Neutrosophic Quadruple Algebraic Codes over Z_2 and their Properties

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Abstract. In this paper we for the first time develop, define and describe a new class of algebraic codes using Neutrosophic Quadruples which uses the notion of known value, and three unknown triplets (T, I, F) where T is the truth value, I is the indeterminate and F is the false value. Using this Neutrosophic Quadruples several researchers have built groups, NQ-semigroups, NQ-vector spaces and NQ-linear algebras. However, so far NQ algebraic codes have not been developed or defined. These NQ-codes have some peculiar properties like the number of message symbols are always fixed as 4-tuples, that is why we call them as Neutrosophic Quadruple codes. Here only the check symbols can vary according to the wishes of the researchers. Further we find conditions for two NQ-Algebraic codewords to be orthogonal. In this paper we study these NQ codes only over the field Z_2. However, it can be carried out as a matter of routine in case of any field Z_p of characteristics p.

Keywords: Neutrosophic Quadruples; NQ-vector spaces; NQ-groups; Neutrosophic Quadruple Algebraic codes (NQ-algebraic codes); Dual NQ-algebraic codes; orthogonal NQ- algebraic codes; NQ generator matrix; parity check matrix; self dual NQ algebraic codes

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1. Introduction

Neutrosophic Quadruples (NQ) was introduced by Smarandache [1] in 2015, it assigns a value to known part in addition to the truth, indeterminate and false values, it happens to be very interesting and innovative. NQ numbers was first introduced by [1] and algebraic operations like addition, subtraction and multiplication were defined. Neutrosophic Quadruple algebraic structures where studied in [2]. Smarandache and et al introduced Neutrosophic triplet groups, modal logic Hedge algebras in [3, 4]. Zhang and et al in [5–7] defined and described Neutrosophic duplet semigroup and triplet loops and strong AG(1, 1) loops. In [8–12],

various structures like Neutrosophic triplet and neutrosophic rings application to mathematical modelling, classical group of neutrosophic triplets on $\{Z_{2p}, \times\}$ and neutrosophic duplets in neutrosophic rings were developed and analyzed.

Algebraic structures of neutrosophic duplets and triplets like quasi neutrosophic triplet loops, AG-groupoids, extended triplet groups and NT-subgroups were studied in $[7, 13, 16, 17]$. Various types of refined neutrosophic sets were introduced, developed and applied to real world problems by $[18$–$24]$. In 2015, $[18]$ has obtained several algebraic structures on refined Neutrosophic sets. Neutrosophy has found immense applications in $[25$–$28]$. Neutrosophic algebraic structures in general were studied in $[29$–$32]$. The algebraic structure of Neutrosophic Quadruples, such as groups, monoids, ideals, BCI-algebras, BCI-positive implicative ideals, hyper structures and BCK/BCI algebras have been developed recently and studied in $[34$–$39]$. In 2016 $[33]$ have developed some algebraic structures using Neutrosophic Quadruples $(NQ, +)$ groups and $(NQ, .)$ monoids and scalar multiplication on Neutrosophic Quadruples. $[41]$ have recently developed the notion of NQ vector spaces over R(reals) (or Complex numbers C or Z_p the field of characteristic p, p a prime). They have also defined NQ dual vector subspaces and proved all these NQ-vectors though are distinctly different, yet they are of dimension 4.

The main aim of this paper is to introduce Neutrosophic Quadruple (NQ) algebraic codes over Z_2. (However it can be extended for any Z_p, p a prime). Any NQ codeword is an ordered quadruple with four message symbols which can be a real or complex value, truth value, indeterminate or complex value and the check symbols are combinations of these four elements. We have built a new class of NQ algebraic codes which can measure the four aspects of any code word.

The proposed work is important for Neutrosophic codes have been studied Neutrosophic codes have been studied by $[42]$ but it has the limitations for it could involve only the indeterminacy present and not all the four factors which are present in Neutrosophic Quadruple codes. Hence when the codes are endowed with all the four features it would give in general a better result of detecting the problems while transmission takes place.

It is to be recalled any classical code gives us only the approximately received code word. However the degrees of truth or false or indeterminacy present in the correctness of the received code word is never studied. So our approach would not only be novel and innovative but give a better result when used in real channels.

The main objective of this study is to assess the quality of the received codeword for the received code word may be partially indeterminate or partially false or all the four, we can by this method assess the presence of these factors and accordingly go for re-transmission or rejection.

Hexi codes were defined in [43,44] which uses 16 symbols, 0 to 9 and A to F. Likewise these NQ codes uses the symbols 0, 1, T, I and F.

This paper is organized into six sections. Section one is introductory in nature. Basic concepts needed to make this paper a self-contained one is given in section two. Neutrosophic Quadruple algebraic codes (NQ-codes) are introduced and some interesting properties about them are given in section three. Section four defines the new notion of special orthogonal NQ codes using the inner product of two NQ codewords. The uses of NQ codes and comparison with classical linear algebraic codes are carried out in section five. The final section gives the conclusions based on our study.

2. Basic Concepts

In this section we first give the basic properties about the NQ algebraic structures needed for this study. Secondly we give some fundamental properties associated with algebraic codes in general. For NQ algebraic structures refer [29, 33].

Definition 2.1. A Neutrosophic quadruple number is of the form (x, yT, zI, wF) where T, I, F are the usual truth value, indeterminate value and the false value respectively and $x, y, z, w \in Z_p$ (or R or C). The set NQ is defined by $NQ = \{(x, yT, zI, wF)|x, y, z, w \in R$ (or Z_p or C); p a prime$\}$ is defined as the Neutrosophic set of quadruple numbers.

A Neutrosophic quadruple number (x, yT, zI, wF) represents any entity or concept which may be a number an idea etc., x is called the known part and (yT, zI, wF) is called the unknown part. Addition, subtraction and scalar multiplication are defined in [33] in the following way. Let $x = (x_1, x_2T, x_3I, x_4F)$ and $y = (y_1, y_2T, y_3I, y_4F) \in NQ$.

$x + y = (x_1 + y_1, (x_2 + y_2)T, (x_3 + y_3)I, (x_4 + y_4)F)$

$x - y = (x_1 - y_1, (x_2 - y_2)T, (x_3 - y_3)I, (x_4 - y_4)F)$

For any $a \in R$ (or C or Z_p) and $x = (x_1, x_2T, x_3I, x_4F)$ where $a \in R$ (or C or Z_P) will be known as scalars and $x \in NQ$ the scalar product of a with x in defined by

$$a.x = a(x_1, x_2T, x_3I, x_4F)$$

$$= (ax_1, ax_2T, ax_3I, ax_4F).$$

If $a = 0$ then $a.x = (0,0,0,0)$. $(0,0,0,0)$ is the additive identity in $(NQ, +)$. For every $x \in NQ$ there exists a unique element $-x = (-x_1, -x_2T, -x_3I, -x_4F)$, in NQ such that $x + (-x) = (0,0,0,0)$. x is called the additive inverse of $-x$ and vice versa.

Finally for $a, b \in C$ (or R or Z_p) and $x, y, \in NQ$ we have $(a + b).x = a.x + b.x$ and $(a \times b).x = a \times (b.x); a(x + y) = a.x + a.y$.

These properties are essential for us to build NQ-algebraic codes.

We use the following results; proofs of which can be had form [33].

Theorem 2.2. $(NQ, +)$ *is an abelian group.*

[33] defines product of any pair of elements $x, y \in NQ$ *as follows.* Let $x = (x_1, x_2T, x_3I, x_4F)$ *and* $y = (y_1, y_2T, y_3I, y_4F) \in NQ$.

$$x.y = (x_1, x_2T, x_3I, x_4F).(y_1, y_2T, y_3I, y_4F)$$

$$(x_1y_1, (x_1y_2 + x_2y_1 + x_2y_2)T,$$

$$(x_1y_3 + x_2y_3 + x_3y_1 + x_3y_2 + x_3y_3)I,$$

$$(x_1y_4 + x_2y_4 + x_3y_4 + x_4y_4 + x_4y_1 + x_4y_2 + x_4y_3)F).$$

Theorem 2.3. $(NQ, .)$ *is a commutative monoid.*

Now we just recall some of the properties associated with basic algebraic codes.

Through out this paper Z_2 will denote the finite field of characteristic two. V a finite dimensional vector space over $F = Z_2$ [40].

We call a n-tuple to be $C = C(n, k)$ codeword if C has k message symbols and $n - k$ check symbols. For $c = (c_1, c_2, \ldots, c_k, c_{k+1}, \ldots c_n)$ where $(c_1, c_2, \ldots, c_k) \in V$ (dimension of V over Z_2) and $c_{k+1}, \ldots, c_n$ are check symbols calculated using the $(c_1, c_2, \ldots, c_k) \in V$. To basically generate the code words we use the concept of generator matrix denoted by G and G is a $k \times n$ matrix with entries from Z_2 and to evaluate the correctness of the received codeword we use the parity check matrix H, which is a $n - k \times n$ matrix with entries from Z_2. We in this paper use only the standard form of the generator matrix and parity check matrix for any $C(n, k)$ code of length n with k message symbols. The standard form of the generator matrix G for an $C(n, k)$ code is as follows:

$$G = (I_k, -A^T)$$

where I_k is a $k \times k$ identity matrix and $-A^T$ is a $k \times n - k$ matrix with entries from Z_2. Here the standard form of the parity check matrix $H = (A, I_{n-k})$ where A is a $n - k \times k$ matrix with entries from Z_2 and I_{n-k} is the $n - k \times n - k$ identity matrix. We have $GH^T = (0)$. In this paper, we use both the generator matrix and the parity check matrix of a NQ code to be only in the standard form.

3. Definition of NQ algebraic codes and their properties

In this section we proceed on to define the new class of algebraic codes called Neutrosophic Quadruple algebraic codes (NQ-algebraic codes) using the NQ vector spaces over the finite field Z_2. We have defined NQ vector spaces over Z_2 in [41].

$$NQ = \{(a, bT, cI, dF) | a, b, c, d \in Z_2\}$$

under $+$ is an abliean group.

Now we proceed on to define $\times$ on NQ. Let

$$x = x_1 + x_2T + x_3I + x_4F$$

and

$$y = y_1 + y_2T + y_3I + y_4F$$

where $x_i, y_i \in R$ or C or Z_p (p a prime) and T, I and F satisfy the following table for product $\times$.

$\times$	T	I	F	0
T	T	0	0	0
I	0	I	0	0
F	0	0	F	0
0	0	0	0	0

So the set $\{T, I, F, 0\}$ under product is an idempotent semigroup. now we find

$$x \times y = (x_1 + x_2T + x_3I + x_4F) \times (y_1 + y_2T + y_3I + y_4F)$$

$$= x_1y_1 + (y_1x_2 + x_1y_2 + x_2y_2)T + (x_3y_1 + y_3x_1 + x_3y_3)I + (x_1y_4 + y_1x_4 + x_4y_4)F \in NQ$$

$\{NQ, \times\}$ is a semigroup which is commutative.

In this section we introduce the new notion of algebraic codes using the set NQ which is a group under '+'

$NQ = \{(0\ 0\ 0\ 0),\ (1\ 0\ 0\ 0),\ (0\ T\ 0\ 0),\ (0\ 0\ I\ 0),\ (0\ 0\ 0\ F),\ (1\ T\ 0\ 0),\ (1\ 0\ I\ 0),\ (1\ 0\ 0\ F),$

$(0\ T\ I\ 0),\ (0\ T\ 0\ F),\ (0\ 0\ I\ F),\ (1\ T\ I\ 0),\ (1\ T\ 0\ F),\ (1\ 0\ I\ F),\ (0\ T\ I\ F),\ (1\ T\ I\ F)\ \}$;

$\{NQ, +\}$ is a NQ vector space over $Z_2 = \{0, 1\}$. NQ coding comprises of transforming a block of message symbols in NQ into a NQ code word $a_1a_2a_3a_4x_5x_6\ldots x_n$, where $a_1a_2a_3a_4 \in NQ$ that is $a_1a_2a_3a_4 = (a_1a_2a_3a_4) \in NQ$ is a quadruple and $x_5, x_6, \ldots, x_n$ belongs to the set $T = \{a + bT + cI + dF/a, b, c, d$ takes its values from $Z_2 = \{0, 1\}\}$. The first four terms $a_1a_2a_3a_4$ symbols are always the message symbols taken from NQ and the remaining $n - 4$ are the check symbols or the control symbols which are from T.

In this paper NQ codewords will be written as $a_1a_2a_3a_4x_5x_6x_7\ldots x_n$, where $(a_1a_2a_3a_4) \in NQ$ and $x_i \in T, 4 < i \leq n$. The check symbols can be obtained from the NQ message symbols in such a way that the NQ code words $a = (a_1a_2a_3a_4)$ satisfy the system of linear equations $Ha^T = (0)$, where H is the $n - 4 \times n$ parity check matrix in the standard form with elements

from Z_2. Throughout this paper we assume $H = (A, I_{n-4})$, with A, a $n - 4 \times 4$ matrix and I_{n-4} the $n - 4 \times n - 4$ identity matrix with entries from Z_2.

The matrix $G = (I_{4\times4}, -A^T)$ is called the canonical generator matrix of the linear $(n, 4)$ NQ code with parity check matrix $H = (A, I_{n-4})$.

We use only standard form of the generator matrix and parity check matrix to generate the NQ-codewords for general matrix of appropriate order will not serve the purpose which is a limitation in this case.

We provide some examples of a HQ linear algebraic code.

Example 3.1. Let $C(7, 4)$ be a NQ code of length 7. G be the NQ generator matrix of the $(7, 4)NQ$ code.

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

G takes the entries from Z_2, over which the NQ vector space is defined and the message symbols are from NQ. Consider the set of NQ message symbols, $P = \{(0\ 0\ 0\ 0), (0\ T\ 0\ 0), (0\ 0\ I\ 0), (0\ 0\ 0\ F), (0\ 0\ I\ F), (0\ T\ I\ F), (1\ 0\ I\ F), (0\ T\ I\ 0), (0\ T\ 0\ F)\} \subseteq NQ$. We now give the NQ code words of

$C(7, 4) = \{$ $(0\ 0\ 0\ 0\ 0\ 0\ 0), (0\ T\ 0\ 0\ 0\ T\ 0), (0\ 0\ 0\ F\ 0\ 0\ F), (0\ 0\ I\ 0\ I\ 0\ 0), (0\ 0\ I\ F\ I\ 0\ F), (0\ T\ I\ F\ I\ T\ F), (1\ 0\ I\ F\ 1 + I\ I\ F)\ (0\ T\ I\ 0\ I\ T\ 0), (0\ T\ 0\ F\ 0\ T\ F)\ \}$ which are associated with $P \subseteq NQ$. The NQ parity check matrix associated with this generator matrix G is as follows;

$$H = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

It is easily verified $Hx^t = (0)$; for all NQ code words $x \in C(7, 4)$. Suppose one receives a NQ code word $y = (0\ I\ 0\ T\ I\ 0\ 0)$; how to find out if the received NQ code word y is a correct one or not. For this we find out Hy^t, if $Hy^t = (0)$, then y is a correct code word; if $Hy^t \neq (0)$, then some error has occurred during transmission. Clearly $Hy^t \neq (0)$. Thus y is not a correct NQ code word.

How to correct it? These NQ code behave differently as these codewords, which is a $1 \times n$ row matrix does not take the values from Z_2, but from NQ and T; message symbols from NQ and check symbols from T. Hence, we cannot use the classical method of coset leader method for error correction, however we use the parity check matrix for error detection.

We have to adopt a special method to find the corrected version of the received NQ code word which has error.

Here we describe the procedure for error correction which is carried out in three steps; Suppose y is the received NQ code word;

(1) We first find Hy^t, if Hy^t is zero no error; on the other hand if Hy^t is not zero there is error so we go to step two for correction.

(2) Now consider the NQ received code word with error. We observe and correct only the first four component in the y that is we correct the message symbols; if the first component is 1 or 0 then it is accepted as the correct component in y; if on the other hand the first component is T (or I or F) and if 1 has occurred in the rest of any three components then replace T (or I or F) by one if 1 has not occurred in the 2nd or 3rd or 4th component replace the first component by 0.

Now observe the second component if it is T accept, if not T but 0 or 1 or I or F, then replace by zero if T has not occurred in the first or third or fourth place. If T has occurred in any of the 3 other components replace it by T. Next observe the third component if it is I accept else replace by I if I has occurred as first or second or fourth component. If in none of the first four places I has occurred, then fill the third place by zero. Now observe the fourth component if it is F accept it, if not replace by 0 if in none of the other places F has occurred or by F if F has occurred in first or second or third place, now the message word is in NQ by this procedure. If the corrected NQ code word z of y is such that $Hz^t = (0)$ then accept it if not we go for the next step. We check only for the correctness of the message symbols.

(3) For check symbols we use the table of codewords or check matrix H and find the check symbols.

Table of NQ codewords related to $P \subset NQ$ given in example 2.

TABLE 1. Table of NQ codewords related to P

Sno	Message symbols in P	NQ Codeword
1	(0 0 0 0)	(0 0 0 0 0 0 0)
2	(0 T 0 0)	(0 T 0 0 0 T 0)
3	(0 0 I 0)	(0 0 I 0 I 0 0)
4	(0 0 0 F)	(0 0 0 F 0 0 F)
5	(0 0 I F)	(0 0 I F I 0 F)
6	(0 T I F)	(0 T I F I T F)
7	(0 T I 0)	(0 T I 0 I T 0)
8	(0 T 0 F)	(0 T 0 F 0 T F)
9	(1 0 I F)	(1 0 I F 1+I 1 F)

We provide one example of the codeword given in Example 2.1. Let $y = (I\ 1\ F\ 0\ 1{+}I\ 1\ F)$, we see Hy^t is not zero, so we have found the error hence we proceed to next step. We see first component cannot be I so replace I by 1 for 1 has occurred as second component. As second component cannot be one we see in none of the four components T has occurred so we replace 1 by zero. In the second place. Third component is F which is incorrect so we replace it by I as I has occurred in the first place. We observe the fourth component it can be 0 or F; 0 only in case F has not occurred in the first three places but F has occurred as the third component so we replace the zero of the fourth component by F. So the corrected message symbol is (1 0 I F). In step three we check from the table of codes the check symbols and the check symbols matches with the check symbols of the corrected message symbols so we take this as the corrected version of corrected code word as (1 0 I F I+I 1 F).

We give the definition of the procedure.

Definition 3.2. Let C(n, 4) be a NQ code of length n defined over Z_2. The message symbols are always from the set NQ; whatever be n there are only 16 codewords only check symbols increase and not the message symbol length, for it is always four. If $y = (A_1\ A_2\ A_3\ A_4\ a_5\ a_6\ a_7 \ldots a_n)$ is a received NQ codeword and it has some error, then we define the rearrangement technique of error correction in the message symbols $A_1\ A_2\ A_3\ A_4$ only, where if $A_1\ A_2\ A_3\ A_4$ is to be in NQ then A_1 can only values 1 or 0, A_2 can take values 0 or T; A_3 can take values 0 or I and A_4 can take values 0 or F. If this is taken care of the message symbol will be correct and will be in NQ.

If not the following rearrangement process is carried out;

Observe if A_1 is different from 0 or 1 then see values in the 2nd, third and the fourth components if 1 has occurred in any one of them replace the first component by 1, if 1 has not occurred in any one of the four components fill the first component by zero. Now go for the second component A_2 if A_2 is T then it is correct ;if not and 1 or 0 or I or F has occurred and T has occurred in any one of the other three places replace the second component by T; if T has not occurred as any one of the four components replace the second component by 0. Inspect the third component if it is I then it is correct, if not I and if T or 0 or 1 or F has occurred and I has occurred in any of the four components replace the third component by I, if I has failed to occur in any of the four places replace the third component by zero. Now for the fourth component if it is F it is correct, if not and if F has occurred in any one of the other three components replace it by F, if not by zero. After this arrangement certainly the message symbols will be in NQ.

This method of getting the correct code word is defined as the rearrangement technique.

4. **Orthogonal NQ codes and special orthogonal NQ codes**

In this section we define the notion of orthogonality of two HQ code words and the special orthogonal HQ code words and suggest some open problems in this direction in the last section of this paper. Now we define first inner product on the NQ code words of the NQ algebraic code $C(n, 4)$ defined over Z_2.

Definition 4.1. Let $C(n, 4)$ be a NQ code of length n defined over Z_2. Let $x = (A_1 \ A_2 \ A_3 \ A_4 \ a_5 \ a_6 \ a_7 \ \dots \ a_n)$ and $y = (B_1 \ B_2 \ B_3 \ B_4 \ b_5 \ b_6 \ b_7 \dots b_n)$ be any two NQ code words from $C(n, 4)$, where $A_i, B_i \in NQ$, $i = 1, 2, 3, 4$ and $a_j, b_j \in T$; $j = 5, 6, \dots n$. We define the dot product of x and y as follows:

$$x.y = A_1 \times B_1 + A_2 \times B_2 + A_3 \times B_3 + A_4 \times B_4 + a_5 \times b_5 + \dots + a_n \times b_n$$

.

If $x.y = 0$ then we say the two NQ codes words are orthogonal or dual with each other.

Example 4.2. Let $C(6, 4)$ be a NQ code of length 4 defined over Z_2; with associated generated matrix G in the standard form with entries from Z_2 given in the following:

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

The $C(6, 4)$ NQ code words generated by G is as follows; $C(6, 4) = \{(0\ 0\ 0\ 0\ 0\ 0)$, $(1\ 0\ 0\ 0\ 1\ 0)$, $(0\ T\ 0\ 0\ 0\ T)$, $(0\ 0\ I\ 0\ I\ I)$, $(0\ 0\ 0\ F\ F\ 0)$, $(1\ T\ 0\ 0\ 1\ T)$, $(1\ 0\ I\ 0\ 1+I\ I)$, $(1\ 0\ 0\ F\ F+1\ 0)$, $(0\ T\ I\ 0\ I\ T+I)$, $(0\ T\ 0\ F\ F\ T)$, $(0\ 0\ I\ F\ I+F\ I)$, $(1\ T\ I\ 0\ 1\ +I\ T+I)$, $(1\ T\ 0\ F\ 1+\ F\ T)$, $(1\ 0\ I\ F\ 1\ +I\ +F\ I)$,$(0\ T\ I\ F\ I+\ F\ I\ +T)$, $(1\ T\ I\ F\ 1\ +I\ +\ F\ I+\ T)\}$.

We see $(0\ 0\ 0\ 0\ 0\ 0)$ is orthogonal with every other NQ code word in the NQ code $(6, 4)$. Consider the NQ code word $(1\ 0\ 0\ 0\ 1\ 0)$ in C $(6, 4)$, NQ code words orthogonal to $(1\ 0\ 0\ 0\ 1\ 0)$ are $\{(1\ 0\ 0\ 0\ 1\ 0)$, $(0\ 0\ 0\ 0\ 0\ 0)$, $(0\ T\ 0\ 0\ 0\ T)$, $(1\ T\ 0\ 0\ 1\ T)$ $\}$. The NQ codes orthogonal to $(0\ T\ 0\ 0\ 0\ T)$ are given by

$\{(0\ 0\ 0\ 0\ 0\ 0)$, $(0\ T\ 0\ 0\ 0\ T)$, $(1\ 0\ 0\ 0\ 1\ 0)$, $(0\ 0\ I\ 0\ I\ I)$, $(0\ 0\ 0\ F\ F\ 0)$, $(1\ T\ 0\ 0\ 1\ T)$, $(1\ 0\ 0\ F\ 1\ +\ F\ 0)$, $(0\ T\ I\ 0\ 1\ +\ I\ T\ +\ I)$, $(1\ 0\ I\ 0\ 1\ +I\ I)$, $(0\ T\ 0\ F\ F\ T)$, $(0\ 0\ I\ F\ I\ +\ F\ I)$, $(1\ I\ T\ 0\ 1\ +I\ T\ +\ I)$, $(1\ T\ 0\ F\ 1\ +\ F\ T)$, $(1\ 0\ I\ F\ 1\ +\ I\ +\ F\ I)$, $(0\ T\ I\ F\ I\ +\ F\ T\ +\ I)$, $(1\ T\ I\ F\ 1\ +\ T\ +F\ T\ +\ I)\ \} = C(6, 4)$.

Thus every element in $C(6, 4)$ is orthogonal with $(0\ T\ 0\ 0\ 0\ T)$. However $(1\ 0\ 0\ 0\ 0\ 1)$ is not orthogonal with every element in $C(6, 4)$. We call all those NQ codes words which are orthogonal to every code word in $C(6, 4)$ including it as the special orthogonal NQ code. A NQ code word which is orthogonal to itself is defined as the self orthogonal NQ code word.

We define them in the following;

Definition 4.3. Let C(n, 4) be a NQ code of length n. We say a NQ code word is self orthogonal if x . x = 0 for x in C(n, 4). A NQ code word x in C(n, 4) is defined as a special orthogonal NQ code word if x is self orthogonal and x is orthogonal with every NQ code word in C(n, 4). (0 0 0 ...0) is a trivial special NQ code word.

We give yet another example of a NQ code which has NQ special orthogonal code word.

Example 4.4. Let C(7, 4) be a NQ code word of length 7. Let G be the associated generator matrix of the NQ code C.

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

It is easily verified that only the NQ code word (0 0 I 0 0 0 I) in C is the special orthogonal NQ code word. We have yet another extreme case where every NQ code word in that NQ code is a special orthogonal NQ code word.

We give examples of them.

Example 4.5. Let C(8, 4) be a NQ code generated by the following generator matrix G

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

It is easily verified every NQ code word in $C(8,4)$ is a special orthogonal NQ code word. We call such NQ codes as special self orthogonal NQ code or self orthogonal NQ code.

Definition 4.6. Let $C = C(n,4)$ be a NQ code word defined over Z_2. We define C to be a NQ special self orthogonal code if every NQ code word in C is a special orthogonal NQ code word of C.

5. **Uses of NQ codes and comparison of NQ codes with classical linear algebraic codes**

NQ codes are best suited for data transmission where one does not require security. They are also very useful in data storage for one can easily retrieve the data even if the data is corrupted. The disadvantage of these NQ codes is that they always have a fixed number of message symbols namely four. They are not compatible in channels were one needs security. The only flexibility is one can have any number of check symbols. NQ codes are entirely different from the classical linear algebraic code ; for these code words take the message

Kandasamy V. et. al, Neutrosophic Quadruple Algebraic Codes

symbols from NQ and the check symbols from T where as the later take their values from Z_2 (or Z_p).

Classical linear algebraic codes takes its code words from Z_p, p a prime or more commonly from Z_2; and are defined over Z_p or Z_2; but in case of NQ codes the code words take their values from NQ for message symbols and from T for their check symbols which is a big difference as we can only use the standard form of the generator matrix and the parity check matrix, in this case also both the matrices take their values from Z_2 (or Z_p) only. The similarity is both the codes take the entries of the matrices from the finite field over which they are defined. All NQ codes are only of a fixed form that is they can have only 4 message symbols from NQ, but the classical codes can have any value from 1 to m , m ¡ n, which is a major difference between the two class of codes. Both NQ codes and the classical linear code use parity matrix to detect the error in the received code word, that is error detection procedure for both of them is the same. For error correction we have to adopt a special technique of rearrangement of the message symbols once an error is detected in the received NQ code word, as the coset leader method of error correction cannot be carried out as the NQ code words do not belong to the field over which the NQ code words are defined.

6. Conclusions

In this paper for the first time we have defined the new class of codes called NQ codes which are distinctly different from the classical algebraic linear codes. All these NQ codes can have only fixed number of message symbols viz four. NQ codes are of the form $C(n, 4)$, n can vary from 5 to any finite integer. We have defined orthogonality of these NQ codes. This has lead us to define NQ special orthogonal code word and NQ special orthogonal codes. We suggest the following problems:

(1) Prove or disprove all NQ codes have a non trivial code word which is orthogonal to all codes in C (n, 4).
(2) Characterize all NQ codes C (n, 4) which are NQ special orthogonal codes.

For future research we would be defining super NQ structures and NQ codes over Z_p, p an odd prime. Also application of these codes can be done in case of Hexi codes [43] in McEliece Public Key crypto-systems [44] and in coding applications like T-Direct codes [45] and multi covering radius with rank metric [46].

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