Neutrosophic General Finite Automata

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Abstract: The constructions of finite switchboard state automata is known to be an extension of finite automata in the view of commutative and switching automata. In this research, the idea of a neutrosophic is incorporated in the general fuzzy finite automata and general fuzzy finite switchboard automata to introduce neutrosophic general finite automata and neutrosophic general finite switchboard automata. Moreover, we define the notion of the neutrosophic subsystem and strong neutrosophic subsystem for both structures. We also establish the relationship between the neutrosophic subsystem and neutrosophic strong subsystem.

Keywords: Neutrosophic set, General fuzzy automata; switchboard; subsystems.

1 Introduction

It is well-known that the simplest and most important type of automata is finite automata. After the introduction of fuzzy set theory by [47] Zadeh in 1965, the first mathematical formulation of fuzzy automata was proposed by[46] Wee in 1967, considered as a generalization of fuzzy automata theory. Consequently, numerous works have been contributed towards the generalization of finite automata by many authors such as Cao and Ezawac [9], Jin et al [18], Jun [20], Li and Qiu [27], Qiu [34], Sato and Kuroki [36], Srivastava and Tiwari [41], Santos [35], Jun and Kavikumar [21], Kavikumar et al, [22, 23, 24] especially the simplest one by Mordeson and Malik [29]. In 2005, the theory of general fuzzy automata was firstly proposed by Doostfatemeh and Kermer [11] which is used to resolve the problem of assigning membership values to active states of the fuzzy automaton and its multi-membership. Subsequently, as a generalization, the concept of intuitionistic general fuzzy automata has been introduced and studied by Shamsizadeh and Zahedi [37], while Abolpour and Zahedi [6] proposed general fuzzy automata theory based on the complete residuated lattice-valued. As a further extension, Kavikumar et al [25] studied the notions of general fuzzy switchboard automata. For more details see the recent literature as [5, 12, 13, 14, 15, 16, 17].

The notions of neutrosophic sets was proposed by Smarandache [38, 39], generalizing the existing ordinary fuzzy sets, intuitionistic fuzzy sets and interval-valued fuzzy set in which each element of the universe has the degrees of truth, indeterminacy and falsity and the membership values are lies in $]0^{-}, 1^{+}[$, the nonstandard unit interval [40] it is an extension from standard interval [0,1]. It has been shown that fuzzy sets provides limited platform for computational complexity but neutrosophic sets is suitable for it. The neutrosophic sets is an appropriate mechanism for interpreting real-life philosophical problems but not for scientific problems since it is difficult to consolidate. In neutrosophic sets, the degree of indeterminacy can be defined independently since it is quantified explicitly which led to different from intuitionistic fuzzy sets. Single-valued neutrosophic set and interval neutrosophic set are the subclasses of the neutrosophic sets which was introduced by Wang et al. [44, 45] in order to examine kind of real-life and scientific problems. The applications of fuzzy sets have been found very useful in the domain of mathematics and elsewhere. A number of authors have been applied the concept of the neutrosophic set to many other structures especially in algebra [19, 28], decision-making [1, 2, 10, 30], medical [3, 4, 8], water quality management [33] and traffic control management [31, 32].

1.1 Motivation

In view of exploiting neutrosophic sets, Tahir et al. [43] introduced and studied the concept of single valued Neutrosophic finite state machine and switchboard state machine. Moreover, the fuzzy finite switchboard state machine is introduced into the context of the interval neutrosophic set in [42]. However, the realm of general structure of fuzzy automata in the neutrosophic environment has not been studied yet in the literature so far. Hence, it is still open to many possibilities for innovative research work especially in the context of neutrosophic general automata and its switchboard automata. The fundamental advantage of incorporating neutrosophic sets into general fuzzy automata is the ability to bring indeterminacy membership and nonmembership in each transitions and active states which help us to overcome the uncertain situation at the time of predicting next active state. Motivated by the work of [11], [36] and [38] the concept of neutrosophic general automata and neutrosophic general switchboard automata are introduced in this paper.

1.2 Main Contribution

The purpose of this paper is to introduce the primary algebraic structure of neutrosophic general finite automata and neutrosophic switchboard finite automata. The subsystem and strong subsystem of neutrosophic general finite automata and neutrosophic general finite switchboard f automata are exhibited. The relationship between these subsystems have been discussed and the characterizations of switching and commutative are discussed in the neutrosophic backdrop. We prove that the implication of a strong subsystem is a subsystem of neutrosophic general finite automata. The remainder of this paper is organised as follows. Section 2 provides the results and definitions concerning the general fuzzy automata. Section 3 describes the algebraic properties of the neutrosophic general finite automata. Finally, in section 4, the notion of the neutrosophic general finite switchboard automata is introduced. The paper concludes with Section 5.

2 Preliminaries

”For a nonempty set X, $\tilde{P}(X)$ denotes the set of all fuzzy sets on X.

Definition 2.1. [11] A general fuzzy automaton (GFA) is an eight-tuple machine $\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ where

(a) Q is a finite set of states, $Q = \{q_1, q_2, \cdots, q_n\}$,

(b) Σ is a finite set of input symbols, $\Sigma = \{a_1, a_2, \cdots, a_m\}$,

(c) $\tilde{R}$ is the set of fuzzy start states, $\tilde{R} \subseteq \tilde{P}(Q)$,

(d) Z is a finite set of output symbols, $Z = \{b_1, b_2, \cdots, b_k\}$,

(e) $\omega : Q \to Z$ is the non-fuzzy output function,

(f) $F_1 : [0,1] \times [0,1] \to [0,1]$ is the membership assignment function,

(g) $\tilde{\delta} : (Q \times [0,1]) \times \Sigma \times Q \overset{F_1(\mu,\delta)}{\longrightarrow} [0,1]$ is the augmented transition function,

(h) $F_2 : [0,1]^* \to [0,1]$ is a multi-membership resolution function.

Noted that the function $F_1(\mu, \delta)$ has two parameters μ and δ, where μ is the membership value of a predecessor and δ is the weight of a transition. In this definition, the process that takes place upon the transition from state q_i to q_j on input a_k is represented as:

$$\mu^{t+1}(q_j) = \tilde{\delta}((q_i, \mu^t(q_i)), a_k, q_j) = F_1(\mu^t(q_i), \delta(q_i, a_k, q_j)).$$

This means that the membership value of the state q_j at time $t + 1$ is computed by function F_1 using both the membership value of q_i at time t and the weight of the transition. The usual options for the function $F(\mu, \delta)$ are $\max\{\mu, \delta\}, \min\{\mu, \delta\}$ and $(\mu + \delta)/2$. The multi-membership resolution function resolves the multi-membership active states and assigns a single membership value to them.

Let $Q_{act}(t_i)$ be the set of all active states at time $t_i, \forall i \geq 0$. We have $Q_{act}(t_0) = \tilde{R}$,

$$Q_{act}(t_i) = \{(q, \mu^{t_i}(q)) : \exists q' \in Q_{act}(t_{i-1}), \exists\, a \in \Sigma, \delta(q', a, q) \in \Delta\}, \forall i \geq 1.$$

Since $Q_{act}(t_i)$ is a fuzzy set, in order to show that a state q belongs to $Q_{act}(t_i)$ and T is a subset of $Q_{act}(t_i)$, we should write: $q \in Domain(Q_{act}(t_i))$ and $T \subset Domain(Q_{act}(t_i))$. Hereafter, we simply denote them as: $q \in Q_{act}(t_i)$ and $T \subset Q_{act}(t_i)$. The combination of the operations of functions F_1 and F_2 on a multi-membership state q_j leads to the multi-membership resolution algorithm.

Algorithm 2.2. [11] (Multi-membership resolution) If there are several simultaneous transitions to the active state q_j at time $t + 1$, the following algorithm will assign a unified membership value to it:

1. Each transition weight $\tilde{\delta}(q_i, a_k, q_j)$ together with $\mu^t(q_i)$, will be processed by the membership assignment function F_1, and will produce a membership value. Call this v_i,

$$v_i = \tilde{\delta}((q_i, \mu^t(q_i)), a_k, q_j) = F_1(\mu^t(q_i), \delta(q_i, a_k, q_j)).$$

2. These membership values are not necessarily equal. Hence, they need to be processed by the multi-membership resolution function F_2.

3. The result produced by F_2 will be assigned as the instantaneous membership value of the active state q_j,

$$\mu^{t+1}(q_j) = F_2{}_{i=1}^{n}[v_i] = F_2{}_{i=1}^{n}[F_1(\mu^t(q_i), \delta(q_i, a_k, q_j))],$$

where

- n is the number of simultaneous transitions to the active state q_j at time $t+1$.
- $\delta(q_i, a_k, q_j)$ is the weight of a transition from q_i to q_j upon input a_k.
- $\mu^t(q_i)$ is the membership value of q_i at time t.
- $\mu^{t+1}(q_j)$ is the final membership value of q_j at time $t+1$.

Definition 2.3. Let $\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ be a general fuzzy automaton, which is defined in Definition 2.1. The max-min general fuzzy automata is defined of the form:

$$\tilde{F}^* = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}^*, \omega, F_1, F_2),$$

where $Q_{act} = \{Q_{act}(t_0), Q_{act}(t_1), \cdots\}$ and for every i, $i \geq 0$:

$$\tilde{\delta}^*((q, \mu^{t_i}(q)), \Lambda, p) = \begin{cases} 1, & q = p \\ 0, & \text{otherwise} \end{cases}$$

and for every i, $i \geq 1$: $\tilde{\delta}^*((q, \mu^{t_{i-1}}(q)), u_i, p) = \tilde{\delta}((q, \mu^{t_{i-1}}(q)), u_i, p)$,

$$\tilde{\delta}^*((q, \mu^{t_{i-1}}(q)), u_i u_{i+1}, p) = \bigvee_{q' \in Q_{act}(t_i)} (\tilde{\delta}((q, \mu^{t_{i-1}}(q)), u_i, q') \wedge \tilde{\delta}((q', \mu^{t_i}(q')), u_{i+1}, p))$$

and recursively

$$\tilde{\delta}^*((q, \mu^{t_0}(q)), u_1 u_2 \cdots u_n, p) = \bigvee \{\tilde{\delta}((q, \mu^{t_0}(q)), u_1, p_1) \wedge \tilde{\delta}((p_1, \mu^{t_1}(p_1)), u_2, p_2) \wedge \cdots \wedge \tilde{\delta}((p_{n-1}, \mu^{t_{n-1}}(p_{n-1})), u_n, p) | p_1 \in Q_{act}(t_1), p_2 \in Q_{act}(t_2), \cdots, p_{n-1} \in Q_{act}(t_{n-1})\},$$

in which $u_i \in \Sigma, \forall 1 \leq i \leq n$ and assuming that the entered input at time t_i be $u_i, \forall 1 \leq i \leq n-1$.

Definition 2.4. [13] Let $\tilde{F}^*$ be a max-min GFA, $p \in Q, q \in Q_{act}(t_i), i \geq 0$ and $0 \leq \alpha < 1$. Then p is called a successor of q with threshold α if there exists $x \in \Sigma^*$ such that $\tilde{\delta}^*((q, \mu^{t_j}(q)), x, p) > \alpha$.

Definition 2.5. [13] Let $\tilde{F}^*$ be a max-min GFA, $q \in Q_{act}(t_i), i \geq 0$ and $0 \leq \alpha < 1$. Also let $S_\alpha(q)$ denote the set of all successors of q with threshold α. If $T \subseteq Q$, then $S_\alpha(T)$ the set of all successors of T with threshold α is defined by $S_\alpha(T) = \bigcup\{S_\alpha(q) : q \in T\}$.

Definition 2.6. [38] Let X be an universe of discourse. The neutrosophic set is an object having the form $A = \{\langle x, \mu_1(x), \mu_2(x), \mu_3(x)\rangle | \forall x \in X\}$ where the functions can be defined by $\mu_1, \mu_2, \mu_3 : X \to]0, 1[$ and μ_1 is the degree of membership or truth, μ_2 is the degree of indeterminacy and μ_3 is the degree of non-membership or false of the element $x \in X$ to the set A with the condition $0 \leq \mu_1(x) + \mu_2(x) + \mu_3(x) \leq 3$."

3 Neutrosophic General Finite Automata

Definition 3.1. An eight-tuple machine $\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ is called neutrosophic general finite automata (*NGFA* for short), where

1. Q is a finite set of states, $Q = \{q_1, q_2, \cdots, q_n\}$,
2. Σ is a finite set of input symbols, $\Sigma = \{u_1, u_2, \cdots, u_m\}$,
3. $\tilde{R} = \{(q, \mu_1^{t_0}(q), \mu_2^{t_0}(q), \mu_3^{t_0}(q))|q \in R\}$ is the set of fuzzy start states, $R \subseteq \tilde{P}(Q)$,
4. Z is a finite set of output symbols, $Z = \{b_1, b_2, \cdots, b_k\}$,
5. $\tilde{\delta} : (Q \times [0,1] \times [0,1] \times [0,1])) \times \Sigma \times Q \xrightarrow{F_1(\mu,\delta)} [0,1] \times [0,1] \times [0,1]$ is the neutrosophic augmented transition function,
6. $\omega : (Q \times [0,1] \times [0,1] \times [0,1]) \to Z$ is the non-fuzzy output function,
7. $F_1 = (F_1^{\wedge}, F_1^{\wedge\vee}, F_1^{\vee})$, where $F_1^{\wedge} : [0,1] \times [0,1] \to [0,1]$, $F_2^{\wedge\vee} : [0,1] \times [0,1] \to [0,1]$ and $F_3^{\vee} : [0,1] \times [0,1] \to [0,1]$ are the truth, indeterminacy and false membership assignment functions, respectively. $F_1^{\wedge}(\mu_1, \tilde{\delta}_1)$, $F_2^{\wedge\vee}(\mu_2, \tilde{\delta}_2)$ and $F_3^{\vee}(\mu_3, \tilde{\delta}_3)$ are motivated by two parameters μ_1, μ_2, μ_3 and $\tilde{\delta}_1$, $\tilde{\delta}_2$, $\tilde{\delta}_3$ where μ_1, μ_2 and μ_3 are the truth, indeterminacy and false membership value of a predecessor and $\tilde{\delta}_1$, $\tilde{\delta}_2$ and $\tilde{\delta}_3$ are the truth, indeterminacy and false membership value of a transition,
8. $F_2 = (F_2^{\wedge}, F_2^{\wedge\vee}, F_2^{\vee})$, where $F_2^{\wedge} : [0,1]^* \to [0,1]$, $F_2^{\wedge\vee} : [0,1]^* \to [0,1]$ and $F_2^{\vee} : [0,1]^* \to [0,1]$ are the truth, indeterminacy and false multi-membership resolution function.

Remark 3.2. In Definition 3.1, the process that takes place upon the transition from the state q_i to q_j on an input u_k is represented by

$$\mu_1^{t_{k+1}}(q_j) = \tilde{\delta}_1((q_i, \mu_1^{t_k}(q_i)), u_k, q_j) = F_1^{\wedge}(\mu_1^{t_k}(q_i), \delta_1(q_i, u_k, q_j)) = \bigwedge(\mu_1^{t_k}(q_i), \delta_1(q_i, u_k, q_j)),$$

$$\mu_2^{t_{k+1}}(q_j) = \tilde{\delta}_2((q_i, \mu_2^{t_k}(q_i)), u_k, q_j) = F_1^{\wedge\vee}(\mu_2^{t_k}(q_i), \delta_2(q_i, u_k, q_j)) = \begin{cases} \bigvee(\mu_2^{t_k}(q_i), \delta_2(q_i, u_k, q_j)) & \text{if } t_k < t_{k+1} \\ \bigwedge(\mu_2^{t_k}(q_i), \delta_2(q_i, u_k, q_j)) & \text{if } t_k \geq t_{k+1} \end{cases},$$

$$\mu_3^{t_{k+1}}(q_j) = \tilde{\delta}_3((q_i, \mu_3^{t_k}(q_i)), u_k, q_j) = F_1^{\vee}(\mu_3^{t_k}(q_i), \delta_3(q_i, u_k, q_j)) = \bigvee(\mu_3^{t_k}(q_i), \delta_3(q_i, u_k, q_j)),$$

where

$$\tilde{\delta}((q_i.\mu^t(q_i)), u_k, q_j) = (\tilde{\delta}_1((q_i, \mu_1^t(q_i)), u_k, q_j), \tilde{\delta}_2((q_i, \mu_2^t(q_i)), u_k, q_j), \tilde{\delta}_3((q_i, \mu_3^t(q_i)), u_k, q_j)) \text{ and}$$

$$\delta(q_i, u_k, q_j) = (\delta_1(q_i, u_k, q_j), \delta_2(q_i, u_k, q_j), \delta_3(q_i, u_k, q_j)).$$

Remark 3.3. The algorithm for truth, indeterminacy and false multi-membership resolution for transition function is same as Algorithm 2.2 but the computation depends (see Remark 3.2) on the truth, indeterminacy and false membership assignment function.

Definition 3.4. Let $\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ be a NGFA. We define the max-min neutrosophic general fuzzy automaton $\tilde{F}^* = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}^*, \omega, F_1, F_2)$, where $\tilde{\delta}^* : (Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma^* \times Q \to [0,1] \times [0,1] \times [0,1]$ and define a neutrosophic set $\tilde{\delta}^* = \langle \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{\delta}_3^* \rangle$ in $(Q \times [0,1] \times [0,1] \times [01]) \times \Sigma^* \times Q$ and for every i, $i \geq 0$:

$$\tilde{\delta}_1^*((q, \mu^{t_i}(q)), \Lambda, p) = \begin{cases} 1, & q = p \\ 0, & q \neq p \end{cases},$$

$$\tilde{\delta}_2^*((q, \mu^{t_i}(q)), \Lambda, p) = \begin{cases} 0, & q = p \\ 1, & q \neq p \end{cases},$$

$$\tilde{\delta}_3^*((q, \mu^{t_i}(q)), \Lambda, p) = \begin{cases} 0, & q = p \\ 1, & q \neq p \end{cases},$$

and for every i, $i \geq 1$:

$$\tilde{\delta}_1^*((q, \mu^{t_{i-1}}(q)), u_i, p) = \tilde{\delta}_1((q, \mu^{t_{i-1}}(q)), u_i, p), \tilde{\delta}_2^*((q, \mu^{t_{i-1}}(q)), u_i, p) = \tilde{\delta}_2((q, \mu^{t_{i-1}}(q)), u_i, p)$$
$$\tilde{\delta}_3^*((q, \mu^{t_{i-1}}(q)), u_i, p) = \tilde{\delta}_3((q, \mu^{t_{i-1}}(q)), u_i, p)$$

and recursively,

$$\tilde{\delta}_1^*((q, \mu^{t_0}(q)), u_1u_2 \cdots u_n, p) = \bigvee\{\tilde{\delta}_1((q, \mu^{t_0}(q)), u_1, p_1) \wedge \tilde{\delta}_1((p_1, \mu^{t_1}(p_1)), u_2, p_2) \wedge \cdots \wedge$$
$$\tilde{\delta}_1((p_{n-1}, \mu^{t_{n-1}}(p_{n-1})), u_n, p) | p_1 \in Q_{act}(t_1), p_2 \in Q_{act}(t_2), \cdots, p_{n-1} \in Q_{act}(t_{n-1})\},$$
$$\tilde{\delta}_2^*((q, \mu^{t_0}(q)), u_1u_2 \cdots u_n, p) = \bigwedge\{\tilde{\delta}_2((q, \mu^{t_0}(q)), u_1, p_1) \vee \tilde{\delta}_2((p_1, \mu^{t_1}(p_1)), u_2, p_2) \vee \cdots \vee$$
$$\tilde{\delta}_2((p_{n-1}, \mu^{t_{n-1}}(p_{n-1})), u_n, p) | p_1 \in Q_{act}(t_1), p_2 \in Q_{act}(t_2), \cdots, p_{n-1} \in Q_{act}(t_{n-1})\},$$
$$\tilde{\delta}_3^*((q, \mu^{t_0}(q)), u_1u_2 \cdots u_n, p) = \bigwedge\{\tilde{\delta}_3((q, \mu^{t_0}(q)), u_1, p_1) \vee \tilde{\delta}_3((p_1, \mu^{t_1}(p_1)), u_2, p_2) \vee \cdots \vee$$
$$\tilde{\delta}_3((p_{n-1}, \mu^{t_{n-1}}(p_{n-1})), u_n, p) | p_1 \in Q_{act}(t_1), p_2 \in Q_{act}(t_2), \cdots, p_{n-1} \in Q_{act}(t_{n-1})\},$$

in which $u_i \in \Sigma, \forall 1 \leq i \leq n$ and assuming that the entered input at time t_i be $u_i, \forall 1 \leq i \leq n-1$.

Example 3.5. Consider the NGFA in Figure 1 with several transition overlaps. Let $\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$, where

- $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}$ be a set of states,
- $\Sigma = \{a, b\}$ be a set of input symbols,
- $\tilde{R} = \{(q_0, 0.7, 0.5, 0.2), (q_4, 0.6, 0.2, 0.45)\}$, set of initial states,
- the operation of $F_1^{\wedge}, F_1^{\wedge\vee}$ and $F_1^{\vee}$ are according to Remark 3.2,
- $Z = \emptyset$ and ω are not applicable (output mapping is not of our interest in this paper),
- $\tilde{\delta} : (Q \times [0,1] \times [0,1] \times [0,1])) \times \Sigma \times Q \overset{F_1(\mu,\delta)}{\longrightarrow} [0,1] \times [0,1] \times [0,1]$, the neutrosophic augmented transition function.

Assuming that $\tilde{F}$ starts operating at time t_0 and the next three inputs are a, b, b respectively (one at a time), active states and their membership values at each time step are as follows:

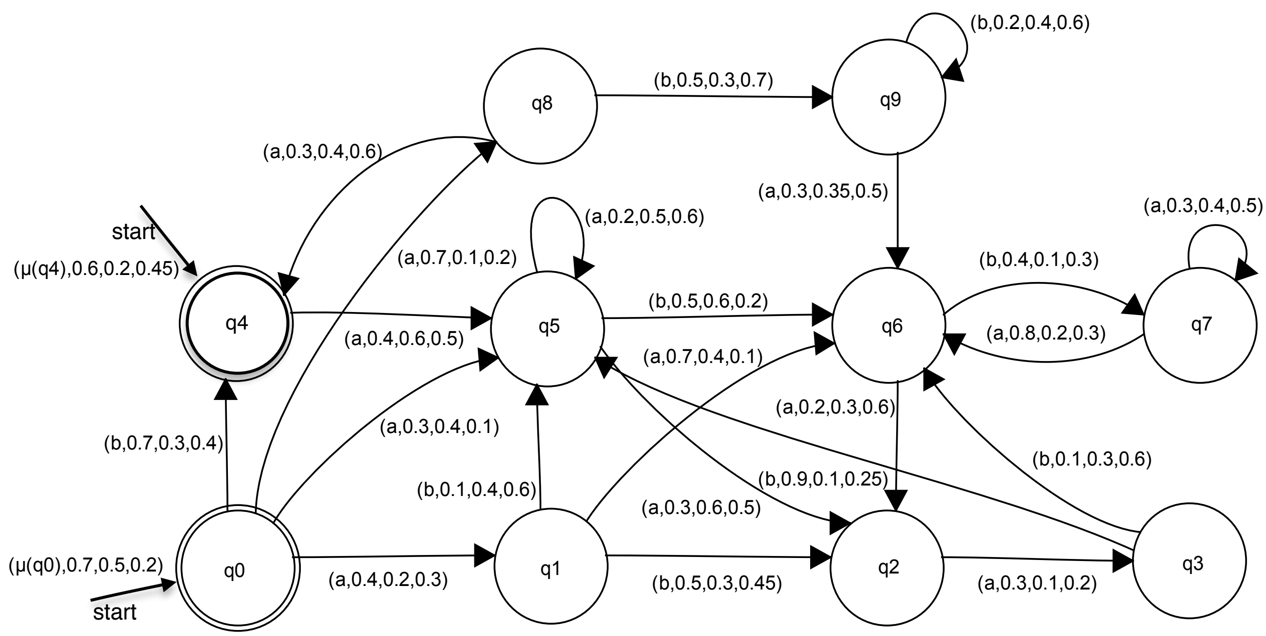

Figure 1: The NGFA of Example 3.5

- At time t_0: $Q_{act}(t_0) = \tilde{R} = \{(q_0, 0.7, 0.5, 0.2), (q_4, 0.6, 0.2, 0.45)\}$

- At time t_1, input is a. Thus q_1, q_5 and q_8 get activated. Then:

$$\begin{aligned}
\mu^{t_1}(q_1) &= \tilde{\delta}((q_0, \mu_1^{t_0}(q_0), \mu_2^{t_0}(q_0), \mu_3^{t_0}(q_0)), a, q_1) \\
&= \left[F_1^{\wedge}(\mu_1^{t_0}(q_0), \delta_1(q_0, a, q_1)), F_1^{\wedge\vee}(\mu_2^{t_0}(q_0), \delta_2(q_0, a, q_1)), F_1^{\vee}(\mu_3^{t_0}(q_0), \delta_3(q_0, a, q_1))\right] \\
&= [F_1^{\wedge}(0.7, 0.4), F_1^{\wedge\vee}(0.5, 0.2), F_1^{\vee}(0.2, 0.3)] = (0.4, 0.2, 0.3),
\end{aligned}$$

$$\begin{aligned}
\mu^{t_1}(q_8) &= \tilde{\delta}((q_0, \mu_1^{t_0}(q_0), \mu_2^{t_0}(q_0), \mu_3^{t_0}(q_0)), a, q_8) \\
&= \left[F_1^{\wedge}(\mu_1^{t_0}(q_0), \delta_1(q_0, a, q_8)), F_1^{\wedge\vee}(\mu_2^{t_0}(q_0), \delta_2(q_0, a, q_8)), F_1^{\vee}(\mu_3^{t_0}(q_0), \delta_3(q_0, a, q_8))\right] \\
&= [F_1^{\wedge}(0.7, 0.7), F_1^{\wedge\vee}(0.5, 0.1), F_1^{\vee}(0.2, 0.2)] = (0.7, 0.1, 0.2),
\end{aligned}$$

but q_5 is multi-membership at t_1. Then

$$\begin{aligned}
\mu^{t_1}(q_5) &= \underset{i=0\&4}{F_2} \left[F_1[\mu^{t_0}(q_i), \delta(q_i, a, q_5)]\right] \\
&= F_2\left[F_1[\mu^{t_0}(q_0), \delta(q_0, a, q_5)], F_1[\mu^{t_0}(q_0), \delta(q_4, a, q_5)]\right] \\
&= F_2\left[F_1[(0.7, 0.5, 0.2), (0.3, 0.4, 0.1)], F_1[(0.6, 0.2, 0.45), (0.4, 0.6, 0.5)]\right] \\
&= (F_2^{\wedge}[F_1^{\wedge}(0.7, 0.3), F_1^{\wedge}(0.6, 0.4)], F_2^{\wedge\vee}[F_1^{\wedge\vee}(0.5, 0.4), F_1^{\wedge\vee}(0.2, 0.6)], \\
&\qquad\qquad F_2^{\vee}[F_1^{\vee}(0.2, 0.1), F_1^{\vee}(0.45, 0.5)]) \\
&= (F_2^{\wedge}(0.3, 0.4), F_2^{\wedge\vee}(0.4, 0.2), F_2^{\vee}(0.2, 0.5)) = (0.3, 0.2, 0.5).
\end{aligned}$$

Then we have:

$$\begin{aligned} Q_{act}(t_1) &= \{(q_1, \mu^{t_1}(q_1)), (q_5, \mu^{t_1}(q_5)), (q_8, \mu^{t_1}(q_8))\} \\ &= \{(q_1, 0.4, 0.2, 0.3), (q_5, 0.3, 0.2, 0.5), (q_8, 0.7, 0.1, 0.2)\}. \end{aligned}$$

- At t_2 input is b. q_2, q_5, q_6 and q_9 get activated. Then

$$\begin{aligned} \mu^{t_2}(q_5) &= \tilde{\delta}((q_1, \mu_1^{t_1}(q_1), \mu_2^{t_1}(q_1), \mu_3^{t_1}(q_1)), b, q_5) \\ &= \left[F_1^{\wedge}(\mu_1^{t_1}(q_1), \delta_1(q_1, b, q_5)), F_1^{\wedge\vee}(\mu_2^{t_1}(q_1), \delta_2(q_1, b, q_5)), F_1^{\vee}(\mu_3^{t_1}(q_1), \delta_3(q_1, b, q_5))\right] \\ &= [F_1^{\wedge}(0.4, 0.1), F_1^{\wedge\vee}(0.2, 0.4), F_1^{\vee}(0.3, 0.6)] = (0.1, 0.2, 0.6), \end{aligned}$$

$$\begin{aligned} \mu^{t_2}(q_6) &= \tilde{\delta}((q_5, \mu_1^{t_1}(q_5), \mu_2^{t_1}(q_5), \mu_3^{t_1}(q_5)), b, q_6) \\ &= \left[F_1^{\wedge}(\mu_1^{t_1}(q_5), \delta_1(q_5, b, q_6)), F_1^{\wedge\vee}(\mu_2^{t_1}(q_5), \delta_2(q_5, b, q_6)), F_1^{\vee}(\mu_3^{t_1}(q_5), \delta_3(q_5, b, q_6))\right] \\ &= [F_1^{\wedge}(0.3, 0.5), F_1^{\wedge\vee}(0.2, 0.6), F_1^{\vee}(0.5, 0.2)] = (0.3, 0.2, 0.5), \end{aligned}$$

$$\begin{aligned} \mu^{t_2}(q_9) &= \tilde{\delta}((q_8, \mu_1^{t_1}(q_8), \mu_2^{t_1}(q_8), \mu_3^{t_1}(q_8)), b, q_9) \\ &= \left[F_1^{\wedge}(\mu_1^{t_1}(q_8), \delta_1(q_8, b, q_9)), F_1^{\wedge\vee}(\mu_2^{t_1}(q_8), \delta_2(q_8, b, q_9)), F_1^{\vee}(\mu_3^{t_1}(q_8), \delta_3(q_8, b, q_9))\right] \\ &= [F_1^{\wedge}(0.7, 0.5), F_1^{\wedge\vee}(0.1, 0.3), F_1^{\vee}(0.2, 0.7)] = (0.5, 0.1, 0.7), \end{aligned}$$

but q_2 is multi-membership at t_2. Then:

$$\begin{aligned} \mu^{t_2}(q_2) &= \underset{i=1\&5}{F_2} \left[F_1[\mu^{t_1}(q_i), \delta(q_i, b, q_2)]\right] \\ &= F_2 \left[F_1[\mu^{t_1}(q_1), \delta(q_1, b, q_2)], F_1[\mu^{t_1}(q_5), \delta(q_5, b, q_2)]\right] \\ &= F_2 \left[F_1[(0.4, 0.2, 0.3), (0.5, 0.3, 0.45)], F_1[(0.3, 0.2, 0.5), (0.1, 0.4, 0.6)]\right] \\ &= (F_2^{\wedge}[F_1^{\wedge}(0.4, 0.5), F_1^{\wedge}(0.3, 0.1)], F_2^{\wedge\vee}[F_1^{\wedge\vee}(0.2, 0.3), F_1^{\wedge\vee}(0.2, 0.4)], \\ &\qquad F_2^{\vee}[F_1^{\vee}(0.3, 0.45), F_1^{\vee}(0.5, 0.6)]) \\ &= (F_2^{\wedge}(0.4, 0.1), F_2^{\wedge\vee}(0.2, 0.2), F_2^{\vee}(0.3, 0.5)) = (0.1, 0.2, 0.5). \end{aligned}$$

Then we have:

$$\begin{aligned} Q_{act}(t_2) &= \{(q_2, \mu^{t_2}(q_2)), (q_5, \mu^{t_2}(q_5)), (q_6, \mu^{t_2}(q_6)), (q_9, \mu^{t_2}(q_9))\} \\ &= \{(q_2, 0.1, 0.2, 0.5), (q_5, 0.1, 0.2, 0.6), (q_6, 0.3, 0.2, 0.5), (q_9, 0.5, 0.1, 0.7)\}. \end{aligned}$$

- At t_3 input is b. q_2, q_6, q_7 and q_9 get activated and none of them is multi-membership. It is easy to verify that:

$$\begin{aligned} Q_{act}(t_3) &= \{(q_2, \mu^{t_3}(q_2)), (q_6, \mu^{t_3}(q_6)), (q_7, \mu^{t_3}(q_7)), (q_9, \mu^{t_3}(q_9))\} \\ &= \{(q_2, 0.1, 0.1, 0.6), (q_6, 0.1, 0.2, 0.6), (q_7, 0.3, 0.1, 0.5), (q_9, 0.3, 0.1, 0.5)\}. \end{aligned}$$

Proposition 3.6. *Let $\tilde{F}$ be a NGFA, if $\tilde{F}^*$ is a max-min NGFA, then for every $i \geq 1$,*

$$\tilde{\delta}_1^*((q, \mu^{t_{i-1}}(q)), xy, p) = \bigvee_{r \in Q_{act}(t_i)} \left[\tilde{\delta}_1^*((p, \mu^{t_{i-1}}(p)), x, r) \wedge \tilde{\delta}_1^*((r, \mu^{t_{i-1}}(r)), y, q)\right],$$
$$\tilde{\delta}_2^*((q, \mu^{t_{i-1}}(q)), xy, p) = \bigwedge_{r \in Q_{act}(t_i)} \left[\tilde{\delta}_2^*((p, \mu^{t_{i-1}}(p)), x, r) \vee \tilde{\delta}_2^*((r, \mu^{t_{i-1}}(r)), y, q)\right],$$
$$\tilde{\delta}_3^*((q, \mu^{t_{i-1}}(q)), xy, p) = \bigwedge_{r \in Q_{act}(t_i)} \left[\tilde{\delta}_3^*((p, \mu^{t_{i-1}}(p)), x, r) \vee \tilde{\delta}_3^*((r, \mu^{t_{i-1}}(r)), y, q)\right],$$

for all $p, q \in Q$ and $x, y \in \Sigma^*$.

Proof. Since $p, q \in Q$ and $x, y \in \Sigma^*$, we prove the result by induction on $|y| = n$. First, we assume that $n = 0$, then $y = \Lambda$ and so $xy = x\Lambda = x$. Thus, for all $r \in Q_{act}(t_i)$

$$\begin{aligned}
\bigvee \left[\tilde{\delta}_1^*((p, \mu^{t_{i-1}}(p)), x, r) \wedge \tilde{\delta}_1^*((r, \mu^{t_{i-1}}(r)), y, q)\right] &= \bigvee \left[\tilde{\delta}_1^*((p, \mu^{t_{i-1}}(p)), x, r) \wedge \tilde{\delta}_1^*((r, \mu^{t_{i-1}}(r)), \Lambda, q)\right] \\
&= \tilde{\delta}_1^*((p, \mu^{t_{i-1}}(p)), x, r) = \tilde{\delta}_1^*((q, \mu^{t_{i-1}}(q)), xy, p), \\
\bigwedge \left[\tilde{\delta}_2^*((p, \mu^{t_{i-1}}(p)), x, r) \vee \tilde{\delta}_2^*((r, \mu^{t_{i-1}}(r)), y, q)\right] &= \bigwedge \left[\tilde{\delta}_2^*((p, \mu^{t_{i-1}}(p)), x, r) \vee \tilde{\delta}_2^*((r, \mu^{t_{i-1}}(r)), \Lambda, q)\right] \\
&= \tilde{\delta}_2^*((p, \mu^{t_{i-1}}(p)), x, r) = \tilde{\delta}_2^*((q, \mu^{t_{i-1}}(q)), xy, p), \\
\bigwedge \left[\tilde{\delta}_3^*((p, \mu^{t_{i-1}}(p)), x, r) \vee \tilde{\delta}_3^*((r, \mu^{t_{i-1}}(r)), y, q)\right] &= \bigwedge \left[\tilde{\delta}_3^*((p, \mu^{t_{i-1}}(p)), x, r) \vee \tilde{\delta}_3^*((r, \mu^{t_{i-1}}(r)), \Lambda, q)\right] \\
&= \tilde{\delta}_3^*((p, \mu^{t_{i-1}}(p)), x, r) = \tilde{\delta}_3^*((q, \mu^{t_{i-1}}(q)), xy, p).
\end{aligned}$$

The result holds for $n = 0$. Now, continue the result is true for all $u \in \Sigma^*$ with $|u| = n - 1$, where $n > 0$. Let $y = ua$, where $a \in \Sigma$ and $u \in \Sigma^*$. Then

$$\begin{aligned}
&\tilde{\delta}_1^*((q, \mu^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_1^*((q, \mu^{t_{i-1}}(q)), xua, p) = \bigvee_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_1((q, \mu^{t_{i-1}}(q)), xu, r) \wedge \tilde{\delta}_1((r, \mu^{t_i}(r)), a, p)\Big) \\
&= \bigvee_{r \in Q_{act}(t_i)} (\bigvee_{s \in Q_{act}(t_i)} (\tilde{\delta}_1((q, \mu^{t_{i-1}}(q)), x, s) \wedge \tilde{\delta}_1((s, \mu^{t_{i-1}}(s)), u, r)) \wedge \tilde{\delta}_1((r, \mu^{t_i}(r)), a, p)) \\
&= \bigvee_{r, s \in Q_{act}(t_i)} (\tilde{\delta}_1((q, \mu^{t_{i-1}}(q)), x, s) \wedge \tilde{\delta}_1((s, \mu^{t_{i-1}}(s)), u, r) \wedge \tilde{\delta}_1((r, \mu^{t_i}(r)), a, p)) \\
&= \bigvee_{s \in Q_{act}(t_i)} (\tilde{\delta}_1((q, \mu^{t_{i-1}}(q)), x, s) \wedge (\bigvee_{r \in Q_{act}(t_i)} (\tilde{\delta}_1((s, \mu^{t_{i-1}}(s)), u, r) \wedge \tilde{\delta}_1((r, \mu^{t_i}(r)), a, p)))) \\
&= \bigvee_{s \in Q_{act}(t_i)} (\tilde{\delta}_1((q, \mu^{t_{i-1}}(q)), x, s) \wedge \tilde{\delta}_1((s, \mu^{t_i}(r)), ua, p))) = \bigvee_{s \in Q_{act}(t_i)} (\tilde{\delta}_1((q, \mu^{t_{i-1}}(q)), x, s) \wedge \tilde{\delta}_1((s, \mu^{t_i}(r)), y, p))),
\end{aligned}$$

$$\begin{aligned}
&\tilde{\delta}_2^*((q, \mu^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_2^*((q, \mu^{t_{i-1}}(q)), xua, p) = \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_2((q, \mu^{t_{i-1}}(q)), xu, r) \vee \tilde{\delta}_2((r, \mu^{t_i}(r)), a, p)\Big) \\
&= \bigwedge_{r \in Q_{act}(t_i)} (\bigwedge_{s \in Q_{act}(t_i)} (\tilde{\delta}_2((q, \mu^{t_{i-1}}(q)), x, s) \vee \tilde{\delta}_2((s, \mu^{t_{i-1}}(s)), u, r)) \vee \tilde{\delta}_2((r, \mu^{t_i}(r)), a, p)) \\
&= \bigwedge_{r, s \in Q_{act}(t_i)} (\tilde{\delta}_2((q, \mu^{t_{i-1}}(q)), x, s) \vee \tilde{\delta}_2((s, \mu^{t_{i-1}}(s)), u, r) \vee \tilde{\delta}_2((r, \mu^{t_i}(r)), a, p)) \\
&= \bigwedge_{s \in Q_{act}(t_i)} (\tilde{\delta}_2((q, \mu^{t_{i-1}}(q)), x, s) \vee (\bigwedge_{r \in Q_{act}(t_i)} (\tilde{\delta}_2((s, \mu^{t_{i-1}}(s)), u, r) \vee \tilde{\delta}_2((r, \mu^{t_i}(r)), a, p)))) \\
&= \bigwedge_{s \in Q_{act}(t_i)} (\tilde{\delta}_2((q, \mu^{t_{i-1}}(q)), x, s) \vee \tilde{\delta}_2((s, \mu^{t_i}(r)), ua, p))) = \bigwedge_{s \in Q_{act}(t_i)} (\tilde{\delta}_2((q, \mu^{t_{i-1}}(q)), x, s) \vee \tilde{\delta}_2((s, \mu^{t_i}(r)), y, p))),
\end{aligned}$$

$$\tilde{\delta}_3^*((q,\mu^{t_{i-1}}(q)),xy,p) = \tilde{\delta}_3^*((q,\mu^{t_{i-1}}(q)),xua,p) = \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_3((q,\mu^{t_{i-1}}(q)),xu,r) \vee \tilde{\delta}_3((r,\mu^{t_i}(r)),a,p)\Big)$$
$$= \bigwedge_{r\in Q_{act}(t_i)} (\bigwedge_{s\in Q_{act}(t_i)} (\tilde{\delta}_3((q,\mu^{t_{i-1}}(q)),x,s) \vee \tilde{\delta}_3((s,\mu^{t_{i-1}}(s)),u,r)) \vee \tilde{\delta}_3((r,\mu^{t_i}(r)),a,p))$$
$$= \bigwedge_{r,s\in Q_{act}(t_i)} (\tilde{\delta}_3((q,\mu^{t_{i-1}}(q)),x,s) \vee \tilde{\delta}_3((s,\mu^{t_{i-1}}(s)),u,r) \vee \tilde{\delta}_3((r,\mu^{t_i}(r)),a,p))$$
$$= \bigwedge_{s\in Q_{act}(t_i)} (\tilde{\delta}_3((q,\mu^{t_{i-1}}(q)),x,s) \vee (\bigwedge_{r\in Q_{act}(t_i)} (\tilde{\delta}_3((s,\mu^{t_{i-1}}(s)),u,r) \vee \tilde{\delta}_3((r,\mu^{t_i}(r)),a,p))))$$
$$= \bigwedge_{s\in Q_{act}(t_i)} (\tilde{\delta}_3((q,\mu^{t_{i-1}}(q)),x,s) \vee \tilde{\delta}_3((s,\mu^{t_i}(r)),ua,p))) = \bigwedge_{s\in Q_{act}(t_i)} (\tilde{\delta}_3((q,\mu^{t_{i-1}}(q)),x,s) \vee \tilde{\delta}_3((s,\mu^{t_i}(r)),y,p))).$$

Hence the result is valid for $|y| = n$. This completes the proof. □

Definition 3.7. Let $\tilde{F}^*$ be a max-min NGFA, $p \in Q, q \in Q_{act}(t_i), i \geq 0$ and $0 \leq \alpha < 1$. Then p is called a successor of q with threshold α if there exists $x \in \Sigma^*$ such that $\tilde{\delta}_1^*((q,\mu_1^{t_j}(q)),x,p) > \alpha$, $\tilde{\delta}_2^*((q,\mu_2^{t_j}(q)),x,p) < \alpha$ and $\tilde{\delta}_3^*((q,\mu_3^{t_j}(q)),x,p) < \alpha$.

Definition 3.8. Let $\tilde{F}^*$ be a max-min NGFA, $q \in Q_{act}(t_i), i \geq 0$ and $0 \leq \alpha < 1$. Also let $S_\alpha(q)$ denote the set of all successors of q with threshold α. If $T \subseteq Q$, then $S_\alpha(T)$ the set of all successors of T with threshold α is defined by $S_\alpha(T) = \bigcup\{S_\alpha(q) : q \in T\}$.

Definition 3.9. Let $\tilde{F}^*$ be a max-min NGFA. Let $\mu = \langle\mu_1,\mu_2,\mu_3\rangle$ and $\tilde{\delta}^* = \langle\tilde{\delta}_1^*,\tilde{\delta}_2^*,\tilde{\delta}_3^*\rangle$ in $(Q\times[0,1]\times[0,1]\times[0,1])\times\Sigma^*\times Q$ be a neutrosophic set in Q. Then μ is a neutrosophic subsystem of $\tilde{F}^*$, say $\mu \subseteq \tilde{F}^*$ if for every j, $1 \leq j \leq k$ such that $\mu_1^{t_j}(p) \geq \tilde{\delta}_1^*((q,\mu_1^{t_j}(q)),x,p)$, $\mu_2^{t_j}(p) \leq \tilde{\delta}_2^*((q,\mu_2^{t_j}(q)),x,p)$, $\mu_3^{t_j}(p) \leq \tilde{\delta}_3^*((q,\mu_3^{t_j}(q)),x,p)$. $\forall q,p \in Q$ and $x \in \Sigma^*$.

Example 3.10. Let $Q = \{p,q\}$, $\Sigma = \{a\}$. Let $\mu = \langle\mu_1,\mu_2,\mu_3\rangle$ and $\tilde{\delta}^* = \langle\tilde{\delta}_1^*,\tilde{\delta}_2^*,\tilde{\delta}_3^*\rangle$ in $(Q\times[0,1]\times[0,1]\times[0,1])\times\Sigma^*\times Q$ be a neutrosophic set in Q such that $\mu_1^{t_j}(p) = 0.8$, $\mu_2^{t_j}(p) = 0.7$, $\mu_3^{t_j}(p) = 0.5$, $\mu_1^{t_j}(q) = 0.5$, $\mu_2^{t_j}(q) = 0.6$, $\mu_3^{t_j}(q) = 0.8$, $\delta_1(q,x,p) = 0.7$, $\delta_2(q,x,p) = 0.9$ and $\delta_3(q,x,p) = 0.7$. Then

$$\tilde{\delta}_1^*((q,\mu_1^{t_j}(q)),x,p) = F_1^{\wedge}(\mu_1^{t_j}(q),\delta_1(q,x,p)) = \min\{0.5,0.7\} = 0.5 \leq \mu_1^{t_j}(p),$$
$$\tilde{\delta}_2^*((q,\mu_2^{t_j}(q)),x,p) = F_2^{\wedge\vee}(\mu_2^{t_j}(q),\delta_2(q,x,p)) = \max\{0.6,0.9\} = 0.9 \geq \mu_2^{t_j}(p), \quad (\text{since } \ t < t_j)$$
$$\tilde{\delta}_3^*((q,\mu_3^{t_j}(q)),x,p) = F_3^{\vee}(\mu_3^{t_j}(q),\delta_3(q,x,p)) = \max\{0.8,0.7\} = 0.8 \geq \mu_3^{t_j}(p).$$

Hence μ is a neutrosophic subsystem of $\tilde{F}^*$.

Theorem 3.11. *Let $\tilde{F}^*$ be a NGFA and let $\mu = \langle\mu_1,\mu_2,\mu_3\rangle$ and $\tilde{\delta}^* = \langle\tilde{\delta}_1^*,\tilde{\delta}_2^*,\tilde{\delta}_3^*\rangle$ in $(Q\times[0,1]\times[0,1]\times[0,1])\times\Sigma^*\times Q$ be a neutrosophic set in Q. Then μ is a neutrosophic subsystem of $\tilde{F}^*$ if and only if $\mu_1^{t_j}(p) \geq \tilde{\delta}_1^*((q,\mu_1^{t_j}(q)),x,p)$, $\mu_2^{t_j}(p) \leq \tilde{\delta}_2^*((q,\mu_2^{t_j}(q)),x,p)$, $\mu_3^{t_j}(p) \leq \tilde{\delta}_3^*((q,\mu_3^{t_j}(q)),x,p)$, for all $q \in Q_{(act)}(t_j)$, $p \in Q$ and $x \in \Sigma^*$.*

Proof. Suppose that μ is a neutrosophic subsystem of $\tilde{F}^*$. Let $q \in Q_{(act)}(t_j)$, $p \in Q$ and $x \in \Sigma^*$. The proof is by induction on $|x| = n$. If $n = 0$, then $x = \Lambda$. Now if $q = p$, then $\tilde{\delta}_1^*((p,\mu_1^{t_i}(p)),\Lambda,p) = F_1^{\wedge}(\mu_1^{t_i}(p),\tilde{\delta}_1(p,\Lambda,p)) = \mu_1^{t_i}(p)$, $\tilde{\delta}_2^*((p,\mu_2^{t_i}(p)),\Lambda,p) = F_1^{\wedge\vee}(\mu_2^{t_i}(p),\tilde{\delta}_2(p,\Lambda,p)) = \mu_2^{t_i}(p)$, $\tilde{\delta}_3^*((p,\mu_3^{t_i}(p)),\Lambda,p) = F_1^{\vee}(\mu_3^{t_i}(p),\tilde{\delta}_3(p,\Lambda,p)) = \mu_3^{t_i}(p)$.

If $q \neq p$, then $\tilde{\delta}_1^*((q,\mu_1^{t_i}(q)),\Lambda,p) = F_1^{\wedge}(\mu_1^{t_i}(q),\tilde{\delta}_1(q,\Lambda,p)) = 0 \leq \mu_1^{t_j}(p)$, $\tilde{\delta}_2^*((q,\mu_2^{t_i}(q)),\Lambda,p) = F_1^{\wedge\vee}(\mu_2^{t_i}(q),\tilde{\delta}_2(q,\Lambda,p)) = 1 \geq \mu_2^{t_j}(p)$, $\tilde{\delta}_3^*((q,\mu_3^{t_i}(q)),\Lambda,p) = F_1^{\vee}(\mu_3^{t_i}(q),\tilde{\delta}_3(q,\Lambda,p)) = 1 \geq \mu_3^{t_j}(p)$.

Hence the result is true for $n = 0$. For now, we assume that the result is valid for all $y \in \Sigma^*$ with $|y| = n-1$, $n > 0$. For the y above, let $x = u_1 \cdots u_n$ where $u_i \in \Sigma, i = 1, 2, \cdots n$. Then

$$\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p) = \tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), u_1 \cdots u_n, p) = \bigvee \Big(\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), u_1, r_1) \wedge \cdots \wedge \tilde{\delta}_1^*((r_{n-1}, \mu_1^{t_{i+n}}(r_{n-1})), u_n, p)\Big)$$
$$\leq \bigvee \Big(\tilde{\delta}_1^*((r_{n-1}, \mu_1^{t_{i+n}}(r_{n-1})), u_n, p) | r_{n-1} \in Q_{(act)}(t_{i+n})\Big) \leq \bigvee \mu_1^{t_j}(p) = \mu_1^{t_j}(p),$$

$$\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p) = \tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), u_1 \cdots u_n, p) = \bigwedge \Big(\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), u_1, r_1) \vee \cdots \vee \tilde{\delta}_2^*((r_{n-1}, \mu_2^{t_{i+n}}(r_{n-1})), u_n, p)\Big)$$
$$\leq \bigwedge \Big(\tilde{\delta}_2^*((r_{n-1}, \mu_2^{t_{i+n}}(r_{n-1})), u_n, p) | r_{n-1} \in Q_{(act)}(t_{i+n})\Big) \leq \bigwedge \mu_2^{t_j}(p) = \mu_2^{t_j}(p),$$

$$\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p) = \tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), u_1 \cdots u_n, p) = \bigwedge \Big(\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), u_1, r_1) \vee \cdots \vee \tilde{\delta}_3^*((r_{n-1}, \mu_3^{t_{i+n}}(r_{n-1})), u_n, p)\Big)$$
$$\leq \bigwedge \Big(\tilde{\delta}_3^*((r_{n-1}, \mu_3^{t_{i+n}}(r_{n-1})), u_n, p) | r_{n-1} \in Q_{(act)}(t_{i+n})\Big) \leq \bigwedge \mu_3^{t_j}(p) = \mu_3^{t_j}(p),$$

where $r_1 \in Q_{(act)}(t_{i+1}) \cdots r_{n-1} \in Q_{(act)}(t_{i+n})$. Hence $\mu_1^{t_j}(p) \geq \tilde{\delta}_1^*((q, \mu_1^{t_j}(q)), x, p)$, $\mu_2^{t_j}(p) \leq \tilde{\delta}_2^*((q, \mu_2^{t_j}(q)), x, p)$, $\mu_3^{t_j}(p) \leq \tilde{\delta}_3^*((q, \mu_3^{t_j}(q)), x, p)$. The converse is trivial. This proof is completed. □

Definition 3.12. Let $\tilde{F}^*$ be a NGFA. Let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ and $\tilde{\delta}^* = \langle \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{\delta}_3^* \rangle$ in $(Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma^* \times Q$ be a neutrosophic set in Q. Then μ is a neutrosophic strong subsystem of $\tilde{F}^*$, say $\mu \subseteq \tilde{F}^*$, if for every i, $1 \leq i \leq k$ such that $p \in S_\alpha(q)$, then for $q, p \in Q$ and $x \in \Sigma$, $\mu_1^{t_j}(p) \geq \mu_1^{t_j}(q)$, $\mu_2^{t_j}(p) \leq \mu_2^{t_j}(q)$, $\mu_3^{t_j}(p) \leq \mu_3^{t_j}(q)$, for every $1 \leq j \leq k$.

Theorem 3.13. *Let $\tilde{F}^*$ be a NGFA and let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ and $\tilde{\delta}^* = \langle \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{\delta}_3^* \rangle$ in $(Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma^* \times Q$ be a neutrosophic set in Q. Then μ is a strong neutrosophic subsystem of $\tilde{F}^*$ if and only if there exists $x \in \Sigma^*$ such that $p \in S_\alpha(q)$, then $\mu_1^{t_j}(p) \geq \mu_1^{t_j}(q)$, $\mu_2^{t_j}(p) \leq \mu_2^{t_j}(q)$, $\mu_3^{t_j}(p) \leq \mu_3^{t_j}(q)$, for all $q \in Q_{(act)}(t_j)$, $p \in Q$.*

Proof. Suppose that μ is a strong neutrosophic subsystem of $\tilde{F}^*$. Let $q \in Q_{(act)}(t_j)$, $p \in Q$ and $x \in \Sigma^*$. The proof is by induction on $|x| = n$. If $n = 0$, then $x = \Lambda$. Now if $q = p$, then $\delta_1^*((p, \mu_1^{t_i}(p)), \Lambda, p) = 1$, $\delta_2^*((p, \mu_2^{t_i}(p)), \Lambda, p) = 0$, $\delta_3^*((p, \mu_3^{t_i}(p)), \Lambda, p) = 0$ and $\mu_1^{t_j}(p) = \mu_1^{t_j}(p)$, $\mu_2^{t_j}(p) = \mu_2^{t_j}(p)$, $\mu_3^{t_j}(p) = \mu_3^{t_j}(p)$. If $q \neq p$, then $\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), \Lambda, p) = F_1^{\wedge}(\mu_1^{t_i}(q), \tilde{\delta}_1(q, \Lambda, p)) = c \leq \mu_1^{t_j}(p)$, $\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), \Lambda, p) = F_1^{\wedge\vee}(\mu_2^{t_i}(q), \tilde{\delta}_2(q, \Lambda, p)) = d \geq \mu_2^{t_j}(p)$, $\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), \Lambda, p) = F_1^{\vee}(\mu_3^{t_i}(q), \tilde{\delta}_3(q, \Lambda, p)) = e \geq \mu_3^{t_j}(p)$. Hence the result is true for $n = 0$. For now, we assume that the result is valid for all $u \in \Sigma^*$ with $|u| = n-1$, $n > 0$. For the u above, let $x = u_1 \cdots u_n$ where $u_i \in \Sigma^*, i = 1, 2, \cdots n$. Suppose that $\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p) > c$, $\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p) < d$, $\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p) < e$. Then

$$\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), u_1 \cdots u_n, p) = \bigvee \Big\{\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), u_1, p_1) \wedge \cdots \wedge \tilde{\delta}_1^*((p_{n-1}, \mu_1^{t_{i+n}}(p_{n-1})), u_n, p)\Big\} > c,$$
$$\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), u_1 \cdots u_n, p) = \bigwedge \Big\{\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), u_1, p_1) \vee \cdots \vee \tilde{\delta}_2^*((p_{n-1}, \mu_2^{t_{i+n}}(p_{n-1})), u_n, p)\Big\} < d,$$
$$\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), u_1 \cdots u_n, p) = \bigwedge \Big\{\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), u_1, p_1) \vee \cdots \vee \tilde{\delta}_3^*((p_{n-1}, \mu_3^{t_{i+n}}(p_{n-1})), u_n, p)\Big\} < e,$$

where $p_1 \in Q_{(act)}(t_i), \cdots, p_{n-1} \in Q_{(act)}(t_{i+n})$.

This implies that $\tilde{\delta}_1^*((q,\mu_1^{t_i}(q)),u_1,p_1) > c, \cdots, \tilde{\delta}_1^*((p_{n-1},\mu_1^{t_{i+n}}(p_{n-1})),u_n,p) > c$, $\tilde{\delta}_2^*((q,\mu_2^{t_i}(q)),u_1,p_1) < d, \cdots, \tilde{\delta}_2^*((p_{n-1},\mu_2^{t_{i+n}}(p_{n-1})),u_n,p) < d$, $\tilde{\delta}_3^*((q,\mu_3^{t_i}(q)),u_1,p_1) < e, \cdots, \tilde{\delta}_3^*((p_{n-1},\mu_3^{t_{i+n}}(p_{n-1})),u_n,p) < e$. Hence $\mu_1^{t_j}(p) \geq \mu_1^{t_{i+n}}(p_{n-1}), \mu_1^{t_{i+n}}(p) \geq \mu_1^{t_{i+n-1}}(p_{n-2}), \cdots, \mu_1^{t_i}(p_1) \geq \mu_1^{t_j}(q)$, $\mu_2^{t_j}(p) \leq \mu_2^{t_{i+n}}(p_{n-1}), \mu_2^{t_{i+n}}(p) \leq \mu_2^{t_{i+n-1}}(p_{n-2}), \cdots, \mu_2^{t_i}(p_1) \leq \mu_2^{t_j}(q)$, $\mu_3^{t_j}(p) \leq \mu_3^{t_{i+n}}(p_{n-1}), \mu_3^{t_{i+n}}(p) \leq \mu_3^{t_{i+n-1}}(p_{n-2}), \cdots, \mu_3^{t_i}(p_1) \leq \mu_3^{t_j}(q)$. Thus $\mu_1^{t_j}(p) \geq \mu_1^{t_j}(q)$, $\mu_2^{t_j}(p) \leq \mu_2^{t_j}(q)$, $\mu_3^{t_j}(p) \leq \mu_3^{t_j}(q)$. The converse is trivial. The proof is completed. □

4 Neutrosophic General Finite Switchboard Automata

Definition 4.1. Let $\tilde{F}^*$ be a max-min NGFA. Let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ and $\tilde{\delta}^* = \langle \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{\delta}_3^* \rangle$ be a neutrosophic set in $(Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma \times Q$ in Q. Then

1. $\tilde{F}^*$ is switching, if it satisfies $\forall p, q \in Q$, $a \in \Sigma$ and for every i, $i \geq 0$,
   $\tilde{\delta}_1^*((q,\mu_1^{t_i}(q)),a,p) = \tilde{\delta}_1^*((p,\mu_1^{t_i}(p)),a,q)$, $\tilde{\delta}_2^*((q,\mu_2^{t_i}(q)),a,p) = \tilde{\delta}_2^*((p,\mu_2^{t_i}(p)),a,q)$,
   $\tilde{\delta}_3^*((q,\mu_3^{t_i}(q)),a,p) = \tilde{\delta}_3^*((p,\mu_3^{t_i}(p)),a,q)$.

2. $\tilde{F}^*$ is commutative, if it satisfies $\forall p, q \in Q$, $a, b \in \Sigma$ and for every i, $i \geq 1$, $\tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),ab,p) = \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),ba,p)$, $\tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),ab,p) = \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),ba,p)$,
   $\tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),ab,p) = \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),ba,p)$.

3. $\tilde{F}^*$ is Neutrosophic General Finite Switchboard Automata (NGFSA, for short), if $\tilde{F}^*$ satisfies both switching and commutative.

Proposition 4.2. *Let $\tilde{F}$ be a NGFA, if $\tilde{F}^*$ is a commutative NGFSA, then for every $i \geq 1$,*

$$\begin{aligned}
\tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),xa,p) &= \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),ax,p),\\
\tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),xa,p) &= \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),ax,p),\\
\tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),xa,p) &= \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),ax,p),
\end{aligned}$$

for all $q \in Q_{act}(t_{i-1}), p \in S_c(q)$, $a \in \Sigma$ and $x \in \Sigma^*$.

Proof. Since $p \in S_c(q)$ then $q \in Q_{act}(t_{i-1})$ and $|x| = n$. If $n = 0$, then $x = \Lambda$. Thus

$$\begin{aligned}
\tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),xa,p) = \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),\Lambda a,p) = \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),a,p) &= \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),a\Lambda,p)\\
&= \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)),ax,p),\\
\tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),xa,p) = \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),\Lambda a,p) = \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),a,p) &= \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),a\Lambda,p)\\
&= \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)),ax,p),\\
\tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),xa,p) = \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),\Lambda a,p) = \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),a,p) &= \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),a\Lambda,p)\\
&= \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)),ax,p).
\end{aligned}$$

Suppose the result is true for all $u \in \Sigma^*$ with $|u| = n-1$, where $n > 0$. Let $x = ub$, where $b \in \Sigma$. Then

$$\tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)), xa, p) = \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)), uba, p) = \bigvee_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_1((q,\mu_1^{t_{i-1}}(q)), u, r) \wedge \tilde{\delta}_1((r,\mu_1^{t_i}(r)), ba, p)\Big)$$
$$= \bigvee_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_1((q,\mu_1^{t_{i-1}}(q)), u, r) \wedge \tilde{\delta}_1((r,\mu_1^{t_i}(r)), ab, p)\Big) = \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)), uab, p)$$
$$= \bigvee_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_1((q,\mu_1^{t_{i-1}}(q)), ua, r) \wedge \tilde{\delta}_1((r,\mu_1^{t_i}(r)), b, p)\Big)$$
$$= \bigvee_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_1((q,\mu_1^{t_{i-1}}(q)), au, r) \wedge \tilde{\delta}_1((r,\mu_1^{t_i}(r)), b, p)\Big) = \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)), aub, p) = \tilde{\delta}_1^*((q,\mu_1^{t_{i-1}}(q)), ax, p),$$

$$\tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)), xa, p) = \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)), uba, p) = \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_2((q,\mu_2^{t_{i-1}}(q)), u, r) \vee \tilde{\delta}_2((r,\mu_2^{t_i}(r)), ba, p)\Big)$$
$$= \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_2((q,\mu_2^{t_{i-1}}(q)), u, r) \vee \tilde{\delta}_2((r,\mu_2^{t_i}(r)), ab, p)\Big) = \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)), uab, p)$$
$$= \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_2((q,\mu_2^{t_{i-1}}(q)), ua, r) \vee \tilde{\delta}_2((r,\mu_2^{t_i}(r)), b, p)\Big)$$
$$= \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_2((q,\mu_2^{t_{i-1}}(q)), au, r) \vee \tilde{\delta}_2((r,\mu_2^{t_i}(r)), b, p)\Big) = \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)), aub, p) = \tilde{\delta}_2^*((q,\mu_2^{t_{i-1}}(q)), ax, p),$$

$$\tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)), xa, p) = \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)), uba, p) = \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_3((q,\mu_3^{t_{i-1}}(q)), u, r) \vee \tilde{\delta}_3((r,\mu_3^{t_i}(r)), ba, p)\Big)$$
$$= \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_3((q,\mu_3^{t_{i-1}}(q)), u, r) \vee \tilde{\delta}_3((r,\mu_3^{t_i}(r)), ab, p)\Big) = \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)), uab, p)$$
$$= \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_3((q,\mu_3^{t_{i-1}}(q)), ua, r) \vee \tilde{\delta}_3((r,\mu_3^{t_i}(r)), b, p)\Big)$$
$$= \bigwedge_{r\in Q_{act}(t_i)} \Big(\tilde{\delta}_3((q,\mu_3^{t_{i-1}}(q)), au, r) \vee \tilde{\delta}_3((r,\mu_3^{t_i}(r)), b, p)\Big) = \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)), aub, p) = \tilde{\delta}_3^*((q,\mu_3^{t_{i-1}}(q)), ax, p).$$

This completes the proof. □

Proposition 4.3. *Let $\tilde{F}$ be a NGFA, if $\tilde{F}^*$ is a switching NGFSA, then for every $i \geq 0$, $\tilde{\delta}_1^*((q,\mu_1^{t_i}(q)), x, p) = \tilde{\delta}_1^*((p,\mu_1^{t_i}(p)), x, q)$, $\tilde{\delta}_2^*((q,\mu_2^{t_i}(q)), x, p) = \tilde{\delta}_2^*((p,\mu_2^{t_i}(p)), x, q)$, $\tilde{\delta}_3^*((q,\mu_3^{t_i}(q)), x, p) = \tilde{\delta}_3^*((p,\mu_3^{t_i}(p)), x, q)$, for all $p, q \in Q_{act}(t_i)$ and $x \in \Sigma^*$.*

Proof. Since $p, q \in Q_{act}(t_i)$ and $x \in \Sigma^*$, we prove the result by induction on $|x| = n$. First, we assume that $x = \Lambda$, whenever $n = 0$. Then we have $\tilde{\delta}_1^*((q,\mu_1^{t_i}(q)), x, p) = \tilde{\delta}_1^*((q,\mu_1^{t_i}(q)), \Lambda, p) = \tilde{\delta}_1^*((p,\mu_1^{t_i}(p)), \Lambda, q) = \tilde{\delta}_1^*((p,\mu_1^{t_i}(p)), x, q)$, $\tilde{\delta}_2^*((q,\mu_2^{t_i}(q)), x, p) = \tilde{\delta}_2^*((q,\mu_2^{t_i}(q)), \Lambda, p) = \tilde{\delta}_2^*((p,\mu_2^{t_i}(p)), \Lambda, q) = \tilde{\delta}_2^*((p,\mu_2^{t_i}(p)), x, q)$ $\tilde{\delta}_3^*((q,\mu_3^{t_i}(q)), x, p) = \tilde{\delta}_3^*((q,\mu_3^{t_i}(q)), \Lambda, p) = \tilde{\delta}_3^*((p,\mu_3^{t_i}(p)), \Lambda, q) = \tilde{\delta}_3^*((p,\mu_3^{t_i}(p)), x, q)$. Thus, the theorem holds for $x = \Lambda$. Now, we assume that the results holds for all $u \in \Sigma^*$ such that $|u| = n-1$ and $n > 0$. Let

$a \in \Sigma$ and $x \in \Sigma^*$ be such that $x = ua$. Then

$$\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p) = \tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), ua, p) = \bigvee_{r \in Q_{act}(t_{i+1})} \Big(\tilde{\delta}_1((q, \mu_1^{t_i}(q)), u, r) \wedge \tilde{\delta}_1((r, \mu_1^{t_{i+1}}(r)), a, p)\Big)$$
$$= \bigvee_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_1((r, \mu_1^{t_i}(r)), u, q) \wedge \tilde{\delta}_1(p, \mu_1^{t_{i+1}}(p)), a, r)\Big) = \bigvee_{r \in Q_{act}(t_{i+1})} \Big(\tilde{\delta}_1((p, \mu_1^{t_i}(p)), a, r) \wedge \tilde{\delta}_1((r, \mu_1^{t_{i+1}}(r)), u, q)\Big)$$
$$= \tilde{\delta}_1^*((p, \mu_1^{t_i}(p)), au, q) = \tilde{\delta}_1^*((p, \mu_1^{t_i}(p)), ua, q) = \tilde{\delta}_1^*((p, \mu_1^{t_i}(p)), x, q),$$

$$\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p) = \tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), ua, p) = \bigwedge_{r \in Q_{act}(t_{i+1})} \Big(\tilde{\delta}_2((q, \mu_2^{t_i}(q)), u, r) \vee \tilde{\delta}_2((r, \mu_2^{t_{i+1}}(r)), a, p)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_2((r, \mu_2^{t_i}(r)), u, q) \vee \tilde{\delta}_2(p, \mu_2^{t_{i+1}}(p)), a, r)\Big) = \bigwedge_{r \in Q_{act}(t_{i+1})} \Big(\tilde{\delta}_2((p, \mu_2^{t_i}(p)), a, r) \vee \tilde{\delta}_2((r, \mu_2^{t_{i+1}}(r)), u, q)\Big)$$
$$= \tilde{\delta}_2^*((p, \mu_2^{t_i}(p)), au, q) = \tilde{\delta}_2^*((p, \mu_2^{t_i}(p)), ua, q) = \tilde{\delta}_2^*((p, \mu_2^{t_i}(p)), x, q),$$

$$\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p) = \tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), ua, p) = \bigwedge_{r \in Q_{act}(t_{i+1})} \Big(\tilde{\delta}_3((q, \mu_3^{t_i}(q)), u, r) \vee \tilde{\delta}_3((r, \mu_3^{t_{i+1}}(r)), a, p)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_3((r, \mu_3^{t_i}(r)), u, q) \vee \tilde{\delta}_3(p, \mu_3^{t_{i+1}}(p)), a, r)\Big) = \bigwedge_{r \in Q_{act}(t_{i+1})} \Big(\tilde{\delta}_3((p, \mu_3^{t_i}(p)), a, r) \vee \tilde{\delta}_3((r, \mu_3^{t_{i+1}}(r)), u, q)\Big)$$
$$= \tilde{\delta}_3^*((p, \mu_3^{t_i}(p)), au, q) = \tilde{\delta}_3^*((p, \mu_3^{t_i}(p)), ua, q) = \tilde{\delta}_3^*((p, \mu_3^{t_i}(p)), x, q).$$

Hence, the result is true for $|u| = n$. This completes the proof. □

Proposition 4.4. *Let $\tilde{F}$ be a NGFA, if $\tilde{F}^*$ is a NGFSA, then for every $i \geq 1$, $\tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_1^*((p, \mu_1^{t_{i-1}}(p)), yx, q)$, $\tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_2^*((p, \mu_2^{t_{i-1}}(p)), yx, q)$, $\tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_3^*((p, \mu_3^{t_{i-1}}(p)), yx, q)$ for all $p, q \in Q$ and $x, y \in \Sigma^*$.*

Proof. Since $p, q \in Q$ and $x, y \in \Sigma^*$, we prove the result by induction on $|x| = n$. First, we assume that $n = 0$, then $x = \Lambda$. Thus
$\tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), x\Lambda, p) = \tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), \Lambda x, p) = \tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), yx, p)$,
$\tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), x\Lambda, p) = \tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), \Lambda x, p) = \tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), yx, p)$,
$\tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), x\Lambda, p) = \tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), \Lambda x, p) = \tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), yx, p)$.

Suppose that
$\tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), xu, p) = \tilde{\delta}_1^*((p, \mu_1^{t_{i-1}}(p)), ux, q)$, $\tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), xu, p) = \tilde{\delta}_2^*((p, \mu_2^{t_{i-1}}(p)), ux, q)$,
$\tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), xu, p) = \tilde{\delta}_3^*((p, \mu_3^{t_{i-1}}(p)), ux, q)$, for every $u \in \Sigma^*$.

Now, continue the result is true for all $u \in \Sigma^*$ with $|u| = n - 1$, where $n > 0$. Let $y = ua$, where $a \in \Sigma$

and $u \in \Sigma^*$. Then

$$\tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), xua, p) = \bigvee_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_1((q, \mu_1^{t_{i-1}}(q)), xu, r) \wedge \tilde{\delta}_1((r, \mu_1^{t_i}(r)), a, p)\Big)$$
$$= \bigvee_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_1((q, \mu_1^{t_{i-1}}(q)), ux, r) \wedge \tilde{\delta}_1((r, \mu_1^{t_i}(r)), a, p)\Big)$$
$$= \bigvee_{r \in Q_{act}(t_{i-1})} \Big(\tilde{\delta}_1((r, \mu_1^{t_{i-1}}(r)), ux, q) \wedge \tilde{\delta}_1((p, \mu_1^{t_i}(p)), a, r)\Big)$$
$$= \bigvee_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_1((p, \mu_1^{t_{i-1}}(p)), a, r) \wedge \tilde{\delta}_1((r, \mu_1^{t_i}(r)), ux, q)\Big)$$
$$= \tilde{\delta}_1^*((p, \mu_1^{t_{i-1}}(p)), aux, q) = \bigvee_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_1((p, \mu_1^{t_{i-1}}(p)), au, r) \wedge \tilde{\delta}_1((r, \mu_1^{t_i}(r)), x, q)\Big)$$
$$= \bigvee_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_1((p, \mu_1^{t_{i-1}}(p)^), ua, r) \wedge \tilde{\delta}_1((r, \mu_1^{t_i}(r)), x, q)\Big)$$
$$= \tilde{\delta}_1^*((p, \mu_1^{t_{i-1}}(p)), uax, q) = \tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), uax, p) = \tilde{\delta}_1^*((q, \mu_1^{t_{i-1}}(q)), yx, p),$$

$$\tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), xua, p) = \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_2((q, \mu_2^{t_{i-1}}(q)), xu, r) \vee \tilde{\delta}_2((r, \mu_2^{t_i}(r)), a, p)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_2((q, \mu_2^{t_{i-1}}(q)), ux, r) \vee \tilde{\delta}_2((r, \mu_2^{t_i}(r)), a, p)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_{i-1})} \Big(\tilde{\delta}_2((r, \mu_2^{t_{i-1}}(r)), ux, q) \vee \tilde{\delta}_2((p, \mu_2^{t_i}(p)), a, r)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_2((p, \mu_2^{t_{i-1}}(p)), a, r) \vee \tilde{\delta}_2((r, \mu_2^{t_i}(r)), ux, q)\Big)$$
$$= \tilde{\delta}_2^*((p, \mu_2^{t_{i-1}}(p)), aux, q) = \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_2((p, \mu_2^{t_{i-1}}(p)), au, r) \wedge \tilde{\delta}_2((r, \mu_2^{t_i}(r)), x, q)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_2((p, \mu_2^{t_{i-1}}(p)^), ua, r) \vee \tilde{\delta}_2((r, \mu_2^{t_i}(r)), x, q)\Big)$$
$$= \tilde{\delta}_2^*((p, \mu_2^{t_{i-1}}(p)), uax, q) = \tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), uax, p) = \tilde{\delta}_2^*((q, \mu_2^{t_{i-1}}(q)), yx, p),$$

$$\tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), xy, p) = \tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), xua, p) = \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_3((q, \mu_3^{t_{i-1}}(q)), xu, r) \vee \tilde{\delta}_3((r, \mu_3^{t_i}(r)), a, p)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_3((q, \mu_3^{t_{i-1}}(q)), ux, r) \vee \tilde{\delta}_3((r, \mu_3^{t_i}(r)), a, p)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_{i-1})} \Big(\tilde{\delta}_3((r, \mu_3^{t_{i-1}}(r)), ux, q) \vee \tilde{\delta}_3((p, \mu_3^{t_i}(p)), a, r)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_3((p, \mu_3^{t_{i-1}}(p)), a, r) \vee \tilde{\delta}_3((r, \mu_3^{t_i}(r)), ux, q)\Big)$$
$$= \tilde{\delta}_3^*((p, \mu_3^{t_{i-1}}(p)), aux, q) = \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_3((p, \mu_3^{t_{i-1}}(p)), au, r) \wedge \tilde{\delta}_3((r, \mu_3^{t_i}(r)), x, q)\Big)$$
$$= \bigwedge_{r \in Q_{act}(t_i)} \Big(\tilde{\delta}_3((p, \mu_3^{t_{i-1}}(p)^), ua, r) \vee \tilde{\delta}_3((r, \mu_3^{t_i}(r)), x, q)\Big)$$
$$= \tilde{\delta}_3^*((p, \mu_3^{t_{i-1}}(p)), uax, q) = \tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), uax, p) = \tilde{\delta}_3^*((q, \mu_3^{t_{i-1}}(q)), yx, p).$$

This completes the proof. □

Definition 4.5. Let $\tilde{F}^*$ be a GNFSA. Let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ and $\tilde{\delta}^* = \langle \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{\delta}_3^* \rangle$ in $(Q \times [0,1]) \times \Sigma^* \times Q$ be a neutrosophic set in Q. Then μ is a neutrosophic switchboard subsystem of $\tilde{F}^*$, say $\mu \subseteq \tilde{F}^*$, if for every j, $1 \leq j \leq k$ such that $\mu_1^{t_j}(p) \geq \tilde{\delta}_1^*((q, \mu_1^{t_j}(q)), x, p)$, $\mu_2^{t_j}(p) \leq \tilde{\delta}_2^*((q, \mu_2^{t_j}(q)), x, p)$, $\mu_3^{t_j}(p) \leq \tilde{\delta}_3^*((q, \mu_3^{t_j}(q)), x, p)$. $\forall q, p \in Q$ and $x \in \Sigma$.

Theorem 4.6. *Let $\tilde{F}^*$ be a NGFSA and let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ and $\tilde{\delta}^* = \langle \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{\delta}_3^* \rangle$ in $(Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma^* \times Q$ be a neutrosophic set in Q. Then μ is a neutrosophic switchboard subsystem of $\tilde{F}^*$ if and only if $\mu_1^{t_j}(p) \geq \tilde{\delta}_1^*((q, \mu_1^{t_j}(q)), x, p)$, $\mu_2^{t_j}(p) \leq \tilde{\delta}_2^*((q, \mu_2^{t_j}(q)), x, p)$, $\mu_3^{t_j}(p) \leq \tilde{\delta}_3^*((q, \mu_3^{t_j}(q)), x, p)$, for all $q \in Q_{(act)}(t_j)$, $p \in Q$ and $x \in \Sigma^*$.*

Proof. The proof of the theorem is similar to Theorem 3.11 and it is clear that μ satisfies switching and commutative, since $\tilde{F}^*$ is NGFSA. This proof is completed. □

Definition 4.7. Let $\tilde{F}^*$ be a NGFSA. Let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ and $\tilde{\delta}^* = \langle \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{\delta}_3^* \rangle$ in $(Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma^* \times Q$ be a neutrosophic set in Q. Then μ is a neutrosophic strong switchboard subsystem of $\tilde{F}^*$, say $\mu \subseteq \tilde{F}^*$, if for every i, $1 \leq i \leq k$ such that $p \in S_\alpha(q)$, then for $q, p \in Q$ and $x \in \Sigma$, $\mu_1^{t_j}(p) \geq \mu_1^{t_j}(q)$, $\mu_2^{t_j}(p) \leq \mu_2^{t_j}(q)$, $\mu_3^{t_j}(p) \leq \mu_3^{t_j}(q)$, for every $1 \leq j \leq k$.

Theorem 4.8. *Let $\tilde{F}^*$ be a NGFA and let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ and $\tilde{\delta}^* = \langle \tilde{\delta}_1^*, \tilde{\delta}_2^*, \tilde{\delta}_3^* \rangle$ in $(Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma^* \times Q$ be a neutrosophic set in Q. Then μ is a strong neutrosophic switchboard subsystem of $\tilde{F}^*$ if and only if there exists $x \in \Sigma^*$ such that $p \in S_\alpha(q)$, then $\mu_1^{t_j}(p) \geq \mu_1^{t_j}(q)$, $\mu_2^{t_j}(p) \leq \mu_2^{t_j}(q)$, $\mu_3^{t_j}(p) \leq \mu_3^{t_j}(q)$, for all $q \in Q_{(act)}(t_j)$, $p \in Q$.*

Proof. The proof of the theorem is similar to Theorem 3.13 and it is clear that μ satisfies switching and commutative, since $\tilde{F}^*$ is NGFSA. The proof is completed. □

Theorem 4.9. *Let $\tilde{F}^*$ be a NGFSA and let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ be a neutrosophic subset of Q. If μ is a neutrosohic switchboard subsystem of $\tilde{F}^*$, then μ is a strong neutrosophic switchboard subsystem of $\tilde{F}^*$.*

Proof. Assume that $\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p) > 0$, $\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p) < 1$ and $\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p) < 1$, for all $x \in \Sigma$. Since μ is a neutrosophic switchboard subsystem of $\tilde{F}^*$, we have

$$\mu_1^{t_j}(p) \geq \tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p), \quad \mu_2^{t_j}(p) \leq \tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p), \quad \mu_3^{t_j}(p) \leq \tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p).$$

for all $q \in Q_{(act)}(t_j)$, $p \in Q$ and $x \in \Sigma$. As μ is switching, then we have

$$\begin{aligned}
\mu_1^{t_j}(p) &\geq \tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p) = \tilde{\delta}_1^*((p, \mu_1^{t_i}(p)), x, q) = \mu_1^{t_j}(q), \\
\mu_2^{t_j}(p) &\leq \tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p) = \tilde{\delta}_2^*((p, \mu_2^{t_i}(p)), x, q) = \mu_2^{t_j}(q), \\
\mu_3^{t_j}(p) &\leq \tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p) = \tilde{\delta}_3^*((p, \mu_3^{t_i}(p)), x, q) = \mu_3^{t_j}(q).
\end{aligned}$$

As μ is commutative, then $x = uv$, we have

$$\begin{aligned}
\mu_1^{t_j}(p) &\geq \tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p) = \tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), uv, p) \\
&= \bigvee \left\{ \tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), u, r) \wedge \tilde{\delta}_1^*((r, \mu_1^{t_{i+1}}(r)), v, p) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \bigvee \left\{ \tilde{\delta}_1^*((r, \mu_1^{t_i}(r)), u, q) \wedge \tilde{\delta}_1^*((p, \mu_1^{t_{i+1}}(p)), v, r) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \bigvee \left\{ \tilde{\delta}_1^*((p, \mu_1^{t_{i+1}}(p)), v, r) \wedge \tilde{\delta}_1^*((r, \mu_1^{t_i}(r)), u, q) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \tilde{\delta}_1^*((p, \mu_1^{t_{i+1}}(p)), vu, q) = \tilde{\delta}_1^*((p, \mu_1^{t_{i+1}}(p)), uv, q) = \tilde{\delta}_1^*((p, \mu_1^{t_{i+1}}(p)), x, q) \geq \mu_1^{t_j}(q), \\
\mu_2^{t_j}(p) &\leq \tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p) = \tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), uv, p) \\
&= \bigwedge \left\{ \tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), u, r) \vee \tilde{\delta}_2^*((r, \mu_2^{t_{i+1}}(r)), v, p) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \bigwedge \left\{ \tilde{\delta}_2^*((r, \mu_2^{t_i}(r)), u, q) \vee \tilde{\delta}_2^*((p, \mu_2^{t_{i+1}}(p)), v, r) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \bigwedge \left\{ \tilde{\delta}_2^*((p, \mu_2^{t_{i+1}}(p)), v, r) \vee \tilde{\delta}_2^*((r, \mu_2^{t_i}(r)), u, q) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \tilde{\delta}_2^*((p, \mu_2^{t_{i+1}}(p)), vu, q) = \tilde{\delta}_2^*((p, \mu_2^{t_{i+1}}(p)), uv, q) = \tilde{\delta}_2^*((p, \mu_2^{t_{i+1}}(p)), x, q) \leq \mu_2^{t_j}(q), \\
\mu_3^{t_j}(p) &\leq \tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p) = \tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), uv, p) \\
&= \bigwedge \left\{ \tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), u, r) \vee \tilde{\delta}_3^*((r, \mu_3^{t_{i+1}}(r)), v, p) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \bigwedge \left\{ \tilde{\delta}_3^*((r, \mu_3^{t_i}(r)), u, q) \vee \tilde{\delta}_3^*((p, \mu_3^{t_{i+1}}(p)), v, r) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \bigwedge \left\{ \tilde{\delta}_3^*((p, \mu_3^{t_{i+1}}(p)), v, r) \vee \tilde{\delta}_3^*((r, \mu_3^{t_i}(r)), u, q) | r \in Q_{1(act)}(t_{i+1}) \right\} \\
&= \tilde{\delta}_3^*((p, \mu_3^{t_{i+1}}(p)), vu, q) = \tilde{\delta}_3^*((p, \mu_3^{t_{i+1}}(p)), uv, q) = \tilde{\delta}_3^*((p, \mu_3^{t_{i+1}}(p)), x, q) \leq \mu_3^{t_j}(q).
\end{aligned}$$

Hence μ is a strong neutrosophic switchboard subsystem of $\tilde{F}^*$. □

Theorem 4.10. *Let $\tilde{F}^*$ be a NGFSA and let $\mu = \langle \mu_1, \mu_2, \mu_3 \rangle$ be a neutrosophic subset of Q. If μ is a strong neutrosophic switchboard subsystem of $\tilde{F}^*$, then μ is a neutrosophic switchboard subsystem of $\tilde{F}^*$.*

Proof. Let $q, p \in Q$. Since μ is a strong neutrosophic switchboard subsystem of $\tilde{F}^*$ and μ is switching, we have for all $x \in \Sigma$, since $\tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p) > 0$, $\tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p) < 1$ and $\tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p) < 1$, $\forall x \in \Sigma$,
$\mu_1^{t_j}(p) \geq \mu_1^{t_j}(q) \geq \tilde{\delta}_1^*((p, \mu_1^{t_i}(p)), x, q) \geq \tilde{\delta}_1^*((q, \mu_1^{t_i}(q)), x, p)$,
$\mu_2^{t_j}(p) \leq \mu_2^{t_j}(q) \leq \tilde{\delta}_2^*((p, \mu_2^{t_i}(p)), x, q) \leq \tilde{\delta}_2^*((q, \mu_2^{t_i}(q)), x, p)$,
$\mu_3^{t_j}(p) \leq \mu_3^{t_j}(q) \leq \tilde{\delta}_3^*((p, \mu_3^{t_i}(p)), x, q) \leq \tilde{\delta}_3^*((q, \mu_3^{t_i}(q)), x, p)$.

It is clear that μ is commutative. Thus μ is a neutrosophic switchboard subsystem of $\tilde{F}^*$. □

5 Conclusions

This paper attempt to develop and present a new general definition for neutrosophic finite automata. The general definition for (strong) subsystem also examined and discussed their properties. A comprehensive analysis and an appropriate methodology to manage the essential issues of output mapping in general fuzzy automata were studied by Doostfatemen and Kremer [11]. Their approach is consistent with the output which is either associated with the states (Moore model) or with the transitions (Mealy model). Interval-valued fuzzy subsets have many applications in several areas. The concept of interval-valued fuzzy sets have been studied in various algebraic structures, see [7, 26]. On the basis [11] and [7], the future work will focus on general interval-valued neutrosophic finite automata with output respond to input strings.

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