Neutrosophic Rough Set Algebra

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ABSTRACT

A rough set is a formal approximation of a crisp set which gives lower and upper approximation of original set to deal with uncertainties. The concept of neutrosophic set is a mathematical tool for handling imprecise, indeterministic and inconsistent data. In this paper, we defne concepts of Rough Neutrosophic algebra and investigate some of their properties.

1 Introduction

Rough set theory is a [7, 8], is an extension of set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. Moreover, it is a mathematical tool for machine learning, information sciences and expert systems and successfully applied in data analysis and data mining. There are two basic elements in rough set theory, crisp set and equivalence relation, which constitute the mathematical basis of rough set. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set . In classical rough set theory partition or equilence relation is the basic concept. Now fuzzy sets are combined with rough sets in a fruitful way and defned by rough fuzzy sets and fuzzy rough sets [3, 4, 5, 6]. Also fuzzy rough sets, generalize fuzzy rough, intuitionistic fuzzy rough sets, rough intuitionistic fuzzy sets, rough vague sets are introduced. The theory of rough sets is based upon the classifcation mechanism, from which the classifcation can be viewed as an equivalence relation and knowledge blocks induced by it be a partition on universe. One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache[9, 10]. Neutrosophic sets described by three functions: Truth function indeterminacy function and false function that are independently related. The theories of neutrosophic set have achieved great success in various areas such as medical diagnosis, database, topology, image processing, and decision making problem. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness.

Neutrosophic sets and rough sets are two different topics, none conicts the other. In this paper we defne neutrosophic rough in an axiomatic approach and neutrosophic set algebra in infnite universe are studied.

2 Preliminaries

Definition 2.1[9]

A Neutrosophic set A on the universe of discourse X is defined as
A = $\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \ / \ x \in X\}$ where
$T, I, F : X \longrightarrow \ ]\, 0^-, 1^+ \,[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$

Definition 2.2[9]

A neutrosophice set A is contained in another neutrosophic set B (ie)
$A \subseteq B \Leftrightarrow T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x),\ F_A(x) \geq F_B(x)$

Definition 2.3[9]

If $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \ /x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \ /x \in X\}$ are any two neutrosophic sets of X then

(i) $A \subseteq B \ \leftrightarrow T_A(x) \leq T_B(x);\ I_A(x) \leq I_B(x)$ and $F_A(x) \geq F_B(x)$

(ii) $A = B \ \leftrightarrow T_A(x) = T_B(x);\ I_A(x) = I_B(x)$ and $F_A(x) = F_B(x)\ \forall x \in X$

(iii) $\bar{A} = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \ /x \in X\}$

(iv) $A \cap B = \{\langle x, T_{(A\cap B)}(x), \quad I_{(A\cap B)}(x), F_{(A\cap B)}(x)\rangle / x \in X\}$ where
$T_{A\cap B}(x) = min\{T_A(x), T_B(x)\}\ \ I_{A\cap B}(x) = min\{I_A(x), I_B(x)\}\ \ F_{A\cap B}(x) = max\{F_A(x), F_B(x)\}$

v) $A \cup B = \{\langle x, T_{(A\cup B)}(x), \quad I_{(A\cup B)}(x), F_{(A\cup B)}(x)\rangle / x \in X\}$ where
$T_{A\cup B}(x) = max\{T_A(x), T_B(x)\}\ \ I_{A\cup B}(x) = max\{I_A(x), I_B(x)\}\ \ F_{A\cup B}(x) = min\{F_A(x), F_B(x)\}$

Definition 2.4[9]

Let $R \subseteq U \times U$ be a crisp binary relation on U. R is referred to as reflexive if $(x, x) \in R$ for all $x \in U$.R is referred to as symmetric if for all (x,y) ∈ U, (x,y) ∈ R implies (y,x) ∈ R and R is referred to as transitive if for all x,y,z ∈ U, (x,y) ∈ R and (y,z) ∈ R imply (x,z) ∈ R.

Definition 2.5[6]

Let U be a non empty universe of discourse and $R \subseteq U \times U$, an arbitrary crisp relation on U. Denote $xR = y \in U/(x, y) \in R \ \ x \in U$
xR is called the R-after set of x (Bandler and kohout 1980) or successor neighbourhood of x with respect to R (Yao 1998 b). The pair (U,R) is called a crisp approximation space.

For any $A \subseteq U$ the upper and lower approximation of A with respect to (U,R) denoted by $\bar{R}$ and $\underline{R}$ are respectively defined as follows

$\bar{R} = \{x \in U/xR \cap A \neq \varphi\}$

$\underline{R} = \{x \in U/xR \subseteq A\}$

The pair $(\underline{R}(A), \bar{R}(A))$ is referred to as crisp rough set of A with respect to (U,R) and $\bar{R}, \underline{R} : \rho(U) \longrightarrow \rho(U)$ are referred to upper and lower crisp approximation operator respectively.

The crisp approximation operator satisfies the following properties[61] for all A, B $\in \rho(U)$

$(L_1)\ \underline{R}(A) = \bar{R}'(A')$	$(U_1)\bar{R} = \ \underline{R}(\ A)$
$(L_2)\underline{R}(U) = U$	$(U_2)\bar{R}\ \ \varphi = \varphi$
$(L_3)\ \underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B)$	$(U_3)\bar{R}(A \cap B) = \bar{R}(A) \cup \bar{R}(B)$
$(L_4)\ A \subseteq B \Rightarrow \underline{R}(A) \subseteq \underline{R}(B)$	$(U_4)A \subseteq B = \bar{R}(A) \subseteq \bar{R}(B)$
$(L_5)\ \underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B)$	$(U_5)\bar{R}(A \cap B) \subseteq \bar{R}(A) \cap \bar{R}(B)$

Properties $(L_1)$and(U_1) show that the approximation operators $\underline{R}$and$\bar{R}$ are dual to each other. Properties with the same number may be considered as a dual properties. If R is equivalence relation in U then the pair (U,R) is called a Pawlak approximation space and $(\underline{R}(A), \bar{R}(A))$ is a Pawlak rough set, in such a case the approximation operators have additional properties.

3 NEUTROSOPHIC ROUGH SET ALGEBRAS IN INFINITE UNIVERSE

In this section we introduce a special lattice on $I \times I \times I$ (where I=[0,1] is the unit interval) Then we define neutrosophic rough set algebras and present some of its properties.

Definition 3.1 Denote

$L^* = \{(x_1, x_2, x_3) \in [0,1] \times [0,1] \times [0,1] \mid x_1 + x_2 + x_3 \leq 3\}$

We define a relation $\leq_{L^*}\ onL^*$ as follows:

$\forall (x_1, x_2, x_3) \in L^*$,the relation $\leq_{L^*}$ is a partial ordering on L^* and the pair $(L^*, \leq_{L^*}\ )$ is a complete lattice with the smallest element $0_{L^*} = (0,0,1)$ and the greatest element $1_{L^*} = (1,1,0)$.

The partial ordering is defined as $\forall (x_1, x_2, x_3), (y_1, y_2, y_3) \in L^*$

$(x_1, x_2, x_3) \bigwedge (y_1, y_2, y_3) = (min(x_1, y_1), min(x_1, y_1), max(x_1, y_1))$

$(x_1, x_2, x_3) \bigvee (y_1, y_2, y_3) = max(x_1, y_1), max(x_1, y_1), min(x_1, y_1))$ We can also introduce the relation $\geq_{L^*}$ on L^* as follows: $\forall (x_1, x_2, x_3), (y_1, y_2, y_3) \in L^*$

$(y_1, y_2, y_3) \geq_{L^*} (x_1, x_2, x_3) \Leftrightarrow (x_1, x_2, x_3) \leq_{L^*} (y_1, y_2, y_3)$

Definition 3.2 Let U be a non empty finite universe of dicourse, a neutrosophic subset $R \in N(U \times U)$ is called a neutrosophic binary relation on U , where $R(x,y) = (T_R(x,y), I_R(x,y), F_R(x,y))$ satisfies the condition $0 \leq T_R(x,y) + I_R(x,y) + F_R(x,y) \leq 3$ for all $(x,y) \in U \times U$. For a neutrosophic binary relation R on U, R is said to be a serial if for each $x \in U$ there exist a $y \in U$ such $R(x,y) = 1_{L^*}$; R is said to be symmetric if $R(x,y) = R(y,x)$ for any $(x,y) \in U \times U$. R is said to be transitive if for any $(x,z) \in U \times U$, $T_R(x,z) \geq \bigvee_{y \in U}[T_R(x,y) \bigwedge T_R(y,z)]$, $I_R(x,z) \geq \bigvee_{y \in U}[I_R(x,y) \bigwedge I_R(y,z)]$ and $F_R(x,z) \leq \bigwedge_{y \in U}[F_R(x,y) \bigvee F_R(y,z)]$. R is referred to as neutrosophic similarity relation if it is reflexive, symmetric and transitive.

Definition 3.3 A neutrosophic unary operator $L : N(U) \to N(U)$ is referred to as neutrosophic rough lower approximation operator iff it satisfies the axioms:$\forall A, B \in N(U)$ and $\alpha, \beta, \gamma \in L^*$
(NL1)$L(A \bigcup (\widehat{\alpha, \beta, \gamma})) = L(A) \bigcup (\widehat{\alpha, \beta, \gamma}) \forall \ (\alpha, \beta, \gamma) \ \in \ [0,1]$
(NL2)$L(\bigcap\limits_{i \in J} A_i) = \bigcap\limits_{i \in J} L(A_i)$
A neutrosophic unary operator $G : N(U) \to N(U)$ is referred to as neutrosophic rough upper approximation operator iff it satisfies the following axioms: $\forall A, B \in N(U)$ and $\alpha, \beta, \gamma \in L^*$
(NG1)$G(A \bigcap (\widehat{\alpha, \beta})) = G(A) \bigcap (\widehat{\alpha, \beta})$
(NG2)$G(\bigcup\limits_{i \in J} A_i) = \bigcup\limits_{i \in J} G(A_i)$
The lower and upper approximation operators L,G: $N(U) \to N(U)$ are said to be dual to operators iff the property (D) $G(A) =\sim L(\sim A)$

Remark 3.4 For the dual neutrosophic rough lower and upper approximation operators L,G: $N(U) \to N(U)$, there exist a neutrosphic binary relation R on U such that
$\underline{R}(A) = L(A), \overline{R}(A) = G(A)$
where
The pair $(\underline{R}(A), \bar{R}(A))$ is called neutrosophic rough set of A with respect to (U,R) and $\underline{R}, \bar{R} : N(U) \longrightarrow N(U)$ are referred to as upper and lower neutrosophic rough approximation operators respectively.

Definition 3.5 If L,G: $N(U) \to N(U)$ are dual neutrosophic lower and upper rough approximation operators, ie L satisfies the axioms (NL1),(NL2) and (D) or equivalently G satisfies (NG1),(NG2) and (D). Then the system $N_L =: (N(U), \bigcap, \bigcup, \sim, L, G)$ is referred to as neutrosophic rough set algebra (NRSA) on U. Moreover if there exist a serial (respectively , a reflexive, a symmetric, a transitive, a similarity) neutrosophic relation R on U such that

$L(A) = \underline{R}(A) and G(A) = \overline{R}(A)$ for all $A \in N(U)$, then N_L is called a a serial (respectively , a reflexive, a symmetric, a transitive, a similarity)algebra. Axiom (D) implies that operators L and G in a NRSA N_L are dual with each other. It can be easily verified that axiom (N-L2) implies the following axiom (NL3)and (NL4) and dually axiom (NG2) implies (NG3)and (NG4):

(NL3) $L(\bigcup_{i\in J} A_i) \supseteq \bigcup_{i\in J} L(A_i) \forall\ A_i \ \in N(U) j \in J, J\ is\ an\ index\ set,$

(NL4) $A \subseteq B \Rightarrow L(A) \subseteq L(B) \forall A, B \in N(U)$

(NG3) $G(\bigcap_{i\in J} A_i) \supseteq \bigcap_{i\in J} G(A_i) \forall\ A_i \ \in N(U) j \in J, J\ is\ an\ index\ set,$

(NG4) $A \supseteq B \Rightarrow L(A) \supseteq L(B) \forall A, B \in N(U)$

Theorem 3.6 Assume that $N_L =: (N(U), \bigcap, \bigcup, \sim, L, G)$ is a NRSA on U. Then N_L is a serial NRSA iff one of the following equivalent axioms holds:

(NL5) $L(\widehat{\alpha, \beta, \gamma}) = \widehat{\alpha, \beta, \gamma}\ \forall\ \alpha, \beta, \gamma \in L^*$.

(NG5) $G(\widehat{\alpha, \beta, \gamma}) = \widehat{\alpha, \beta, \gamma}\ \forall \alpha, \beta, \gamma\ \in L^*$.

(NL6) $L(\emptyset) = \emptyset\ \forall \alpha, \beta, \gamma\ \in L^*$.

(NG6)$G(U) = U\ \forall\ \alpha, \beta, \gamma\ \in L^*$.

(NLG)$L(A) \subseteq G(A), \forall A \in N(U)$.

Proof: For NRA N_L we can find a neutrosophic binary relation R on U such that $\underline{R}(A) = L(A), \overline{R}(A) = G(A)$ holds.

Now it is enough to prove that R is a serial if and only if $\underline{R}(A) \subseteq \overline{R}(A)$ for all N(U).

Assume that R is a serial neutrosophic binary relation on U. For any $A \in N(U)$ and $x \in U$, we find a find a $y_0 \in U$ such that $T_R(x, y_0) = 1, I_R(x, y_0) = 1$ and $F_R(x, y_0) = 0$.

Then $T_{\underline{R}(A)}(x) = \bigwedge_{y\in U} [\ F_R(x, y) \vee T_A(y)\ ] \leq \bigwedge_{y\in U} [\ F_R(x, y_0) \vee T_A(x, y_0)\ ] = 0 \vee T_A(x, y_0) = T_A(x, y_0) = 1 \wedge T_A(x, y_0) = T_R(x, y_0) \wedge T_A(x, y_0) \leq \bigvee_{y\in U} (T_R(x, y) \wedge T_A(x, y)). = T_{\overline{R}(A)}(x)$

Similarly we can prove $I_{\underline{R}(A)}(x) \leq I_{\overline{R}(A)}(x)$.

Thus, $\underline{R}(A) \subseteq \overline{R}(A)$.

Conversely, if $I_{\underline{R}(A)}(x) \leq I_{\overline{R}(A)}(x)$.

we have,$\underline{R}(A) \subseteq \overline{R}(A)$ holds for all $A \subseteq N(U)$.

Then by [1] we have $\underline{R}(\phi) = \overline{R}(\phi) = \phi$, ie $\underline{R}(\phi) = \phi$.

Hence we conclude R is a serial.

Theorem 3.7 Assume that $N_L = (N(U), \bigcap, \bigcup, \sim, L, G)$ is a NRSA on U. Then

(1)N_L is reflexive NRA iff one of the following equivalent axioms holds:

(NL7) $L(A) \subseteq A \forall A \in N(U)$

(NG7) $A \subseteq G(A) \forall A \in N(U)$

(2) N_L is symmetric NRA if and only if one of the following equivalent axioms holds:

(NL8) $L(1_{U-x})(y) = L(1_{U-y})(x) \forall (x, y) \in U \times U$

(NG8) $G(1_x)(y) = G(1_y)(x) \forall (x, y) \in U \times U$

(3) N_L is transitive NRA if and only if one of the following equivalent axioms holds:

(NL9) $L(A) \subseteq LL(A) \forall A \in N(U)$

(NG9) $G(G(A)) \subseteq G(A) \forall A \in N(U)$

$LL(A) = L(L(A)), GL(A) = G(L(A)), A \in N(U)$

$GG(A) = G(L(A)), LG(A) = L(G(A)), A \in N(U)$

$N_{LL} = (N(U), \bigcap, \bigcup, \sim, L, G)$

Theorem 3.8 If $N_L = (N(U), \bigcap, \bigcup, \sim, L, G)$ is a NRA then N_{LL} is also NRA. Proof: It is enough to prove that GG satisfies the axioms (NGI), (NG2) and (ND). For any $A \in N(U) and (\alpha, \beta, \gamma) \in L^*$, by axiom(NG1), we have

$$GG(A \bigcap \widehat{(\alpha, \beta, \gamma)}) = G(G(A \bigcap \widehat{(\alpha, \beta, \gamma)}))$$
$$= G(G(A) \bigcap \widehat{(\alpha, \beta, \gamma)})$$
$$= G(G(A)) \bigcap \widehat{(\alpha, \beta, \gamma)}$$
$$= GG(A) \bigcap \widehat{(\alpha, \beta, \gamma)}$$

Thus operator GG satisfies the axiom(NG1). Since G satisfies the axioms (NG2) and (ND), it is easy to verify that GG also satisfies axioms (NG2) and (ND).

SPECIAL CLASSES OF NRAS In this section we discuss properties of approximation operators in special classes of NRAs and investigate the relationships between $NRAN_L$ and its inducing system N_{LL}.

SERIAL NRAS In a serial NRA N_L, L(A) is a subset of G(A) for all $A \in N(U)$ and L and G map any constant neutrosophic set into itself. We then have the following relationship between the approximtion operators: for $A \in N(U)$, $LL(A) \subseteq LG(A)$ and $GL(A)$ and $GL(A) \subseteq GG(A)$

Thus , operators LL and GG obey axioms (NLG), then by Theorem (3.7) and (3.9), We can obtain the following.

Theorem 3.9 If N_L is a serial NRA on U, then N_{LL} is also a serial NRA on U.

REFLEXIVE NRAS In a reflexive NRA N_L,L and G satisfy the axioms (NRL7) and (N-RG7). It is easily verified that LL and GG also satisfies the axiom (NRL5) and (NRG5) respectively, and by Theorem(3.9), the following is established.

Theorem 3.10 If N_L is a reflexive NRA on U, then N_{LL} is also a reflexive NRA on U. for$A \in N(U)$

$LL(A) \subseteq L(A) \subseteq A \subseteq G(A) \subseteq GG(A)$.

$LL(A) \subseteq L(A) \subseteq LG(A) \subseteq G(A) \subseteq GG(A)$.

$LL(A) \subseteq L(A) \subseteq GL(A) \subseteq G(A) \subseteq GG(A)$. AS L and G satisfies monotonicity, axioms (NRL7) and (NRG7) imply the following properties, $\forall A, B \in N(U)$.

$A \subseteq L(B) \Rightarrow L(A) \subseteq L(B)$

$G(A) \subseteq B \Rightarrow G(A) \subseteq G(B)$

$L(A) \subseteq B \Rightarrow L(A) \subseteq G(B)$

$A \subseteq G(B) \Rightarrow L(A) \subseteq L(B)$

$L(A) \subseteq L(B) \Rightarrow L(A) \subseteq B$

$G(A) \subseteq G(B) \Rightarrow A \subseteq G(B)$

$G(A) \subseteq L(B) \Rightarrow A \subseteq L(B), G(A) \subseteq B$.

SYMMETRIC NRAS In a symmetric NRA N_L, approximation operators L and G satisfies (NL8)and (NG8).

Theorem 3.1.11 If N_L is a symmetric NRA on U then N_L on U.

Proof: We haveN_{LL} is a NRA. Since N_L is a NRA, by Remark 3.5,
we can find there exist neutrosophic binary relation R on U such that,
$\forall A \in N(U), \forall x \in U$

$$G(A)(x) = \overline{R}(A)(x), L(A)(x) = \underline{R}(A)(x).$$

It can be easily verified that $\forall (x, y) \in U \times U$

$$G(1_y)(x) = R(x, y) = (T_R(x, y), I_R(x, y), F_R(x, y)).$$

Since N_L is symmetric NRA on U, we conclude that for all $(x, y) \in U \times U$,
$R(x, y) = G(1_y)(x) = G(1_x)(y) = R(y, x)$.
For any $(x, y) \in U \times U$, by Equation (D1) and (D2) ,
we then have
$GG(1_y)(x) = G(G(1_y))(x) = \overline{R}(\overline{R}(1_y))(x)$
$= (T_{\overline{R}\overline{R}(1_y)})(x), I_{\overline{R}\overline{R}(1_y)})(x), F_{\overline{R}\overline{R}(1_y)})(x)$
$= (\bigvee_{z \in U} [T_R(x, z) \wedge T_{\overline{R}(1_y)}(z)], \bigvee_{z \in U} [I_R(x, z) \wedge I_{\overline{R}(1_y)}(z)], \bigwedge_{z \in U} [F_R(x, z) \vee F_{\overline{R}(1_y)}(z)])$
$= (\bigvee_{z \in U} [T_R(x, z) \wedge T_{R(1_y)}(z, y)], \bigvee_{z \in U} [I_R(x, z) \wedge I_{R(1_y)}(z, y)], \bigwedge_{z \in U} [F_R(x, z) \vee F_{R(1_y)}(z, y)])$
similarly we can obtain
$GG(1_x)(y) = (\bigvee_{z \in U} [T_R(y, z) \wedge T_{R(1_y)}(z, x)], \bigvee_{z \in U} [I_R(y, z) \wedge I_{R(1_y)}(z, x)], \bigwedge_{z \in U} [F_R(y, z) \vee F_{R(1_y)}(z, x)])$
since S_L is a symmetric NRA, we have for all $z \in U$,

$T_R(x,z) \wedge T_R(z,y) = T_R(y,z) \wedge T_R(x,z)$
$I_R(x,z) \wedge I_R(z,y) = I_R(y,z) \wedge I_R(x,z)$
$F_R(x,z) \vee F_R(z,y) = F_R(y,z) \vee F_R(x,z)$
Thus, we conclude that
$GG(1_y)(x) = GGGG(1_x)(y)$ for every $(x,y) \in U \times U$
similarly we can prove
$LL(1_{U-y})(x) = GGGG(1_{U-x})(y)$ for every $(x,y) \in U \times U$
Hence the operators LL and GG respectively satisfies the axioms (NL8) and (NG8).
Therefore, N_{LL} is a symmetric NRA.

TRANSITIVE NRAS In a NRA N_L, L and G respectively satisfies (NL9) and (NG9), we then have $A \in N(U)$. $GGGG(A) \subseteq GGG(A) \subseteq GG(A)$. Thus, GG satisfies axiom (NG7)

Theorem 3.2.12 If N_L is a transitive NRA on U, then N_{LL} is also a transitive NRA on U. In a transitive NRA, by employing the monotonicity of L and G, and in terms of axioms (NR7) and (NG7), we can conclude following properties: for $A, B \in N(U)$.
$L(A) \subseteq B \Rightarrow L(A) \subseteq L(B)$ $A \subseteq G(B) \Rightarrow G(A) \subseteq G(B)$.

TOPOLOGICAL NRAS: If N_L is a serial and transitive NRA, the operator L in N_L is characterized by axioms (ND), (NL1), (NL2), (NLG0) and (NL7), by theorems 4 and 7, N_{LL} is also a serial and transitive NRA. It is easy to verify that, $\forall A \in N(U)$,
(D1) $L(A) \subseteq LL(A) \subseteq GL(A) \subseteq GG(A) \subseteq G(A)$
(D2) $L(A) \subseteq LL(A) \subseteq LG(A) \subseteq GG(A) \subseteq G(A)$
Moreover, if N_L is a reflexive NRA, by Theorem 2, D1 and D2, we see that L and G respectively obey following axioms (NL9) and (NG9):
(NL9) $L(A) = LL(A) \forall A \in N(U)$
(NG9) $G(A) = GG(A) \forall A \in N(U)$
In such a case, two systems N_L and N_{LL} become the same one. Obviously, a reflexive NRA is a serial one, and, thus, operators L and G respectively obey axioms (NL0) and (NG0). It should be noted that axioms (NL0), (NL2), (NL5), and (NL9) of L, and (NG0), (NG2), (NG5), and (NG9) of G are the axioms of interior and closure operators in an N topological space. Such an N rough set algebra is thus called a topological NRA. with the topological NRA N_L, a neutrosophic set A is said to be open if $L(A) = A$, and closed if $G(A) = A$. It follows from axioms (NL9) and (NG9) that L and G respectively map each neutrosophic set into an open neutrosophic set and a closed neutrosophic set. By axioms (NL9) and (NU9), we conclude the following.

Theorem 3.13 $If N_L$ is a topological NRA on U and N_{LL} is also a topological NRA on

U. Operators L and G in a topological $NRAN_L$ have relationhips:$\forall A \in N(U)$,
$L(A) = LL(A) \subseteq A \subseteq GG(A) = G(A)$,
$L(A) = LL(A) \subseteq GL(A) \subseteq GG(A) = G(A)$,
$L(A) = LL(A) \subseteq LG(A) \subseteq GG(A) = G(A)$.
we can easily obtain

Theorem 3.14 If N_L is a similarity NRA on U, then N_{LL} is also a similarity NRA on U.

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