Neutrosophic Geometric Programming (NGP) Problems Subject to $(\vee, .)$ Operator; the Minimum Solution

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Abstract. This paper comes as a second step serves the purpose of constructing a neutrosophic optimization model for the relation geometric programming problems subject to (max, product) operator in its constraints. This essay comes simultaneously with my previous paper entitled (Neutrosophic Geometric Programming (NGP) with (max-product) Operator, An Innovative Model) which contains the structure of the maximum solution. The purpose of this article is to set up the minimum solution for the (RNGP) problems, the author faced many difficulties, where the feasible region for this type of problems is already non-convex; furthermore, the negative signs of the exponents with neutrosophic variables $x_j \in [0,1] \cup I$. A new technique to avoid the divided by the indeterminacy component (I) was introduced; Separate the neutrosophic geometric programming into two optimization models, introducing two new matrices named as the distinguishing matrix and the facilitation matrix. All these notions were important for finding the minimum solution of the program. Finally, two numerical examples were presented to enable the reader to understand this work.

Keyword: Relational Neutrosophic Geometric Programming (RNGP); $(\vee, .)$ Operator; Neutrosophic Relation Equations; Distinguishing Matrix; Facilitation Matrix; Minimum Solution; Incompatible Problem.

1. Introduction

As of 1995 so far, dozens of mathematicians and researchers in many fields of sciences trying to study and understand the neutrosophic theory, the first mathematician who set up and put forward the neutrosophic theory was Smarandache F. at 1995 [2,11], he is in the neutrosophic theory as Lotfi A. Zadeh [12] in fuzzy theory and as K. Atanasov [10] in intuitionistic fuzzy theory. The importance of the neutrosophic logic comes from its ability to deal with the indeterminacy component (I), this component makes scholars generalize the fuzzy and intuitionistic fuzzy logics, give them the ability to put the paradoxes in a new framework, and it makes the researchers deal with contradicted information in more relaxation. This paper comes as an establishing article in the relational neutrosophic programming problems (RNGP) with $(\vee, .)$ in its constraints. This kind of problems has many applications in real-world problems, like communication system, civil engineering, mechanical engineering, structural design and optimization, business management …etc. The author published previous articles [1,3,4,6,7,9] to expand the fuzzy theory to be fit with neutrosophic theory, this essay was one of the series of these articles.

This publication includes three original sections, despite the second section goes to the basic concepts, but these pure concepts were originated by the author at the

simultaneously published paper, which focused on the form of the maximum solution in the (RNGP) with (∨,.) operator, the third section was dedicated to many unprecedented mathematical formulas such as pre-distinguishing matrices, pre-facilitation matrices, a new technique to separate the optimization model into two models depending upon the sign of terms powers in the objective function, and a technique to filter all minimum solutions, the forth section was for two numerical examples, they are the same examples that presented in the article [8] which assigned to the maximum solution, the last section includes the conclusion.

2. Basic Concepts

We call

$$
\left.
\begin{array}{ll}
\min f(x) = \left(c_1. x_1^{\gamma_1}\right) \vee \left(c_2. x_2^{\gamma_2}\right) \vee \ldots \vee \left(c_n. x_n^{\gamma_n}\right) \\
s.t. \qquad\quad Aox = b \\
\quad x_j \in [0,1] \cup I, \quad 1 \le j \le n
\end{array}
\right\} \qquad (1)
$$

A ($\vee,.$) (max- product) neutrosophic geometric programming, where $A = (a_{ij})$, $1 \le i \le m$, $1 \le j \le n$, $a_{ij} \in [0,1]$ is $(m \times n)$ dimensional neutrosophic matrix, $x = (x_1, x_2, \ldots, x_n)^T$ an n-dimensional variable vector, $b = (b_1, b_2, \ldots, b_m)^T$ $(b_i \in [0,1] \cup I)$ an m- dimensional constant vector, $c = (c_1, c_2, \ldots, c_n)^T$ $(c_j \ge 0)$ an n- dimensional constant vector, γ_j is an arbitrary real number, and the composition operator "o" is ($\vee,.$), i.e. $\vee_{j=1}^{n}(a_{ij}. x_j) = b_i$. Note that the program (1) is undefined and has no minimal solution in the case of $\gamma_j < 0$ with all $x_j's$ taking indeterminacy value.

2.1. Definition [8]

$$
a_{ij} \bowtie b_i = \begin{cases}
\frac{b_i}{a_{ij}}, & if \;\; a_{ij} > b_i \, , \; a_{ij} \in [0,1], \; b_i \in [0,1] \\
1, & if \;\; a_{ij} \le b_i \, , \; a_{ij} \in [0,1], \; b_i \in [0,1] \\
1 \, , & if \qquad a_{ij} \in [0,1], \; b_i = nI, n \in (0,1]
\end{cases} \qquad (2)
$$

$$
a_{ij} \Theta b_i = \begin{cases}
\frac{nI}{a_{ij}}, & if \;\; a_{ij} > n \, , \; a_{ij} \in [0,1], b_i = nI, n \in (0,1] \\
1, & if \;\; a_{ij} \le n \, , \; a_{ij} \in [0,1], b_i = nI, n \in (0,1] \\
not \; comp. & if \qquad a_{ij} = mI \, , m \in (0,1] \, , b_i \in [0,1] \cup I \\
1 & if \qquad\qquad\qquad a_{ij}, b_{ij} \in [0,1]
\end{cases} \qquad (3)
$$

Where $\bowtie$ is an operator defined at [0,1], while the operator Θ is defined at $[0,1] \cup I$. Let

$$
\hat{x}_j = \wedge_{i=1}^{m}(a_{ij} \bowtie b_i), \qquad (1 \le j \le n) \qquad (4)
$$

be the components of the pre maximum solution $\hat{x}_{v1}$.(i.e. $\hat{x}_{v1} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$)

Let $\hat{x}_j = \wedge_{i=1}^{m}(a_{ij} \Theta b_i), \qquad (1 \le j \le n)$, $\qquad (5)$

be the components of the pre maximum solution $\hat{x}_{v2}$. (i.e. $\hat{x}_{v2} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$)

Now the following question will be raised,

Which one $\hat{x}_{v1}$ or $\hat{x}_{v2}$ should be the exact maximum solution?

Neither $\hat{x}_{v1}$ nor $\hat{x}_{v2}$ will be the exact solution! The exact solution is integrated between them.

Before solving $Ao\hat{x} = b$, we first define the matrices A_{v1}, A_{v2}.

Let A_{v1} be a matrix has the same dimension and the same rows elements of A except for those rows of the indexes $i = i_o$ corresponding to those indexes of $b_{i_o} = nI$, those special rows of

A_{v1} will be zeros. Let A_{v2} be a matrix has the same dimension and the same rows elements of A except for those rows of the indexes $i = i_o$ corresponding to those indexes of $b_{i_o} \in [0,1]$, those special rows of A_{v2} will be zeros. Consequently,

$$Ao\hat{x} = b = (A_{v1}o\hat{x}_{v1}) + (A_{v2}o\hat{x}_{v2}) \tag{6}$$

The formula (6) is the greatest solution in $X(A,b)$.

The maximum value of the objective function $f(\hat{x}) = f(\hat{x}_{v1}) \vee f(\hat{x}_{v2})$.

2.2. Theorem [8]

If $\gamma_j < 0$ $(1 \leq j \leq n)$, then the greatest solution to the problem (1) is an optimal solution.

2.3. Definition [5]

If there exists a solution to $x = b$, it's called compatible. Suppose $X(A,b) = \{(x_1, x_2, \dots, x_m)^T \in [0,1]^n \cup I, \ I^n = I, n > 0 | xoA = b, x_i \in [0,1] \cup I\}$ is a solution set of $Aox = b$, we define $x^1 \leq x^2 \Leftrightarrow x_j^1 \leq x_j^2$ $(1 \leq j \leq n), \forall x^1, x^2 \in X(A,b)$. Where " $\leq$ " is a partial order relation on $X(A,b)$.

3. The Structure of the Minimum Solution $\check{x}$.

The feasible region of the solution domain for the neutrosophic geometric programming (NGP) problems subject to (max-product) operator in its constraints is a solution to $Aox = b$, therefore the definition of the solution set $X(A,b)$ and the shape of the maximum and the minimum solutions are very important to optimize the (NGP) model.

The structure of the maximum solution was introduced by Huda E. Khalid in [8].

The definition (2.3) was constructed by Huda E. Khalid at 2016 [5], this definition was dedicated for (RNGP) problems subject to (max-min) operator, this definition is also appropriate for (RNGP) problems with (max, product) operator.

3.1. Definition

If there exists a minimum solution in the solution set $X(A,b)$, then the numbers of the minimum solutions are not lonesome such as the maximum solution. If we denote all minimum elements by $\check{X}(A,b)$, then another version of $X(A,b)$ can be presented depending upon the minimum and the maximum solutions as follows:

$$X(A,b) = \cup_{\check{x} \in \check{X}(A,b)} \ \{x \mid \check{x} \leq x \leq \hat{x}, x \in X\} \tag{7}$$

The following definitions introduce some important new matrices that were constructed by the author for using them in the filtering rule for finding the minimum solution.

3.2. Definition

Let $S_1 = (s_{ij}{}^1)_{m \times n}$, $S_2 = (s_{ij}{}^2)_{m \times n}$ be two pre - distinguishing matrices of A, where

$$s_{ij}{}^1 = \begin{cases} a_{ij}, & a_{ij}.\hat{x}_j = b_i \\ 0, & a_{ij}.\hat{x}_j \neq b_i \end{cases} \tag{8}$$

In (8), the $\hat{x}_j$'s are the components of the pre - maximum solution $\hat{x}_{v1}$ which supports the fuzzy part of the problem, while the elements a_{ij} are the elements of the matrix A_{v1}.

$$s_{ij}{}^2 = \begin{cases} a_{ij}, & a_{ij}.\hat{x}_j = b_i \\ 0, & a_{ij}.\hat{x}_j \neq b_i \end{cases} \tag{9}$$

In (9), the $\hat{x}_j$'s are the components of the pre - maximum solution $\hat{x}_{v2}$ which supports the neutrosophic part of the problem, while a_{ij} are the elements of the matrix A_{v2}.

Let

$$S = (s_{ij})_{m\times n} = (s_{ij}{}^1)_{m\times n} + (s_{ij}{}^2)_{m\times n} = S_1 + S_2 \tag{10}$$

The matrix S is called the distinguishing matrix of A. It is obvious that the constraints system $Aox = b$ has a solution if and only if the distinguishing matrix S of A has non zero rows (i.e. S has at least a nonzero element in each row).

3.3. Definition

Let $F_1 = (f_{ij}{}^1)_{m\times n}$, $F_2 = (f_{ij}{}^2)_{m\times n}$ be two pre - facilitation matrices of A, where

$$f_{ij}{}^1 = \begin{cases} \hat{x}_{ij}, & a_{ij}.\hat{x}_j = b_i \\ 0, & a_{ij}.\hat{x}_j \neq b_i \end{cases} \tag{11}$$

In (11), the $\hat{x}_j$'s are the components of the pre- maximum solution $\hat{x}_{v1}$ which supports the fuzzy part of the problem, while the elements a_{ij} are the entries of A_{v1}.

$$f_{ij}{}^2 = \begin{cases} \hat{x}_{ij}, & a_{ij}.\hat{x}_j = b_i \\ 0, & a_{ij}.\hat{x}_j \neq b_i \end{cases} \tag{12}$$

In (12), the $\hat{x}_j$'s are the components of the pre - maximum solution $\hat{x}_{v2}$ which supports the neutrosophic part of the problem,

Let

$$F = (f_{ij})_{m\times n} = (f_{ij}{}^1)_{m\times n} + (f_{ij}{}^2)_{m\times n} = F_1 + F_2 \tag{13}$$

The matrix F is called the Facilitation matrix of A.

Both matrices S and F are first introduced in this paper and they have a key role in finding the set of all quasi-minimum solutions and then the optimal solution for NGP problems.

3.4 The Filtration Method for Finding Minimum Solutions

1. Delete the $i - th$ row of F, for which $b_i = 0$

2. At $b_i > 0$, find an index $z \in \{1,2, \dots, m\}$ such that $z > i$, if for all $j = 1,2, \dots, n$ we find $f_{zj} \neq 0 \Longleftrightarrow f_{ij} \neq 0$, then delete the $i - th$ row of F.

3. Denote $\tilde{F}$ for the matrix that gained from the above steps (i.e steps 1&2).

4. To each row of $\tilde{F}$, in each time, the only nonzero value is selected in every row with all entries of the rest seen as zero, perhaps all of the matrices are denoted by $\tilde{F}_1, \tilde{F}_2, \dots, \tilde{F}_p$.

5. To each column of $\tilde{F}_k$ $(1 \leq k \leq p)$, the maximum element is selected, a quasi-minimum solution $\check{x}_j$ can be obtained through such a method

The set composed of all $\check{x}_j$ is called a quasi-minimum solution, and it includes all minimum solutions to $Aox = b$. Delete all repeated solutions, and then all minimum solutions $\check{X}(A, b)$ can be obtained.

As an integrated study for all cases of the exponents (γ_j) of the terms in the objective function $f(x)$, we saw that the theorem (2.2) covered the negative exponents, while the following theorem will cover the positive exponents for the terms of $f(x)$.

3.5 Theorem

If $\gamma_j \geq 0$ $(1 \leq j \leq n)$, then a certain minimum solution $\check{x}$ to $Aox = b$ is an optimal one to the program (1).

Proof

Since $\gamma_j \geq 0$ $(1 \leq j \leq n)$, then $\frac{d(x_j^{\gamma_j})}{dx_j} = \gamma_j x_j^{\gamma_j - 1} \geq 0$.

We have $x_j \in [0,1] \cup I$, so $x_j^{\gamma_j}$ is a monotone increasing function concerning x_j, so is $c_j x_j^{\gamma_j}$ concerning x_j. Hence, $\forall x \in X(A, b)$, depending on formula (7), then there exists $\check{x} \in \check{X}(A, b)$, such that $x \geq \check{x}$ (i.e. $x_j \geq \check{x}_j$) $\Longrightarrow c_j. x_j^{\gamma_j} \geq c_j. \check{x}_j^{\gamma_j}$ $(1 \leq j \leq n) \Longrightarrow f(x) \geq f(\check{x})$, this means that the optimal solution to the program (1) must exist in

$\breve{X}(A,b).f(\breve{x}^*) = \min\{f(\breve{x}) \mid \breve{x} \in \breve{X}(A,b)\}$. Then $\forall\, x \in X(A,b)$, there exists $f(x) \geq f(\breve{x}^*)$, so $\breve{x}^* \in \breve{X}(A,b)$ is an optimal solution to the program (1).

3.6 Two Optimization Models Based on the Sign of γ_j

Let $M_1 = \{j \mid \gamma_j < 0, 1 < j < n\}$, $M_2 = \{j \mid \gamma_j > 0, 1 < j < n\}$, then $M_1 \cap M_2 = \emptyset$, $M_1 \cup M_2 = J$, here $J = \{1,2,\dots,n\}$. It is evident that the terms of the objective function $f(x)$ in the program (1) having negative powers is

$$f_1(x) = \vee_{j \in M_1}\{(c_j.x_j^{\gamma_j})\} \tag{14}$$

While the terms of $f(x)$ that having positive exponents is

$$f_2(x) = \vee_{j \in M_2}\{(c_j.x_j^{\gamma_j})\} \tag{15}$$

Based on (14) and (15), we have the following two optimization models,

$$\begin{aligned} &\min f_1(x) \\ &s.t.\, Aox = b \\ &x_j \in [0,1] \cup I \end{aligned} \tag{16}$$

$$\begin{aligned} &\min f_2(x) \\ &s.t.\, Aox = b \\ &x_j \in [0,1] \cup I \end{aligned} \tag{17}$$

Using theorem (2.2), $\hat{x}$ is an optimal solution for (16). By theorem (3.5), there exists $\breve{x}^* \in \breve{X}(A,b)$, where $\breve{x}^*$ is an optimal solution for (17).

3.7 Important Notes

1. In this type of problems, the first step is to search for the maximum solution which is lonesome for every problem. If the purpose of the program (1) is to optimize it, with the restriction that all powers of the variables x_j are negative, then the greatest solution is the optimal one {i.e. $f(x^*) = f(\hat{x}) = f(\hat{x}_{v1}) \wedge f(\hat{x}_{v2})$}.

2. The second step is to search for the minimum solution which is the set of all minimal solutions $\breve{X}(A,b)$. When the purpose of the program (1) is to optimize it, with the restriction that some of the exponents are negative and others are positive, then $f(x^*) = f_1(\hat{x}) \wedge f_2(\breve{x})$.

3. It should be noticed that the components of $\hat{x}_{v2}$ containing indeterminate values (I) raised to the negative powers of $f(x)$ must be neglected, otherwise, it will be undefined program.

The upcoming section covering numerical examples, those examples are the same that discussed in [8] for its maximal solution, we could not be remote far away from the paper [8], present paper regarded as the complement of [8] which contained the formula of the maximum solution, while this present paper introduces the set of all minimum solutions.

4   Numerical examples

We now gaze the (max, product) neutrosophic relation geometric programming examples as follows

3.1   Example

Solve

$$\min f(x) = (0.3. x_1^2) \vee (1.8I . x_2^{\frac{1}{3}}) \vee (I . x_3^{\frac{1}{4}})$$

s. t. $Aox = b$

$x_j \in [0,1] \cup I \quad (1 \leq j \leq n)$

Where $b = (1, \frac{1}{3}I, \frac{1}{5}I)^T$, $A = \begin{pmatrix} .6 & 1 & .2 \\ .5 & .2 & .1 \\ .3 & .5 & .1 \end{pmatrix}_{3 \times 3}$.

Solution:

$\hat{x}_{v1} = (\hat{x}_1, \hat{x}_2, \hat{x}_3)^T = (1,1,1)^T$, $\hat{x}_{v2} = (\hat{x}_1, \hat{x}_2, \hat{x}_3)^T = \left(\frac{2}{3}I, \frac{2}{5}I, 1\right)^T$,

$A_{v1} = \begin{pmatrix} .6 & 1 & .2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $A_{v2} = \begin{pmatrix} 0 & 0 & 0 \\ .5 & .2 & .1 \\ .3 & .5 & .1 \end{pmatrix}$,

It is easy to notice that all exponents of $f(x)$ terms are positive. Therefore there will not be a need to separate $f(x)$ into f_1 and f_2.

$f(\hat{x}) = f(\hat{x}_{v1}) \vee f(\hat{x}_{v2}) = 1.8I$ is the maximum solution.

Using theorem (3.5), it is essential to find the set of all minimum solutions for $f(x)$, where the optimal solution occurs at the minimal solution.

$$S_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ S_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0.3 & 0.5 & 0 \end{bmatrix}, S = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0 \\ 0.3 & 0.5 & 0 \end{bmatrix}.$$

$$F_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3}I & 0 & 0 \\ \frac{2}{3}I & \frac{2}{5}I & 0 \end{bmatrix}, F = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2}{3}I & 0 & 0 \\ \frac{2}{3}I & \frac{2}{5}I & 0 \end{bmatrix}.$$

Using the filtration rule stated in section (3.4),

$$\tilde{F} = \begin{bmatrix} \frac{2}{3}I & 0 & 0 \\ \frac{2}{3}I & \frac{2}{5}I & 0 \end{bmatrix} \Rightarrow \tilde{F}_1 = \begin{bmatrix} \frac{2}{3}I & 0 & 0 \\ \frac{2}{3}I & 0 & 0 \end{bmatrix}, \tilde{F}_2 = \begin{bmatrix} \frac{2}{3}I & 0 & 0 \\ 0 & \frac{2}{5}I & 0 \end{bmatrix},$$

so the minimum solutions that related to $\tilde{F}_1$ and $\tilde{F}_2$ are $\check{x}_1 = [\frac{2}{3}I, 0,0]$, $\check{x}_2 =$

$$[\frac{2}{3}I, \frac{2}{5}I, 0].$$

$f(\check{x}_1) = f(\check{x}_2) = \frac{2}{15}I$ is the minimum solution.

3.2 Example

Let $\min f(x) = \left(0.2I. x_1^{-\frac{2}{3}}\right) \vee \left(1.3. x_2^{\frac{1}{3}}\right) \vee (I . x_3^{\frac{1}{2}}) \vee (0.35. x_4^{-2})$

s. t. $Aox = b$

$x_j \in [0,1] \cup I \quad (1 \le j \le n)$

Where $\quad b = (0.3, 0.7I, 0.5, 0.2I)^T$, $A = \begin{pmatrix} .2 & .3 & .4 & .6 \\ .3 & .2 & .9 & .8 \\ 1 & 0 & .1 & 1 \\ 0 & .5 & 1 & 0 \end{pmatrix}_{4\times4}$.

Solution

$\hat{x}_{v1} = (\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4)^T = \left(0.5, 1, \frac{3}{4}, 0.5\right)^T$, $\hat{x}_{v2} = (\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4)^T =$

$\left(\frac{2}{5}I, 1, 0.2I, 0.875I\right)^T$,

The greatest solution for this problem is $f(\hat{x}) = f(\hat{x}_{v1}) \vee f(\hat{x}_{v2}) = 1.3$.

The following calculations are for finding the minimum solution.

$$A_{v1} = \begin{pmatrix} .2 & .3 & .4 & .6 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & .1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, A_{v2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ .3 & .2 & .9 & .8 \\ 0 & 0 & 0 & 0 \\ 0 & .5 & 1 & 0 \end{pmatrix}.$$

$$S_1 = \begin{pmatrix} 0 & .3 & .4 & .6 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, S_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \Rightarrow S = \begin{pmatrix} 0 & .3 & .4 & .6 \\ 0 & 0 & 0 & .8 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

$$F_1 = \begin{bmatrix} 0 & 1 & \frac{3}{4} & .5 \\ 0 & 0 & 0 & 0 \\ .5 & 0 & 0 & .5 \\ 0 & 0 & 0 & 0 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .875I \\ 0 & 0 & 0 & 0 \\ 0 & 1 & .2I & 0 \end{bmatrix}, \Rightarrow F = \begin{bmatrix} 0 & 1 & \frac{3}{4} & .5 \\ 0 & 0 & 0 & .875I \\ .5 & 0 & 0 & .5 \\ 0 & 1 & .2I & 0 \end{bmatrix}.$$

$$\tilde{F} = \begin{bmatrix} 0 & 0 & 0 & .875I \\ .5 & 0 & 0 & .5 \\ 0 & 1 & .2I & 0 \end{bmatrix},$$

$$\tilde{F}_1 = \begin{bmatrix} 0 & 0 & 0 & .875I \\ .5 & 0 & 0 & 0 \\ 0 & 1 & .2I & 0 \end{bmatrix} \Longrightarrow \check{x}_1 = (.5,1,.2I,.875I)^T,$$

$$\tilde{F}_2 = \begin{bmatrix} 0 & 0 & 0 & .875I \\ 0 & 0 & 0 & .5 \\ 0 & 1 & .2I & 0 \end{bmatrix} \Longrightarrow \check{x}_2 = (0,1,.2I,.875I)^T,$$

$$\tilde{F}_3 = \begin{bmatrix} 0 & 0 & 0 & .875I \\ .5 & 0 & 0 & .5 \\ 0 & 1 & 0 & 0 \end{bmatrix} \Longrightarrow \check{x}_3 = (.5,1,0,.875I)^T,$$

$$\tilde{F}_4 = \begin{bmatrix} 0 & 0 & 0 & .875I \\ .5 & 0 & 0 & .5 \\ 0 & 0 & .2I & 0 \end{bmatrix} \Longrightarrow \check{x}_4 = (.5,0,.2I,.875I)^T,$$

$$\tilde{F}_5 = \begin{bmatrix} 0 & 0 & 0 & .875I \\ .5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \Longrightarrow \check{x}_5 = (.5,1,0,.875I)^T,$$

$$\tilde{F}_6 = \begin{bmatrix} 0 & 0 & 0 & .875I \\ 0 & 0 & 0 & .5 \\ 0 & 1 & 0 & 0 \end{bmatrix} \Longrightarrow \check{x}_6 = (0,1,0,.875I)^T,$$

$$\tilde{F}_7 = \begin{bmatrix} 0 & 0 & 0 & .875I \\ .5 & 0 & 0 & 0 \\ 0 & 0 & .2I & 0 \end{bmatrix} \Longrightarrow \check{x}_7 = (.5,0,.2I,.875I)^T,$$

$$\tilde{F}_8 = \begin{bmatrix} 0 & 0 & 0 & .875I \\ 0 & 0 & 0 & .5 \\ 0 & 0 & .2I & 0 \end{bmatrix} \Longrightarrow \check{x}_8 = (0,0,.2I,.875I)^T.$$

It is clear that there are two repeated solution,

$\check{x}_5 = (.5,1,0,.875I)^T = \check{x}_3$, and $\check{x}_7 = (.5,0,.2I,.875I)^T = \check{x}_4$, after deleting all repeated solutions, the set of all quasi- minimum solutions $\check{X}(A,b) = \{\check{x}_1, \check{x}_2, \check{x}_3, \check{x}_4, \check{x}_6, \check{x}_8\}$.

Since the powers of some terms in $f(x)$ are positive while others are negative, we separate the objective function $f(x)$ into

$$f_1(x) = (0.2I.x_1^{-\frac{2}{3}}) \vee (0.35 . x_4^{-2}) , \quad f_2(x) = (1.3.x_2^{\frac{1}{3}}) \vee (I . x_3^{\frac{1}{2}}),$$

First, solve for optimizing

$$\min f_1(x)$$
$$s.t. Aox = b$$
$$x_j \in [0,1] \cup I$$

By theorem (2.2), we have $f_1(x^*) = f_1(\hat{x}) = f_1(\hat{x}_{v1}) \wedge f_1(\hat{x}_{v2}) = 1.4$, take care of those terms of $\hat{x}_{v2}$ that holding indeterminate components must be neglected and avoid apply them in the terms of $f_1(x)$.

Second, solve for optimizing

$\min f_2(x)$

$s.t. Aox = b$

$x_j \in [0,1] \cup I$

$f_2(\check{x}_1) = 1.3, f_2(\check{x}_2) = 1.3, f_2(\check{x}_3) = 1.3, f_2(\check{x}_4) = .447I, f_2(\check{x}_6) = 1.3,$

$f_2(\check{x}_8) = 0.447I,$

$\check{x}_4, \check{x}_8$ are the optimal for $f_2(x)$, (i.e. $f_2(x^*) = 0.447I$).

$\therefore \mathbf{f}(x^*) = \mathbf{f_1}(x^*) \wedge \mathbf{f_2}(x^*) = \mathbf{0.447I}$

5 Conclusion

The importance of this work comes from the unprecedented notions that were firstly introduced in this article which are essential mathematical tools to establish the structure of neutrosophic geometric programming (NGP) problems with $(\vee, .)$ operator. Any optimization problem needs to specify its minimum and maximum solution, in this article the author introduced an effective technique to find the set of all quasi- minimum solution $\check{X}(A, b)$, side by side with the structure of the maximum solution $\hat{x}$. This work contains the theoretical rules with two numerical examples to enable the readers to understand the pure mathematical concepts.

Reference

[1] F. Smarandache & Huda E. Khalid "Neutrosophic Precalculus and Neutrosophic Calculus". Second enlarged edition, Pons asbl 5, Quai du Batelage, Brussels, Belgium, European Union, 2018.

[2] F. Smarandache, H. E. Khalid & A. K. Essa, "Neutrosophic Logic: the Revolutionary Logic in Science and Philosophy", Proceedings of the National Symposium, EuropaNova, Brussels, 2018.

[3] F. Smarandache, H. E. Khalid, A. K. Essa, M. Ali, "The Concept of Neutrosophic Less Than or Equal To: A New Insight in Unconstrained Geometric Programming", Critical Review, Volume XII, 2016, pp. 72-80.

[4] H. E. Khalid, "An Original Notion to Find Maximal Solution in the Fuzzy Neutrosophic Relation Equations (FNRE) with Geometric Programming (GP)", Neutrosophic Sets and Systems, vol. 7, 2015, pp. 3-7.

[5] H. E. Khalid, "The Novel Attempt for Finding Minimum Solution in Fuzzy Neutrosophic Relational Geometric Programming (FNRGP) with (max, min) Composition", Neutrosophic Sets and Systems, vol. 11, 2016, pp. 107-111.

[6]. H. E. Khalid, F. Smarandache, & A. K. Essa, (2018). The Basic Notions for (over, off, under) Neutrosophic Geometric Programming Problems. Neutrosophic Sets and Systems, 22, 50-62.

[7] H. E. Khalid, (2020). Geometric Programming Dealt with a Neutrosophic Relational Equations Under the $(max - min)$ Operation. Neutrosophic Sets in Decision Analysis and Operations Research, chapter four. IGI Global Publishing House.

[8] H. E. Khalid, "Neutrosophic Geometric Programming (NGP) with (max-product) Operator, An Innovative Model", Neutrosophic Sets and Systems, vol. 32, 2020.

[9] H. E. Khalid, F. Smarandache, & A. K. Essa, (2016). A Neutrosophic Binomial Factorial Theorem with their Refrains. Neutrosophic Sets and Systems, 14, 50-62.

[10] K. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets System 20 (1):87–96

[11] V. Kandasamy, F. Smarandache, "Fuzzy Relational Maps and Neutrosophic Relational Maps", American Research Press, Rehoboth,2004.

[12] Zadeh LA (1965) Fuzzy sets. Inf. control 8(3):338–353.