(0, -a_3T, a_3I, 0) \mid a_3 \neq 0\}$	$(0, -T, I, 0)$	$c_1 + c_3 + c_4 = \frac{1}{a_3}, c_2 \in \mathbb{R}$
$\{(0, a_2T, a_3I, a_4F) \mid a_3 \neq 0, a_2 + a_3 \neq 0\}$	$(0, 0, I, 0)$	$c_1 + c_3 + c_4 = \frac{1}{a_3}$, $c_2 = -\frac{a_2}{a_3(a_2+a_3)}$
$\{(0, 0, -a_4I, a_4F) \mid a_4 \neq 0\}$	$(0, 0, -I, F)\}$	$c_1 + c_4 = \frac{1}{a_4}, c_2, c_3 \in \mathbb{R}$
$\{(0, a_2T, -a_4I, a_4F) \mid a_2 \neq 0, a_4 \neq 0\}$	$(0, T, -I, F)$	$c_1 + c_4 = \frac{1}{a_4}, c_2 + c_3 = \frac{1}{a_2} - \frac{1}{a_4}$
$\{(0, a_2T, a_3I, a_4F) \mid a_4 \neq 0, a_3 + a_4 \neq 0, a_2 + a_3 + a_4 = 0\}$	$(0, -T, 0, F)$	$c_1 + c_4 = \frac{1}{a_4}, c_3 = -\frac{a_3}{a_4(a_3+a_4)}, c_2 \in \mathbb{R}$
$\{(0, a_2T, a_3I, a_4F) \mid a_4 \neq 0, a_3 + a_4 \neq 0, a_2 + a_3 + a_4 \neq 0\}$	$(0, 0, 0, F)$	$c_1 + c_4 = \frac{1}{a_4}, c_3 = -\frac{a_3}{a_4(a_3+a_4)}$, $c_2 = -\frac{a_2}{(a_3+a_4)(a_2+a_3+a_4)}$
$\{(a_1, 0, 0, -a_1F) \mid a_1 \neq 0\}$	$(1, 0, 0, -F)\}$	$c_1 = -\frac{1}{a_1}, c_2, c_3, c_4 \in \mathbb{R}$
$\{(a_1, a_2T, 0, -a_1F) \mid a_1 \neq 0, a_2 \neq 0\}$	$(1, T, 0, -F)$	$c_1 = \frac{1}{a_1}, c_2 + c_3 + c_4 = \frac{1}{a_2} - \frac{1}{a_1}$
$\{(a_1, -a_3T, a_3I, -a_1F) \mid a_1 \neq 0, a_3 \neq 0\}$	$(1, -T, I, -F)$	$c_1 = \frac{1}{a_1}, c_3 + c_4 = \frac{1}{a_3} - \frac{1}{a_1}, c_4 \in \mathbb{R}$
$\{(a_1, a_2T, a_3I, -a_1F) \mid a_1 \neq 0, a_3 \neq 0, a_2 + a_3 \neq 0\}$	$(1, 0, I, -F)$	$c_1 = \frac{1}{a_1}, c_3 + c_4 = \frac{1}{a_3} - \frac{1}{a_1}$, $c_2 = -\frac{a_2}{a_3(a_2+a_3)}$
$\{(a_1, 0, a_3I, a_4F) \mid a_1 \neq 0, a_1 + a_4 \neq 0, a_1 + a_3 + a_4 = 0\}$	$(1, 0, -I, 0)$	$c_1 = \frac{1}{a_1}, c_4 = -\frac{a_4}{a_1(a_1+a_4)}, c_2, c_3 \in \mathbb{R}$
$\{(a_1, a_2T, a_3I, a_4F) \mid a_1 \neq 0, a_1 + a_4 \neq 0, a_1 + a_3 + a_4 = 0, a_2 \neq 0\}$	$(1, T, -I, 0)$	$c_1 = \frac{1}{a_1}, c_4 = -\frac{a_4}{a_1(a_1+a_4)}$, $c_2 + c_3 = \frac{1}{a_2} - \frac{1}{a_1+a_4}$
$\{(a_1, a_2T, a_3I, a_4F) \mid a_1 \neq 0, a_1 + a_4 \neq 0, a_1 + a_3 + a_4 \neq 0, a_1 + a_2 + a_3 + a_4 = 0\}$	$(1, -T, 0, 0)$	$c_1 = \frac{1}{a_1}, c_4 = -\frac{a_4}{a_1(a_1+a_4)}$, $c_3 = -\frac{a_3}{(a_1+a_4)(a_1+a_3+a_4)}, c_2 \in \mathbb{R}$
$\{(a_1, a_2T, a_3I, a_4F) \mid a_1 \neq 0, a_1 + a_4 \neq 0, a_1 + a_3 + a_4 \neq 0, a_1 + a_2 + a_3 + a_4 \neq 0\}$	$(1, 0, 0, 0)$	$c_1 = \frac{1}{a_1}, c_4 = -\frac{a_4}{a_1(a_1+a_4)}$, $c_3 = -\frac{a_3}{(a_1+a_4)(a_1+a_3+a_4)}$, $c_2 = -\frac{a_2}{(a_1+a_3+a_4)(a_1+a_2+a_3+a_4)}$

Theorem 5. *For an algebra system $(NQ,\star)$, let $V_c = \{a|a \in NQ \wedge neut(a) = c\}$, $V_{c\star d} = \{a \star b|a,b \in NQ \wedge neut(a) = c, neut(b) = d\}$, we have:*
(1) V_c is a neutrosophic triplet subgroup of NQ.
(2) $V_{c\star d}$ is a neutrosophic triplet subgroup of NQ.

Theorem 6. *For algebra system $(NQ,\star)$, Let $V_c = \{a|a \in NQ \wedge neut(a) = c\}$, we have:*
(1) $V_c \cap V_d = \emptyset$ if $c \neq d$.
(2) $NQ = \cup_{c\in NS} V_c$. So $\cup_{c\in NS} V_c$ is a partition of NQ, where NS is a set, which contains all the neutral elements of $(NQ,\star)$.

4. Two Kinds of Degenerate Systems of Neutrosophic Quadruple Numbers

The neutrosophic quadruple numbers consider (T, I, F) to solve real problems. In this section, we will explore two kinds of degenerate systems about neutrosophic quadruple numbers. The first system is only consider logical true, and the second system is only consider logical true and logical indeterminacy.

4.1. The Neutrosophic Binary Numbers

Definition 8. *A neutrosophic binary number is a number of the form (a, bT), where T have their usual neutrosophic logic true and $a, b \in \mathbb{R}$ or $\mathbb{C}$. The set NB defined by*

$$NB = \{(a, bT) : a, b \in \mathbb{R} \text{ or } \mathbb{C}\}. \tag{7}$$

is called a neutrosophic set of binary numbers. For a neutrosophic binary number (a, bT), a is called the known part and (bT) is called the unknown part.

Definition 9. *Let $a = (a_1, a_2T), b = (b_1, b_2T) \in NB$, the multiplication operation is defined as following:*

$$a * b = (a_1, a_2T) * (b_1, b_2T) = (a_1b_1,(a_1b_2 + a_2b_1 + a_2b_2)T). \tag{8}$$

We have the following results similar to $(NQ, *)$.

Theorem 7. *For the algebra system $(NB, *)$, for every element $a \in NB$, there exists the neutral element $neut(a)$ and opposite element $anti(a)$.*

For algebra system $(NB, *)$, Table 3 gives all the subset, which has the same neutral element, and the corresponding neutral element and opposite elements.

Table 3. The corresponding neutral element and opposite elements for $(NB, *)$.

The Subset	Neutral Elements	Opposite Element (c_1, c_2T)
$\{(0, 0)\}$	$(0, 0)$	$c_i \in \mathbb{R}$
$\{(0, a_2T)\|a_2 \neq 0\}$	$(0, T)$	$c_1 + c_2 = \frac{1}{a_2}$
$\{(a_1, -a_1T)\|a_1 \neq 0\}$	$(1, 0)$	$c_1 = \frac{1}{a_1}, c_2 \in \mathbb{R}$
$\{(a_1, a_2T)\|a_1 \neq 0$	$(1, -T)$	$c_1 = \frac{1}{a_1}, c_2 = -\frac{a_2}{a_1(a_1+a_2)}$

Theorem 8. *For algebra system $(NB, *)$, let $V_c = \{a|a \in NB \wedge neut(a) = c\}$, $V_{c*d} = \{a * b|a,b \in NB \wedge neut(a) = c, neut(b) = d\}$, we have:*
(1) V_c is a neutrosophic triplet subgroup of NB.
(2) V_{c*d} is a neutrosophic triplet subgroup of NB.

Theorem 9. *For an algebra system $(NB,*)$, let $V_c = \{a|a \in NB \wedge neut(a) = c\}$, we have:*

(1) $V_c \cap V_d = \emptyset$ if $c \neq d$.

(2) $NB = \cup_{c \in NS} V_c$. So $\cup_{c \in NS} V_c$ is a partition of NB, where NS is a set, which contains all the neutral elements of $(NB,*)$.

4.2. The Neutrosophic Triple Numbers

Definition 10. *A neutrosophic triple number is a number of the form (a, bT, cI), where T, I have their usual neutrosophic logic meanings and $a, b, c \in \mathbb{R}$ or $\mathbb{C}$. The set NT defined by*

$$NT = \{(a, bT, cI) : a, b, c \in \mathbb{R} \text{ or } \mathbb{C}\} \tag{9}$$

is called a neutrosophic set of triple numbers. For a neutrosophic triple number (a, bT, cI), a is called the known part and (bT, cI) is called the unknown part.

Definition 11. *Let $a = (a_1, a_2T, a_3I), b = (b_1, b_2T, b_3I) \in NT$, suppose in an pessimistic way, the neutrosophic expert considers the prevalence order $T \prec I$. Then the multiplication operation is defined as following:*

$$\begin{aligned} a * b = & \ (a_1, a_2T, a_3I) * (b_1, b_2T, b_3I) \\ & = (a_1b_1, (a_1b_2 + a_2b_1 + a_2b_2)T, (a_1b_3 + a_2b_3 + a_3b_1 + a_3b_2 + a_3b_3)I. \end{aligned} \tag{10}$$

Suppose in an optimistic way the neutrosophic expert considers the prevalence order $T \succ I$. Then:

$$\begin{aligned} a \star b = & \ (a_1, a_2T, a_3I) * (b_1, b_2T, b_3I) \\ & = (a_1b_1, (a_1b_2 + a_2b_1 + a_2b_2 + a_2b_2 + a_3b_2)T, (a_1b_3 + a_3b_1 + a_3b_3)I). \end{aligned} \tag{11}$$

Theorem 10. *For the algebra system $(NT,*)$, for every element $a \in NT$, there exists the neutral element $neut(a)$ and opposite element $anti(a)$.*

For algebra system $(NT,*)$, Table 4 gives all the subset, which has the same neutral element, and the corresponding neutral element and opposite elements.

Table 4. The corresponding neutral element and opposite elements for $(NT,*)$.

The Subset	Neutral Elements	Opposite Element (c_1, c_2T, c_3I)
$\{(0,0,0)\}$	$(0,0,0)$	$c_i \in \mathbb{R}$
$\{(0,0,a_3I)\|a_3 \neq 0\}$	$(0,0,I)$	$c_1 + c_2 + c_3 = \frac{1}{a_3}$
$\{(0,a_2T,-a_2I)\|a_2 \neq 0, a_2 + a_3 = 0\}$	$(0,T,-I)$	$c_1 + c_2 = \frac{1}{a_2}, c_3 \in \mathbb{R}$
$\{(0,a_2T,a_3I)\|a_2 \neq 0, a_2 + a_3 \neq 0\}$	$(0,T,0)$	$c_1 + c_2 = \frac{1}{a_2}, c_3 = -\frac{a_3}{a_2(a_2+a_3)}$
$\{(a_1,-a_1T,0)\|a_1 \neq 0\}$	$(1,-T,0)\}$	$c_1 = \frac{1}{a_1}, c_2, c_3 \in \mathbb{R}$
$\{(a_1,-a_1T,a_3I)\|a_1 \neq 0, a_3 \neq 0\}$	$(1,-T,I)$	$c_1 = \frac{1}{a_1}, c_2 + c_3 = \frac{1}{a_3} - \frac{1}{a_1}$
$\{(a_1,a_2T,a_3I)\|a_1 \neq 0, a_1 + a_2 \neq 0, a_1 + a_2 + a_3 = 0\}$	$(1,0,-I)$	$c_1 = \frac{1}{a_1}, c_2 = -\frac{a_2}{a_1(a_1+a_2)}, c_3 \in \mathbb{R}$
$\{(a_1,a_2T,a_3I)\|a_1 \neq 0, a_1 + a_2 \neq 0, a_1 + a_2 + a_3 \neq 0\}$	$(1,0,0)$	$c_1 = \frac{1}{a_1}, c_2 = -\frac{a_2}{a_1(a_1+a_2)}$, $c_3 = -\frac{a_3}{(a_1+a_2)(a_1+a_2+a_3)}$

Theorem 11. *For an algebra system $(NT, *)$, let $V_c = \{a|a \in NT \wedge neut(a) = c\}$, $V_{c*d} = \{a * b|a, b \in NT \wedge neut(a) = c, neut(b) = d\}$, we have:*

(1) V_c is a neutrosophic triplet subgroup of NT.

(2) V_{c*d} is a neutrosophic triplet subgroup of NT.

Theorem 12. *For an algebra system $(NT, *)$, let $V_c = \{a|a \in NT \wedge neut(a) = c\}$, we have:*

(1) $V_c \cap V_d = \emptyset$ if $c \neq d$.

(2) $NT = \cup_{c \in NS} V_c$. So $\cup_{c \in NS} V_c$ is a partition of NT, where NS is a set, which contains all the neutral elements of $(NT, *)$.

Theorem 13. *For the algebra system $(NT, \star)$, for every element $a \in NT$, there exists the neutral element $neut(a)$ and opposite element $anti(a)$.*

For an algebra system $(NT, \star)$, Table 5 gives all the subset, which has the same neutral element, and the corresponding neutral element and opposite elements.

Table 5. The corresponding neutral element and opposite elements for $(NT, \star)$.

The Subset	Neutral Elements	Opposite Element (c_1, c_2T, c_3I)
$\{(0,0,0)\}$	$(0,0,0)$	$c_i \in \mathbb{R}$
$\{(0, a_2T, 0)\|a_2 \neq 0\}$	$(0, T, 0)$	$c_1 + c_2 + c_3 = \frac{1}{a_2}$
$\{(0, a_3T, -a_3I)\|a_3 \neq 0, a_2 + a_3 = 0\}$	$(0, -T, I)$	$c_1 + c_3 = \frac{1}{a_3}, c_2 \in \mathbb{R}$
$\{(0, a_2T, a_3I)\|a_3 \neq 0, a_2 + a_3 \neq 0\}$	$(0, 0, I)$	$c_1 + c_3 = \frac{1}{a_3}, c_2 = -\frac{a_2}{a_3(a_2+a_3)}$
$\{(a_1, 0, -a_1I)\|a_1 \neq 0\}$	$(1, 0, -I)\}$	$c_1 = \frac{1}{a_1}, c_2, c_3 \in \mathbb{R}$
$\{(a_1, a_2T, -a_1I)\|a_1 \neq 0, a_2 \neq 0\}$	$(1, T, -I)$	$c_1 = \frac{1}{a_1}, c_2 + c_3 = \frac{1}{a_2} - \frac{1}{a_1}$
$\{(a_1, a_2T, a_3I)\|a_1 \neq 0, a_1 + a_3 \neq 0, a_1 + a_2 + a_3 = 0\}$	$(1, -T, 0)$	$c_1 = \frac{1}{a_1}, c_3 = -\frac{a_3}{a_1(a_1+a_3)}, c_2 \in \mathbb{R}$
$\{(a_1, a_2T, a_3I)\|a_1 \neq 0, a_1 + a_3 \neq 0, a_1 + a_2 + a_3 \neq 0\}$	$(1, 0, 0)$	$c_1 = \frac{1}{a_1}, c_3 = -\frac{a_3}{a_1(a_1+a_3)}$, $c_2 = -\frac{a_2}{(a_1+a_3)(a_1+a_2+a_3)}$

Theorem 14. *For algebra system $(NT, \star)$, let $V_c = \{a|a \in NT \wedge neut(a) = c\}$, $V_{c\star d} = \{a \star b|a, b \in NT \wedge neut(a) = c, neut(b) = d\}$, we have:*

(1) V_c is a neutrosophic triplet subgroup of NT.

(2) $V_{c\star d}$ is a neutrosophic triplet subgroup of NT.

Theorem 15. *For an algebra system $(NT, \star)$, let $V_c = \{a|a \in NT \wedge neut(a) = c\}$, we have:*

(1) $V_c \cap V_d = \emptyset$ if $c \neq d$.

(2) $NT = \cup_{c \in NS} V_c$. So $\cup_{c \in NS} V_c$ is a partition of NT, where NS is a set, which contains all the neutral elements of $(NT, \star)$.

5. Conclusions

In the paper, we prove that $(NQ, *)$(or $NQ, \star$) is a neutrosophic extended triplet group, and provide new examples of a neutrosophic extended triplet group. We also explore the algebra structure properties of neutrosophic quadruple numbers. Moreover, we discuss two kinds of degenerate systems of neutrosophic quadruple numbers. For neutrosophic quadruple numbers, the results in the paper can be extended to general fields. In the following, we will explore the relation of neutrosophic quadruple numbers and other algebra systems [24–26]. Moreover, on the one hand, we will discuss the neutrosophic quadruple numbers based on some particular ring which can form a neutrosophic extended triplet group, on the other hand, we will introduce a new operation $\circ$ in order to guarantee $(NQ, *, \circ)$ is a neutrosophic triplet ring.

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