mathematics

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Neutrosophic Soft Expert Multiset and Their Application to Multiple Criteria Decision Making

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Received: 11 December 2018; Accepted: 2 January 2019; Published: 6 January 2019

Abstract: In this paper, we have investigated neutrosophic soft expert multisets (NSEMs) in detail. The concept of NSEMs is introduced. Several operations have been defined for them and their important algebraic properties are studied. Finally, we define a NSEMs aggregation operator to construct an algorithm for a NSEM decision-making method that allows for a more efficient decision-making process.

Keywords: aggregation operator; decision making; neutrosophic soft expert sets; neutrosophic soft expert multiset

1. Introduction

Multiple criteria decision making (MCDM) is an important part of modern decision science and relates to many complex factors, such as economics, psychological behavior, ideology, military and so on. For a proper description of objects in an uncertain and ambiguous environment, indeterminate and incomplete information has to be properly handled. Intuitionistic fuzzy sets were introduced by Atanassov [1], followed by Molodtsov [2] on soft set and neutrosophy logic [3] and neutrosophic sets [4] by Smarandache. The term neutrosophy means knowledge of neutral thought and this neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic and set. Presently, work on soft set theory is progressing rapidly. Various operations and applications of soft sets were developed rapidly, including multi-adjoint t-concept lattices [5], signatures, definitions, operators and applications to fuzzy modelling [6], fuzzy inference system optimized by genetic algorithm for robust face and pose detection [7], fuzzy multi-objective modeling of effectiveness and user experience in online advertising [8], possibility fuzzy soft set [9], soft multiset theory [10], multiparameterized soft set [11], soft intuitionistic fuzzy sets [12], Q-fuzzy soft sets [13–15], and multi Q-fuzzy sets [16–18], thereby opening avenues to many applications [19,20]. Later, Maji [21] introduced a more generalized concept, which is a combination of neutrosophic sets and soft sets and studied its properties. Alkhazaleh and Salleh [22] defined the concept of fuzzy soft expert sets, which were later extended to vague soft expert set theory [23], generalized vague soft expert set [24], and multi Q-fuzzy soft expert set [25]. Şahin et al. [26] introduced neutrosophic soft expert sets, while Hassan et al. [27] extended it further to Q-neutrosophic soft expert sets. Broumi et al. [28] defined neutrosophic parametrized soft set theory and its decision making. Deli [29] introduced refined neutrosophic sets and refined neutrosophic soft sets.

Since membership values are inadequate for providing complete information in some real problems which has different membership values for each element, different generalizations of fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets have been introduced called the multi fuzzy set [30], intuitionistic fuzzy multiset [31] and neutrosophic multiset [32,33], respectively. In the multisets, an element of a universe can be constructed more than once with possibly the same or different membership values. Some work on the multi fuzzy set [34,35], on the intuitionistic fuzzy multiset [36–39] and on the neutrosophic multiset [40–43] have been studied. The above set theories have been applied to many different areas including real decision-making problems [44–47]. The aim of this paper is allow the neutrosophic set to handle problems involving incomplete, indeterminacy and awareness of inconsistency knowledge, and this is further developed to neutrosohic soft expert sets.

The initial contributions of this paper involve the introduction of various new set-theoretic operators on neutrosophic soft expert multisets (NSEMs) and their properties. Later, we intend to extend the discussion further by proposing the concept of NSEMs and its basic operations, namely complement, union, intersection AND and OR, along with a definition of a NSEMs-aggregation operator to construct an algorithm of a NSEMs decision method. Finally we provide an application of the constructed algorithm to solve a decision-making problem.

2. Preliminaries

In this section we review the basic definitions of a neutrosophic set, neutrosophic soft set, soft expert sets, neutrosophic soft expert sets, and NP-aggregation operator required as preliminaries.

Definition 1 ([4])**.** *A neutrosophic set A on the universe of discourse $\mathcal{U}$ is defined as $A = \{\langle u, (\mu_A(u), v_A(u), w_A(u))\rangle : u \in U, \mu_A(u), v_A(u), w_A(u) \in [0,1]\}$. There is no restriction on the sum of $\mu_A(u)$; $v_A(u)$ and $w_A(u)$, so $0^- \leq \mu_A(u) + v_{\mathcal{A}}(u) + w_{\mathcal{A}}(u) \leq 3^+$.*

Definition 2 ([21])**.** *Let $\mathcal{U}$ be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $NS(\mathcal{U})$ denotes the set of all neutrosophic sets of $\mathcal{U}$. The collection (F, A) is termed to be the neutrosophic soft set over $\mathcal{U}$, where F is a mapping given by $F : A \to NS(\mathcal{U})$.*

Definition 3 ([22])**.** *$\mathcal{U}$ is an initial universe, E is a set of parameters X is a set of experts (agents), and $O = \{agree = 1, disagree = 0\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$. A pair (F, A) is called a soft expert set over $\mathcal{U}$, where F is mapping given by $F : A \to P(\mathcal{U})$ where $P(\mathcal{U})$ denote the power set of $\mathcal{U}$.*

Definition 4 ([26])**.** *A pair (F, A) is called a neutrosophic soft expert set over $\mathcal{U}$, where F is mapping given by*

$$F : A \to P(\mathcal{U}) \tag{1}$$

where $P(\mathcal{U})$ denotes the power neutrosophic set of U.

Definition 5 ([26])**.** *The complement of a neutrosophic soft expert set (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, A)$ where $F^c = \neg A \to P(\mathcal{U})$ is mapping given by $F^c(x)$ = neutrosophic soft expert complement with $\mu_{F^c(x)} = w_{F(x)}$, $v_{F^c(x)} = v_{F(x)}$, $w_{F^c(x)} = \mu_{F(x)}$.*

Definition 6 ([26])**.** *The agree-neutrosophic soft expert set $(F, A)_1$ over $\mathcal{U}$ is a neutrosophic soft expert subset of (F, A) is defined as*

$$(F, A)_1 = \{F_1(m) : m \in E \times X \times \{1\}\}. \tag{2}$$

Definition 7 ([26])**.** *The disagree-neutrosophic soft expert set $(F, A)_0$ over $\mathcal{U}$ is a neutrosophic soft expert subset of (F, A) is defined as*

$$(F, A)_0 = \{F_0(m) : m \in E \times X \times \{0\}\}. \tag{3}$$

Definition 8 ([26])**.** *Let (H, A) and (G, B) be two NSESs over the common universe U. Then the union of (H, A) and (G, B) is denoted by "$(H, A) \tilde{\cup} (G, B)$" and is defined by $(H, A) \tilde{\cup} (G, B) = (K, C)$, where*

$C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$\mu_{K(e)}(m) = \begin{cases} \mu_{H(e)}(m), & if\ e \in A - B, \\ \mu_{G(e)}(m), & if\ e \in B - A, \\ \max\Big(\mu_{H(e)}(m), \mu_{G(e)}(m)\Big), & if\ e \in AB. \end{cases}$$
$$v_{K(e)}(m) = \begin{cases} v_{H(e)}(m), & if\ e \in A - B, \\ v_{G(e)}(m), & if\ e \in B - A, \\ \frac{v_{H(e)}(m)+v_{G(e)}(m)}{2}, & if\ e \in AB. \end{cases} \tag{4}$$
$$w_{K(e)}(m) = \begin{cases} w_{H(e)}(m), & if\ e \in A - B, \\ w_{G(e)}(m), & if\ e \in B - A, \\ \min\Big(w_{H(e)}(m), w_{G(e)}(m)\Big), & if\ e \in AB. \end{cases}$$

Definition 9 ([26]). *Let (H, A) and (G, B) be two NSESs over the common universe U. Then the intersection of (H, A) and (G, B) is denoted by "$(H, A) \tilde{\cap} (G, B)$" and is defined by $(H, A) \tilde{\cap} (G, B) = (K, C)$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:*

$$\begin{aligned} c\mu_{K(e)}(m) &= \min\Big(\mu_{H(e)}(m), \mu_{G(e)}(m)\Big), \\ v_{K(e)}(m) &= \frac{v_{H(e)}(m) + v_{G(e)}(m)}{2}, \\ w_{K(e)}(m) &= \max\Big(w_{H(e)}(m), w_{G(e)}(m)\Big), \end{aligned} \tag{5}$$

$if\ e \in AB$.

Definition 10 ([29]). *Let $\mathcal{U}$ be a universe. A neutrosophic multiset set (Nms) A on $\mathcal{U}$ can be defined as follows:*

$$A = \Big\{\prec u, \Big(\mu_A^1(u), \mu_A^2(u), \ldots, \mu_A^p(u)\Big), \Big(v_A^1(u), v_A^2(u), \ldots, v_A^p(u)\Big), \Big(w_A^1(u), w_A^2(u), \ldots, w_A^p(u)\Big) \succ : u \in \mathcal{U}\Big\}$$

where,

$$c\,\mu_A^1(u), \mu_A^2(u), \ldots, \mu_A^p(u) : \mathcal{U} \to [0, 1],$$
$$v_A^1(u), v_A^2(u), \ldots, v_A^p(u) : \mathcal{U} \to [0, 1],$$

and

$$w_A^1(u), w_A^2(u), \ldots, w_A^p(u) : \mathcal{U} \to [0, 1],$$

such that

$$0 \leq sup\mu_A^i(u) + supv_A^i(u) + supw_A^i(u) \leq 3$$

$(i = 1, 2, \ldots, P)$ and

$$\Big(\mu_A^1(u), \mu_A^2(u), \ldots, \mu_A^p(u)\Big), \Big(v_A^1(u), v_A^2(u), \ldots, v_A^p(u)\Big) \text{ and } \Big(w_A^1(u), w_A^2(u), \ldots, w_A^p(u)\Big)$$

This is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element u, respectively. Also, P is called the dimension (cardinality) of Nms A, denoted $d(A)$. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order.

The set of all neutrosophic multisets on $\mathcal{U}$ is denoted by NMS($\mathcal{U}$).

Definition 11 ([28]). *Let $\Psi_K \in$ NP-soft set. Then an NP-aggregation operator of Ψ_K, denoted by Ψ_K^{agg} is defined by*

$$\Psi_K^{agg} = \left\{ \left(\langle u, \mathrm{T}_K^{agg}, \mathrm{I}_K^{agg}, \mathrm{F}_K^{agg} \rangle \right) : u \in U \right\},$$

which is a neutrosophic set over U,

$$\begin{array}{ll} \mathrm{T}_K^{agg} : U \to [0,1] & \mathrm{T}_K^{agg}(u) = \frac{1}{|U|} \sum\limits_{\substack{e \in E \\ u \in U}} \mathrm{T}_K(u).\lambda f_{K(x)}(u), \\ \mathrm{I}_K^{agg} : U \to [0,1] & \mathrm{I}_K^{agg}(u) = \frac{1}{|U|} \sum\limits_{\substack{e \in E \\ u \in U}} \mathrm{I}_K(u).\lambda f_{K(x)}(u), \\ \mathrm{F}_K^{agg} : U \to [0,1] & \mathrm{F}_K^{agg} = \frac{1}{|U|} \sum\limits_{\substack{e \in E \\ u \in U}} \mathrm{F}_K(u).\lambda f_{K(x)}(u) \end{array} \tag{6}$$

and where,

$$\lambda f_{K(x)}(u) = \begin{cases} 1, & x \in f_{K(x)}(u), \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

$|U|$ is the cardinality of U.

3. Neutrosophic Soft Expert Multiset (NSEM) Sets

This section introduces neutrosophic soft expert multiset as a generalization of neutrosophic soft expert set. Throughout this paper, V is an initial universe, E is a set of parameters X is a set of experts (agents), and $O = \{\text{agree} = 1, \text{disagree} = 0\}$ a set of opinions. Let $Z = E \times X \times O$ and $G \subseteq Z$ and u is a membership function of G; that is, $\Omega : G \to= [0,1]$.

Definition 12. *A pair (F^{Ω}, G) is called a neutrosophic soft expert multiset over V, where F^{Ω} is mapping given by*

$$F^{\Omega} : G \to \mathcal{N}(V) \times, \tag{8}$$

where $\mathcal{N}(V)$ be the set of all neutrosophic soft expert subsets of U. For any parameter $e \in G$, $F(e)$ is referred as the neutrosophic value set of parameter e, i.e.,

$$F(e) = \left\{ \langle \frac{v}{\left(D^1_{F(e)}(v), \ldots, D^n_{F(e)}\right), \left(I^1_{F(e)}(v), \ldots, I^n_{F(e)}\right), \left(Y^1_{F(e)}(v), \ldots, Y^n_{F(e)}\right)} \rangle \right\}, \tag{9}$$

where $D^i, ^i, Y^i : U \to [0,1]$ are the membership sequence of truth, indeterminacy and falsity respectively of the element $v \in V$. For any $v \in V$, $e \in G$ and $i = 1, 2, \ldots, n$.

$$0 \leq D^i{}_{F(e)}(v) +^i {}_{F(e)}(v) + Y^i{}_{F(e)}(v) \leq 3$$

In fact F^{Ω} is a parameterized family of neutrosophic soft expert multisets on V, which has the degree of possibility of the approximate value set which is prepresented by $\Omega(e)$ for each parameter e. So we can write it as follows:

$$F^{\Omega}(e) = \left\{ \left(\frac{v_1}{F(e)(v_1)}, \frac{v_2}{F(e)(v_2)}, \frac{v_3}{F(e)(v_3)}, \cdots, \frac{v_n}{F(e)(v_n)} \right), \Omega(e) \right\}. \tag{10}$$

Example 1. *Suppose that $V = \{v_1\}$ is a set of computers and $E = \{e_1, e_2\}$ is a set of decision parameters. Let $X = \{p, r\}$ be set of experts. Suppose that*

$$cF^{\Omega}(e_1, p, 1) = \left\{\left(\frac{v_1}{(0.4, 0.3, \ldots, 0.2), (0.5, 0.7, \ldots, 0.2), (0.6, 0.1, \ldots, 0.3)}\right), 0.4\right\}$$
$$F^{\Omega}(e_1, r, 1) = \left\{\left(\frac{v_1}{(0.3, 0.2, \ldots, 0.5), (0.8, 0.1, \ldots, 0.4), (0.5, 0.6, \ldots, 0.2)}\right), 0.8\right\}$$
$$F^{\Omega}(e_2, p, 1) = \left\{\left(\frac{v_1}{(0.7, 0.3, \ldots, 0.6), (0.3, 0.2, \ldots, 0.6), (0.8, 0.2, \ldots, 0.1)}\right), 0.5\right\}$$
$$F^{\Omega}(e_2, r, 1) = \left\{\left(\frac{v_1}{(0.8, 0.3, \ldots, 0.4), (0.3, 0.1, \ldots, 0.5), (0.2, 0.3, \ldots, 0.4)}\right), 0.4\right\}$$
$$F^{\Omega}(e_1, p, 0) = \left\{\left(\frac{v_1}{(0.5, 0.1, \ldots, 0.2), (0.6, 0.3, \ldots, 0.4), (0.7, 0.2, \ldots, 0.6)}\right), 0.1\right\}$$
$$F^{\Omega}(e_1, r, 0) = \left\{\left(\frac{v_1}{(0.4, 0.2, \ldots, 0.1), (0.6, 0.1, \ldots, 0.3), (0.7, 0.2, \ldots, 0.4)}\right), 0.4\right\}$$
$$F^{\Omega}(e_2, p, 0) = \left\{\left(\frac{v_1}{(0.8, 0.1, \ldots, 0.5), (0.2, 0.1, \ldots, 0.4), (0.6, 0.3, \ldots, 0.1)}\right), 0.6\right\}$$
$$F^{\Omega}(e_2, r, 0) = \left\{\left(\frac{v_1}{(0.7, 0.2, \ldots, 0.3), (0.4, 0.1, \ldots, 0.6), (0.3, 0.2, \ldots, 0.1)}\right), 0.2\right\}$$

The neutrosophic soft expert multiset (F, Z) is a parameterized family $\{F(e_i),\ i = 1, 2, \ldots\}$ of all neutrosophic multisets of V and describes a collection of approximation of an object.

Definition 13. *For two neutrosophic soft expert multisets (NSEMs) $\left(F^{\Omega}, G\right)$ and (H^{η}, R) over U, $\left(F^{\Omega}, G\right)$ is called a neutrosophic soft expert subset of (H^{η}, R) if*

i. $R \subseteq G$,
ii. *for all $\varepsilon \in H$, $H^{\eta}(\varepsilon)$ is neutrosophic soft expert subset $F^{\Omega}(\varepsilon)$.*

Example 2. *Consider Example 1. Suppose that G and R are as follows.*

$$cG = \{(e_1, p, 1), (e_2, p, 1), (e_2, p, 0), (e_2, r, 1)\}$$
$$R = \{(e_1, p, 1), (e_2, r, 1)\}$$

Since R is a neutrosophic soft expert subset of G, clearly $R \subset G$. Let (H^{η}, R) and $\left(F^{\Omega}, G\right)$ be defined as follows:

$$\begin{aligned} c\left(F^{\Omega}, G\right) = \Bigg\{ & \left[(e_1, p, 1), \left(\frac{v_1}{(0.4, 0.3, \ldots, 0.2), (0.5, 0.7, \ldots, 0.2), (0.6, 0.1, \ldots, 0.3)}\right), 0.4\right], \\ & \left[(e_2, p, 1), \left(\frac{v_1}{(0.7, 0.3, \ldots, 0.6), (0.3, 0.2, \ldots, 0.6), (0.8, 0.2, \ldots, 0.1)}\right), 0.5\right], \\ & \left[(e_2, p, 0), \left(\frac{v_1}{(0.8, 0.1, \ldots, 0.5), (0.2, 0.1, \ldots, 0.4), (0.6, 0.3, \ldots, 0.1)}\right), 0.6\right], \\ & \left[(e_2, r, 1), \left(\frac{v_1}{(0.8, 0.3, \ldots, 0.4), (0.3, 0.1, \ldots, 0.5), (0.2, 0.3, \ldots, 0.4)}\right), 0.4\right] \Bigg\}. \\ (H^{\eta}, R) = \Bigg\{ & \left[(e_1, p, 1), \left(\frac{v_1}{(0.4, 0.3, \ldots, 0.2), (0.5, 0.7, \ldots, 0.2), (0.6, 0.1, \ldots, 0.3)}\right), 0.4\right], \\ & \left[(e_2, r, 1), \left(\frac{v_1}{(0.8, 0.3, \ldots, 0.4), (0.3, 0.1, \ldots, 0.5), (0.2, 0.3, \ldots, 0.4)}\right), 0.4\right] \Bigg\}. \end{aligned}$$

Therefore $(H^{\eta}, R) \subseteq \left(F^{\Omega}, G\right)$.

Definition 14. *Two NSEMs* (F^{Ω}, G) *and* (G^{η}, B) *over* V *are said to be equal if* (F^{Ω}, G) *is a NSEM subset of* (H^{η}, R) *and* (H^{η}, R) *is a NSEM subset of* (F^{Ω}, G).

Definition 15. *Agree-NSEMs* $(F^{\Omega}, G)_1$ *over* V *is a NSEM subset of* (F^{Ω}, G) *defined as follows.*

$$\left(F^{\Omega}, G\right)_1 = \{F_1(\Delta) : \Delta \in E \times X \times \{1\}\}. \tag{11}$$

Example 3. *Consider Example 1. The agree- neutrosophic soft expert multisets* $(F^{\Omega}, Z)_1$ *over* V *is*

$$\begin{aligned} c(F^{\Omega}, Z)_1 = \Bigg\{ & \left[(e_1, p, 1), \left(\frac{v_1}{(0.4, 0.3, \ldots, 0.2), (0.5, 0.7, \ldots, 0.2), (0.6, 0.1, \ldots, 0.3)}\right), 0.4\right], \\ & \left[(e_1, r, 1), \left(\frac{v_1}{(0.3, 0.2, \ldots, 0.5), (0.8, 0.1, \ldots, 0.4), (0.5, 0.6, \ldots, 0.2)}\right), 0.8\right], \\ & \left[(e_2, p, 1), \left(\frac{v_1}{(0.7, 0.3, \ldots, 0.6), (0.3, 0.2, \ldots, 0.6), (0.8, 0.2, \ldots, 0.1)}\right), 0.5\right], \\ & \left[(e_2, r, 1), \left(\frac{v_1}{(0.8, 0.3, \ldots, 0.4), (0.3, 0.1, \ldots, 0.5), (0.2, 0.3, \ldots, 0.4)}\right), 0.4\right] \Bigg\}. \end{aligned}$$

Definition 16. *A disagree-NSEMs* $(F^{\Omega}, G)_0$ *over* V *is a NSES subset of* (F^{Ω}, G) *is defined as follows:*

$$(F^{\Omega}, A)_0 = \{F_0(\Delta) : \Delta \in E \times X \times \{0\}\}. \tag{12}$$

Example 4. *Consider Example 1. The disagree- neutrosophic soft expert multisets* $(F^{\Omega}, Z)_0$ *over* V *are*

$$\begin{aligned} (F^{\Omega}, Z)_0 = \Bigg\{ & \left[(e_1, p, 0), \left(\frac{v_1}{(0.5, 0.1, \ldots, 0.2), (0.6, 0.3, \ldots, 0.4), (0.7, 0.2, \ldots, 0.6)}\right), 0.1\right], \\ & \left[(e_1, r, 0), \left(\frac{v_1}{(0.4, 0.2, \ldots, 0.1), (0.6, 0.1, \ldots, 0.3), (0.7, 0.2, \ldots, 0.4)}\right), 0.4\right], \\ & \left[(e_2, p, 0), \left(\frac{v_1}{(0.8, 0.1, \ldots, 0.5), (0.2, 0.1, \ldots, 0.4), (0.6, 0.3, \ldots, 0.1)}\right), 0.6\right], \\ & \left[(e_2, r, 0), \left(\frac{v_1}{(0.7, 0.2, \ldots, 0.3), (0.4, 0.1, \ldots, 0.6), (0.3, 0.2, \ldots, 0.1)}\right), 0.2\right] \Bigg\}. \end{aligned}$$

4. Basic Operations on NSEMs

Definition 17. *The complement of a neutrosophic soft expert multiset* (F^{Ω}, G) *is denoted by* $(F^{\Omega}, G)^c$ *and is defined by* $(F^{\Omega}, G)^c = \left(F^{\Omega(c)}, \neg G\right)$ *where* $F^{u(c)} : \neg G \to \mathcal{N}(V)\times$ *is mapping given by*

$$F^{\Omega(c)}(\Delta) = \left\{ {D^i}_{F(\Delta)^{(c)}} = {Y^i}_{F(\Delta)},\ {I^i}_{F(\Delta)^{(c)}} = \bar{1} - {I^i}_{F(\Delta)},\ {Y^i}_{F(\Delta)^{(c)}} = {D^i}_{F(\Delta)} \text{ and } \Omega^c(\Delta) = \bar{1} - \Omega(\Delta) \right\} \tag{13}$$

for each $\Delta \in E$.

Example 5. *Consider Example 1. The complement of the neutrosophic soft expert multiset F^{Ω} denoted by $F^{\Omega(c)}$ is given by as follows:*

$$c(F^{\Omega(c)}, Z) = \Bigg\{ \left[(\neg e_1, p, 1), \left(\frac{v_1}{(0.2, 0.7, \ldots, 0.4), (0.2, 0.3, \ldots, 0.5), (0.3, 0.9, \ldots, 0.6)} \right), 0.6 \right],$$
$$\left[(\neg e_1, r, 1), \left(\frac{v_1}{(0.5, 0.8, \ldots, 0.3), (0.4, 0.9, \ldots, 0.8), (0.2, 0.4, \ldots, 0.5)} \right), 0.2 \right],$$
$$\left[(\neg e_2, p, 1), \left(\frac{v_1}{(0.6, 0.7, \ldots, 0.7), (0.6, 0.8, \ldots, 0.3), (0.1, 0.8, \ldots, 0.8)} \right), 0.5 \right],$$
$$\left[(\neg e_2, r, 1), \left(\frac{v_1}{(0.4, 0.7, \ldots, 0.8), (0.5, 0.9, \ldots, 0.3), (0.4, 0.7, \ldots, 0.2)} \right), 0.6 \right],$$
$$\left[(\neg e_1, p, 0), \left(\frac{v_1}{(0.2, 0.9, \ldots, 0.5), (0.4, 0.7, \ldots, 0.6), (0.6, 0.8, \ldots, 0.7)} \right), 0.9 \right],$$
$$\left[(\neg e_1, r, 0), \left(\frac{v_1}{(0.1, 0.8, \ldots, 0.4), (0.3, 0.9, \ldots, 0.6), (0.4, 0.8, \ldots, 0.7)} \right), 0.6 \right],$$
$$\left[(\neg e_2, p, 0), \left(\frac{v_1}{(0.5, 0.9, \ldots, 0.8), (0.4, 0.9, \ldots, 0.2), (0.1, 0.7, \ldots, 0.6)} \right), 0.4 \right],$$
$$\left[(\neg e_2, r, 0), \left(\frac{v_1}{(0.3, 0.8, \ldots, 0.7), (0.6, 0.9, \ldots, 0.4), (0.1, 0.8, \ldots, 0.3)} \right), 0.8 \right] \Bigg\}.$$

Proposition 1. *If (F^{Ω}, G) is a neutrosophic soft expert multiset over V, then*

1. $((F^{\Omega}, G)^c)^c = (F^{\Omega}, G)$
2. $((F^{\Omega}, G)_1)^c = (F^{\Omega}, G)_0$
3. $((F^{\Omega}, G)_0)^c = (F^{\Omega}, G)_1$

Proof. (1) From Definition 17, we have $(F^{\Omega}, G)^c = \left(F^{\Omega(c)}, \neg G\right)$ where $F^{\Omega(c)}(\Delta) = D^i_{F(\Delta)^{(c)}} = Y^i_{F(\Delta)}$, $I^i_{F(\Delta)^{(c)}} = \bar{1} - I^i_{F(\Delta)}$, $Y^i_{F(\Delta)^{(c)}} = D^i_{F(\Delta)}$ and $\Omega^c(\Delta) = \bar{1} - \Omega(\Delta)$ for each $\Delta \in E$. Now $((F^{\Omega}, G)^c)^c = \left(\left(F^{\Omega(c)}\right)^c, G\right)$ where

$$\begin{array}{llllllll}
\left(F^{\Omega(c)}\right)^c(\Delta) & = [D^i_{F(\Delta)^{(c)}} = Y^i_{F(\Delta)}, & I^i_{F(\Delta)^{(c)}} & = \bar{1} - I^i_{F(\Delta)}, & Y^i_{F(\Delta)^{(c)}} & = D^i_{F(\Delta)}, & (\Omega^i)^c(\Delta) & = \bar{1} - \Omega^i(\Delta)]^c \\
 & = D^i_{F(\Delta)} = Y^i_{F(\Delta)^{(c)}}, & I^i_{F(\Delta)} & = \bar{1} - I^i_{F(\Delta)^{(c)}}, & Y^i_{F(\Delta)} & = D^i_{F(\Delta)^{(c)}}, & \Omega^i(\Delta) & = \bar{1} - (\Omega^i)(\Delta) \\
 & & & = \bar{1} - (\bar{1} - I^i_{F(\Delta)}) & & & & = \bar{1} - (\bar{1} - \Omega^i(\Delta)) \\
 & & & = I^i_{F(\Delta)} & & & & = \Omega^i(\Delta)
\end{array}$$

Thus $\left((F^{\Omega}, G)^c\right)^c = \left(\left(F^{\Omega(c)}\right)^c, G\right) = (F^{\Omega}, G)$, for all $\Delta \in E$.
The Proofs (2) and (3) can proved similarly. □

Definition 18. *The union of two NSEMs (F^{Ω}, G) and (K^{ρ}, L) over V, denoted by $(F^{\Omega}, G) \,\tilde{\cup}\, (K^{\rho}, L)$ is a NSEMs (H^{σ}, C) where $C = G \cup L$ and $\forall\, e \in C$,*

$$(H^{\sigma}, C) = \begin{cases} max\left(D^i{}_{(F^{\Omega}(e)}(m), D^i{}_{(K^{\rho}(e)}(m)\right) & if\ \Delta \in G \cap L \\ min\left(I^i{}_{(F^{\Omega}(e)}(m), I^i{}_{(K^{\rho}(e)}(m)\right) & if\ \Delta \in G \cap L \\ min\left(Y^i{}_{(F^{\Omega}(e)}(m), Y^i{}_{(K^{\rho}(e)}(m)\right) & if\ \Delta \in G \cap L \end{cases} \tag{14}$$

where $\sigma(m) = max\left(\Omega_{(e)}(m), \rho_{(e)}(m)\right)$.

Example 6. *Suppose that (F^{Ω}, G) and (K^{ρ}, L) are two NSEMs over V, such that*

$$c(F^{\Omega}, G) = \left\{ \left[(e_1, p, 1), \left(\frac{v_1}{(0.7, 0.3, \ldots, 0.6), (0.5, 0.2, \ldots, 0.4), (0.7, 0.6, \ldots, 0.3)} \right), 0.3 \right], \right.$$
$$\left[(e_2, q, 1), \left(\frac{v_1}{(0.4, 0.3, \ldots, 0.6), (0.8, 0.2, \ldots, 0.4), (0.5, 0.1, \ldots, 0.7)} \right), 0.6 \right],$$
$$\left. \left[(e_3, r, 1), \left(\frac{v_1}{(0.8, 0.2, \ldots, 0.3), (0.6, 0.3, \ldots, 0.7), (0.4, 0.2, \ldots, 0.8)} \right), 0.5 \right] \right\}.$$
$$(K^{\rho}, L) = \left\{ \left[(e_1, p, 1), \left(\frac{v_1}{(0.4, 0.3, \ldots, 0.1), (0.7, 0.2, \ldots, 0.3), (0.5, 0.4, \ldots, 0.7)} \right), 0.6 \right], \right.$$
$$\left. \left[(e_3, r, 1), \left(\frac{v_1}{(0.8, 0.3, \ldots, 0.2), (0.6, 0.1, \ldots, 0.2), (0.3, 0.5, \ldots, 0.3)} \right), 0.7 \right] \right\}$$

Then $(F^{\Omega}, G) \,\widetilde{\cup}\, (K^{\rho}, L) = (H^{\sigma}, C)$ *where*

$$c(H^{\sigma}, C) = \left\{ \left[(e_1, p, 1), \left(\frac{v_1}{(0.7, 0.3, \ldots, 0.1), (0.7, 0.2, \ldots, 0.3), (0.7, 0.4, \ldots, 0.3)} \right), 0.6 \right], \right.$$
$$\left[(e_2, q, 1), \left(\frac{v_1}{(0.4, 0.3, \ldots, 0.6), (0.8, 0.2, \ldots, 0.4), (0.5, 0.1, \ldots, 0.7)} \right), 0.6 \right],$$
$$\left. \left[(e_3, r, 1), \left(\frac{v_1}{(0.8, 0.2, \ldots, 0.2), (0.6, 0.1, \ldots, 0.2), (0.4, 0.2, \ldots, 0.3)} \right), 0.7 \right] \right\}.$$

Proposition 2. *If (F^{Ω}, G), (K^{ρ}, L) and $\left(H^{\Omega}, C\right)$ are three NSEMs over V, then*

1. $\left((F^{\Omega}, G) \,\widetilde{\cup}\, (K^{\rho}, L)\right) \widetilde{\cup}\, (H^{\sigma}, C) = (F^{\Omega}, G) \,\widetilde{\cup} \left((K^{\rho}, L) \,\widetilde{\cup}\, (H^{\sigma}, C)\right)$
2. $(F^{\Omega}, G)(F^{\Omega}, G) \subseteq (F^{\Omega}, G)$.

Proof. (1) We want to prove that

$$\left((F^{\Omega}, G) \,\widetilde{\cup}\, (K^{\rho}, L)\right) \widetilde{\cup}\, (H^{\sigma}, C) = (F^{\Omega}, G) \,\widetilde{\cup} \left((K^{\rho}, L) \,\widetilde{\cup}\, (H^{\sigma}, C)\right)$$

by using Definition 18, we consider the case when if $e \in G \cap L$ as other cases are trivial. We will have

$$\begin{aligned}
&(F^{\Omega}, G) \,\widetilde{\cup}\, (K^{\rho}, L) \\
&= \left\{ \left(v / max\left(D^i{}_{F^{\Omega}(e)}(m), D^i{}_{G^{\rho}(e)}(m) \right), min\left(I^i{}_{F^{\Omega}(e)}(m), I^i{}_{G^{\rho}(e)}(m) \right), min\left(Y^i{}_{F^{\Omega}(e)}(m), Y^i{}_{G^{\rho}(e)}(m) \right) \right), \right. \\
&\qquad\qquad \left. max\left(\Omega_{(e)}(m), \rho_{(e)}(m) \right), v \in V \right\}
\end{aligned}$$

Also consider the case when $e \in H$ as the other cases are trivial. We will have

$$\begin{aligned}
&\left((F^u, A) \,\widetilde{\cup}\, (G^{\eta}, B) \right) \widetilde{\cup}\, (H^{\Omega}, C) \\
&= \left\{ \left(v / max\left(D^i{}_{F^{\Omega}(e)}(m), D^i{}_{G^{\rho}(e)}(m) \right), min\left(I^i{}_{F^{\Omega}(e)}(m), I^i{}_{G^{\rho}(e)}(m) \right), min\left(Y^i{}_{F^{\Omega}(e)}(m), Y^i{}_{G^{\rho}(e)}(m) \right) \right), \right. \\
&\left. \left(v / D^i{}_{H^{\Omega}(e)}(m), I^i{}_{H^{\Omega}(e)}(m), Y^i{}_{H^{\Omega}(e)}(m) \right), max\left(u_{(e)}(m), \eta_{(e)}(m), \Omega(m) \right), v \in V \right\} \\
&= \left\{ \begin{array}{c} \left(v / D^i{}_{F^{\Omega}(e)}(m), I^i{}_{F^{\Omega}(e)}(m), Y^i{}_{F^{\Omega}(e)}(m) \right), \\ \left(v / max\left(D^i{}_{G^u(e)}(m), D^i{}_{H^{\eta}(e)}(m) \right), min\left(I^i{}_{G^u(e)}(m), I^i(m) \right), \; min\left(Y^i{}_{G^u(e)}(m), Y^i(m) \right) \right) \end{array} \right. \\
&\left. max\left(\Omega_{(e)}(m), \rho_{(e)}(m), \sigma(m) \right), v \in V \right\} \\
&= (F^{\Omega}, G) \,\widetilde{\cup} \left((K^{\rho}, L) \,\widetilde{\cup}\, (H^{\sigma}, C) \right).
\end{aligned}$$

(2) The proof is straightforward. □

Definition 19. *The intersection of two NSEMs* (F^{Ω}, G) *and* (K^{ρ}, L) *over* V*, denoted by* $(F^{\Omega}, G) \tilde{\cap} (K^{\rho}, L) = \left(P^{\delta}, C\right)$ *where* $C = G \cap L$ *and* $\forall\, e \in C$*,*

$$\left(P^{\delta}, C\right) = \begin{cases} min\Big(D^{i}{}_{(F^{\Omega}(e)}(m), D^{i}{}_{(K^{\rho}(e)}(m)\Big) & if\ e \in G \cap L \\ max\Big(I^{i}{}_{(F^{\Omega}(e)}(m), I^{i}{}_{(K^{\rho}(e)}(m)\Big) & if\ e \in G \cap L \\ max\Big(Y^{i}{}_{(F^{\Omega}(e)}(m), Y^{i}{}_{(K^{\rho}(e)}(m)\Big) & if\ e \in G \cap L \end{cases} \tag{15}$$

where $\delta(m) = min\Big(\Omega_{(e)}(m), \rho_{(e)}(m)\Big)$.

Example 7. *Suppose that* (F^{Ω}, G) *and* (K^{ρ}, L) *are two NSEMs over* V*, such that*

$$c(F^{\Omega}, G) = \Bigg\{\Bigg[(e_3, r, 1), \left(\frac{v_1}{(0.8, 0.3, \ldots, 0.2), (0.6, 0.1, \ldots, 0.2), (0.3, 0.5, \ldots, 0.3)}\right), 0.4\Bigg],$$
$$\Bigg[(e_1, q, 1), \left(\frac{v_1}{(0.8, 0.2, \ldots, 0.2), (0.7, 0.3, \ldots, 0.2), (0.4, 0.2, \ldots, 0.3)}\right), 0.7\Bigg],$$
$$\Bigg[(e_3, q, 0), \left(\frac{v_1}{(0.4, 0.3, \ldots, 0.6), (0.8, 0.2, \ldots, 0.4), (0.5, 0.1, \ldots, 0.7)}\right), 0.6\Bigg]\Bigg\}.$$
$$(K^{\rho}, L) = \Bigg\{\Bigg[(e_1, p, 1), \left(\frac{v_1}{(0.7, 0.3, \ldots, 0.1), (0.7, 0.2, \ldots, 0.3), (0.7, 0.4, \ldots, 0.3)}\right), 0.3\Bigg],$$
$$\Bigg[(e_3, r, 1), \left(\frac{v_1}{(0.4, 0.7, \ldots, 0.8), (0.5, 0.9, \ldots, 0.3), (0.4, 0.7, \ldots, 0.2)}\right), 0.8\Bigg]\Bigg\}$$

Then $(F^{\Omega}, G) \tilde{\cap} (K^{\rho}, L) = \left(P^{\delta}, C\right)$ *where*

$$\left(P^{\delta}, C\right) = \Bigg\{\Bigg[(e_3, r, 1), \left(\frac{v_1}{(0.4, 0.3, \ldots, 0.2), (0.6, 0.9, \ldots, 0.3), (0.4, 0.7, \ldots, 0.3)}\right), 0.4\Bigg]\Bigg\}.$$

Proposition 3. *If* (F^{Ω}, G)*,* (K^{ρ}, L) *and* $\left(H^{\Omega}, C\right)$ *are three NSEMs over* V*, then*

1. $\Big((F^{\Omega}, G) \tilde{\cap} (K^{\rho}, L)\Big) \tilde{\cap} (H^{\sigma}, C) = (F^{\Omega}, G) \tilde{\cap} \Big((K^{\rho}, L) \tilde{\cap} (H^{\sigma}, C)\Big)$
2. $(F^{\Omega}, G) \tilde{\cap} (F^{\Omega}, G) \subseteq (F^{\Omega}, G)$.

Proof. (1) We want to prove that

$$\Big((F^{\Omega}, G) \tilde{\cap} (K^{\rho}, L)\Big) \tilde{\cap} (H^{\sigma}, C) = (F^{\Omega}, G) \tilde{\cap} \Big((K^{\rho}, L) \tilde{\cap} (H^{\sigma}, C)\Big)$$

by using Definition 19, we consider the case when if $e \in G \cap L$ as other cases are trivial. We will have

$$\begin{aligned} &(F^{\Omega}, G) \tilde{\cap} (K^{\rho}, L) \\ &= \Big\{\Big(v/min\Big(D^{i}{}_{F^{\Omega}(e)}(m), D^{i}{}_{G^{\rho}(e)}(m)\Big), max\Big(I^{i}{}_{F^{\Omega}(e)}(m), I^{i}{}_{G^{\rho}(e)}(m)\Big), max\Big(Y^{i}{}_{F^{\Omega}(e)}(m), Y^{i}{}_{G^{\rho}(e)}(m)\Big)\Big), \end{aligned}$$
$$min\Big(\Omega_{(e)}(m), \rho_{(e)}(m)\Big), v \in V\Big\}$$

Also consider the case when $\Delta \in H$ as the other cases are trivial. We will have

$$\Big((F^u, A) \,\widetilde{\cap}\, (G^\eta, B)\Big) \,\widetilde{\cap}\, (H^\Omega, C)$$
$$= \Big\{\Big(v/max\Big(D^i{}_{F^{\Omega}(e)}(m), D^i{}_{G^{\rho}(e)}(m)\Big), min\Big(I^i{}_{F^{\Omega}(e)}(m), I^i{}_{G^{\rho}(e)}(m)\Big), min\Big(Y^i{}_{F^{\Omega}(e)}(m), Y^i{}_{G^{\rho}(e)}(m)\Big)\Big),$$
$$\Big(v/D^i{}_{H^{\Omega}(e)}(m), I^i{}_{H^{\Omega}(e)}(m), Y^i{}_{H^{\Omega}(e)}(m)\Big), min\Big(u_{(e)}(m), \eta_{(e)}(m), \Omega(m)\Big), v \in V\Big\}$$
$$= \left\{\begin{array}{c} \Big(v/D^i{}_{F^{\Omega}(e)}(m), I^i{}_{F^{\Omega}(e)}(m), Y^i{}_{F^{\Omega}(e)}(m)\Big), \\ \Big(v/min\Big(D^i{}_{G^{u}(e)}(m), D^i{}_{H^{\eta}(e)}(m)\Big), max\Big(I^i{}_{G^{u}(e)}(m), I^i(m)\Big), max\Big(Y^i{}_{G^{u}(e)}(m), Y^i(m)\Big)\Big) \end{array}\right.$$
$$min\Big(\Omega_{(e)}(m), \rho_{(e)}(m), \sigma(m)\Big), v \in V\Big\}$$
$$= (F^\Omega, G) \,\widetilde{\cap}\, \Big((K^\rho, L) \,\widetilde{\cap}\, (H^\sigma, C)\Big).$$

(2) The proof is straightforward. □

Proposition 4. *If (F^Ω, G), (K^ρ, L) and $\big(H^\Omega, C\big)$ are three NSEMs over V. Then*

1. $\Big((F^\Omega, G) \,\widetilde{\cup}\, (K^\rho, L)\Big) \,\widetilde{\cap}\, (H^\sigma, C) = \Big((F^\Omega, G) \,\widetilde{\cap}\, (H^\sigma, C)\Big) \,\widetilde{\cup}\, \Big((K^\rho, L) \,\widetilde{\cap}\, (H^\sigma, C)\Big).$
2. $\Big((F^\Omega, G) \,\widetilde{\cap}\, (K^\rho, L)\Big) \,\widetilde{\cup}\, (H^\sigma, C) = \Big((F^\Omega, G) \,\widetilde{\cup}\, (H^\sigma, C)\Big) \,\widetilde{\cap}\, \Big((K^\rho, L) \,\widetilde{\cup}\, (H^\sigma, C)\Big).$

Proof. The proofs can be easily obtained from Definitions 18 and 19. □

5. AND and OR Operations

Definition 20. *Let (F^Ω, G) and (K^ρ, L) be any two NSEMs over V, then $(F^\Omega, G)AND(K^\rho, L)''$ denoted $(F^\Omega, G) \wedge (K^\rho, L)$ is defined by*

$$(F^\Omega, G) \wedge (K^\rho, L) = (H^\sigma, G \times L) \tag{16}$$

where $(H^\sigma, G \times L) = H^\sigma(\alpha, \beta)$ such that $H^\sigma(\alpha, \beta) = F^\Omega(\alpha) \cap K^\rho(\beta)$ for all $(\alpha, \beta) \in G \times L$ where $\cap$ represent the basic intersection.

Example 8. *Suppose that (F^Ω, G) and (K^ρ, L) are two NSEMs over V, such that*

$$c(F^\Omega, G) = \left\{\left[(e_1, p, 1), \left(\frac{v_1}{(0.2, 0.3, \ldots, 0.6), (0.2, 0.1, \ldots, 0.8), (0.3, 0.2, \ldots, 0.6)}\right), 0.1\right],\right.$$
$$\left.\left[(e_2, r, 0), \left(\frac{v_1}{(0.5, 0.3, \ldots, 0.4), (0.6, 0.5, \ldots, 0.4), (0.2, 0.4, \ldots, 0.3)}\right), 0.5\right]\right\}.$$
$$(K^\rho, L) = \left\{\left[(e_1, p, 1), \left(\frac{v_1}{(0.3, 0.2, \ldots, 0.1), (0.5, 0.2, \ldots, 0.3), (0.8, 0.3, \ldots, 0.4)}\right), 0.2\right],\right.$$
$$\left.\left[(e_2, q, 0), \left(\frac{v_1}{(0.6, 0.4, \ldots, 0.7), (0.3, 0.4, \ldots, 0.2), (0.6, 0.1, \ldots, 0.5)}\right), 0.6\right]\right\}.$$

Then $(F^\Omega, G) \wedge (K^\rho, L) = (H^\sigma, G \times L)$ where

$$c(H^\sigma, G \times L) = \left\{\left[(e_1, p, 1), (e_1, p, 1)\left(\frac{v_1}{(0.2, 0.2, \ldots, 0.1), (0.5, 0.2, \ldots, 0.8), (0.8, 0.3, \ldots, 0.6)}\right), 0.1\right],\right.$$
$$\left[(e_1, p, 1), (e_2, q, 0), \left(\frac{v_1}{(0.2, 0.3, \ldots, 0.6), (0.3, 0.4, \ldots, 0.8), (0.6, 0.2, \ldots, 0.6)}\right), 0.1\right],$$
$$\left[(e_2, r, 0), (e_1, p, 1), \left(\frac{v_1}{(0.3, 0.2, \ldots, 0.1), (0.6, 0.5, \ldots, 0.4), (0.8, 0.4, \ldots, 0.4)}\right), 0.2\right],$$
$$\left.\left[(e_2, r, 0), (e_2, q, 0), \left(\frac{v_1}{(0.5, 0.3, \ldots, 0.4), (0.6, 0.5, \ldots, 0.4), (0.6, 0.4, \ldots, 0.5)}\right), 0.5\right]\right\}.$$

Definition 21. *Let (F^{Ω}, G) and (K^{ρ}, L) be any two NSEMs over V, then $(F^{\Omega}, G)OR(K^{\rho}, L)$" denoted $(F^{\Omega}, G) \vee (K^{\rho}, L)$ is defined by*

$$(F^{\Omega}, G) \vee (K^{\rho}, L) = (H^{\sigma}, G \times L) \tag{17}$$

where $(H^{\sigma}, G \times L) = H^{\sigma}(\alpha, \beta)$ such that $H^{\sigma}(\alpha, \beta) = F^{\Omega}(\alpha) \cup K^{\rho}(\beta)$ for all $(\alpha, \beta) \in G \times L$ where $\cup$ represent the basic union.

Example 9. *Suppose that (F^{Ω}, G) and (K^{ρ}, L) are two NSEMs over V, such that*

$$c(F^{\Omega}, G) = \left\{ \left[(e_1, p, 1), \left(\frac{v_1}{(0.2, 0.3, \ldots, 0.6), (0.2, 0.1, \ldots, 0.8), (0.3, 0.2, \ldots, 0.6)} \right), 0.1 \right], \right.$$
$$\left. \left[(e_2, r, 0), \left(\frac{v_1}{(0.5, 0.3, \ldots, 0.4), (0.6, 0.5, \ldots, 0.4), (0.2, 0.4, \ldots, 0.3)} \right), 0.5 \right] \right\}.$$
$$(K^{\rho}, L) = \left\{ \left[(e_1, p, 1), \left(\frac{v_1}{(0.3, 0.2, \ldots, 0.1), (0.5, 0.2, \ldots, 0.3), (0.8, 0.3, \ldots, 0.4)} \right), 0.2 \right], \right.$$
$$\left. \left[(e_2, q, 0), \left(\frac{v_1}{(0.6, 0.4, \ldots, 0.7), (0.3, 0.4, \ldots, 0.2), (0.6, 0.1, \ldots, 0.5)} \right), 0.6 \right] \right\}.$$

Then $(F^{\Omega}, G) \vee (K^{\rho}, L) = (H^{\sigma}, G \times L)$ where

$$c(H^{\sigma}, G \times L) = \left\{ \left[(e_1, p, 1), (e_1, p, 1) \left(\frac{v_1}{(0.3, 0.3, \ldots, 0.6), (0.2, 0.1, \ldots, 0.3), (0.3, 0.2, \ldots, 0.4)} \right), 0.2 \right], \right.$$
$$\left[(e_1, p, 1), (e_2, q, 0), \left(\frac{v_1}{(0.6, 0.4, \ldots, 0.7), (0.2, 0.1, \ldots, 0.2), (0.3, 0.1, \ldots, 0.5)} \right), 0.6 \right],$$
$$\left[(e_2, r, 0), (e_1, p, 1), \left(\frac{v_1}{(0.5, 0.3, \ldots, 0.4), (0.5, 0.2, \ldots, 0.3), (0.2, 0.3, \ldots, 0.3)} \right), 0.2 \right],$$
$$\left. \left[(e_2, r, 0), (e_2, q, 0), \left(\frac{v_1}{(0.6, 0.4, \ldots, 0.7), (0.3, 0.4, \ldots, 0.2), (0.2, 0.1, \ldots, 0.3)} \right), 0.6 \right] \right\}.$$

Proposition 5. *Let (F^{Ω}, G) and (K^{ρ}, L) be NSEMs over V. Then*

1. $((F^{\Omega}, G) \wedge (K^{\rho}, L))^c = (F^u, A)^c \vee (G^{\eta}, B)^c$
2. $((F^{\Omega}, G) \vee (K^{\rho}, L))^c = (F^u, A)^c \wedge (G^{\eta}, B)^c$

Proof. (1) Suppose that (F^{Ω}, G) and (K^{ρ}, L) be NSEMs over V defined as:

$$\begin{aligned}
(F^{\Omega}, G) \wedge (K^{\rho}, L) &= \left(F^{\Omega}(\alpha) \wedge K^{\rho}(\beta)\right)^c \\
&= \left(F^{\Omega}(\alpha) \cap K^{\rho}(\beta)\right)^c \\
&= \left(F^{\Omega}(\alpha) \cap K^{\rho}(\beta)\right)^c \\
&= \left(F^{\Omega(c)}(\alpha) \cup K^{\rho(c)}(\beta)\right) \\
&= \left(F^{\Omega(c)}(\alpha) \vee K^{\rho(c)}(\beta)\right) \\
&= (F^u, A)^c \vee (G^{\eta}, B)^c
\end{aligned}$$

(2) The proofs can be easily obtained from Definitions 20 and 21. □

6. NSEMs-Aggregation Operator

In this section, we define a NSEMs-aggregation operator of NSEMs to construct a decision method by which approximate functions of a soft expert set are combined to produce a neutrosophic set that can be used to evaluate each alternative.

Definition 22. *Let $\Gamma_G \in NSEMs$. Then NSEMs-aggregation operator of Γ_G, denoted by Γ_G^{agg}, is defined by*

$$\Gamma_G^{agg} = \left\{ \left(\langle v, \left(\mathrm{D}^i\right)_G^{agg}(v), \left(\mathrm{I}^i\right)_G^{agg}(v), \left(\mathrm{Y}^i\right)_G^{agg}(v) \rangle \right) : v \in V \right\},$$

which are NSEMs over V,

$$\begin{aligned} \left(\mathrm{D}^i\right)_G^{agg} : V \to [0,1] \quad & \left(\mathrm{D}^i\right)_G^{agg}(v) = \left(\frac{1}{|V|} \sum_{\substack{e \in E \\ v \in V}} \mathrm{D}^i{}_G(v) \right) .\Omega, \\ \left(\mathrm{Y}^i\right)_G^{agg} : V \to [0,1] \quad & \left(\mathrm{Y}^i\right)_G^{agg}(v) = \left(\frac{1}{|V|} \sum_{\substack{e \in E \\ v \in V}} \mathrm{Y}^i{}_G(v) \right) .\Omega, \\ \left(\mathrm{I}^i\right)_G^{agg} : V \to [0,1] \quad & \left(\mathrm{I}^i\right)_G^{agg}(v) = \left(\frac{1}{|V|} \sum_{\substack{e \in E \\ v \in V}} \mathrm{I}^i{}_G(v) \right) .\Omega \end{aligned} \tag{18}$$

where $|V|$ is the cardinality of V and Ω^i is defined below

$$\Omega = \frac{1}{n}.\sum_{i=1}^{n} \Omega(e_i). \qquad (e_i,\ i = 1,2,3,\ldots,n) \tag{19}$$

Definition 23. *Let $\Gamma_G \in NSEMs$, Γ_G^{agg} be NSEMs. Then a reduced fuzzy set of Γ_G^{agg} is a fuzzy set over is denoted by*

$$\Gamma_G^{agg} = \left\{ \frac{\lambda\Gamma_G^{agg}(v)}{v} : v \in V \right\}, \tag{20}$$

where $\lambda\Gamma_G^{agg}(v) : V \to [0,1]$ and $v_i = \left| \left(D^i\right)_{G_i}^{agg} - \left(Y^i\right)_{G_i}^{agg} - \left(I^i\right)_{G_i}^{agg} \right|$.

7. An Application of NSEMs

In this section, we present an application of NSEMs theory in a decision-making problem. Based on Definitions 22 and 23, we construct an algorithm for the NSEMs decision-making method as follows:

Step 1-Choose a feasible subset of the set of parameters.
Step 2-Construct the NSEMs for each opinion (agree, disagree) of expert.
Step 3-Compute the aggregation NSEMS Γ_G^{agg} of Γ_G and the reduced fuzzy set $\left(\mathrm{D}^i\right)_{G_i}^{agg}, \left(\mathrm{Y}^i\right)_{G_i}^{agg}, \left(\mathrm{I}^i\right)_{G_i}^{agg}$ of Γ_G^{agg}
Step 4-Score $(v_{\mathrm{i}}) = (\max - agree(v_i)) - (\min - disagree(v_i))$
Step 5-Choose the element of v_i that has maximum membership. This will be the optimal solution.

Example 10. *In the architectural design process, let us assume that the design outputs used in the design of moving structures are taken by a few experts at certain time intervals. So, let us take the samples at three different timings in a day (in 08:30, 14:30 and 20:30) The design of moving structures consists of the architectural design, the design of the mechanism and the design of the surface covering membrane. Architectural design will be evaluated from these designs., $V = \{v_1, v_2, v_3\}$. Suppose there are three parameters $E = \{e_1, e_2, e_3\}$ where the parameters e_i $(i = 1, 2, 3)$ stand for "time", "temperature" and "spatial needs" respectively. Let $X = \{p, q\}$ be a set of experts. After a serious discussion, the experts construct the following NSEMs.*

Step 1-Choose a feasible subset of the set of parameters:

$$
\begin{aligned}
&(F^{\Omega}, Z) = \\
&\Big\{\Big[(e_1,p,1),\Big(\tfrac{v_1}{(0.3,0.1,0.4),(0.2,0.1,0.5),(0.5,0.2,0.6)}\Big),\Big(\tfrac{v_2}{(0.4,0.2,0.3),(0.7,0.1,0.6),(0.3,0.2,0.6)}\Big),\Big(\tfrac{v_3}{(0.5,0.3,0.4),(0.2,0.1,0.8),(0.4,0.2,0.3)}\Big),0.7\Big],\\
&\Big[(e_1,q,1),\Big(\tfrac{v_1}{(0.4,0.2,0.5),(0.3,0.1,0.2),(0.6,0.3,0.4)}\Big),\Big(\tfrac{v_2}{(0.5,0.3,0.2),(0.8,0.2,0.4),(0.5,0.3,0.2)}\Big),\Big(\tfrac{v_3}{(0.6,0.3,0.8),(0.3,0.2,0.1),(0.5,0.4,0.3)}\Big),0.6\Big],\\
&\Big[(e_2,p,1),\Big(\tfrac{v_1}{(0.6,0.4,0.2),(0.3,0.1,0.4),(0.8,0.2,0.5)}\Big),\Big(\tfrac{v_2}{(0.8,0.3,0.4),(0.2,0.1,0.5),(0.4,0.3,0.5)}\Big),\Big(\tfrac{v_3}{(0.8,0.3,0.2),(0.3,0.1,0.4),(0.2,0.1,0.4)}\Big),0.8\Big],\\
&\Big[(e_2,q,1),\Big(\tfrac{v_1}{(0.5,0.2,0.4),(0.3,0.2,0.5),(0.6,0.1,0.3)}\Big),\Big(\tfrac{v_2}{(0.6,0.4,0.7),(0.5,0.3,0.2),(0.6,0.2,0.4)}\Big),\Big(\tfrac{v_3}{(0.6,0.5,0.4),(0.1,0.3,0.2),(0.6,0.2,0.3)}\Big),0.4\Big],\\
&\Big[(e_3,p,1),\Big(\tfrac{v_1}{(0.8,0.1,0.5),(0.2,0.3,0.4),(0.5,0.2,0.3)}\Big),\Big(\tfrac{v_2}{(0.7,0.2,0.5),(0.1,0.2,0.3),(0.3,0.2,0.1)}\Big),\Big(\tfrac{v_3}{(0.4,0.3,0.7),(0.3,0.1,0.4),(0.5,0.3,0.2)}\Big),0.3\Big],\\
&\Big[(e_3,q,1),\Big(\tfrac{v_1}{(0.7,0.2,0.4),(0.3,0.1,0.5),(0.6,0.2,0.1)}\Big),\Big(\tfrac{v_2}{(0.9,0.4,0.5),(0.2,0.4,0.5),(0.1,0.2,0.3)}\Big),\Big(\tfrac{v_3}{(0.6,0.8,0.9),(0.2,0.1,0.6),(0.3,0.1,0.4)}\Big),0.4\Big],\\
&\Big[(e_1,q,0),\Big(\tfrac{v_1}{(0.7,0.1,0.4),(0.3,0.2,0.1),(0.4,0.2,0.5)}\Big),\Big(\tfrac{v_2}{(0.6,0.5,0.4),(0.4,0.2,0.1),(0.8,0.2,0.6)}\Big),\Big(\tfrac{v_3}{(0.9,0.4,0.5),(0.2,0.1,0.3),(0.6,0.2,0.3)}\Big),0.7\Big],\\
&\Big[(e_2,p,0),\Big(\tfrac{v_1}{(0.6,0.5,0.7),(0.3,0.5,0.4),(0.6,0.3,0.4)}\Big),\Big(\tfrac{v_2}{(0.5,0.2,0.3),(0.2,0.1,0.3),(0.4,0.3,0.5)}\Big),\Big(\tfrac{v_3}{(0.6,0.3,0.4),(0.1,0.2,0.4),(0.5,0.3,0.2)}\Big),0.8\Big],\\
&\Big[(e_2,q,0),\Big(\tfrac{v_1}{(0.3,0.1,0.2),(0.4,0.1,0.3),(0.5,0.2,0.6)}\Big),\Big(\tfrac{v_2}{(0.7,0.2,0.4),(0.4,0.3,0.6),(0.5,0.1,0.6)}\Big),\Big(\tfrac{v_3}{(0.7,0.3,0.5),(0.2,0.4,0.3),(0.5,0.2,0.3)}\Big),0.4\Big],\\
&\Big[(e_3,p,0),\Big(\tfrac{v_1}{(0.8,0.5,0.4),(0.2,0.4,0.3),(0.6,0.3,0.4)}\Big),\Big(\tfrac{v_2}{(0.5,0.2,0.3),(0.4,0.1,0.2),(0.2,0.1,0.4)}\Big),\Big(\tfrac{v_3}{(0.4,0.3,0.2),(0.2,0.1,0.6),(0.7,0.3,0.2)}\Big),0.5\Big],\\
&\Big[(e_3,q,0),\Big(\tfrac{v_1}{(0.6,0.1,0.4),(0.2,0.1,0.5),(0.4,0.2,0.3)}\Big),\Big(\tfrac{v_2}{(0.7,0.2,0.5),(0.4,0.3,0.2),(0.1,0.2,0.3)}\Big),\Big(\tfrac{v_3}{(0.5,0.3,0.4),(0.3,0.2,0.4),(0.4,0.2,0.3)}\Big),0.1\Big]\Big\}.
\end{aligned}
$$

Step 2-Construct the neutrosophic soft expert tables for each opinion (agree, disagree) of expert.

Step 3-Now calculate the score of agree (v_i) by using the data in Table 1 to obtain values in Table 2.

$$
\begin{aligned}
\left(\mathrm{D}^1\right)^{agg}_{G_1} &= \left(\frac{D^1_{G_1}+D^1_{G_2}+D^1_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.3+0.6+0.8}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.34 \\
\left(\mathrm{D}^2\right)^{agg}_{G_1} &= \left(\frac{D^2_{G_1}+D^2_{G_2}+D^2_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.1+0.4+0.1}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.12 \\
\left(\mathrm{D}^3\right)^{agg}_{G_1} &= \left(\frac{D^3_{G_1}+D^3_{G_2}+D^3_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.4+0.2+0.5}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.22 \\
(\mathrm{D})^{agg}_{G_1}(p, v_1) &= \frac{0.34+0.12+0.22}{3} = 0.2267 \\
\left(\mathrm{I}^1\right)^{agg}_{G_1} &= \left(\frac{I^1_{G_1}+I^1_{G_2}+I^1_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.2+0.3+0.2}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.1404 \\
\left(\mathrm{I}^2\right)^{agg}_{G_1} &= \left(\frac{I^2_{G_1}+I^2_{G_2}+I^2_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.1+0.1+0.3}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.1002 \\
\left(\mathrm{I}^3\right)^{agg}_{G_1} &= \left(\frac{I^3_{G_1}+I^3_{G_2}+I^3_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.5+0.4+0.4}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.2604 \\
(\mathrm{I})^{agg}_{G_1}(p, v_1) &= \frac{0.1404+0.1002+0.2604}{3} = 0.167 \\
\left(\mathrm{Y}^1\right)^{agg}_{G_1} &= \left(\frac{Y^1_{G_1}+Y^1_{G_2}+Y^1_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.5+0.8+0.5}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.36 \\
\left(\mathrm{Y}^2\right)^{agg}_{G_1} &= \left(\frac{Y^2_{G_1}+Y^2_{G_2}+Y^2_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.2+0.2+0.2}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.12 \\
\left(\mathrm{Y}^3\right)^{agg}_{G_1} &= \left(\frac{Y^3_{G_1}+Y^3_{G_2}+Y^3_{G_3}}{3}\right).\left(\frac{\Omega_1+\Omega_2+\Omega_3}{3}\right) \\
&= \left(\frac{0.6+0.5+0.3}{3}\right).\left(\frac{0.7+0.8+0.3}{3}\right) \\
&= 0.2802 \\
(\mathrm{Y})^{agg}_{G_1}(p, v_1) &= \frac{0.36+0.12+0.2802}{3} = 0.2534 \\
v_1 &= \left|(\mathrm{D})^{agg}_{G_1} - (\mathrm{I})^{agg}_{G_1} - (\mathrm{Y})^{agg}_{G_1}\right| = |0.2267 - 0.167 - 0.2534| = 0.1937
\end{aligned}
$$

Table 1. Agree-neutrosophic soft expert multiset.

	v_1	v_2	v_3	Ω
(e_1, p)	$\langle(0.3, 0.1, 0.4), (0.2, 0.1, 0.5), (0.5, 0.2, 0.6)\rangle$	$\langle(0.4, 0.2, 0.3), (0.7, 0.1, 0.6), (0.3, 0.2, 0.6)\rangle$	$\langle(0.5, 0.3, 0.4), (0.2, 0.1, 0.8), (0.4, 0.2, 0.3)\rangle$	0.7
(e_2, p)	$\langle(0.6, 0.4, 0.2), (0.3, 0.1, 0.4), (0.8, 0.2, 0.5)\rangle$	$\langle(0.8, 0.3, 0.4), (0.2, 0.1, 0.5), (0.4, 0.3, 0.5)\rangle$	$\langle(0.8, 0.3, 0.2), (0.3, 0.1, 0.4), (0.2, 0.1, 0.4)\rangle$	0.8
(e_3, p)	$\langle(0.8, 0.1, 0.5), (0.2, 0.3, 0.4), (0.5, 0.2, 0.3)\rangle$	$\langle(0.7, 0.2, 0.5), (0.1, 0.2, 0.3), (0.3, 0.2, 0.1)\rangle$	$\langle(0.4, 0.3, 0.7), (0.3, 0.1, 0.4), (0.5, 0.3, 0.2)\rangle$	0.3
(e_1, q)	$\langle(0.4, 0.2, 0.5), (0.3, 0.1, 0.2), (0.6, 0.3, 0.4)\rangle$	$\langle(0.5, 0.3, 0.2), (0.8, 0.2, 0.4), (0.5, 0.3, 0.2)\rangle$	$\langle(0.6, 0.3, 0.8), (0.3, 0.2, 0.1), (0.5, 0.4, 0.3)\rangle$	0.6
(e_2, q)	$\langle(0.5, 0.2, 0.4), (0.3, 0.2, 0.5), (0.6, 0.1, 0.3)\rangle$	$\langle(0.6, 0.4, 0.7), (0.5, 0.3, 0.2), (0.6, 0.2, 0.4)\rangle$	$\langle(0.6, 0.5, 0.4), (0.1, 0.3, 0.2), (0.6, 0.2, 0.3)\rangle$	0.4
(e_3, q)	$\langle(0.7, 0.2, 0.4), (0.3, 0.1, 0.5), (0.6, 0.2, 0.1)\rangle$	$\langle(0.9, 0.4, 0.5), (0.2, 0.4, 0.5), (0.1, 0.2, 0.3)\rangle$	$\langle(0.6, 0.8, 0.9), (0.2, 0.1, 0.6), (0.3, 0.1, 0.4)\rangle$	0.4

Table 2. Degree table of agree- neutrosophic soft expert multiset.

	v_1	v_2	v_3
p	0.1136	0.1267	0.093
q	0.1142	0.0933	0.015

Now calculate the score of disagree (v_i) by using the data in Table 3 to obtain values in Table 4.

Table 3. Disagree-neutrosophic soft expert multiset.

	v_1	v_2	v_3	Ω
(e_1, p)	$\langle(0.5, 0.1, 0.7), (0.4, 0.2, 0.3), (0.5, 0.4, 0.1)\rangle$	$\langle(0.8, 0.2, 0.3), (0.2, 0.1, 0.4), (0.3, 0.4, 0.5)\rangle$	$\langle(0.5, 0.2, 0.6), (0.3, 0.4, 0.1), (0.2, 0.3, 0.1)\rangle$	0.9
(e_2, p)	$\langle(0.6, 0.5, 0.7), (0.3, 0.5, 0.4), (0.6, 0.3, 0.4)\rangle$	$\langle(0.5, 0.2, 0.3), (0.2, 0.1, 0.3), (0.4, 0.3, 0.5)\rangle$	$\langle(0.6, 0.3, 0.4), (0.1, 0.2, 0.4), (0.5, 0.3, 0.2)\rangle$	0.8
(e_3, p)	$\langle(0.8, 0.5, 0.4), (0.2, 0.4, 0.3), (0.6, 0.3, 0.4)\rangle$	$\langle(0.5, 0.2, 0.3), (0.4, 0.1, 0.2), (0.2, 0.1, 0.4)\rangle$	$\langle(0.4, 0.3, 0.2), (0.2, 0.1, 0.6), (0.7, 0.3, 0.2)\rangle$	0.5
(e_1, q)	$\langle(0.7, 0.1, 0.4), (0.3, 0.2, 0.1), (0.4, 0.2, 0.5)\rangle$	$\langle(0.6, 0.5, 0.4), (0.4, 0.2, 0.1), (0.8, 0.2, 0.6)\rangle$	$\langle(0.9, 0.4, 0.5), (0.2, 0.1, 0.3), (0.6, 0.2, 0.3)\rangle$	0.7
(e_2, q)	$\langle(0.3, 0.1, 0.2), (0.4, 0.1, 0.3), (0.5, 0.2, 0.6)\rangle$	$\langle(0.7, 0.2, 0.4), (0.4, 0.3, 0.6), (0.5, 0.1, 0.6)\rangle$	$\langle(0.7, 0.3, 0.5), (0.2, 0.4, 0.3), (0.5, 0.2, 0.3)\rangle$	0.4
(e_3, q)	$\langle(0.6, 0.1, 0.4), (0.2, 0.1, 0.5), (0.4, 0.2, 0.3)\rangle$	$\langle(0.7, 0.2, 0.5), (0.4, 0.3, 0.2), (0.1, 0.2, 0.3)\rangle$	$\langle(0.5, 0.3, 0.4), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3)\rangle$	0.1

Table 4. Degree table of disagree-neutrosophic soft expert multiset.

	v_1	v_2	v_3
p	0.1631	0.1468	0.1386
q	0.1155	0.0933	0.04

Step 4-The final score of v_i is computed as follows:

$$\text{Score}(v_1) = 0.1142 - 0.1155 = -0.0013,$$
$$\text{Score}(v_2) = 0.1267 - 0.0933 = 0.0334,$$
$$\text{Score}(v_3) = 0.093 - 0.04 = 0.053.$$

Step 5-Clearly, the maximum score is the score 0.053, shown in the above for the v_3. Hence the best decision for the experts is to select worker v_2 as the company's employee.

8. Comparison Analysis

The NSEMs model give more precision, flexibility and compatibility compared to the classical, fuzzy and/or neutrosophic models.

In order to verify the feasibility and effectiveness of the proposed decision-making approach, a comparison analysis using neutrosophic soft expert decision method, with those methods used by Alkhazaleh and Salleh [18], Maji [17], Sahin et al. [22], Hassan et al. [23] and Ulucay et al. [40] are given in Table 5, based on the same illustrative example as in An Application of NSEMs. Clearly, the ranking order results are consistent with those in [17,18,22,23,40].

Table 5. Comparison of fuzzy soft set and its extensive set theory.

	Fuzzy Soft Expert	**Neutrosophic Soft Set**	**Neutrosophic Soft Expert**	**Q-Neutrosophic Soft Expert**	**Generalized Neutrosophic Soft Expert**	**NSEMs**
Methods	Alkhazaleh and Salleh [22]	Maji [21]	Sahin et al. [26]	Hassan et al. [27]	Ulucay et al. [48]	Proposed Method in this paper
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse
True	Yes	Yes	Yes	Yes	Yes	Yes
Falsity	No	Yes	Yes	Yes	No	No
Indeterminacy	No	Yes	Yes	Yes	No	No
Expert	Yes	No	Yes	Yes	Yes	No
Q	No	No	No	Yes	Yes	Yes
Ranking	$v_1 > v_3 > v_2$	$v_1 > v_3 > v_2$	$v_1 > v_2 > v_3$	$v_1 > v_3 > v_2$	$v_1 > v_3 > v_2$	$v_3 > v_2 > v_1$
Membershipvalued	Membership-valued	Single-valued	single-valued	Single-valued	Single-valued	Multi-valued

9. Conclusions

In this paper, we reviewed the basic concepts of neutrosophic set, neutrosophic soft set, soft expert sets, neutrosophic soft expert sets and NP-aggregation operator before establishing the concept of neutrosophic soft expert multiset (NSEM). The basic operations of NSEMs, namely complement, union, intersection AND and OR were defined. Subsequently a definition of NSEM-aggregation operator is proposed to construct an algorithm of a NSEM decision method. Finally an application of the constructed algorithm to solve a decision-making problem is provided. This new extension will provide a significant addition to existing theories for handling indeterminacy, and spurs more developments of further research and pertinent applications.

Author Contributions: All authors contributed equally.

Conflicts of Interest: The authors declare no conflict of interest.

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