NEUTROSOPHIC NANO IDEAL TOPOLOGICAL STRUCTURES

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Abstract: *Neutrosophic nano topology and Nano ideal topological spaces induced the authors to propose this new concept. The aim of this paper is to introduce a new type of structural space called neutrosophic nano ideal topological spaces and investigate the relation between neutrosophic nano topological space and neutrosophic nano ideal topological spaces. We define some closed sets in these spaces to establish their relationships. Basic properties and characterizations related to these sets are given.*

Keywords and Phrases: neutrosophic nano ideal, neutrosophic nano local function, topological ideal, neutrosophic nano topological ideal.

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1.Introduction and Preliminaries

The fuzzy set was introduced by Zadeh [17] in the year 1965, where each element had a degree of membership. In 1983, K. Atanassov [1] introduced the concept of intuitionstic fuzzy set which was a generalization of fuzzy set, where each element

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had the degree of membership and the degree of non- membership. Smarandache [15] introduced the concept of neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminacy function and nonmembership function that are independently related. Lellis Thivagar [8], introduced the notation of neutrosophic nano topology, which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined neutrosophic nano closed sets, neutrosophic nano interior and neutrosophic nano closure. There have been wide range of studies on neutrosophic sets, ideals and nano ideals [9-14]. Kuratowski [7] and Vaidyanathaswamy [16], introduced the concept of ideal in topological space and defined local function in ideal topological space. Further Hamlett and Jankovic in [5,6] studied the properties of ideal topological spaces.

In this paper, we introduce the new concept of neutrosophic nano ideal topological structures, which is a generalized concept of neutrosophic nano and ideal topological structure. Also defined the codense ideal in neutrosophic nano topological structure.

Definition 1.1. [8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let F be a neutrosophic set in U with the membership function μ_F, the indeterminancy function σ_F and the nonmembership function ν_F. The neutrosophic nano upper, neutrosophic nano lower approximation and neutrosophic nano boundary of F in the approximation (U, R) denoted by $\overline{N}(F), \underline{N}(F)$ and $B(F)$ are respectively. Then,

(i) The lower approximation of F with respect to R is denoted by $\overline{N}(F)$. That is,
$\overline{N}(F) = \left\{ \left\langle x, \mu_{\overline{R}(F)}(x), \sigma_{\overline{R}(F)}(x), \nu_{\overline{R}(F)}(x) \right\rangle | y \in [x]_R, x \in U \right\}$

(ii) The upper approximation of F with respect to R is the set is denoted by

$\underline{N}(F)$. That is, $\underline{N}(F) = \left\{\left\langle x, \mu_{\underline{R}(F)}(x), \sigma_{\underline{R}(F)}(x), \nu_{\underline{R}(F)}(x)\right\rangle | y \in [x]_R, x \in U\right\}$

(iii) The boundary region of F with respect to R is the set of all objects which can be classified neither as F nor as not F with respect to R and is denoted by $B(F)$. $B(F) = \overline{N}(F) - \underline{N}(F)$.

where, $\mu_{\overline{R}(F)}(x) = \vee_{y\in[x]_R}\mu_F(y), \sigma_{\overline{R}(F)}(x) = \vee_{y\in[x]_R}\sigma_F(y), \nu_{\overline{R}(F)}(x) = \wedge_{y\in[x]_R}\nu_F(y)$.
$\mu_{\underline{R}(F)}(x) = \wedge_{y\in[x]_R}\mu_F(y), \sigma_{\underline{R}(F)}(x) = \wedge_{y\in[x]_R}\sigma_F(y), \nu_{\underline{R}(F)}(x) = \vee_{y\in[x]_R}\nu_F(y)$.

Definition 1.2. [15] Let U be a nonempty set and the neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle, x \in U\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle, x \in U\}$. Then the following statements hold:

(i) $0_N = \{\langle x, 0, 0, 1\rangle, x \in U\}$ and $1_N = \{\langle x, 1, 1, 0\rangle, x \in U\}$.

(ii) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x), \nu_A(x) \ge \nu_B(x)$ for all $x \in U$.

(iii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(iv) $A^C = \{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle, x \in U\}$.

(v) $A \cap B$ if and only if $\mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)$ for all $x \in U$.

(vi) $A \cup B$ if and only if $\mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)$ for all $x \in U$.

(vii) $A - B$ if and only if $\mu_A(x) \wedge \nu_B(x), \sigma_A(x) \wedge 1 - \sigma_B(x), \nu_A(x) \vee \mu_B(x)$ for all $x \in U$.

Definition 1.3. [8] Let U be the universe, R be an equivalence relation on U and $\tau_{\mathcal{N}}(X) = \left\{U, 0_\sim, \overline{N}(F), \underline{N}(F), B(F)\right\}$ where $F \subseteq U$. Then $\tau_{\mathcal{N}}(F)$ satisfies the following axioms:

(i) U and $0_\sim \in \tau_{\mathcal{N}}(F)$.

(ii) The union of the elements of any sub-collection of $\tau_{\mathcal{N}}(F)$ is in $\tau_{\mathcal{N}}(F)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_{\mathcal{N}}(F)$ is in $\tau_{\mathcal{N}}(F)$.

Then $\tau_{\mathcal{N}}(F)$ is a topology on U called the neutrosophic nano topology on U with respect to F. $(U, \tau_{\mathcal{N}}(F))$ is called the neutrosophic nano topological space. Elements of the neutrosophic nano topology are known as neutrosophic nano open sets in U and neutrosophic nano topology is said to be a neutrosophic nano closed set if the complement is neutrosophic nano open set. Elements of $[\tau_{\mathcal{N}}(F)]^c$ being called dual neutrosophic nano topology of $\tau_{\mathcal{N}}(F)$.

Remark 1.4. [8] If $\tau_{\mathcal{N}}(F)$ is the neutrosophic nano topology on U with respect to X, then the set $B = \{U, \underline{N}(F), B(F)\}$ is the basis for $\tau_{\mathcal{N}}(F)$.

Definition 1.5. [9] If $(U, \tau_{\mathcal{N}}(F))$ is a neutrosophic nano topological space with respect to F where $F \subseteq U$ and if $A \subseteq U$, then

(i) The neutrosophic nano interior of the set A is defined as the union of all neutrosophic nano open subsets contained in A and is denoted by $\mathcal{N}int(A)$. $\mathcal{N}int(A)$ is the largest neutrosophic nano open subset of A.

(ii) The neutrosophic nano closure of the set A is defined as the intersection of all neutrosophic nano closed sets containing A and is denoted by $\mathcal{N}cl(A)$. $\mathcal{N}cl(A)$ is the smallest neutrosophic nano closed set closed set containing A.

Definition 1.6. [8] Let X is non-empty set and I a non-empty family of Neutrosophic Sets. We will call I is a neutrosophic ideal (NI for short) on X if

(i) $A \in I$ and $B \subseteq A \Rightarrow B \in I$ [heredity],

(ii) $A \in I$ and $B \in I \Rightarrow A \vee B \in I$ [finite additivity].

2. Neutrosophic nano ideal topological spaces

In this section we shall introduce a new type of local function in neutrosophic nano topological space. Before starting the discussion we shall consider the following concepts.

A neutrosophic nano topological space $(U, \tau_{\mathcal{N}}(F))$ with an ideal I on U is called a neutrosophic nano ideal topological space and is denoted by $(U, \tau_{\mathcal{N}}(F), I)$.

Definition 2.1. Let $(U, \tau_{\mathcal{N}}(F), I)$ be a neutrosophic nano ideal topological space with an ideal I on U and $(.)^*_{\mathcal{N}}$ be a set operator from $P(U)$ to $P(U)$ ($P(U)$ is the set of all subsets of U). For a subset $A \subset U$, the neutrosophic nano local function $A^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F))$ of A is the union of all neutrosophic nano points (NNP, for short) $C(\alpha, \beta, \gamma)$ such that

$A^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F)) = \vee\{C(\alpha, \beta, \gamma) \in U : A \cap G \notin I \text{for all } G \in N(C(\alpha, \beta, \gamma))\}$. We will simply write $A^*_{\mathcal{N}}$ for $A^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F))$.

Example 2.2. Let $(U, \tau_{\mathcal{N}}(F))$ be a neutrosophic nano topological space with an ideal I on U and for every $A \subseteq U$.

(i) If $I = \{0_\sim\}$, then $A^*_{\mathcal{N}} = \mathcal{N}cl(A)$,

(ii) If $I = P(U)$, then $A^*_{\mathcal{N}} = 0_\sim$.

Theorem 2.3. Let $(U, \tau_{\mathcal{N}}(F))$ be a neutrosophic nano topological space with ideals I, I' on U and A, B be subsets of U. Then

(i) $A \subseteq B \Rightarrow A^*_{\mathcal{N}} \subseteq B^*_{\mathcal{N}}$,

(ii) $I \subseteq I' \Rightarrow A^*_{\mathcal{N}}(I') \subseteq A^*_{\mathcal{N}}(I)$,

(iii) $A^*_{\mathcal{N}} = \mathcal{N}cl(A^*_{\mathcal{N}}) \subseteq \mathcal{N}cl(A)$ ($A^*_{\mathcal{N}}$ is a neutrosophic nano closed subset of $\mathcal{N}cl(A)$),

(iv) $(A^*_{\mathcal{N}})^*_{\mathcal{N}} \subseteq A^*_{\mathcal{N}}$,

(v) $A^*_{\mathcal{N}} \cup B^*_{\mathcal{N}} = (A \cup B)^*_{\mathcal{N}}$,

(vi) $A^*_{\mathcal{N}} - B^*_{\mathcal{N}} = (A - B)^*_{\mathcal{N}} - B^*_{\mathcal{N}} \subseteq (A - B)^*_{\mathcal{N}}$,

(vii) $V \in \tau_{\mathcal{N}}(F) \Rightarrow V \cap A^*_{\mathcal{N}} = V \cap (V \cap A)^*_{\mathcal{N}} \subseteq (V \cap A)^*_{\mathcal{N}}$ and

(viii) $J \in I \Rightarrow (A \cup J)^*_{\mathcal{N}} = A^*_{\mathcal{N}} = (A - J)^*_{\mathcal{N}}$.

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Proof. (i) Let $A \subset B$ and $x \in A^*_{\mathcal{N}}$. Assume that $x \notin B^*_{\mathcal{N}}$. We have $G_{\mathcal{N}} \cap B \in I$ for some $G_{\mathcal{N}} \in G_{\mathcal{N}}(x)$. Since $G_{\mathcal{N}} \cap A \subseteq G_{\mathcal{N}} \cap B$ and $G_{\mathcal{N}} \cap B \in I$, we obtain $G_{\mathcal{N}} \cap A \in I$ from the definition of ideal. Thus, we have $x \notin A^*_{\mathcal{N}}$. This is a contradiction. Clearly, $A^*_{\mathcal{N}} \subseteq B^*_{\mathcal{N}}$.
(ii) Let $I \subseteq I'$ and $x \in A^*_{\mathcal{N}}(I')$. Then we have $G_{\mathcal{N}} \cap A \notin I'$ for every $G_{\mathcal{N}} \in G_{\mathcal{N}}(x)$. By hypothesis, we obtain $G_{\mathcal{N}} \cap A \notin I$. So $x \in A^*_{\mathcal{N}}(I)$.
(iii) Let $x \in A^*_{\mathcal{N}}$. Then for every $G_{\mathcal{N}} \in G_{\mathcal{N}}(x), G_{\mathcal{N}} \cap A \notin I$. This implies that $G_{\mathcal{N}} \cap A \neq 0_{\sim}$. Hence $x \in \mathcal{N}cl(A)$.
(iv) From (iii), $(A^*_{\mathcal{N}})^*_{\mathcal{N}} \subseteq \mathcal{N}cl(A^*_{\mathcal{N}}) = A^*_{\mathcal{N}}$, since $A^*_{\mathcal{N}}$ is a neutrosophic nano closed set.
The proofs of the other conditions are also obvious.

The converse implications of (i), (ii) and (iii) of Theorem 2.3. do not hold in general.

Theorem 2.4. If $(U, \tau_{\mathcal{N}}(F), I)$ is a neutrosophic nano topological space with an ideal I and $A \subseteq A^*_{\mathcal{N}}$, then $A^*_{\mathcal{N}} = \mathcal{N}cl(A^*_{\mathcal{N}}) = \mathcal{N}cl(A)$.

Proof. For every subset A of U, we have $A^*_{\mathcal{N}} = \mathcal{N}cl(A^*) \subseteq \mathcal{N}cl(A)$, by Theorem 2.3. (iii) $A \subseteq A^*_{\mathcal{N}}$ implies that $\mathcal{N}cl(A) \subseteq Ncl(A^*_{\mathcal{N}})$ and so $A^*_{\mathcal{N}} = \mathcal{N}cl(A^*_{\mathcal{N}}) = \mathcal{N}cl(A)$.

Definition 2.5. Let $(U, \tau_{\mathcal{N}}(F))$ be a neutrosophic nano topological space with an ideal I on U. The set operator $\mathcal{N}cl^*$ is called a neutrosophic nano$*$-closure and is defined as $\mathcal{N}cl^*(A) = A \cup A^*_{\mathcal{N}}$ for $A \subseteq X$.

Theorem 2.6. The set operator $\mathcal{N}cl^*$ satisfies the following conditions:

(i) $A \subseteq \mathcal{N}cl^*(A)$,

(ii) $\mathcal{N}cl^*(0_{\sim}) = 0_{\sim}$ and $\mathcal{N}cl^*(1_{\sim}) = 1_{\sim}$,

(iii) If $A \subset B$, then $\mathcal{N}cl^*(A) \subseteq \mathcal{N}cl^*(B)$,

(iv) $\mathcal{N}cl^*(A) \cup \mathcal{N}cl^*(B) = \mathcal{N}cl^*(A \cup B)$.

(v) $\mathcal{N}cl^*(\mathcal{N}cl^*(A)) = \mathcal{N}cl^*(A)$.

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Proof. The proofs are clear from Theorem 2.3 and the definition of $\mathcal{N}cl^*$.

Now, $\tau_{\mathcal{N}}(F)^*(I,\tau_{\mathcal{N}}(F)) = \{V \subset U : \mathcal{N}cl^*(U-V) = U-V\}$. $\tau_{\mathcal{N}}(F)^*(I,\tau_{\mathcal{N}}(F))$ is called neutrosophic nano$*$-topology which is finer than $\tau_{\mathcal{N}}(F)$ (we simply write $\tau_{\mathcal{N}}(F)^*$ for $\tau_{\mathcal{N}}(F)^*(I,\tau_{\mathcal{N}}(F))$). The elements of $\tau_{\mathcal{N}}(F)^*(I,\tau_{\mathcal{N}}(F))$ are called neutrosophic nano$*$-open (briefly, $\mathcal{N}*$-open) and the complement of an $\mathcal{N}*$-open set is called neutrosophic nano$*$-closed (briefly, $\mathcal{N}*$-closed). Here $\mathcal{N}cl^*(A)$ and $\mathcal{N}int^*(A)$ will denote the closure and interior of A respectively in $(U,\tau_{\mathcal{N}}(F)^*)$.

Remark 2.7. (i) We know from Example 2.2 that if $I = \{0_\sim\}$ then $A^*_{\mathcal{N}} = \mathcal{N}cl(A)$. In this case, $\mathcal{N}cl^*(A) = \mathcal{N}cl(A)$.

(ii) If $(U,\tau_{\mathcal{N}}(F),I)$ is a neutrosophic nano ideal topological space with $I = \{0_\sim\}$, then $\tau_{\mathcal{N}}(F)^* = \tau_{\mathcal{N}}(F)$.

Definition 2.8. A basis $\beta(I,\tau_{\mathcal{N}}(F))$ for $\tau_{\mathcal{N}}(F)^*$ can be described as follows: $\beta(I,\tau_{\mathcal{N}}(F)) = \{A - B : A \in \tau_{\mathcal{N}}(F), B \in I\}$.

Theorem 2.9. Let $(U,\tau_{\mathcal{N}}(F))$ be a neutrosophic nano topological space and I be an ideal on U. Then $\beta(I,\tau_{\mathcal{N}}(F))$ is a basis for $\tau_{\mathcal{N}}(F)^*$.

Proof. We have to show that for a given space $(U,\tau_{\mathcal{N}}(F))$ and an ideal I on U, $\beta(I,\tau_{\mathcal{N}}(F))$ is a basis for $\tau_{\mathcal{N}}(F)^*$. If $\beta(I,\tau_{\mathcal{N}}(F))$ is itself a neutrosophic nano topology, then we have $\beta(I,\tau_{\mathcal{N}}(F)) = \tau_{\mathcal{N}}(F)^*$ and all the open sets of $\tau_{\mathcal{N}}(F)^*$ are of simple form $A - B$ where $A \in \tau_{\mathcal{N}}(F)$ and $B \in I$.

Theorem 2.10. Let $(U,\tau_{\mathcal{N}}(F),I)$ be a neutrosophic nano topological space with an ideal I on U and $A \subseteq U$. If $A \subseteq A^*_{\mathcal{N}}$, then

(i) $\mathcal{N}cl(A) = \mathcal{N}cl^*(A)$,

(ii) $\mathcal{N}int(U - A) = \mathcal{N}int^*(U - A)$.

Proof. (i) Follows immediately from Theorem 2.4.

(ii) If $A \subseteq A^*_{\mathcal{N}}$, then $\mathcal{N}cl(A) = \mathcal{N}cl^*(A)$ by (i) and so $X - \mathcal{N}cl(A) = X - \mathcal{N}cl^*(A)$. Therefore, $\mathcal{N}int(X - A) = \mathcal{N}int^*(X - A)$.

Theorem 2.11. Let $(U,\tau_{\mathcal{N}}(F),I)$ be a neutrosophic nano topological space with an ideal I on U and $A \subseteq X$. If $A \subseteq A^*_{\mathcal{N}}$, then $A^*_{\mathcal{N}} = \mathcal{N}cl(A^*_{\mathcal{N}}) = n$-$cl(A) = \mathcal{N}cl^*(A)$.

Definition 2.12. A subset A of a neutrosophic nano ideal topological space $(U, \tau_{\mathcal{N}}(F), I)$ is $\mathcal{N}*$-dense in itself (resp. $\mathcal{N}*$-perfect) if $A \subseteq A^*_{\mathcal{N}}$ (resp. $A = A^*_{\mathcal{N}}$).

Remark 2.13. A subset A of a neutrosophic nano ideal topological space $(U, \tau_{\mathcal{N}}(F), I)$ is $\mathcal{N}^*$-closed if and only if $A^*_{\mathcal{N}} \subseteq A$.

For the relationship related to several sets defined in this paper, we have the following implication:

$$\mathcal{N}*\text{-dense in itself} \Leftarrow \mathcal{N}*\text{-perfect} \Rightarrow \mathcal{N}^*\text{-closed}$$

The following example show that the converse implication are not satisfied.

Example 2.14. Let U be the universe, $X = \{P_1, P_2, P_3, P_4, P_5\} \subset U$, $U/R = \{\{P_1, P_2\}, \{P_3\}, \{P_4, P_5\}\}$ and $\tau_{\mathcal{N}}(F) = \{1_\sim, 0_\sim, \overline{N}, \underline{N}, B\}$ and the ideal $I = 0_\sim, 1_\sim$. For $A = \{< P_1, (.5, .4, .7) >, < P_2, (.6, .4, .5) >, < P_3, (.4, .5, .4) >, < P_4, (.7, .3, .4) >, < P_5, (.8, .5, .2) >\}$, $\underline{N}(A) = \{\frac{P_1, P_2}{.5,.4,.7}, \frac{P_3}{.4,.5,.4}, \frac{P_4, P_5}{.7,.3,.4}\}$, $\overline{N}(A) = \{\frac{P_1, P_2}{.6,.4,.5}, \frac{P_3}{.4,.5,.4}, \frac{P_4, P_5}{.8,.5,.2}\}$, $B(A) = \{\frac{P_1, P_2}{.6,.4,.5}, \frac{P_3}{.4,.5,.4}, \frac{P_4, P_5}{.4,.3,.7}\}$. If $I = 0_\sim$ then $A^*_{\mathcal{N}} = Ncl(a)$. Thus $A \subseteq A^*_{\mathcal{N}}$. Hence A is $\mathcal{N}^*$-dense but not $\mathcal{N}^*$-perfect.
If $I = 1_\sim$ then $A^*_{\mathcal{N}} = 0_\sim$. Thus $A \supseteq A^*_{\mathcal{N}}$. Hence $A^*_{\mathcal{N}}$ is $\mathcal{N}^*$-closed but not $\mathcal{N}^*$-perfect.

Lemma 2.15. Let $(U, \tau_{\mathcal{N}}(F), I)$ be an neutrosophic nano ideal topological space and $A \subseteq U$. If A is $\mathcal{N}*$-dense in itself, then $A^*_{\mathcal{N}} = \mathcal{N}cl(A^*_{\mathcal{N}}) = \mathcal{N}cl(A) = \mathcal{N}cl^*(A)$.

Proof. Let A be $\mathcal{N}*$-dense in itself. Then we have $A \subseteq A^*_{\mathcal{N}}$ and using Theorem 2.11 we get$A^*_{\mathcal{N}} = \mathcal{N}cl(A^*_{\mathcal{N}}) = \mathcal{N}cl(A) = \mathcal{N}cl^*(A)$.

Lemma 2.16. If $(U, \tau_{\mathcal{N}}(F), I)$ is a neutrosophic nano topological space with an ideal I and $A \subseteq X$, then $A^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F)) = A^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F)^*)$ and hence $\tau_{\mathcal{N}}(F)^* = \tau_{\mathcal{N}}(F)^{**}$.

3. $\tau_{\mathcal{N}}(F)$-codence ideal

The study of ideal got new dimension when codence ideal [5] has been incorporated in ideal topological space. In this section we introduce similar concept in neutrosophic nano ideal topological spaces.

Definition 3.1. An ideal I in a space $(U, \tau_{\mathcal{N}}(F), I)$ is called $\tau_{\mathcal{N}}(F)$-codense ideal if $\tau_{\mathcal{N}}(F) \cap I = \{0_\sim\}$.

Following theorems are related to $\tau_{\mathcal{N}}(F)$-codense ideal.

Theorem 3.2. Let $(U, \tau_{\mathcal{N}}(F), I)$ be an neutrosophic nano ideal topological space and I is $\tau_{\mathcal{N}}(F)$-codense with $\tau_{\mathcal{N}}(F)$. Then $U = U^*_{\mathcal{N}}$.

Proof. It is obvious that $U^*_{\mathcal{N}} \subseteq U$. For converse, suppose $x \in U$ but $x \notin U^*_{\mathcal{N}}$. Then there exists $G_x \in \tau_{\mathcal{N}}(F)(x)$ such that $G_x \cap U \in I$. That is $G_x \in I$, a contradiction to the fact that $\tau_{\mathcal{N}}(F) \cap I = \{0_\sim\}$. Hence $U = U^*_{\mathcal{N}}$.

Theorem 3.3. Let $(U, \tau_{\mathcal{N}}(F), I)$ be a neutrosophic nano ideal topological space. Then the following conditions are equivalent:

(i) $U = U^*_{\mathcal{N}}$.

(ii) $\tau_{\mathcal{N}}(F) \cap I = \{0_\sim\}$.

(iii) If $J \in I$, then $\mathcal{N}int(J) = 0_\sim$.

(iv) For every $A \in \tau_{\mathcal{N}}(F)$, $A \subseteq A^*_{\mathcal{N}}$.

Proof. By Lemma 2.16, we may replace '$\tau_{\mathcal{N}}(F)$' by '$\tau_{\mathcal{N}}(F)^*$' in (ii), '$\mathcal{N}int(J) = 0_\sim$' by '$\mathcal{N}int^*(J) = 0_\sim$' in (iii) and '$A \in \tau_{\mathcal{N}}(F)$' by '$A \in \tau_{\mathcal{N}}(F)^*$' in (iv).

Conclusions and Future work: In this paper, we introduced the notion of neutrosophic nano ideal topological structures and investigated some relations over neutrosophic nano topology and neutrosophic nano ideal topological structures and studied some of its basic properties. In future, it motivates to apply this concepts in graph structures.

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