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Neutrosophic Soft Sets Applied on Incomplete Data

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Abstract: A neutrosophic set is a part of neutrosophy that studies the origin, nature and scope of neutralities as well as their interactions with different ideational spectra. In this present paper first we have introduced the concept of a neutrosophic soft set having incomplete data with suitable examples. Then we have tried to explain the consistent and inconsistent association between the parameters. We have introduced few new definitions, namely- consistent association number between the parameters, consistent association degree, inconsistent association number between the parameters and inconsistent association degree to measure these associations. Lastly we have presented a data filling algorithm. An illustrative example is employed to show the feasibility and validity of our algorithm in practical situation.

Keywords: Soft set, neutrosophic set, neutrosophic soft set, data filling.

1. Introduction

In 1999, Molodstov **[01]** initiated the concept of soft set theory as a new mathematical tool for modelling uncertainty, vague concepts and not clearly defined objects. Although various traditional tools, including but not limited to rough set theory **[02]**, fuzzy set theory **[03]**, intuitionistic fuzzy set theory **[04]** etc. have been used by many researchers to extract useful information hidden in the uncertain data, but there are immanent complications connected with each of these theories. Additionally, all these approaches lack in parameterizations of the tools and hence they couldn't be applied effectively in real life problems, especially in areas like environmental, economic and social problems. Soft set theory is standing uniquely in the sense that it is free from the above mentioned impediments and obliges approximate illustration of an object from the beginning, which makes this theory a natural mathematical formalism for approximate reasoning.

The Theory of soft set has excellent potential for application in various directions some of which are reported by Molodtsov in his pioneer work. Later on Maji et al. **[05]** introduced some new annotations on soft sets such as subset, complement, union and intersection of soft sets and discussed in detail its applications in decision making problems. Ali et al. **[06]** defined some new operations on soft sets and shown that De Morgan's laws holds in soft set theory with respect to these newly defined operations. Atkas and Cagman **[07]** compared soft sets with fuzzy sets and rough sets to show that every fuzzy set and every rough set may be considered as a soft set. Jun **[08]** connected soft sets to the theory of BCK/BCI-algebra and introduced the concept of soft BCK/BCI-algebras. Feng et al. **[09]** characterized soft semi rings and a few related notions to establish a relation between soft sets and semi rings. In 2001, Maji et al. **[10]** defined the concept of fuzzy soft set by combining of fuzzy sets and soft sets . Roy and Maji **[11]** proposed a fuzzy soft set based decision making method. Xiao et al. **[12]** presented a combined forecasting method based on fuzzy soft set. Feng et al. **[13]** discussed the validity of the

Roy-Maji method and presented an adjustable decision-making method based on fuzzy soft set. Yang et al. **[14]** initiated the idea of interval valued fuzzy soft set (IVFS-set) and analyzed a decision making method using the IVFS-sets. The notion of intuitionistic fuzzy set (IFS) was initiated by Atanassov as a significant generalization of fuzzy set. Intuitionistic fuzzy sets are very useful in situations when description of a problem by a linguistic variable, given in terms of a membership function only, seems too complicated. Recently intuitionistic fuzzy sets have been applied to many fields such as logic programming, medical diagnosis, decision making problems etc. Smarandache **[15]** introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Thao and Smaran **[16]** proposed the concept of divergence measure on neutrosophic sets with an application to medical problem. Song et al. **[17]** applied neutrosophic sets to ideals in BCK/BCI algebras. Some recent applications of neutrosophic sets can be found in **[18]**, **[19]**, **[20]**, **[21]**, **[22]**, **[23]** and **[24]**. Maji **[25]** introduced the concept of neutrosophic soft set and established some operations on these sets. Mukherjee et al **[26]** introduced the concept of interval valued neutrosophic soft sets and studied their basic properties. In 2013, Broumi and Smarandache **[27, 28]** combined the intuitionistic neutrosophic and soft set which lead to a new mathematical model called "intuitionistic neutrosophic soft set". They studied the notions of intuitionistic neutrosophic soft set union, intuitionistic neutrosophic soft set intersection, complement of intuitionistic neutrosophic soft set and several other properties of intuitionistic neutrosophic soft set along with examples and proofs of certain results. Also, in **[29]** S. Broumi presented the concept of "generalized neutrosophic soft set" by combining the generalized neutrosophic sets and soft set models, studied some properties on it, and presented an application of generalized neutrosophic soft set in decision making problem. Recently, Deli **[30]** introduced the concept of interval valued neutrosophic soft set as a combination of interval neutrosophic set and soft set. In 2014, S. Broumi et al. **[31]** initiated the concept of relations on interval valued neutrosophic soft sets.

The soft sets mentioned above are based on complete information. However, incomplete information widely exists in various real life problems. Soft sets under incomplete information become incomplete soft sets. H. Qin et al **[32]** studied the data filling approach of incomplete soft sets. Y. Zou et al **[33]** investigated data analysis approaches of soft sets under incomplete information. In this paper first we have introduced the concept of a neutrosophic soft set with incomplete data supported by examples. Then we have introduced few new definitions to measure the consistent and inconsistent association between the parameters. Lastly we have presented a data filling algorithm supported by an illustrative example to show the feasibility and validity of our algorithm.

2. Preliminaries:

2.1 Definition: **[03]** Let U be a non empty set. Then a fuzzy set τ on U is a set having the form $\tau=\left\{\left(x,\ \mu_\tau\left(x\right)\right): x\in U\right\}$ where the function $\mu_\tau : U\to[0,\ 1]$ is called the membership function and $\mu_\tau\left(x\right)$ represents the degree of membership of each element $x\in U$.

2.2 Definition: **[04]** Let U be a non empty set. Then an intuitionistic fuzzy set (IFS for short) τ is an object having the form $\tau=\left\{\left\langle x,\ \mu_\tau\left(x\right),\ \gamma_\tau\left(x\right)\right\rangle:\ x\in U\right\}$ where the functions $\mu_\tau : U\to[0,\ 1]$ and $\gamma_\tau : U\to[0,\ 1]$ are called membership function and non-membership function respectively.

$\mu_\tau\left(x\right)$ and $\gamma_\tau\left(x\right)$ represent the degree of membership and the degree of non-membership respectively of each element $x\in U$ and $0\le\mu_\tau\left(x\right)+\gamma_\tau\left(x\right)\le 1$ for each $x\in U$. We denote the class of all intuitionistic fuzzy sets on U by IFS^U.

2.3 Definition: **[01]** Let U be a universe set and E be a set of parameters. Let $\mathrm{P}(\mathrm{U})$ denotes the power set of U and $\mathrm{A} \subseteq \mathrm{E}$. Then the pair (F, A) is called a soft set over U, where F is a mapping given by $\mathrm{F}: \mathrm{A} \rightarrow \mathrm{P}(\mathrm{U})$.

In other words, the soft set is not a kind of set, but a parameterized family of subsets of U. For $\mathrm{e} \in \mathrm{A}$, $F(\mathrm{e}) \subseteq \mathrm{U}$ may be considered as the set of e-approximate elements of the soft set (F, A).

2.4 Definition: **[10]** Let U be a universe set, E be a set of parameters and $\mathrm{A} \subseteq \mathrm{E}$. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $\mathrm{F}: \mathrm{A} \rightarrow \mathrm{FS}^{\mathrm{U}}$.

2.5 Definition: **[34]** Let U be a universe set, E be a set of parameters and $\mathrm{A} \subseteq \mathrm{E}$. Then the pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $\mathrm{F}: \mathrm{A} \rightarrow \mathrm{IFS}^{\mathrm{U}}$.

For $\mathrm{e} \in \mathrm{A}$, $F(\mathrm{e})$ is an intuitionistic fuzzy subset of U and is called the intuitionistic fuzzy value set of the parameter 'e'.

Let us denote $\mu_{F(\mathrm{e})}(\mathrm{x})$ by the membership degree that object 'x' holds parameter 'e' and $\gamma_{F(\mathrm{e})}(\mathrm{x})$ by the membership degree that object 'x' doesn't hold parameter 'e', where $\mathrm{e} \in \mathrm{A}$ and $\mathrm{x} \in \mathrm{U}$. Then $F(\mathrm{e})$ can be written as an intuitionistic fuzzy set such that $F(\mathrm{e}) = \left\{\left(\mathrm{x}, \mu_{F(\mathrm{e})}(\mathrm{x}), \gamma_{F(\mathrm{e})}(\mathrm{x})\right): \mathrm{x} \in \mathrm{U}\right\}$.

2.6 Definition: **[15]** A neutrosophic set A on the universe of discourse U is defined as $A = \left\{\left\langle x, \mu_A(x), \gamma_A(x), \delta_A(x)\right\rangle : x \in U\right\}$, where $\mu_A, \gamma_A, \delta_A : U \rightarrow]^{-}0, 1^{+}[$ are functions such that the condition: $\forall x \in U, {}^{-}0 \leq \mu_A(x) + \gamma_A(x) + \delta_A(x) \leq 3^{+}$ is satisfied.

Here $\mu_A(x), \gamma_A(x), \delta_A(x)$ represent the truth-membership, indeterminacy-membership and falsity-membership respectively of the element $x \in U$.

Smarandache **[15]** applied neutrosophic sets in many directions after giving examples of neutrosophic sets. Then he introduced the neutrosophic set operations namely-complement, union, intersection, difference, Cartesian product etc.

2.7 Definition: **[21]** Let U be an initial universe, E be a set of parameters and $A \subseteq E$. Let $NP(U)$ denotes the set of all neutrosophic sets of U. Then the pair (f, A) is termed to be the neutrosophic soft set over U, where f is a mapping given by $f : A \rightarrow NP(U)$.

2.8 Example: Let us consider a neutrosophic soft set (f, A) which describes the "attractiveness of the house". Suppose $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be the set of six houses under consideration and $E = \{e_1 \text{(beautiful)}, e_2 \text{(expensive)}, e_3 \text{(cheap)}, e_4 \text{(good location)}, e_5 \text{(wooden)}\}$ be the set of parameters. Then a neutrosophic soft set (f, A) over U can be given by:

U	e_1	e_2	e_3	e_4	e_5
u_1	(0.8,0.5,0.2)	(0.3,0.4,0.6)	(0.1,0.6,0.4)	(0.7,0.3,0.6)	(0.3,0.4,0.6)
u_2	(0.4,0.1,0.7)	(0.8,0.2,0.4)	(0.4,0.1,0.7)	(0.2,0.4,0.4)	(0.1,0.1,0.3)
u_3	(0.2,0.6,0.4)	(0.5,0.5,0.5)	(0.8,0.1,0.7)	(0.5,0.3,0.5)	(0.5,0.5,0.5)
u_4	(0.3,0.4,0.4)	(0.1,0.3,0.3)	(0.3,0.4,0.4)	(0.6,0.6,0.6)	(0.1,0.1,0.5)

u_5	(0.1,0.1,0.7)	(0.2,0.6,0.7)	(0.4,0.2,0.1)	(0.8,0.6,0.1)	(0.6,0.7,0.7)
u_6	(0.5,0.3,0.9)	(0.3,0.6,0.6)	(0.1,0.5,0.5)	(0.3,0.6,0.5)	(0.4,0.4,0.4)

3. Neutrosophic soft sets with incomplete (missing) data:

Suppose that (f,E) is a neutrosophic soft set over U, such that $\$ x_i \hat{I} U$ and $e_j \hat{I} E$ so that none of $m_{f(e_j)}(x_i), g_{f(e_j)}(x_i)$ and $d_{f(e_j)}(x_i)$ is known. In this case, in the tabular representation of the neutrosophic soft set (f,E), we write $\left(m_{f(e_j)}(x_i), g_{f(e_j)}(x_i), d_{f(e_j)}(x_i)\right) = *$. Here we say that the data for $f(e_j)$ is missing and the neutrosophic soft set (f,E) over U has incomplete data.

3.1 Example: Suppose Tech Mahindra is recruiting some new Graduate Trainee for the session 2019-2020 and suppose that eight candidates have applied for the job. Assume that $U = \{u_1, u_2, u_3, ..., u_8\}$ be the set of candidates and $E = \{e_1(\text{communication skill}), e_2(\text{domain knowledge}), e_3(\text{experienced}), e_4(\text{young}), e_5(\text{highest academic degree}), e_6(\text{professional attitute})\}$ be the set of parameters. Then a neutrosophic soft set over U having missing data can be given by Table-1.

Table-1

U	e_1	e_2	e_3	e_4	e_5	e_6
u_1	(0.8,0.5,0.2)	(0.3,0.4,0.6)	(0.1,0.6,0.4)	(0.7,0.3,0.6)	(0.3,0.4,0.6)	(0.2,0.5,0.5)
u_2	(0.4,0.1,0.7)	(0.8,0.2,0.4)	(0.4,0.1,0.7)	(0.2,0.4,0.4)	*	(0.6,0.6,0.4)
u_3	(0.2,0.6,0.4)	(0.5,0.5,0.5)	*	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.3,0.4,0.6)
u_4	(0.3,0.4,0.4)	(0.1,0.3,0.3)	(0.3,0.4,0.4)	(0.6,0.6,0.6)	(0.1,0.1,0.5)	(0.3,0.4,0.4)
u_5	(0.1,0.1,0.7)	*	(0.4,0.2,0.1)	(0.8,0.6,0.1)	(0.6,0.7,0.7)	(0.3,0.4,0.3)
u_6	(0.5,0.3,0.9)	(0.3,0.6,0.6)	(0.1,0.5,0.5)	(0.3,0.6,0.6)	(0.4,0.4,0.4)	(0.3,0.6,0.6)
u_7	(0.2,0.4,0.6)	(0.4,0.4,0.5)	(0.5,0.5,0.6)	*	(0.7,0.5,0.8)	(0.4,0.4,0.5)
u_8	(0.2,0.3,0.1)	(0.6,0.6,0.1)	(0.8,0.3,0.8)	(0.4,0.3,0.4)	(0.5,0.6,0.3)	(0.9,0.3,0.3)

In case of soft set theory, there always exist some obvious or hidden associations between parameters. Let us focus on this to find the associations between the parameters of a neutrosophic soft set.

In example **2.8**, one can easily find that if a house is expensive, the house is not cheap and vice versa. Thus there is an inconsistent association between the parameters 'expensive' and 'cheap'. Generally, if a house is beautiful or situated in a good location, the house is expensive. Thus there is a consistent association between the parameters 'beautiful' and 'expensive' or the parameters 'good location' and 'expensive'.

In example **3.1**, we find that if a candidate is experienced or have highest academic degree, he/she is not young. Thus there is an inconsistent association between parameters 'experienced' and 'young' or between 'highest academic degree' and 'young'.

The above two examples reveal the interior relations of parameters. In a neutrosophic soft set, these associations between parameters will be very useful for filling incomplete data. If it is found that the parameters e_i and e_j are associated and the data for $f(e_i)$ is missing, then we can fill the missing data according to the corresponding data in $f(e_j)$. To measure these associations, let us define the notion of association degree and some relevant concepts.

For the rest of the paper we shall assume that *U* be the universe set and *E* be the set of parameters.

Let U_{ij} denotes the set of objects that have specified values in the form of an ordered triplet (*a*, *b*, *c*) where *a*, *b*, *c*∈[0, 1] on both parameters e_i and e_j such that

$$U_{ij} = \left\{ x \in U : \left(m_{f(e_i)}(x), g_{f(e_i)}(x), d_{f(e_i)}(x)\right) \neq *, \left(m_{f(e_j)}(x), g_{f(e_j)}(x), d_{f(e_j)}(x)\right) \neq * \right\}$$

In other words U_{ij} is the collection of those objects that have known data both on e_i and e_j .

3.2 Definition: Let $e_i, e_j \in E$. Then the consistent association number between the parameters e_i and e_j is denoted by CAN_{ij} and is defined as:

$$CAN_{ij} = \left| \left\{ x \in U_{ij} : m_{f(e_i)}(x) = m_{f(e_j)}(x), g_{f(e_i)}(x) = g_{f(e_j)}(x), d_{f(e_i)}(x) = d_{f(e_j)}(x) \right\} \right|$$

where $|.|$ denotes the cardinality of a set.

3.3 Definition: Let $e_i, e_j \in E$. Then the consistent association degree between the parameters e_i and e_j is denoted by CAD_{ij} and is defined as: $CAD_{ij} = \dfrac{CAN_{ij}}{|U_{ij}|}$ where $|.|$ denotes the cardinality of a set.

It can be easily verified that the value of CAD_{ij} lies in [0, 1]. Actually CAD_{ij} measures the extent to which the value of parameter e_i keeps consistent with that of parameter e_j over U_{ij} . Next we define inconsistent association number and inconsistent association degree as follows:

3.4 Definition: Let $e_i, e_j \in E$. Then the inconsistent association number between the parameters e_i and e_j is denoted by $ICAN_{ij}$ and is defined as

$$ICAN_{ij} = \left| \left\{ x \in U_{ij} : m_{f(e_i)}(x) \neq m_{f(e_j)}(x) \text{ or } g_{f(e_i)}(x) \neq g_{f(e_j)}(x) \text{ or } d_{f(e_i)}(x) \neq d_{f(e_j)}(x) \right\} \right|$$

where $|.|$ denotes the cardinality of a set.

3.5 Definition: Let $e_i, e_j \in E$. Then the inconsistent association degree between the parameters e_i and e_j is denoted by $ICAD_{ij}$ and is defined as: $ICAD_{ij} = \dfrac{ICAN_{ij}}{|U_{ij}|}$ where $|.|$ denotes the cardinality of a set.

It can be easily verified that the value of $ICAD_{ij}$ lies in [0, 1]. Actually $ICAD_{ij}$ measures the extent to which the parameters e_i and e_j is inconsistent.

3.6 Definition: Let $e_i, e_j \in E$. Then the association degree between the parameters e_i and e_j is denoted by AD_{ij} and is defined by $AD_{ij} = \max\left\{CAD_{ij}, ICAD_{ij}\right\}$.

If $CAD_{ij} > ICAD_{ij}$, then $AD_{ij} = CAD_{ij}$, which means that most of the objects over U_{ij} have consistent values on parameters e_i and e_j. If $CAD_{ij} < ICAD_{ij}$, then $AD_{ij} = ICAD_{ij}$, which means that most of the objects over U_{ij} have inconsistent values on parameters e_i and e_j. Again if $CAD_{ij} = ICAD_{ij}$, then it means that there is the lowest association degree between the parameters e_i and e_j.

3.7 Theorem: For parameters e_i and e_j, $AD_{ij} \geq 0.5$ for all *i, j*.

Proof: Follows from the fact that $CAD_{ij} + ICAD_{ij} = 1$.

3.8 Definition: If $e_i \in E$, then the maximal association degree of parameter e_i is denoted by MAD_i and is defined by $MAD_i = \max_j AD_{ij}$.

4. DATA Filling Algorithm for a neutrosophic soft set:

<u>**Step-1:**</u> Input the neutrosophic soft set (f, E) which has incomplete data.

<u>**Step-2:**</u> Find all parameters e_i for which data is missing.

<u>**Step-3:**</u> Compute AD_{ij} for *j*=1,2,3….,*m* (where '*m*' is the number of parameters in *E*).

<u>**Step-4:**</u> Compute MAD_i.

<u>**Step-5:**</u> Find out all parameters e_j which have the maximal association degree MAD_i with the parameter e_i.

<u>**Step-6:**</u> In case of consistent association between the parameter e_i and e_j 's (*j*=1,2,3,….) $\left(m_{f(e_i)}(x), g_{f(e_i)}(x), d_{f(e_i)}(x)\right) = \left(\max_j m_{f(e_j)}(x), \max_j g_{f(e_j)}(x), \max_j d_{f(e_j)}(x)\right)$. In case of inconsistent association between the parameter e_i and e_j 's (*j*=1,2,3,….) $\left(m_{f(e_i)}(x), g_{f(e_i)}(x), d_{f(e_i)}(x)\right) = \left(1 - \max_j m_{f(e_j)}(x), 1 - \max_j g_{f(e_j)}(x), 1 - \max_j d_{f(e_j)}(x)\right)$.

<u>**Step-7:**</u> If all the missing data are filled then stop else go to step-2.

- **An Illustrative example:** Consider the neutrosophic soft set given in example 3.1.

<u>**Step-1:**</u>

U	e_1	e_2	e_3	e_4	e_5	e_6
u_1	(0.8,0.5,0.2)	(0.3,0.4,0.6)	(0.1,0.6,0.4)	(0.7,0.3,0.6)	(0.3,0.4,0.6)	(0.2,0.5,0.5)
u_2	(0.4,0.1,0.7)	(0.8,0.2,0.4)	(0.4,0.1,0.7)	(0.2,0.4,0.4)	*	(0.6,0.6,0.4)
u_3	(0.2,0.6,0.4)	(0.5,0.5,0.5)	*	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.3,0.4,0.6)
u_4	(0.3,0.4,0.4)	(0.1,0.3,0.3)	(0.3,0.4,0.4)	(0.6,0.6,0.6)	(0.1,0.1,0.5)	(0.3,0.4,0.4)
u_5	(0.1,0.1,0.7)	*	(0.4,0.2,0.1)	(0.8,0.6,0.1)	(0.6,0.7,0.7)	(0.3,0.4,0.3)
u_6	(0.5,0.3,0.9)	(0.3,0.6,0.6)	(0.1,0.5,0.5)	(0.3,0.6,0.6)	(0.4,0.4,0.4)	(0.3,0.6,0.6)
u_7	(0.2,0.4,0.6)	(0.4,0.4,0.5)	(0.5,0.5,0.6)	*	(0.7,0.5,0.8)	(0.4,0.4,0.5)
u_8	(0.2,0.3,0.1)	(0.6,0.6,0.1)	(0.8,0.3,0.8)	(0.4,0.3,0.4)	(0.5,0.6,0.3)	(0.9,0.3,0.3)

Step-2: Clearly there are missing data in $f(e_2), f(e_3), f(e_4), f(e_5)$. We shall fill these missing data.

Step-3:

(a) **For the parameter** e_2 **.**

$\backslash\ U_{21} = \{u_1,u_2,u_3,u_4,u_6,u_7,u_8\}, U_{23} = \{u_1,u_2,u_4,u_6,u_7,u_8\}, U_{24} = \{u_1,u_2,u_3,u_4,u_6,u_8\},$
$U_{25} = \{u_1,u_3,u_4,u_6,u_7,u_8\}, U_{26} = \{u_1,u_2,u_3,u_4,u_6,u_7,u_8\}.$

Now $CAN_{21} = |\{\ \}| = 0$ and so $CAD_{21} = 0$. Again $ICAN_{21} = |\{u_1,u_2,u_3,u_4,u_6,u_7,u_8\}| = 7$ and so $ICAD_{21} = \frac{ICAN_{21}}{|U_{21}|} = \frac{7}{7} = 1$. Hence $AD_{21} = \max\{CAD_{21}, ICAD_{21}\} = \max\{0,1\} = 1.$

$CAN_{23} = |\{\ \}| = 0$ and so $CAD_{23} = 0$. Again $ICAN_{23} = |\{u_1,u_2,u_4,u_6,u_7,u_8\}| = 6$ and so $ICAD_{23} = \frac{ICAN_{23}}{|U_{23}|} = \frac{6}{6} = 1$. Hence $AD_{23} = \max\{CAD_{23}, ICAD_{23}\} = \max\{0,1\} = 1.$

$CAN_{24} = |\{u_3,u_6\}| = 2$ and so $CAD_{24} = \frac{2}{6} = 0.33$. Again $ICAN_{24} = |\{u_1,u_2,u_4,u_8\}| = 4$ and so $ICAD_{24} = \frac{ICAN_{24}}{|U_{24}|} = \frac{4}{6} = 0.66$. Hence $AD_{24} = \max\{CAD_{24}, ICAD_{24}\} = \max\{0.33, 0.66\} = 0.66.$

$CAN_{25} = |\{u_3,u_1\}| = 2$ and so $CAD_{25} = \frac{2}{6} = 0.33$. Again $ICAN_{25} = |\{u_4,u_6,u_7,u_8\}| = 4$ and so $ICAD_{25} = \frac{ICAN_{24}}{|U_{24}|} = \frac{4}{6} = 0.66$. Hence $AD_{25} = \max\{CAD_{25}, ICAD_{25}\} = \max\{0.33, 0.66\} = 0.66.$

$CAN_{26} = |\{u_4\}| = 1$ and so $CAD_{26} = \frac{1}{7} = 0.14$. Again $ICAN_{26} = |\{u_1,u_2,u_3,u_6,u_7,u_8\}| = 6$ and so $ICAD_{26} = \frac{ICAN_{26}}{|U_{26}|} = \frac{6}{7} = 0.85$. Hence $AD_{26} = \max\{CAD_{26}, ICAD_{26}\} = \max\{0.14, 0.85\} = 0.85.$

Thus $MAD_2 = \max_j AD_{2j} = \max\{AD_{21}, AD_{23}, AD_{24}, AD_{25}, AD_{26}\} = \max\{1,1,0.66,0.66,0.85\} = 1.$.

(b) **For the parameter** e_3 **.**

$\backslash\ U_{31} = \{u_1,u_2,u_4,u_6,u_7,u_8\}, U_{32} = \{u_1,u_2,u_4,u_6,u_7,u_8\}, U_{34} = \{u_1,u_2,u_4,u_5,u_6,u_8\},$
$U_{35} = \{u_1,u_4,u_5,u_6,u_7,u_8\}, U_{36} = \{u_1,u_2,u_4,u_5,u_6,u_7,u_8\}.$

Now $CAN_{31} = |\{u_2,u_4\}| = 2$ and so $CAD_{31} = \frac{2}{6} = 0.33$. Again $ICAN_{31} = |\{u_1,u_6,u_7,u_8\}| = 4$ and so $ICAD_{31} = \frac{ICAN_{31}}{|U_{31}|} = \frac{4}{6} = 0.66$. Hence $AD_{31} = \max\{CAD_{31}, ICAD_{31}\} = \max\{0.33, 0.66\} = 0.66.$

$CAN_{32} = |\{\ \}| = 0$ and so $CAD_{32} = 0$. Again $ICAN_{32} = |\{u_1,u_2,u_4,u_6,u_7,u_8\}| = 6$ and so $ICAD_{32} = \frac{ICAN_{32}}{|U_{32}|} = \frac{6}{6} = 1$. Hence $AD_{32} = \max\{CAD_{32}, ICAD_{32}\} = \max\{0,1\} = 1.$

$CAN_{34} = |\{\ \}| = 0$ and so $CAD_{34} = 0$. Again $ICAN_{34} = |\{u_1,u_2,u_4,u_5,u_6,u_8\}| = 6$ and so $ICAD_{34} = \frac{ICAN_{34}}{|U_{34}|} = \frac{4}{6} = 0.66$. Hence $AD_{34} = \max\{CAD_{34}, ICAD_{34}\} = \max\{0, 0.66\} = 0.66.$

$CAN_{35} = |\{\ \}| = 0$ and so $CAD_{35} = 0$. Again $ICAN_{35} = |\{u_1, u_4, u_5, u_6, u_7, u_8\}| = 6$ and so $ICAD_{35} = \frac{ICAN_{35}}{|U_{35}|} = \frac{6}{6} = 1$. Hence $AD_{35} = \max\{CAD_{35}, ICAD_{35}\} = \max\{0,1\} = 1$.

$CAN_{36} = |\{u_4\}| = 1$ and so $CAD_{36} = \frac{1}{7} = 0.14$. Again $ICAN_{36} = |\{u_1, u_2, u_5, u_6, u_7, u_8\}| = 6$ and so $ICAD_{36} = \frac{ICAN_{36}}{|U_{36}|} = \frac{6}{7} = 0.85$. Hence $AD_{36} = \max\{CAD_{36}, ICAD_{36}\} = \max\{0.14, 0.85\} = 0.85$.

Thus $MAD_3 = \max_j AD_{3j} = \max\{AD_{31}, AD_{32}, AD_{34}, AD_{35}, AD_{36}\} = \max\{0.66, 1, 0.66, 1, 0.85\} = 1$.

(c) **For the parameter** e_4 **.**

\ $U_{41} = \{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}, U_{42} = \{u_1, u_2, u_3, u_4, u_6, u_8\}, U_{43} = \{u_1, u_2, u_4, u_5, u_6, u_8\}$,
$U_{45} = \{u_1, u_3, u_4, u_5, u_6, u_8\}, U_{46} = \{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}$.

Now $CAN_{41} = |\{\ \}| = 0$ and so $CAD_{41} = 0$. Again $ICAN_{41} = |\{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}| = 7$ and so $ICAD_{41} = \frac{ICAN_{41}}{|U_{41}|} = \frac{7}{7} = 1$. Hence $AD_{41} = \max\{CAD_{41}, ICAD_{41}\} = \max\{0,1\} = 1$.

$CAN_{42} = |\{u_3, u_6\}| = 2$ and so $CAD_{42} = \frac{2}{6} = 0.33$. Again $ICAN_{42} = |\{u_1, u_2, u_4, u_8\}| = 4$ and so $ICAD_{42} = \frac{ICAN_{42}}{|U_{42}|} = \frac{4}{6} = 0.66$. Hence $AD_{42} = \max\{CAD_{42}, ICAD_{42}\} = \max\{0.33, 0.66\} = 0.66$.

$CAN_{43} = |\{\ \}| = 0$ and so $CAD_{43} = 0$. Again $ICAN_{43} = |\{u_1, u_2, u_4, u_5, u_6, u_8\}| = 6$ and so $ICAD_{43} = \frac{ICAN_{43}}{|U_{43}|} = \frac{6}{6} = 1$. Hence $AD_{43} = \max\{CAD_{43}, ICAD_{43}\} = \max\{0,1\} = 1$.

$CAN_{45} = |\{u_3\}| = 1$ and so $CAD_{45} = \frac{1}{6} = 0.16$. Again $ICAN_{45} = |\{u_1, u_4, u_5, u_6, u_8\}| = 5$ and so $ICAD_{45} = \frac{ICAN_{45}}{|U_{45}|} = \frac{5}{6} = 0.83$. Hence $AD_{45} = \max\{CAD_{45}, ICAD_{35}\} = \max\{0.16, 0.83\} = 0.83$.

$CAN_{46} = |\{u_6\}| = 1$ and so $CAD_{46} = \frac{1}{7} = 0.14$. Again $ICAN_{46} = |\{u_1, u_2, u_3, u_4, u_5, u_8\}| = 6$ and so $ICAD_{46} = \frac{ICAN_{46}}{|U_{46}|} = \frac{6}{7} = 0.85$. Hence $AD_{46} = \max\{CAD_{46}, ICAD_{46}\} = \max\{0.14, 0.85\} = 0.85$.

Thus $MAD_4 = \max_j AD_{4j} = \max\{AD_{41}, AD_{42}, AD_{43}, AD_{45}, AD_{46}\} = \max\{1, 0.66, 1, 0.83, 0.85\} = 1$.

(d) **For the parameter** e_5 **.**

\ $U_{51} = \{u_1, u_3, u_4, u_5, u_6, u_7, u_8\}, U_{52} = \{u_1, u_3, u_4, u_6, u_7, u_8\}, U_{53} = \{u_1, u_4, u_5, u_6, u_7, u_8\}$,
$U_{54} = \{u_1, u_3, u_4, u_5, u_6, u_8\}, U_{56} = \{u_1, u_3, u_4, u_5, u_6, u_7, u_8\}$.

Now $CAN_{51} = |\{\ \}| = 0$ and so $CAD_{51} = 0$. Again $ICAN_{51} = |\{u_1, u_3, u_4, u_5, u_6, u_7, u_8\}| = 7$ and so $ICAD_{51} = \frac{ICAN_{51}}{|U_{51}|} = \frac{7}{7} = 1$. Hence $AD_{51} = \max\{CAD_{51}, ICAD_{51}\} = \max\{0,1\} = 1$.

$CAN_{52} = |\{u_1, u_3\}| = 2$ and so $CAD_{52} = \frac{2}{6} = 0.33$. Again $ICAN_{52} = |\{u_4, u_6, u_7, u_8\}| = 4$ and so $ICAD_{52} = \frac{ICAN_{52}}{|U_{52}|} = \frac{4}{6} = 0.66$. Hence $AD_{52} = \max\{CAD_{52}, ICAD_{52}\} = \max\{0.33, 0.66\} = 0.66.$

$CAN_{53} = |\{\ \}| = 0$ and so $CAD_{53} = 0$. Again $ICAN_{53} = |\{u_1, u_4, u_5, u_6, u_7, u_8\}| = 6$ and so $ICAD_{53} = \frac{ICAN_{53}}{|U_{53}|} = \frac{6}{6} = 1$. Hence $AD_{53} = \max\{CAD_{53}, ICAD_{53}\} = \max\{0,1\} = 1.$

$CAN_{54} = |\{u_3\}| = 1$ and so $CAD_{54} = \frac{1}{6} = 0.16$. Again $ICAN_{54} = |\{u_1, u_4, u_5, u_6, u_8\}| = 5$ and so $ICAD_{54} = \frac{ICAN_{54}}{|U_{54}|} = \frac{5}{6} = 0.83$. Hence $AD_{54} = \max\{CAD_{54}, ICAD_{54}\} = \max\{0.16, 0.83\} = 0.83.$

$CAN_{56} = |\{\ \}| = 0$ and so $CAD_{56} = 0$. Again $ICAN_{56} = |\{u_1, u_3, u_4, u_5, u_6, u_7, u_8\}| = 7$ and so $ICAD_{56} = \frac{ICAN_{56}}{|U_{56}|} = \frac{7}{7} = 1$. Hence $AD_{56} = \max\{CAD_{56}, ICAD_{56}\} = \max\{0,1\} = 1.$

Thus $MAD_5 = \max_j AD_{5j} = \max\{AD_{51}, AD_{52}, AD_{53}, AD_{54}, AD_{56}\} = \max\{1, 0.66, 1, 0.83, 1\} = 1.$

The association degree table for the neutrosophic soft set (f, E) is given below:

	e_1	e_2	e_3	e_4	e_5	e_6
e_2	1	_	1	0.66	0.66	0.85
e_3	0.66	1	_	0.66	1	0.85
e_4	1	0.66	1	_	0.83	0.85
e_5	1	0.66	1	0.83	_	1

Step-4: From step-3, we have, $MAD_2 = 1, MAD_3 = 1, MAD_4 = 1, MAD_5 = 1$.

Step-5: The parameters e_1 and e_3 have the maximal association degree AD_{21} and AD_{23} respectively with the parameter e_2.

The parameters e_2 and e_5 have the maximal association degree AD_{32} and AD_{35} respectively with the parameter e_3.

The parameters e_1 and e_3 have the maximal association degree AD_{41} and AD_{43} respectively with the parameter e_4.

The parameters e_1, e_3 and e_6 have the maximal association degree AD_{51}, AD_{53} and AD_{56} respectively with the parameter e_5.

Step-6: There is a consistent association between the parameters e_2 and e_1, e_2 and e_3, e_5 and e_1, e_3 and e_5; while there is an inconsistent association between the parameters e_4 and e_1, e_4 and e_3.So we have,

$$\left(m_{f(e_2)}(u_5), g_{f(e_2)}(u_5), d_{f(e_2)}(u_5)\right)$$
$$= \left(\max\left(m_{f(e_1)}(u_5), m_{f(e_3)}(u_5)\right), \max\left(g_{f(e_1)}(u_5), g_{f(e_3)}(u_5)\right), \max\left(d_{f(e_1)}(u_5), d_{f(e_3)}(u_5)\right)\right)$$
$$= (\max(0.1, 0.4), \max(0.1, 0.2), \max(0.7, 0.1)) = (0.4, 0.2, 0.7),$$
$$\left(m_{f(e_3)}(u_3), g_{f(e_3)}(u_3), d_{f(e_3)}(u_3)\right)$$
$$= \left(\max\left(m_{f(e_2)}(u_3), m_{f(e_5)}(u_3)\right), \max\left(g_{f(e_2)}(u_3), g_{f(e_5)}(u_3)\right), \max\left(d_{f(e_2)}(u_3), d_{f(e_5)}(u_3)\right)\right)$$
$$= (\max(0.5, 0.5), \max(0.5, 0.5), \max(0.5, 0.5)) = (0.5, 0.5, 0.5),$$
$$\left(m_{f(e_4)}(u_7), g_{f(e_4)}(u_7), d_{f(e_4)}(u_7)\right)$$
$$= \left(1- \max\left(m_{f(e_1)}(u_7), m_{f(e_3)}(u_7)\right), 1- \max\left(g_{f(e_1)}(u_7), g_{f(e_3)}(u_7)\right), 1- \max\left(d_{f(e_1)}(u_7), d_{f(e_3)}(u_7)\right)\right)$$
$$= (\max(0.2, 0.5), \max(0.4, 0.5), \max(0.6, 0.6)) = (0.5, 0.5, 0.6),$$
$$\left(m_{f(e_5)}(u_2), g_{f(e_5)}(u_2), d_{f(e_5)}(u_2)\right)$$
$$= \left(\max\left(m_{f(e_1)}(u_2), m_{f(e_3)}(u_2)\right), \max\left(g_{f(e_1)}(u_2), g_{f(e_3)}(u_2)\right), \max\left(d_{f(e_1)}(u_2), d_{f(e_3)}(u_2)\right)\right)$$
$$= (\max(0.4, 0.4), \max(0.1, 0.1), \max(0.7, 0.7)) = (0.4, 0.1, 0.7).$$

Thus we have the following table which gives the tabular representation of the filled neutrosophic soft set:

U	e_1	e_2	e_3	e_4	e_5	e_6
u_1	(0.8,0.5,0.2)	(0.3,0.4,0.6)	(0.1,0.6,0.4)	(0.7,0.3,0.6)	(0.3,0.4,0.6)	(0.2,0.5,0.5)
u_2	(0.4,0.1,0.7)	(0.8,0.2,0.4)	(0.4,0.1,0.7)	(0.2,0.4,0.4)	**(0.4,0.1,0.7)**	(0.6,0.6,0.4)
u_3	(0.2,0.6,0.4)	(0.5,0.5,0.5)	**(0.5,0.5,0.5)**	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.3,0.4,0.6)
u_4	(0.3,0.4,0.4)	(0.1,0.3,0.3)	(0.3,0.4,0.4)	(0.6,0.6,0.6)	(0.1,0.1,0.5)	(0.3,0.4,0.4)
u_5	(0.1,0.1,0.7)	**(0.4,0.2,0.7)**	(0.4,0.2,0.1)	(0.8,0.6,0.1)	(0.6,0.7,0.7)	(0.3,0.4,0.3)
u_6	(0.5,0.3,0.9)	(0.3,0.6,0.6)	(0.1,0.5,0.5)	(0.3,0.6,0.6)	(0.4,0.4,0.4)	(0.3,0.6,0.6)
u_7	(0.2,0.4,0.6)	(0.4,0.4,0.5)	(0.5,0.5,0.6)	**(0.5,0.5,0.6)**	(0.7,0.5,0.8)	(0.4,0.4,0.5)
u_8	(0.2,0.3,0.1)	(0.6,0.6,0.1)	(0.8,0.3,0.8)	(0.4,0.3,0.4)	(0.5,0.6,0.3)	(0.9,0.3,0.3)

Conclusion: Incomplete information or missing data in a neutrosophic soft set restricts the usage of the neutrosophic soft set. To make the neutrosophic soft set (with missing / incomplete data) more useful, in this paper, we have proposed a data filling approach, where missing data is filled in terms of the association degree between the parameters. We have validated the proposed algorithm by an example and drawn the conclusion that relation between parameters can be applied to fill the missing data.

References:

1. D. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications, 37 (1999), 19-31.
2. Z. Pawlak, Rough sets, International Journal of Computing and Information Sciences, 11 (1982), 341-356.
3. L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965), 338-353.

4. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87-96.
5. P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Computers and Mathematics with Applications , 45 (2003), 555-562.
6. M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications, 57 (2009), 1547-1553.
7. H. Aktas and N. Cagman, Soft sets and soft groups, Information Sciences, 177 (2007), 2726-2735.
8. Y.B. Jun; Soft BCK/BCI algebras; Computers and Mathematics with Applications; 56 (2008), 1408-1413.
9. F. Feng, Y.B. Jun and X.Zhao; Soft semi rings; Computers and Mathematics with Applications; 56 (2008), 2621-2628.
10. P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, The Journal Fuzzy Mathematics, 9 (2001), 589-602.
11. A.R. Roy and P.K. Maji; A fuzzy soft theoretic approach to decision making problems; Journal of Computational and Applied Mathematics; 203 (2007), 412-418.
12. Z. Xiao, K. Gong and Y. Zou; A combined forecasting approach based on fuzzy soft sets; Journal of Computational and Applied Mathematics; 228 (2009), 326-333.
13. F. Feng, Y.B. Jun, x. Liu and L.Li; An adjustable approach to fuzzy soft set based decision making; Journal of Computational and Applied Mathematics; 234 (2010), 10-20.
14. X.B. Yang, T.Y. Lin, J.Y. Yang, Y.Li and D.J. Yu; Combination of interval valued fuzzy set and soft set; Computers and Mathematics with Applications; 58 (2009), 521-527.
15. F. Smarandache, Neutrosophic set- a generalisation of the intuitionistic fuzzy sets, Int. J. Pure Appl. Math. 24 (2005) 287–297.
16. N. X. Thao and F. S Smarandache, Divergence measure of neutrosophic sets and applications, Neutrosophic sets and systems, 21 (2018), 142-152.
17. S. Z. Song, M. Khan, F. Smarandache and Y. B. Jun, Interval neutrosophic sets applied to ideals in BCK/BCI algebras, Neutrosophic sets and systems, 18 (2017), 16-26.
18. M. Abdel-Baset, V. Chang, and A. Gamal, Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry, 108 (2019), 210-220.
19. M. Abdel-Baset, V. Chang, A. Gamal and F. Smarandache, An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field. Computers in Industry,106(2019).,94-110.
20. M. Abdel-Basset, G. Manogaran, A. Gamal and F. Smarandache, A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. Journal of medical systems, 43(2) (2019), 38-54.
21. M. Abdel-Basset, M. Ali and A. Atef., Uncertainty assessments of linear time-cost tradeoffs using neutrosophic set. Computers & Industrial Engineering, 141 (2020): 106286.
22. Abdel-Basset, M., Mohamed, M., Elhoseny, M., Chiclana, F., & Zaied, A. E. N. H. (2019). Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. Artificial Intelligence in Medicine, 101, 101735.
23. Gafar, M. G., Elhoseny, M., & Gunasekaran, M. (2018). Modeling neutrosophic variables based on particle swarm optimization and information theory measures for forest fires. The Journal of Supercomputing, 1-18.
24. Abdel-Basset, M., El-hoseny, M., Gamal, A., & Smarandache, F. (2019). A novel model for evaluation Hospital medical care systems based on plithogenic sets. Artificial intelligence in medicine, 100, 101710.P.K. Maji, Neutrosophic soft set, Annals of Fuzzy Mathematics and Information, 5(1) (2013) 157-168.
25. A. Mukherjee, M. Dutta, A. Saha , Interval valued neutrosophic soft sets, The Journal of Fuzzy Mathematics, 23 (2015), 283-294.
26. S. Broumi and F. Smarandache, More on Intuitionistic Neutrosophic Soft Sets”, Computer Science and Information Technology, 1(4) (2013) 257-268.
27. S. Broumi and F. Smarandache, Intuitionistic Neutrosophic Soft Set, Journal of Information and Computing Science, 8(2) (2013) 130-140.
28. S. Broumi, “Generalized Neutrosophic Soft Set”, Int. Journal of Computer Science, Engineering and Information Tech., 3(2) (2013) 17-30.
29. I. Deli, Interval-valued neutrosophic soft sets and its decision making, http://arxiv.org/abs/1402.3130.

30. S. Broumi, I. Deli and F. Smarandache, Relations on interval valued neutrosophic soft sets, Journal of New Results in Science, 5 (2014), 1-20.
31. H. Qin, X. Ma, T. Herawan, J. M. Zain, Data Filling Approach of Soft Sets under Incomplete Information,Intelligent Information and Data Base Systems, 2011, 302-311.
32. Y. Zou, Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge based systems, 21 (2008), 941-945.
33. P. K. Maji, R. Biswas and A. R. Roy, Intuitionistic fuzzy soft sets, The Journal of Fuzzy Mathematics; 12 (2004), 677-692.

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