NEUTROSOPHIC REGULAR GENERALIZED STAR b-CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract:

Smarandache introduced and developed the new concept of Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama introduced Neutrosophic topological spaces by using the Neutrosophic crisp sets. In this paper we introduce and study about Neutrosophic Regular Generalized Star b Closed sets in Neutrosophic Topological Spaces and its properties are discussed.

1. Introduction

Topology is a classical subject, as a generalization topological spaces many type of topological spaces introduced over the year. C.L. Chang[3] was introduced and developed fuzzy topological space by using L.A. Zadeh's[12] fuzzy sets. Coker[4] introduced the concepts of Intuitionistic fuzzy topological spaces by using Atanassov's[1] Intuitionistic fuzzy set Neutrality the degree of indeterminacy, as an independent concept, was introduced by Smarandache [6] in 1998. He also defined the Neutrosophic set on three component Neutrosophic topological spaces (T- Truth, F -Falsehood ,I- Indeterminacy). Neutrosophic topological spaces(N-T-S) introduced by Salama [10]et al.In 1996 D. Andrijevic [2] introduced b open sets in topological space, R.Dhavaseelan[5],SaiedJafari are introduced Neutrosophic generalized closed sets. Aim of this paper is to introduce Neutrosophic rg*b-closed sets in Neutrosophic topological space and to discuss the properties of Neutrosophic rg*b-interior and Neutrosophic rg*b-closure in Neutrosophic topological spaces(N-T-S)

2. Preliminaries

In the Second section, we recall needed basic definition and operation of Neutrosophic sets and then fundamental results

Definition 2.1 [10]

Let X be a non-empty fixed set. A Neutrosophic set P is an object having the form $P = \{\langle x, \mu P(x),\sigma P(x),\gamma P(x)\rangle : x \in X\}$ where $\mu P(x)$-represents the degree of membership function, $\sigma P(x)$- represents degree indeterminacy and then $\gamma P(x)$- represents the degree of non-membership function

Remark 2.2 [10]

Neutrosophic set $P=\{\langle x, \mu P(x), \sigma P(x), \gamma P(x)\rangle : x \in X\}$ can be write to an ordered triple lies in the interval in $]^{-}0,1^{+}[$ on X.

Remark 2.3[10]

Symbolically, the Neutrosophic set $P = \{\langle x, \mu P(x), \sigma P(x), \gamma P(x)\rangle : x \in X\}$ can be written briefly as $P= \langle x, \mu P, \sigma P, \gamma P \rangle$

Definition 2.4 [10]

In N-T-S, 0_N may be defined like as:

$x \in X$
$0_1 = \langle x, 0, 0, 1 \rangle$
$0_2 = \langle x, 0, 1, 1 \rangle$
$0_3 = \langle x, 0, 1, 0 \rangle$
$0_4 = \langle x, 0, 0, 0 \rangle$
1_N may be defined like as:
$\forall x \in X$
$1_1 = \langle x, 1, 0, 0 \rangle$
$1_2 = \langle x, 1, 0, 1 \rangle$
$1_3 = \langle x, 1, 1, 0 \rangle$
$1_4 = \langle x, 1, 1, 1 \rangle$

Definition 2.5 [10]

Neutrosophic set $P=\{\langle x, \mu P(x), \sigma P(x), \gamma P(x)\rangle\}$ on X and
$\forall x \in X$ then complement of P is
$P^C = \{\langle x, \gamma P(x), 1 - \sigma P(x), \mu P(x)\rangle\}$

Definition 2.6 [10]

Let P and Q are two Neutrosophic sets
$\forall x \in X$ $P=\{\langle x, \mu P(x), \sigma P(x), \gamma P(x)\rangle\}$ and
$Q=\{\langle x, \mu Q(x), \sigma Q(x),$
$\gamma Q(x)\rangle\}$. Then
$P \subseteq Q \Leftrightarrow \mu P(x) \leq \mu Q(x), \sigma P(x) \leq \sigma Q(x)$ and $\gamma P(x) \geq \gamma Q(x)$

Proposition 2.6 [10]

The following results are true for any Neutrosophic set P
(i) $0_N \subseteq P, 0_N \subseteq 0_N$
(ii) $P \subseteq 1_N, 1_N \subseteq 1_N$

Definition 2.7 [10]

Let X be a non-empty set, and
Let P and Q be two Neutrosophic sets
are $P=\langle x, \mu P(x), \sigma P(x), \gamma P(x)\rangle$,
$Q = \langle x, \mu Q(x), \sigma Q(x), \gamma Q(x)\rangle$ Then
(i) $P \cap Q = \langle x, \mu P(x) \wedge \mu Q(x), \sigma P(x) \wedge \sigma Q(x)$ & $\gamma P(x) \vee \gamma Q(x)\rangle$
(ii) $P \cup Q = \langle x, \mu P(x) \vee \mu Q(x), \sigma P(x) \vee \sigma Q(x)$ & $\gamma P(x) \wedge \gamma Q(x)\rangle$

Proposition 2.8 [10]

The following conditions are true for all two Neutrosophic sets P and Q are
(i) $(P \cap Q)^C = P^C \cup Q^C$
(ii) $(P \cup Q)^C = P^C \cap Q^C$.

Definition 2.9 [10]

Let X be non-empty set and τ_N be the collection of Neutrosophic subsets of X satisfying the following properties :
(i) $0_N, 1_N \in \tau_N$,
(ii) $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$,
(iii) $\cup T_i \in \tau_N$ for every $\{T_i : i \in J\} \subseteq \tau_N$

Then the space (X, τ_N) is called a Neutrosophic topological space(N-T-S). The element of τ_N are called Neu-OS (Neutrosophic open set)
and its complement is Neu-CS(Neutrosophic closed set)

Example 2.10 [10]

Let $X=\{x\}$ and $\forall x \in X$
$A_1 = \langle x, 0.6, 0.6, 0.5 \rangle$
$A_2 = \langle x, 0.5, 0.7, 0.9 \rangle$
$A_3 = \langle x, 0.6, 0.7, 0.5 \rangle$

$A_4 = \langle$ x, 0.5, 0.6, 0.9 $\rangle$

Then the collection $\tau N = \{0_N, A_1, A_2, A_3, A_4, 1_N\}$ is called a N-T-S on X.

Definition 2.11 [10]

(X, τ_N) be N-T-S and $\forall x \in X$

$P = \{\langle x, \mu P(x), \sigma P(x), \gamma P(x)\rangle\}$be a Neutrosophic set in X. Then the Neutrosophic closure and Then the Neutrosophic closure of P is

Neu-Cl(P)=∩{ H:H is a Neutrosophic closed set in X and P⊆H} Neutrosophic interior of P is

Neu-Int(P)=∪{M:M is a Neutrosophic open set in X and M⊆P}.

Then

(i) P is Neutrosophic open set iff P=Neu-Int(P) .
(ii) P is Neutrosophic closed set iff P=Neu-Cl(P).

Proposition 2.12 [10]

Let(X, τ_N) be a Neutrosophic topological spaces ,Then for any Neutrosophic set P
(i) Neu-Cl($(P)^C$)= $(\text{Neu-Int}(P))^C$,
(ii) Neu-Int((P^C))= $(\text{Neu-Cl}(P))^C$.

Proposition 2.13 [10]

Let P, Q be two Neutrosophic sets in N-T-S (X, τ_N). Then the following results are true:
(i) Neu-Int(P)⊆P,
(ii) P⊆Neu-Cl(P),
(iii) P⊆Q⇒Neu-Int(P)⊆Neu-Int(Q),
(iv) P⊆Q⇒Neu-Cl(P)⊆Neu-Cl(Q),
(v) Neu-Int(Neu-Int(P))=Neu-Int(P),
(vi) Neu-Cl(Neu-Cl(P))=Neu-Cl(P),
(vii) Neu-Int(P∩Q))=Neu-Int(P)∩Neu-Int(Q),
(viii) Neu-Cl(P∪Q)=Neu-Cl(P)∪Neu-Cl(Q),
(ix) Neu-Int(0_N)=0_N ,
(x) Neu-Int(1_N)=1_N ,
(xi) Neu-Cl(0_N)=0_N ,
(xii) Neu-Cl(1_N)=1_N ,
(xiii) $P \subseteq Q \Rightarrow Q^C \subseteq P^C$,
(xiv) Neu-Cl(P∩Q)⊆Neu-Cl(P)∩Neu-Cl(Q),
(xv) Neu-Int(P∪Q)⊇Neu-Int(P)∪Neu-Int(Q).

Definition:2.14[5]

Neutrosophic generalized closed set (Neu-g closed) if Neutrosophic- cl(P)⊆G whenever P⊆G and G is Neutrosophic open set in (X,τ_N).

3. Neutrosophic Regular Generalized Star b - Closed Sets

In this section, we introduce and study the new concept of Neutrosophic regular generalized star b-closed sets in N-T-S

Definition 3.1: A subset A of a Neutrosophic topological space (X,τ_N) is called Neutrosophic Regular generalized star b-closed set (briefly Neu-rg*b-closed) if Neu-bcl(A) ⊆ U whenever A ⊆ U and U is Neu-rg-open in X.

The family of all Neu-rg*b-closed subsets of X is denoted by Neu-RG*BC(X,τ_N).

Theorem 3.2: Every Neu-closed set is Neu-rg*b-closed.

Proof: Let A be any Neu-closed set in X and A ⊆ U, where U is Neu-rg-open. Since A is Neu-closed, Neu-bcl(A) ⊆ Neu-cl(A) = A. Therefore Neu-bcl(A) ⊆ A ⊆ U. Hence A is Neu-rg*b-closed set in X.

The converse is not true as seen in the following Example.

Example 3.3:
Let X ={a , b , c} with τ_N={ 0_N, A_1, A_2, 1_N } and $(\tau_N)^C$ ={ 1_N, A_3, A_4, 0_N }
where A_1 = ⟨(0.6, 0.6, 0.4), (0.2, 0.7, 1), (1, 0.6, 0.5) ⟩
A_2 = ⟨(0.1, 0.4, 0.8), (0.2, 0.6, 1), (0.6, 0.5, 0.9)⟩
A_3 = ⟨(0.4, 0.4, 0.6), (1, 0.3, 0.2), (0.5, 0.4, 1)⟩
A_4 = ⟨(0.8, 0.6, 0.1), (1, 0.4, 0.2), (0.9, 0.5, 0.6)⟩.
A_5 = ⟨(0.3, 0.4, 1), (0.1, 0.2, 1), (0.4, 0.2, 1)⟩.
Here the A_5 is Neu-rg*b closed set but not Neu- closed set.

Remark 3.4: Finite intersection of Neu-rg*b-closed sets need not be Neu-rg*b-closed.

Remark 3.5: Finite union of Neu-rg*b-closed sets need not be Neu-rg*b-closed.

Definition 3.6: For a subset A of (X,τ_N), the intersection of all Neu-rg*b-closed sets containing A is called the Neu-rg*b-closure of A and is denoted by Neu-rg*b-cl(A).

That is, Neu-rg*b-cl(A) = ∩ {F : A⊂ F, F is Neu-rg*b-closed in X}.

Remark 3.7: If A and B are subsets of (X,τ_N), then

(a) Neu-rg*b-cl(ϕ) = ϕ and Neu-rg*b-cl(X) = X.

(b) A⊂ B ⇒ Neu-rg*b-cl(A) ⊂ Neu-rg*b-cl(B). Neu-rg*b-cl(Neu-rg*b-cl(A)) = Neu-rg*b-cl(A).

(c) Neu-rg*b-cl(A ∪ B) ⊇ Neu-rg*b-cl(A) ∪ Neu-rg*b-cl(B).

(d) Neu-rg*b-cl(A ∩ B) ⊆ Neu-rg*b-cl(A) ∩ Neu-rg*b-cl(B).

4. Neutrosophic Regular generalized star b-open sets and Neutrosophic rg*b-Neighbourhoods

In this section the notion of Neu-rg*b-open sets are introduced and using them the characterizations of Neu-rg*b-neighbourhoods are obtained.

Definition 4.1: A subset A of a topological space (X,τ_N) is called Neutrosophic Regular generalized star b- open set (briefly Neu-rg*b-open) if A^c is Neu-rg*b-closed in X. The family of all Neu-rg*b-open sets in X is denoted by Neu-RG*BO(X,τ_N).

Remark 4.2:

(i) Finite intersection of Neu-rg*b-open sets need not be Neu-rg*b-open.

(ii) Finite union of Neu-rg*b-open sets need not be Neu-rg*b-open.

Definition 4.3: Let x be a point in a N-T-S, X. A subset N of X is said to be an Neu-rg*b-neighbourhood of x if and only if there exists an Neu-rg*b-open set G such that x ∈ G ⊆ N.

Definition 4.4: A subset N of a space X is called an Neu-rg*b-neighbourhood of A ⊂ X iff there exists an Neu-rg*b-open set G such that A ⊆ G ⊆ N.

Theorem 4.5: Every neighbourhood N of x ∈ X is an Neu-rg*b-neighbourhood of x.

Definition 4.6: Let x be a point in a Neutrosophic topological space X. The set of all Neu-rg*b-neighbourhood of x is called the Neu-rg*b-neighbourhood system at x and is denoted by Neu-rg*b-N(x).

Theorem 4.7: Let X be a Neutrosophic topological space and for each x ∈ X, let Neu-rg*b-N(X) be the collection of all Neu-rg*b-neighbourhoods of x. Then,

(i) for all x ∈ X, Neu-rg*b-N(x) ≠ ϕ.

(ii) N ∈ Neu-rg*b-N(x) implies x ∈ N.

(iii)N ∈ Neu-rg*b-N(x), M ⊃ N implies M ∈ Neu-rg*b-N(x).

(iv)N ∈ Neu-rg*b-N(x) implies there exists M ∈ Neu-rg*b-N(x) such that M ⊂ N and M ∈ Neu-rg*b-N(y) for every y ∈ M.

Definition 4.8: Let A be a subset of a Neutrosophic topological space X. A point x∈X is called an Neu-rg*b-interior point of A if there exists an Neu-rg*b-open set G such that x ∈G ⊆ A. The set of all Neu-rg*b-interior points of A is called the Neu-rg*b-interior of A and is denoted by Neu-rg*b-int(A).

Theorem 4.9: For subsets A and B of X, the following assertions are valid.

(1) Neu-rg*b-int(A) is the union of all Neu-rg*b-open subsets of A.

(2) If A is Neu-rg*b-open, then A = Neu-rg*b-int(A).

(3) Neu-rg*b-int(Neu-rg*b-int(A)) = Neu-rg*b-int(A).

(4) Neu-rg*b-int(A) = A \ Neu-D $_{rg*b}$ (X \ A).

(5) X \ Neu-rg*b-int(A) = Neu-rg*b-cl(X \ A).

(6) X \ Neu-rg*b-cl(A) = Neu-rg*b-int(X \ A).

(7) A ⊆B implies Neu-rg*b-int(A) ⊆Neu-rg*b-int(B).

(8) Neu-rg*b-int(A) ∪Neu-rg*b-int(B) ⊆Neu-rg*b-int(A∪B).

(9) Neu-rg*b-int(A∩B) ⊆Neu-rg*b-int(A)∩ Neu-rg*b-int(B).

Definition 4.10: For any subset A of X, the set Neu-b $_{rg*b}$ (A) = A \ Neu-rg*b-int(A) is called the Neu-rg*b-border of A, and the set Neu-Fr $_{rg*b}$ (A) = Neu-rg*b-cl(A) \ Neu-rg*b-int(A) is called the Neu-rg*b-frontier of A.

Remark 4.11: If A is an Neu-rg*b-closed subset of X, then Neu-b $_{rg*b}$ (A) = Neu-Fr $_{rg*b}$ (A).

Theorem 4.12: For a subset A of X, the following statements hold:

(1) A = Neu-rg*b-int(A) ∪ Neu-b $_{rg*b}$ (A).

(2) Neu-rg*b-int(A) ∩ Neu-b_{rg*b} (A) = ϕ.

(3) If A is an Neu-rg*b-open set then Neu-b_{rg*b} (A) = ϕ.

(4) Neu-b $_{rg*b}$ (Neu-rg*b-int(A)) = ϕ.

(5) Neu-rg*b-int(Neu-b $_{rg*b}$ (A)) = ϕ.

(6) Neu-b $_{rg*b}$ (Neu-b $_{rg*b}$ (A)) = Neu- b $_{rg*b}$ (A).

(7) Neu-b $_{rg*b}$ (A) = A ∩ Neu-rg*b-cl(X \ A).

(8) Neu-b $_{rg*b}$ (A) = A ∩ Neu-D $_{rg*b}$ (X \ A).

Theorem 4.13: For a subset A of X, the following assertions are valid:

1) Neu-rg*b-cl(A) = rg*b-int(A) ∪ Neu-Fr $_{rg*b}$ (A).

2) Neu-rg*b-int(A) ∩ Neu-Fr $_{rg*b}$ (A) = ϕ.

3) Neu-b $_{rg*b}$ (A) ⊆ Neu-Fr $_{rg*b}$ (A).

4) Neu-Fr $_{rg*b}$ (A) = Neu-b $_{rg*b}$ (A) ∪ (Neu-D $_{rg*b}$ (A) \ Neu-rg*b-int(A)).

5) If A is an Neu-rg*b-open set then Neu-Fr $_{rg*b}$ (A) = Neu-b $_{rg*b}$ (X \ A).

6) Neu-Fr $_{rg*b}$ (A) = Neu-rg*b-cl(A) ∩ Neu-rg*b-cl(X \ A).

7) Neu-Fr $_{rg*b}$ (A) = Neu-Fr $_{rg*b}$ (X \ A).

8) Neu-Fr $_{rg*b}$ (A) is Neu-rg*b-closed.

9) Neu-Fr $_{rg*b}$ (Neu-Fr $_{rg*b}$ (A)) ⊆ Neu-Fr $_{rg*b}$ (A).

10) Neu-Fr $_{rg*b}$ (Neu-rg*b-int(A)) ⊆ Neu-Fr $_{rg*b}$ (A).

11) Neu-Fr $_{rg*b}$ (Neu-rg*b-cl(A)) ⊆ Neu-Fr $_{rg*b}$ (A).

12) Neu-rg*b-int(A) = A \ Neu-Fr $_{rg*b}$ (A).

Definition 4.14: For a subset A of X, the Neu-rg*b-interior of X \ A is called the Neu-rg*b-exterior of A, and is denoted by Neu-Ext $_{rg*b}$ (A). That is, Neu-Ext $_{rg*b}$ (A) = Neu-rg*b-int(X \ A).

Theorem 4.15: For subsets A and B of X, the following assertions are valid.

(1) Neu-Ext $_{rg*b}$ (A) is Neu-rg*b-open.

(2) Neu-Ext $_{rg*b}$ (A) = X \ Neu-rg*b-cl(A).

(3) Neu-Ext $_{rg*b}$ (Neu-Ext $_{rg*b}$ (A)) = Neu-rg*b-int (Neu-rg*b-cl(A)) ⊇ Neu-rg*b-int (A).

(4) If A ⊆ B then Neu-Ext $_{rg*b}$ (B) ⊆ Neu-Ext $_{rg*b}$ (A).

(5) Neu-Ext $_{rg*b}$ (A ∪ B) ⊆ Neu-Ext $_{rg*b}$ (A) ∩ Neu-Ext $_{rg*b}$ (B).

(6) Neu-Ext $_{rg*b}$ (A ∩ B) ⊇ Neu-Ext $_{rg*b}$ (A) ∪ Neu-Ext $_{rg*b}$ (B).

(7) Neu-Ext $_{rg*b}$ (X) = ϕ, Neu-Ext $_{rg*b}$ (ϕ) = X.

(8) Neu-Ext $_{rg*b}$ (A) = Neu-Ext $_{rg*b}$ (X \ Neu-Ext $_{rg*b}$ (A)).

(9) X = Neu-rg*b-int (A) ∪ Neu-Ext $_{rg*b}$ (A) ∪ Neu-Fr $_{rg*b}$ (A).

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