International Mathematical Forum, Vol. 14, 2019, no. 2, 95 - 106
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/imf.2019.918

Neutrosophic Lie Algebras

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Abstract

We introduce the concept of neutrosophic Lie subalgebras of a Lie algebra is introduced and investigate some of their properties are investigated. The Cartesian product of neutrosophic Lie subalgebras will be discussed. In particular, the homomorphisms of neutrosophic Lie algebras is introduced and investigated some of their properties.

Keywords: Lie algebra, subalgebra, neutrosophic set, neutrosophic Lie Algebras

1. Introduction

The concept of fuzzy sets was introduced by Lotfi A. Zadeh in 1965 [17]. Since then the fuzzy sets and fuzzy logic have been applied in many real life problems in uncertain, ambiguous environment. Among these new concepts, the concept of intuitionistic fuzzy sets given by Atanasov [2] in 1983 is the most important and interesting one because it is simple an extension of fuzzy sets,. The elements of the intuitionistic fuzzy sets are featured by an additional degree which is called the degree of uncertainty. This kind of fuzzy sets have now gained a wide

recognition as useful tool, in modeling of some uncertain phenomena, computer science, mathematics, medicine, chemistry, economics, astronomy etc. Smarandache [13] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data.
Lie algebras were first discovered by Sophus Lie (1842-1899) when he attempted to classify certain smooth subgroups of general linear groups. The groups he considered are now called Lie groups. By taking the tangent space at the identity element of such a group, he obtained the Lie algebra and hence the problems on groups can be reduced to problems on Lie algebras so that it becomes more tractable. There are many applications of Lie algebras in many branches of mathematics and physics. In [1, 5, 11, 12, 14] there is introduction the concept of fuzzy Lie subalgebras and investigation of some of their properties.

In this paper we have introduce the concept of neutrosophic Lie subalgebras of a Lie algebra is introduced and investigate some of their properties are investigated. The Cartesian product of neutrosophic Lie subalgebras will be discussed. In particular, the homomorphisms of neutrosophic Lie algebras is introduced and investigated some of their properties.

2. Preliminaries

In this section, we first review some elementary aspects that are necessary for this paper.

A *Lie algebra* is a vector space L over a field F (equal to R or C) on which $L \times L \to L$ denoted by $(x, y) \to [x, y]$ is defined satisfying the following axioms:

(L1) $[x, y]$ is bilinear,
(L2) $[x, x] = 0 \; for \; all \; x \epsilon L$,
(L3) $[[x, y], z] + \big[[y, z], x\big] + \big[[z, x]; y\big] = 0 \;\; for \; all \;\; x, y, z \; \epsilon L \; (Jacobi \; identity)$.

In this paper, we will always use L to denote a Lie algebra. We note that the multiplication in a Lie algebra is not associative, i.e., it is not true in general that $\big[[x, y], z\big] = \big[x, [y, z]\big]$. But it is *anti commutative*, i.e. $[x, y] = -[y, x]$.

We call a subspace H of L closed under $[\cdot,\ \cdot]$ a *Lie subalgebra*. A subspace I of L with the property $[I, L] \subseteq I$ is called a *Lie ideal* of L. Obviously, any Lie ideal is a subalgebra.

Definition 2.1. [14] *An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ on L is called an intuitionistic fuzzy Lie subalgebra if the following conditions are satisfied:*

(1) $\mu_A(x + y) \geq min(\mu_A(x),\ \mu_A(y))$ *and* $\lambda_A(x + y) \leq max\,(\lambda_A(x), \lambda_A(y))$,

(2) $\mu_A(\alpha x) \geq \mu_A(x)$ *and* $\lambda_A(\alpha x)) \leq \lambda_A(x)$

(3) $\mu_A([x, y]) \geq min\{\mu_A(x), \mu_A(y)\}$ *and* $\lambda_A([x, y]) \leq max\,\{\lambda_A(x), \lambda_A(y)\}$

for all $x, y \epsilon L$ *and* $\alpha \epsilon F$

Definition 2.2. ([14])*An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ on L is called an intuitionistic fuzzy Lie ideal if it satisfied the conditions (1), (2) and the following additional condition:*

(4) $\mu_A([x, y]) \geq \mu_A(x)$ *and* $\lambda_A([x, y]) \leq \lambda_A(x)$

for all $x, y \epsilon L$.
From (2). it follows that:

(5) $\mu_A(0) \geq \mu_A(x)$, $\quad \lambda_A(0) \leq \lambda_A(x)$,
(6) $\mu_A(-x) \geq \mu_A(x)$, $\quad \lambda_A(-x) \leq \lambda_A(x)$

Definition 2.3.([15])**.** A neutrosophic set A on the universe of X is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >, x\epsilon X\}$, where $T, I, F: X \to]^-0, 1^+[$ and
$^-0 \leq 0 \leq T_A\,(x) + I_A\,(x) + F_A\,(x) \leq 3^+$

Definition 2.4.([5,15])**.** A neutrosophic set A is contained in another neutrosophic set B i.e. $\quad A \subseteq B\ if\ \forall x \in X, T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$.

Definition 2.5. [9] Let X be an initial universe set and E be a set of parametres. Let $P(X)$ denote the set of all neutrosophic sets of X. Then, a neutrosophic soft set $(\tilde{F}\,, E)$ over X is a set defined by a set valued function $\tilde{F}$ representing a mapping $\tilde{F}$: $E \to P(X)$ where $\tilde{F}$ is called approximate function of the neutrosophic soft set

$(\tilde{F}, E)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs,
$(\tilde{F} : E) = \{(e, \langle x,\ T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \rangle : x \epsilon X) : e \epsilon E\}$
where $T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \epsilon [0,1]$ respectively called the truth-membership,indeterminacy-membership, falsity-membership function of $\tilde{F}(e)$.Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \leq 3$ is obvious.

Definition 2.6. [5, 15] Let A and B be two neutrosophic soft sets over the common universe X. Then their union is denoted by $A \cup B = C$ and is defined by :

$$C = \{\langle x, T_C(x), I_{\tilde{F}_C}(x), F_C(x) \rangle : x \epsilon X)\}$$

where

$$T_C(x) = max\,\{T_A(x), T_B(x)\}, I_C(x) = max\,\{I_A(x), I_B(x)\}, F_C(x) = min\,\{F_A(x), F_B(x)\}$$

Definition 2.7. [5, 15] Let A and B be two neutrosophic soft sets over the common universe X. Then their intersection is denoted by $A \cap B = C$ and is defined by :

$$C = \{(\langle x, T_C(x), I_C(x), F_C(x) \rangle : x \in X)\},$$

where
$T_C(x) = min\,\{T_A(x), T_{\tilde{F}_B}(x)\}, I_C(x) = min\,\{I_A(x), I_B(x)\}, F_C(x) = max\,\{F_A(x), F_B(x)\}$.

Definition 2.8 [5, 15] A neutrosophic set A over X is said to be null neutrosophic set if
$T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1, \forall x \epsilon X$. It is denoted by $0_{X.}$
(2) A neutrosophic soft set A over X is said to be absolute neutrosophic set if
$T_A(x) = 1,\ I_A(x) = 1, F_A(x) = 0, for, \forall x \epsilon X$. It is denoted by $1_{\times}$.
Clearly, $0_{\mathrm{X}}^c = 1_{\mathrm{X}}\ and\ 1^c{}_X = 0_{X.}$

3. Neutrosophic Lie Algebras

Definition 3.1. *A neutrosophic set $A = (T, I, F)$ on L is called a neustrosophic Lie subalgebra if the following conditions are satisfied:*

(1) $T_A(x + y) \geq \min(T_A(x), T_A(y)), I_A(x + y) \geq \min(I_A(x), I_A(y)),$

$$F_A(x+y) \leq \max(F_A(x), F(y))$$

(2) $T_A(\alpha x) \geq T_A(x)$, $I_A(\alpha x) \geq I_A(x)$, $F_A(\alpha x) \leq F(x)$

(3) $T_A([x+y]) \geq \min\{T_A(x), T_A(y)\}$, $I_A([x+y]) \geq \min\{I_A(x), I(y)\}$,

$$F_A([x+y]) \leq \leq \max\{F_A(x), F(y)\}$$

for all $x, y \epsilon L$ and $\alpha \epsilon F$

Definition 3.2. *A neutrosophic set* $A = (T, I, A)$ *on* L *is called a neustrosophic Lie ideal if it satisfies the conditions (1), (2) and the following additional condition:*

(4) $T_A([x, y]) \geq T_A(x)$, $I_A([x, y]) \geq I_A(x)$ *and* $F_A([x, y]) \leq F_A(x)$

for all $x, y \epsilon L$

From (3.1) it follows that:

(5) $T_A(0) \geq T_A(x)$, $I_A(0) \geq I_A(x)$, $F_A(0) \leq F_A(x)$

(6) $T_A(-x) \geq T_A(x)$, $I_A(-x) \geq I_A(x)$, $F_A(-x) \leq F_A(x)$

Remarc 3.3. *Every neutrosophic Lie ideal is a neutrosophic Lie subalgebra.*

Theorem 3.4. *Let* $A = (T, I, F)$ *be a neutrosophic set on a Lie algebra* L. *Then* $A = (T, I, F)$ *is a neutrosophic Lie subalgebra* L *if and only if the non-empty upper s-level cut*

$$U_T(s) = \{x \epsilon L \mid T(x) \geq s\}, U_I(s) = \{x \epsilon L \mid I(x) \geq s\}$$

and the non-empty lower s- lewel cut

$$V_F(s) = \{x \epsilon L \mid F(x) \leq s\}$$

are Lie subalgebras of L, *for all* $s \epsilon [0,1]$.

Proof. Let $A = (T, I, F)$ be a neutrosophic Lie subalgebra on L and $s \epsilon [0,1]$ be such that $U_T(s) \neq \varnothing$. Let x, $y \epsilon L$ be such that $x \epsilon U_T(s)$ and $y \epsilon U_T(s)$. It follows that

$$T_A(x+y) \geq min\big(T_A(x), T_A(y)\big) \geq s,$$

$$T_A(\alpha x) \geq T_A(x) \geq s,$$

$$T_A([x, y]) \geq min\,(T_A(x), T_A(y)) \geq s$$

and hence , $x + y \epsilon U_T(s)$,

$\alpha x \epsilon U_T(s)$, and $[x, y] \epsilon U_T$, Thus, $U_T(s)$ forms a Lie subalgebra of L. For the case $V_T(s)$, $V_I(s)$ and $V_F(s)$, the proof is analogously.

Conversely, suppose that $U_T(s) \neq \emptyset$ is a Lie subalgebra of L for every $s \epsilon [0,1]$. Assume that

$$T_A(x+y) < min\{T_A(x), T_A(y)\}$$

For same $x, y \epsilon L$. Now, taking

$s_O \coloneqq \frac{1}{2}\{T_A(x+y) + \min\{T_A(x), T_A(Y)\}\}$,

Then we have

$$T_A(x+y) < s_0 < min\{T_A(x), T_A(y)\}$$

And hence $+y \notin U_T(s)$, $x \in U_T(s)$ and $y \in U_T(s)$.

However , this is clearly a contradiction. Therefore,

$$T_A(x+y) \geq min\{T_A(x), T_A(y)\}$$

for all $x, y \epsilon L$ similarly we can show that:

$$T_A(\alpha\, x) \geq T_A(x),$$

$$T_A([x+y]) \geq min\{T_A(x), T_A(y)\}$$

For the case $U_I(s)$ *and* $V_F(s)$ the proof is similar.

Theorem 3.13 If $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be two neutrosophic Lie subalgebra over L, then the intersection $A \cap B = C = \langle T_C, I_C, F_C \rangle$ is a neutrosophic Lie subalgebra over L.

Proof. For each $x, y \in L$ and $\alpha \in F$

$$\begin{aligned} T_C(x+y) &= \min\{T_A(x+y), T_B(x+y)\} \\ &\geq min\{\min\{T_A(x), T_A(y)\}, \min\{T_B(x), T_B(y)\} \\ &= \min\{\min\{T_A(x), T_B(x)\}, \min\{T_A(y), T_B(y)\} \\ &= \min\{T_C(x), T_C(y)\} \end{aligned}$$

$$\begin{aligned} I_C(x+y) &= \min\{I_A(x+y), I_B(x+y)\} \\ &\geq min\{\min\{I_A(x), I_A(y)\}, \min\{I_B(x), I_B(y)\} \\ &= \min\{\min\{I_A(x), I_B(x)\}, \min\{I_A(y), I_B(y)\} \\ &= \min\{I_C(x), I_C(y)\} \end{aligned}$$

$$\begin{aligned} F_C(x+y) &= \max\{F_A(x+y), F_B(x+y)\} \\ &\leq max\{\max\{F_A(x), F_A(y)\}, \max\{F_B(x), F_B(y)\} \\ &= \max\{\max\{F_A(x), F_B(x)\}, \max\{F_A(y), F_B(y)\} \\ &= \max\{F_C(x), F_C(y)\} \end{aligned}$$

$$T_C(\alpha x) = \min\{T_A(\alpha x), T_B(\alpha x)\} \geq min\{T_A(x), T_B(x)\} = T_C(x)$$

$$I_C(\alpha x) = \min\{I_A(\alpha x), I_B(\alpha x)\} \geq min\{I_A(x), I_B(x)\} = I_C(x)$$

$$F_C(\alpha x) = \max\{F_A(\alpha x), F_B(\alpha x)\} \leq max\{F_A(x), F_B(x)\} = F_C(x)$$

Definition 3.14. *Let* $A = (T^1, I^1, F^1)$ *and* $B = (T^2, I^2, F^2)$ *be two neutrosophic sets on a set L. Then the generalized Cartesian product* $A \times B$ *is defined as follow:*

$$A \times B = (T^1, I^1, F^1) \times (T^2, I^2, F^2) = (T^1 \times T^2, I^1 \times I^2, F^1 \times F^2),$$

where $(T^1 \times T^2)(x, y) = \min\big(T^1(x), T^2(y)\big), (I^1 \times I^2)(x, y) = \min\big(I^1(x), I^2(y)\big)$ *and*

$$(F^1 \times F^2)(x, y) = \max\,(F^1(x), F^2(y)).$$

We note that the generalized Cartesian product $A \times B$ is always and a neutrosophic set in $L \times L$ if

$$\min\big(T^1(x), T^2(y)\big) + \min\big(I^1(x), I^2(y)\big) + \max\,(F^1(x), F^2(y)) \leq 3.$$

Theorem 3.15. *Let A*$= (T^1, I^1, F^1)$ *and* $B = (T^2, I^2, F^2)$ *be two neutrosophic Lie subalgebras of L, then* $A \times B$ *is neutrosophic Lie subalgebra of* $L \times L$

Proof. Let $x = (x_1 x_2)$ and $y = (y_1 y_2) \in L \times L$. Then

$$\begin{aligned}
&(T^1 \times T^2)(x + y) = (T^1 \times T^2)\big((x_1, x_2) + (y_1, y_2)\big) = \\
&= (T^1 \times T^2)\big((x_1 + y_1, x_2 + y_2)\big) = \min(T^1(x_1 + y_1), (T^2(x_2 + y_2), \\
&\geq \min(\min T^1\,(x_1), T^1(y_1), \min T^2\,(x_2), T^2(y_2))) \\
&= \min\,((T^1 \times T^2)(x_1, x_2)), ((T^1 \times T^2)(y_1, y_2)) \\
&= \min\,(\big((T^1 \times T^2)(x), (T^1 \times T^2)(y)\big), \\
&(I^1 \times I^2)(x + y) = (I^1 \times I^2)\big((x_1, x_2) + (y_1, y_2)\big) \\
&= (I^1 \times I^2)\big((x_1 + y_1, x_2 + y_2)\big) = \min(I^1(x_1 + y_1), (I^2(x_2 + y_2) \\
&\geq \min(\min I^1\,(x_1), I^1(y_1), \min I^2\,(x_2), I^2(y_2))) \\
&= \min\,((I^1 \times I^2)(x_1, x_2)), ((I^1 \times I^2)(y_1, y_2)) \\
&= \min\,(\big((I^1 \times I^2)(x), (I^1 \times I^2)(y)\big), \\
&(F^1 \times F^2)(x + y) = (F^1 \times F^2)\big((x_1, x_2) + (y_1, y_2)\big) \\
&= (F^1 \times F^2)\big((x_1 + y_1, x_2 + y_2)\big) = \max(F^1(x_1 + y_1), (F^2(x_2 + y_2) \\
&\leq \max(\max F^1\,(x_1), F^1(y_1), \max F^2\,(x_2), F^2(y_2))) \\
&= \max\,((F^1 \times F^2)(x_1, x_2)), ((F^1 \times F^2)(y_1, y_2)) \\
&= \max\,(\big((F^1 \times F^2)(x), (F^1 \times F^2)(y)\big).
\end{aligned}$$

$$(T^1 \times T^2)(\alpha x) = (T^1 \times T^2)(\alpha(x_1, x_2))$$
$$= (T^1 \times T^2)\Big((\alpha x_1, \alpha x_2)) = \min\big(T^1(\alpha x_1), T^2(\alpha x_1)\big)$$
$$\geq \min(\quad T^1\,(x_1), T^2(x_2)\Big) = (T^1 \times T^2)(x_1, x_2)$$
$$= (T^1 \times T^2)(x), (I^1 \times I^2)(\alpha x) = (I^1 \times I^2)(\alpha(x_1, x_2)$$
$$= (I^1 \times I^2)\Big((\alpha x_1, \alpha x_2) = \min\big(I^1(\alpha x_1), I^2(\alpha x_1)\big)$$
$$\geq \min(\quad I^1\,(x_1), I^2(x_2)\Big) = (I^1 \times I^2)(x_1, x_2)$$
$$= (I^1 \times I^2)(x), (F^1 \times F^2)(\alpha x) = (F^1 \times F^2)(\alpha(x_1, x_2)$$
$$= (F^1 \times F^2)\Big((\alpha x_1, \alpha x_2) = \max\big(F^1(\alpha x_1), F^2(\alpha x_1)\big)$$
$$\leq \max(\quad F^1\,(x_1), F^2(x_2)\Big) = (F^1 \times F^2)(x_1, x_2) = (F^1 \times F^2)(x).$$

$$(T^1 \times T^2)([x, y]) = (T^1 \times T^2)([(x_1, x_2) + (y_1, y_2)])$$
$$\geq \min(\min T^1\,(x_1), T^1(x_2), \min T^1\,(y_1), T^2(y_2))$$
$$= \min\,((T^1 \times T^2)(x_1, x_2)), ((T^1 \times T^2)(y_1, y_2))$$
$$= \min\big((T^1 \times T^2)(x), (T^1 \times T^2)(y)\big),\ (I^1 \times I^2)([x, y])$$
$$= (I^1 \times I^2)([(x_1, x_2) + (y_1, y_2)])$$
$$\geq \min(\min I^1\,(x_1), I^1(x_2), \min I^1\,(y_1), I^2(y_2))$$
$$= \min\,((I^1 \times I^2)(x_1, x_2)), ((I^1 \times I^2)(y_1, y_2))$$
$$= \min\big((I^1 \times I^2)(x), (I^1 \times I^2)(y)\big), (F^1 \times F^2)([x, y])$$
$$= (F^1 \times F^2)([(x_1, x_2) + (y_1, y_2)])$$
$$\leq \max(\max F^1\,(x_1), F^1(x_2), \max F^1\,(y_1), F^2(y_2))$$
$$= \max\big((F^1 \times F^2)(x_1, x_2)\big), \big((F^1 \times F^2)(y_1, y_2)\big).$$

This shows that $A \times B$ is a neutrosophic Lie subalgebra of $L \times L$.

4. Homomorphisms of Neutrosophic Lie algebras

Definition 4.1. *Let L_1 and L_2 be two Lie algebras over a field F. Then a linear transformation $f: L_1 \to L_2$ is called a Lie homomorphism if $f([x, y]) = [f(x), f(y)]$ holds for all $x, y \epsilon L_1$.*

For the Lie algebras L_1 and L_2 it can be easily obsenved that if $f: L_1 \to L_2$ is a Lie homomorphism and $A = (T, I, F)$ is neutrosophic Lie subalgebra of L_2, then the neutrosophic set $f^{-1}(A)$ of L_1 is also a neutrosophic Lie subalgebra, where

$$f^{-1}(T_A)(x) = T_A\big(f(x)\big), f^{-1}(I_A)(x) = I_A\big(f(x)\big), f^{-1}(F_A)(x) = F_A\big(f(x)\big).$$

Definition 4.2. *Let L_1 and L_2 be two Lie algebras. The a Lie homomorphism $f: L_1 \to L_2$ is said to have a natural extension $f: I^{L_1} \to I^{L_2}$ defined by for all $A = (T, I, F) \in I^{L_1}, y \in L_2$:*

$$f(T_A)(y) = \sup\{\, T_A(x) : x \in f^{-1}(y)\}$$
$$f(I_A)(y) = \sup\{\, I_A(x) : x \in f^{-1}(y)\}$$
$$f(F_A)(y) = \inf\{\, F_A(x) : x \in f^{-1}(y)\}$$

We now call these sets the homomorphic images of the NS $A = (T, I, F)$.

We now formulate the following theorems:

Theorem 4.3 Let $f: L_1 \to L_2$ epimorfizm of Lie algebras and $A = (T, I, F)$ neutrosophic Lie subalgebra of L_1,then the homomorphic image of A is neutrosophic Lie subalgebra of L_2.

Proof. Let $y_1, y_2 \in L_2$. Then
$\{x | x \in f^{-1}(y_1 + y_2)\} \supseteq \{x_1 + x_2 |\, x_1 \in f^{-1}(y_1)\ and\ x_2 \in f^{-1}(y_2)\}$.
Now, we have
$f(T_A)(y_1 + y_2) = \sup \{\, T_A(x) \mid x \in f^{-1}(y_1 + y_2)\}$
$\geq \{T_A(x_1 + x_2), \mid x_1 \in f^{-1}(y_1)\ and\ x_2 \in f^{-1}(y_2)\}$
$\geq sup\{\min\{T_A(x_1), T_A(x_2)\} |\, x_1 \in f^{-1}(y_1)\ and\ x_2 \in f^{-1}(y_2)\}$
$= \min\{\sup\{\, T_A(x_1) \mid x_1 \in f^{-1}(y_1)\,\}, \sup\{\, T_A(x_2) \mid x_2 \in f^{-1}(y_2)\,\}\}$
$= \min\{f(T_A)(y_1), f(T_A)(y_2)\}$
For $y_2 \in L_2$ and $\alpha \in F$ we have

$$\{x | x \in f^{-1}(\alpha y)\} \supseteq \{\alpha x | x \in f^{-1}(y)\,\}.$$

$f(T_A)(\alpha y) = \sup \{\, T_A(\alpha x) \mid x \in f^{-1}(y)\}$
$\geq sup\{T_A(\alpha x), \mid x \in f^{-1}(\alpha y)\,\}$
$\geq sup\{T_A(x), | x \in f^{-1}(y)\}$
$= f(T_A)(y),$
If $y_1, y_2 \in L_2$ then
$\{x | x \in f^{-1}([y_1, y_2])\} \supseteq \{[x_1, x_2] |\, x_1 \in f^{-1}(y_1)\,,\ x_2 \in f^{-1}(y_2)\}$.
Now
$f(T_A)([y_1, y_2] = sup\ \{\, T_A(x) \mid x \in f^{-1}([y_1, y_2])\}$
$\geq sup\{T_A[x_1, x_2], \mid x_1 \in f^{-1}(y_1)\ and\ x_2 \in f^{-1}(y_2)\}$
$\geq sup\{\min\{T_A(x_1), T_A(x_2)\} |\, x_1 \in f^{-1}(y_1)\ and\ x_2 \in f^{-1}(y_2)\}$

$= \min\{\sup\{ T_A(x_1) \mid x_1 \in f^{-1}(y_1) \}, \sup\{ T_A(x_2) \mid x_2 \in f^{-1}(y_2) \}\}$
$= \min\{f(T_A)(y_1), f(T_A)(y_2)\}$.

Now , we can easily proof for

$$f(I_A)(y_1 + y_2) \geq \min\{f(I_A)(y_1), f(I_A)(y_2)\}$$

$f(I_A)(\alpha y) \geq f(I_A)(y)$
$f(I_A)([y_1, y_2]) \geq min\{f(I_A)(y_1), f(I_A)(y_2)\}$
$f(F_A)(y_1 + y_2) = \inf\{ F_A(x) \mid x \in f^{-1}(y_1 + y_2)\}$
$\leq inf\{F_A(x_1 + x_2), \mid x_1 \in f^{-1}(y_1) , x_2 \in f^{-1}(y_2)\}$
$\leq inf\{\max\{F_A(x_1), F_A(x_2)\} \mid x_1 \in f^{-1}(y_1) , x_2 \in f^{-1}(y_2)\}$
$= \max\{inf\{ F_A(y_1), F_A(y_2) \mid y_1 \in f^{-1}(y_1), y_2 \in f^{-1}(y_2) \}$
$= \max\{f(F_A)(y_1), f(F_A)(y_2)\}$
For $y \in L_2$ and $\alpha \in F$ we have

$$f(F_A)(\alpha y) = \inf\{ F(\alpha x) \mid x \in f^{-1}(y)\}$$

$\leq inf\{F_A(x), |x \in f^{-1}(y)\}$
$= f(F_A)(y)$

Now
$f(F_A)([y_1, y_2] = inf\{ F_A(x) \mid x \in f^{-1}([y_1, y_2]\}$
$\leq inf\{F_A[x_1, x_2], \mid x_1 \in f^{-1}(y_1) , x_2 \in f^{-1}(y_2)\}$
$\leq inf\{\max\{F_A(x_1), F_A(x_2)\} \mid x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$
$= \max\{\inf\{ F_A(x_1), F_A(x_2) \mid x_1 \in f^{-1}(y_1) \}, x_2 \in f^{-1}(y_2)\}$
$= \max\{F_A(y_1), F_A(y_2)\}$
Thus $f(A) = (f(T_A), f(I_A), f(F_A))$ is a *neutrosophic Lie algebra of* L_2 .

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Received: February 12, 2019; Published: April 20, 2019