Neutrosophic Soft Topological K-Algebras

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Abstract: In this paper, we propose the notion of single-valued neutrosophic soft topological K-algebras. We discuss certain concepts, including interior, closure, C_5-connected, super connected, Compactness and Hausdorff in single-valued neutrosophic soft topological K-algebras. We illustrate these concepts with examples and investigate some of their related properties. We also study image and pre-image of single-valued neutrosophic soft topological K-algebras.

Keywords: K-algebras, Single-valued neutrosophic soft sets, Compactness, C_5-connectedness, Super connectedness, Hausdorff.

1 Introduction

A K-algebra $(G, \cdot, \odot, e)$ is a new class of logical algebra, introduced by Dar and Akram [1] in 2003. A K-algebra is constructed on a group $(G, \cdot, e)$ by adjoining an induced binary operation $\odot$ on G and attached to an abstract K-algebra $(G, \cdot, \odot, e)$. This system is, in general, non-commutative and non-associative with a right identity e. If the given group G is not an elementary abelian 2-group, then the K-algebra is proper . Therefore, a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$ is abelian and non-abelian, proper and improper purely depends upon the base group G. In 2004, a K-algebra renamed as $K(G)$-algebra due to its structural basis G and characterized by left and right mappings when the group G is abelian and non-abelian by Dar and Akram in [2, 3] . In 2007, Dar and Akram [4] investigated the K-homomorphisms of K-algebras.
Non-classical logic leads to classical logic due to various aspects of uncertainty. It has become a conventional tool for computer science and engineering to deal with fuzzy information and indeterminate data and executions. In our daily life, the most frequently encountered uncertainty is incomparability. Zadeh's fuzzy set theory [5] revolutionized the systems, accomplished with vagueness and uncertainty. A number of researchers extended the conception of Zadeh and presented different theories regarding uncertainty which includes intuitionistic fuzzy set theory, interval-valued intuitionistic fuzzy set theory [6] and so on. In addition, Smarandache [7] generalized intuitionistic fuzzy set by introducing the concept of neutrosophic set in 1998. It is such a branch of philosophy which studies the origin, nature, and scope of neutralities as well as their interactions with different ideational spectra. To have real life applications of neutrosophic sets such as in engineering and science, Wang et al. [8] introduced the single-valued neutrosophic set in 2010. In 1999, Molodtsov [9] introduced another mathematical approach to deal with ambiguous data, called soft set theory. Soft set theory gives a parameterized outlook to uncertainty. Maji [10] defined the notion of neutrosophic soft set by unifying

the fundamental theories of neutrosophic set and soft set to deal with inconsistent data in a much-unified mode. A large number of theories regarding uncertainty with their respective topological structures have been introduced. In 1968, Chang [11] introduced the concept of fuzzy topology. Chattopadhyay and Samanta [12], Pu and Liu [13] and Lowan [14] defined some certain notions related to fuzzy topology. Recently, Tahan et al. [15] presented the notion of topological hypergroupoids. Onasanya and Hoskova-Mayerova [16] discussed some topological and algebraic properties of $\alpha-$level subsets of fuzzy subsets. Coker [17] considered the notion of an intuitionistic fuzzy topology. Salama and Alblowi [18] studied the notion of neutrosophic topological spaces. In 2017, Bera and Mahapatra [19] described neutrosophic soft topological spaces. Akram and Dar [20, 21] considered fuzzy topological K-algebras and intuitionistic topological K-algebras. Recently, Akram et al. [22, 23, 24, 25] presented some notions, including single-valued neutrosophic K-algebras, single-valued neutrosophic topological K-algebras and single-valued neutrosophic Lie algebras. In this research article, In this paper, we propose the notion of single-valued neutrosophic soft topological K-algebras. We discuss certain concepts, including interior, closure, C_5-connected, super connected, Compactness and Hausdorff in single-valued neutrosophic soft topological K-algebras. We illustrate these concepts with examples and investigate some of their related properties. We also study image and pre-image of single-valued neutrosophic soft topological K-algebras.

The rest of the paper is organized as follows: In Section 2, we review some elementary concepts related to K-algebras, single-valued neutrosophic soft sets and their topological structures. In Section 3, we define the concept of single-valued neutrosophic soft topological K-algebras and discuss certain concepts with some numerical examples. In Section 4, we present concluding remarks.

2 Preliminaries

This section consists of some basic definitions and concepts, which will be used in the next sections.

Definition 2.1. [1] A K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$ is an algebra of the type (2, 2, 0) defined on the group $(G, \cdot, e)$ in which each non-identity element is not of order 2 with the following $\odot-$ axioms:

(K1) $(x \odot y) \odot (x \odot z) = (x \odot (z^{-1} \odot y^{-1})) \odot x = (x \odot ((e \odot z) \odot (e \odot y))) \odot x$,

(K2) $x \odot (x \odot y) = (x \odot y^{-1}) \odot x = (x \odot (e \odot y)) \odot x$,

(K3) $(x \odot x) = e$,

(K4) $(x \odot e) = x$,

(K5) $(e \odot x) = x^{-1}$

for all x, y, $z \in G$.

Definition 2.2. [1] A nonempty set $\mathcal{S}$ in a K-algebra $\mathcal{K}$ is called a *subalgebra* of $\mathcal{K}$ if for all $x, y \in \mathcal{S}$, $x \odot y \in \mathcal{S}$.

Definition 2.3. [1] Let $\mathcal{K}_1$ and $\mathcal{K}_2$ be two K-algebras. A mapping $f : \mathcal{K}_1 \to \mathcal{K}_2$ is called a *homomorphism* if $f(x \odot y) = f(x) \odot f(y)$ for all $x, y \in \mathcal{K}$.

Definition 2.4. [7] Let Z be a nonempty set of objects. A single-valued neutrosophic set H in Z is of the form $H = \{s \in Z : \mathcal{T}_H(s), \mathcal{I}_H(s), \mathcal{F}_H(s)\}$, where $\mathcal{T}, \mathcal{I}, \mathcal{F} : Z \to [0, 1]$ for all $s \in Z$ with $0 \leq \mathcal{T}_H(s) + \mathcal{I}_H(s) + \mathcal{F}_H(s) \leq 3$.

Definition 2.5. [22] Let $H = (\mathcal{T}_H, \mathcal{I}_H, \mathcal{F}_H)$ be a single-valued neutrosophic set in $\mathcal{K}$, then H is said to be a *single-valued neutrosophic K-subalgebra* of $\mathcal{K}$ if it possess the following properties:

(a) $\mathcal{T}_H(s \odot t) \geq \min\{\mathcal{T}_H(s), \mathcal{T}_H(t)\}$,

(b) $\mathcal{I}_H(s \odot t) \geq \min\{\mathcal{I}_H(s), \mathcal{I}_H(t)\}$,

(c) $\mathcal{F}_H(s \odot t) \leq \max\{\mathcal{F}_H(s), \mathcal{F}_H(t)\}$ for all $s, t \in \mathcal{K}$.

A K-subalgebra also satisfies the following conditions:
$\mathcal{T}_H(e) \geq \mathcal{T}_H(s), \mathcal{I}_H(e) \geq \mathcal{I}_H(s), \mathcal{F}_H(e) \leq \mathcal{F}_H(s)$ for all $s \neq e \in \mathcal{K}$.

Definition 2.6. [26] A *t-norm* is a two-valued function defined by a binary operation $*$, where $* : [0,1] \times [0,1] \to [0,1]$. A t-norm is an associative, monotonic and commutative function possess the following properties, for all $a, b, c, d \in [0,1]$,

(i) $*$ is a commutative binary operation.

(ii) $*$ is an associative binary operation.

(iii) $*(0,0) = 0$ and $*(a,1) = *(1,a) = a$.

(iv) If $a \leq c$ and $b \leq d$, then $*(a,b) \leq *(c,d)$.

Definition 2.7. [26] A *t-conorm (s-norm)* is a two-valued function defined by a binary operation $\circ$ such that $\circ : [0,1] \times [0,1] \to [0,1]$. A t-conorm is an associative, monotonic and commutative two-valued function, possess the following properties, for all $a, b, c, d \in [0,1]$,

(i) $\circ$ is a commutative binary operation.

(ii) $\circ$ is an associative binary operation.

(iii) $\circ(1,1) = 1$ and $\circ(a,0) = \circ(0,a) = a$.

(iv) If $a \leq c$ and $b \leq d$, then $\circ(a,b) \leq \circ(c,d)$.

Definition 2.8. [23] Let $\chi_{\mathcal{K}}$ be a single-valued neutrosophic topology over $\mathcal{K}$. Let H be a single-valued neutrosophic K-algebra of $\mathcal{K}$ and χ_H be a single-valued neutrosophic topology on H. Then H is called a *single-valued neutrosophic topological K-algebra* over $\mathcal{K}$ if the self map $\rho_a : (H, \chi_H) \to (H, \chi_H)$ for all $a \in \mathcal{K}$, defined as $\rho_a(s) = s \odot a$, is relatively single-valued neutrosophic continuous.

Definition 2.9. [9] Let Z be a universe of discourse and E be a universe of parameters. Let $P(Z)$ denotes the set of all subsets of Z and $A \subseteq E$. Then a *soft set* F_A over Z is represented by a set-valued function ζ_A, where $\zeta_A : E \to P(Z)$ such that $\zeta_A(\theta) = \emptyset$ if $\theta \in E - A$. In other words, F_A can be represented in the form of a collection of parameterized subsets of Z such as $F_A = \{(\theta, \zeta_A(\theta)) : \theta \in E, \zeta_A(\theta) = \emptyset \text{ if } \theta \in E - A\}$.

Definition 2.10. [27] Let Z be a universe of discourse and E be a universe of parameters. A *single-valued neutrosophic soft set* H in Z is defined by a set-valued function ζ_H, where $\zeta_H : E \to P(Z)$ and $P(Z)$ denotes the power set set of Z. In other words, a single-valued neutrosophic soft set is a parameterized family of single-valued neutrosophic sets in Z and therefore can be written as:
$H = \{(\theta, \langle u, \mathcal{T}_{\zeta_H(\theta)}(u), \mathcal{I}_{\zeta_H(\theta)}(u), \mathcal{F}_{\zeta_H(\theta)}(u) \rangle : u \in Z) : \theta \in E\}$, where $\mathcal{T}_{\zeta_H(\theta)}, \mathcal{I}_{\zeta_H(\theta)}, \mathcal{F}_{\zeta_H(\theta)}$ are called truth , indeterminacy and falsity membership functions of $\zeta_H(\theta)$, respectively.

Definition 2.11. [27] Let H be a single-valued neutrosophic soft set. The *compliment* of H, denoted by H^c, is defined as follows:

$$H^c = \{(\theta,\ \langle u, \mathcal{F}_{\zeta_H(\theta)}(u), \mathcal{I}_{\zeta_H(\theta)}(u), \mathcal{T}_{\zeta_H(\theta)}(u)\rangle\ : u \in Z) : \theta \in E\}.$$

Definition 2.12. [27] Let H and J be two single-valued neutrosophic soft sets over (Z, E). Then H is called a *neutrosophic soft subset* of J, denoted by $H \subseteq J$, if the following conditions hold:

(i) $\mathcal{T}_{\zeta_H(\theta)}(u) \leq \mathcal{T}_{\eta_J(\theta)}(u)$,

(ii) $\mathcal{I}_{\zeta_H(\theta)}(u) \leq \mathcal{I}_{\eta_J(\theta)}(u)$,

(iii) $\mathcal{F}_{\zeta_H(\theta)}(u) \geq \mathcal{F}_{\eta_J(\theta)}(u)$ for all $\theta \in E, u \in Z$.

Throughout this article, we take the t-norm $(*)$ as $\min(a, b)$ and t-conorm $(\circ)$ as $\max(a, b)$ for intersection of two single-valued neutrosophic soft sets and $(*)$ as $\max(a, b)$ and t-conorm $(\circ)$ as $\min(a, b)$ for union of two single-valued neutrosophic soft sets. The union and the intersection for two single-valued neutrosophic soft sets are defined as follows.

Definition 2.13. [27] Let H and J be two single-valued neutrosophic soft sets over (Z, E). Then the *union* of H and J is denoted by $H \cup J = L$ and defined as:

$$L = \Big\{\Big(\theta,\ \langle u, \mathcal{T}_{\vartheta_L(\theta)}(u), \mathcal{I}_{\vartheta_L(\theta)}(u), \mathcal{F}_{\vartheta_L(\theta)}(u)\rangle\ : u \in Z\Big) : \theta \in E\Big\},$$

where

$$\begin{aligned}
\mathcal{T}_{\vartheta_L(\theta)}(u) &= \{\mathcal{T}_{\zeta_H(\theta)}(u) * \mathcal{T}_{\eta_J(\theta)}(u)\} = \max\{\mathcal{T}_{\zeta_H(\theta)}(u), \mathcal{T}_{\eta_J(\theta)}(u)\},\\
\mathcal{I}_{\vartheta_L(\theta)}(u) &= \{\mathcal{I}_{\zeta_H(\theta)}(u) * \mathcal{T}_{\eta_J(\theta)}(u)\} = \max\{\mathcal{I}_{\zeta_H(\theta)}(u), \mathcal{I}_{\eta_J(\theta)}(u)\},\\
\mathcal{F}_{\vartheta_L(\theta)}(u) &= \{\mathcal{F}_{\zeta_H(\theta)}(u) \circ \mathcal{F}_{\eta_J(\theta)}(u)\} = \min\{\mathcal{F}_{\zeta_H(\theta)}(u), \mathcal{F}_{\eta_J(\theta)}(u)\}.
\end{aligned}$$

Definition 2.14. [27] Let H and J be two single-valued neutrosophic soft sets over (Z, E). Then their *intersection* is denoted by $H \cap J = L$ and defined as:

$$L = \Big\{\Big(\theta,\ \langle u, \mathcal{T}_{\vartheta_L(\theta)}(u), \mathcal{I}_{\vartheta_L(\theta)}(u), \mathcal{F}_{\vartheta_L(\theta)}(u)\rangle\ : u \in Z\Big) : \theta \in E\Big\},$$

where

$$\begin{aligned}
\mathcal{T}_{\vartheta_L(\theta)}(u) &= \{\mathcal{T}_{\zeta_H(\theta)}(u) * \mathcal{T}_{\eta_J(\theta)}(u)\} = \min\{\mathcal{T}_{\zeta_H(\theta)}(u), \mathcal{T}_{\eta_J(\theta)}(u)\},\\
\mathcal{I}_{\vartheta_L(\theta)}(u) &= \{\mathcal{I}_{\zeta_H(\theta)}(u) * \mathcal{T}_{\eta_J(\theta)}(u)\} = \min\{\mathcal{I}_{\zeta_H(\theta)}(u), \mathcal{I}_{\eta_J(\theta)}(u)\},\\
\mathcal{F}_{\vartheta_L(\theta)}(u) &= \{\mathcal{F}_{\zeta_H(\theta)}(u) \circ \mathcal{F}_{\eta_J(\theta)}(u)\} = \max\{\mathcal{F}_{\zeta_H(\theta)}(u), \mathcal{F}_{\eta_J(\theta)}(u)\}.
\end{aligned}$$

Definition 2.15. [27] A single-valued neutrosophic soft set H over the universe Z is termed to be an *empty or null single-valued neutrosophic soft set* with respect to the parametric set E if $\mathcal{T}_{\zeta_H(\theta)}(u) = 0$, $\mathcal{I}_{\zeta_H(\theta)}(u) = 0$, $\mathcal{F}_{\zeta_H(\theta)}(u) = 1$, for all $u \in Z$, $\theta \in E$, denoted by $\emptyset_E$ and can be written as:

$$\emptyset_E(u) = \{u \in Z : \mathcal{T}_{\zeta_H(\theta)}(u) = 0, \mathcal{I}_{\zeta_H(\theta)}(u) = 0, \mathcal{F}_{\zeta_H(\theta)}(u) = 1 : \theta \in E\}.$$

Definition 2.16. [27] A single-valued neutrosophic soft set H over the universe Z is called an *absolute or a whole single-valued neutrosophic soft set* if $\mathcal{T}_{\zeta_{H(\theta)}}(u) = 1$, $\mathcal{I}_{\zeta_{H(\theta)}}(u) = 1$, $\mathcal{F}_{\zeta_{H(\theta)}}(u) = 0$, for all $u \in Z$, $\theta \in E$, denoted by 1_E and can be written as:

$$1_E(u) = \{u \in Z : \mathcal{T}_{\zeta_{H(\theta)}}(u) = 1, \mathcal{I}_{\zeta_{H(\theta)}}(u) = 1, \mathcal{F}_{\zeta_{H(\theta)}}(u) = 0 : \theta \in E\}.$$

Definition 2.17. [10] Let (Z_1, E) and (Z_2, E) be two initial universes. Then a pair (φ, ρ) is called a *single-valued neutrosophic soft function* from (Z_1, E) into (Z_2, E), where $\varphi : Z_1 \to Z_2$ and $\rho : E \to E$, and E is a parametric set of Z_1 and Z_2.

Definition 2.18. [10] Let (H, E) and (J, E) be two single-valued neutrosophic soft sets over G_1 and G_2, respectively. If (φ, ρ) is a single-valued neutrosophic soft function from (G_1, E) into (G_2, E), then under this single-valued neutrosophic soft function (φ, ρ), *image* of (H, E) is a single-valued neutrosophic soft set on $\mathcal{K}_2$, denoted by $(\varphi, \rho)(H, E)$ and defined as follows:
for all $m \in \rho(E)$ and $y \in G_2$, $(\varphi, \rho)(H, E) = (\varphi(H), \rho(E))$, where

$$\mathcal{T}_{\varphi(\zeta)_m}(y) = \begin{cases} \bigvee_{\varphi(x)=y} \bigvee_{\rho(a)=m} \zeta_a(x) & \text{if } x \in \rho^{-1}(y), \\ 1, & \text{otherwise,} \end{cases}$$
$$\mathcal{I}_{\varphi(\zeta)_m}(y) = \begin{cases} \bigvee_{\varphi(x)=y} \bigvee_{\rho(a)=m} \zeta_a(x) & \text{if } x \in \rho^{-1}(y), \\ 1, & \text{otherwise,} \end{cases}$$
$$\mathcal{F}_{\varphi(\zeta)_m}(y) = \begin{cases} \bigwedge_{\varphi(x)=y} \bigwedge_{\rho(a)=m} \zeta_a(x) & \text{if } x \in \rho^{-1}(y), \\ 0, & \text{otherwise.} \end{cases}$$

The *preimage* of (J, E), denoted by $(\varphi, \rho)^{-1}(J, E)$, is defined as $\forall\, l \in \rho^{-1}(E)$ and for all $x \in G_1$, $(\varphi, \rho)^{-1}(J, E) = (\varphi^{-1}(J), \rho^{-1}(E))$, where

$$\mathcal{T}_{\varphi^{-1}(\eta)_l}(x) = \mathcal{T}_{\eta_{\rho(l)}}(\varphi(x)),$$
$$\mathcal{I}_{\varphi^{-1}(\eta)_l}(x) = \mathcal{I}_{\eta_{\rho(l)}}(\varphi(x)),$$
$$\mathcal{F}_{\varphi^{-1}(\eta)_l}(x) = \mathcal{F}_{\eta_{\rho(l)}}(\varphi(x)).$$

Proposition 2.19. Let Z_1 and Z_2 be two initial universes with parametric set E_1 and E_2, respectively. Let H, $(H_i, i \in I)$ be a single-valued neutrosophic soft set in Z_1 and J be a single-valued neutrosophic soft set in Z_2. Let $f : Z_1 \to Z_2$ be a function. Then

(i) $f(1_{E_1}) = 1_{E_2}$, if f is a surjective function.

(ii) $f(\emptyset_{E_1}) = \emptyset_{E_2}$.

(iii) $f^{-1}(1_{E_2}) = 1_{E_1}$.

(iv) $f^{-1}(\emptyset_{E_2}) = \emptyset_{E_1}$.

(v) $f^{-1}(\bigcup_{i=1}^{n} H_i) = \bigcup_{i=1}^{n} f^{-1}(H_i)$.

Through out this article, Z is considered as initial universe, E is a parametric set and $\theta \in E$ an arbitrary parameter.

3 Single-Valued Neutrosophic Soft Topological K-Algebras

Definition 3.1. Let Z be a nonempty set and E be a universe of parameters. A collection χ of single-valued neutrosophic soft sets is called a *single-valued neutrosophic soft topology* if the following properties hold:

(1) $\emptyset_E, 1_E \in \chi$.

(2) The intersection of any two single-valued neutrosophic soft sets of χ belongs to χ.

(3) The union of any collection of single-valued neutrosophic soft sets of χ belongs to χ.

The triplet (Z, E, χ) is called a single-valued neutrosophic soft topological space over (Z, E). Each element of χ is called a single-valued neutrosophic soft open set and compliment of each single-valued neutrosophic soft open set is a single-valued neutrosophic soft closed set in χ . A single-valued neutrosophic soft topology which contains all single-valued neutrosophic soft subsets of Z is called a discrete single-valued neutrosophic soft topology and indiscrete single-valued neutrosophic soft topology if it consists of $\emptyset_E$ and 1_E.

Definition 3.2. Let H be a single-valued neutrosophic soft set over a K-algebras $\mathcal{K}$. Then H is called a *single-valued neutrosophic soft K-subalgebra* of $\mathcal{K}$ if the following conditions hold:

$$\begin{aligned}
&\text{(i)} \quad \mathcal{T}_{\zeta_\theta}(s \odot t) \geq \min\{\mathcal{T}_{\zeta_\theta}(s), \mathcal{T}_{\zeta_\theta}(t)\},\\
&\text{(ii)} \quad \mathcal{I}_{\zeta_\theta}(s \odot t) \geq \min\{\mathcal{I}_{\zeta_\theta}(s), \mathcal{I}_{\zeta_\theta}(t)\},\\
&\text{(iii)} \; \mathcal{F}_{\zeta_\theta}(s \odot t) \leq \max\{\mathcal{F}_{\zeta_\theta}(s), \mathcal{F}_{\zeta_\theta}(t)\} \text{ for all } s, t \in G \text{ and } \theta \in E.
\end{aligned}$$

Note that

$$\begin{aligned}
&\mathcal{T}_{\zeta_\theta}(e) \geq \mathcal{T}_{\zeta_\theta}(s),\\
&\mathcal{I}_{\zeta_\theta}(e) \geq \mathcal{I}_{\zeta_\theta}(s),\\
&\mathcal{F}_{\zeta_\theta}(e) \leq \mathcal{F}_{\zeta_\theta}(s), \text{ for all } s \neq e \in G.
\end{aligned}$$

Example 3.3. Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$ on a group $(G, \cdot)$, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7\}$ is the cyclic group of order 8 and $\odot$ is given by the following Cayley's table as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7
e	e	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^7	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^7	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^7	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e	x^7
x^7	x^7	x^6	x^5	x^4	x^3	x^2	x	e

Let E be a set of parameters defined as $E = \{l_1, l_2\}$. We define single-valued neutrosophic soft sets H, J and L in $\mathcal{K}$ as:

$$\begin{aligned}
\zeta_H(l_1) &= \{(e, 0.8, 0.7, 0.2), (h, 0.6, 0.5, 0.4)\},\\
\zeta_H(l_2) &= \{(e, 0.7, 0.7, 0.2), (h, 0.6, 0.6, 0.5)\},
\end{aligned}$$

$$\zeta_J(l_1) = \{(e, 0.7, 0.7, 0.2), (h, 0.4, 0.1, 0.5)\},$$
$$\zeta_J(l_2) = \{(e, 0.4, 0.6, 0.6), (h, 0.3, 0.5, 0.7)\},$$

$$\zeta_L(l_1) = \{(e, 0.9, 0.8, 0.1), (h, 0.7, 0.6, 0.4)\},$$
$$\zeta_L(l_2) = \{(e, 0.9, 0.7, 0.1), (h, 0.7, 0.6, 0.4)\}$$

for all $h \neq e \in G$.
The collection $\chi_{\mathcal{K}} = \{\emptyset_E, 1_E, H, J, L\}$ is a single-valued neutrosophic soft topology on $\mathcal{K}$ and the triplet $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is a single-valued neutrosophic soft topological space over $\mathcal{K}$. It is interesting to note that corresponding to each parameter $\theta \in E$, we get a single-valued neutrosophic topology over $\mathcal{K}$ which means that a single-valued neutrosophic soft topological space gives a parameterized family of single-valued neutrosophic topological space on $\mathcal{K}$. Now, we define a single-valued neutrosophic soft set Q in $\mathcal{K}$ as:

$$\zeta_Q(l_1) = \{(e, 0.8, 0.5, 0.1), (h, 0.6, 0.4, 0.3)\},$$
$$\zeta_Q(l_2) = \{(e, 0.5, 0.6, 0.5), (h, 0.3, 0.4, 0.6)\}.$$

Clearly, by Definition 3.2, Q is a single-valued neutrosophic soft K-subalgebra over $\mathcal{K}$.

Proposition 3.4. Let $(\mathcal{K}, E, \chi'_{\mathcal{K}})$ and $(\mathcal{K}, E, \chi''_{\mathcal{K}})$ be two single-valued neutrosophic topological spaces over $\mathcal{K}$. If $\chi'_{\mathcal{K}} \cap \chi''_{\mathcal{K}} = M'$, where M' is a single-valued neutrosophic soft set from the set of all single-valued neutrosophic soft sets in $\mathcal{K}$, then $\chi'_{\mathcal{K}} \cap \chi''_{\mathcal{K}}$ is also a single-valued neutrosophic soft topology on $\mathcal{K}$.

Remark 3.5. The union of two single-valued neutrosophic soft topologies over $\mathcal{K}$ may not be a single-valued neutrosophic soft topology over $\mathcal{K}$.

Example 3.6. Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7\}$ is the cyclic group of order 8 and Cayley's table for $\odot$ is given in Example 3.3. We take $E = \{l_1, l_2\}$ and two single-valued neutrosophic soft topological spaces $\chi'_{\mathcal{K}} = \{\emptyset_E, 1_E, H, J\}$, $\chi''_{\mathcal{K}} = \{\emptyset_E, 1_E, R, S\}$ on $\mathcal{K}$, where $R = H$ and single-valued neutrosophic soft set S is defined as:

$$\zeta_S(l_1) = \{(e, 0.7, 0.6, 0.2), (h, 0.5, 0.5, 0.6)\},$$
$$\zeta_S(l_2) = \{(e, 0.9, 0.8, 0.2), (h, 0.7, 0.7, 0.3)\}.$$

Suppose that $\chi'''_{\mathcal{K}} = \chi'_{\mathcal{K}} \cup \chi''_{\mathcal{K}} = \{\emptyset_E, 1_E, H, J, S\}$. We see that $\chi'''_{\mathcal{K}}$ is not a single-valued neutrosophic soft topology over $\mathcal{K}$ since $S \cap J \notin \chi'''_{\mathcal{K}}$.

Definition 3.7. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space over $\mathcal{K}$, where $\chi_{\mathcal{K}}$ is a single-valued neutrosophic soft topology over $\mathcal{K}$. Let F be a single-valued neutrosophic soft set in $\mathcal{K}$, then $\chi_F = \{F \cap H : H \in \chi_{\mathcal{K}}\}$ is called a single-valued neutrosophic soft topology on F and (F, E, χ_F) is called a *single-valued neutrosophic soft subspace* of $(\mathcal{K}, E, \chi_{\mathcal{K}})$.

Definition 3.8. Let $(\mathcal{K}_1, E, \chi_1)$ and $(\mathcal{K}_2, E, \chi_2)$ be two single-valued neutrosophic soft topological spaces, where $\mathcal{K}_1$ and $\mathcal{K}_2$ are two K-algebras. Then, a mapping $f : (\mathcal{K}_1, E, \chi_1) \to (\mathcal{K}_2, E, \chi_2)$ is called *single-valued neutrosophic soft continuous mapping* of single-valued neutrosophic soft topological spaces if it the following properties hold:

(i) For each single-valued neutrosophic soft set $H \in \chi_2$, $f^{-1}(H) \in \chi_1$.

(ii) For each single-valued neutrosophic soft K-subalgebra $H \in \chi_2$, $f^{-1}(H)$ is a single-valued neutrosophic soft K-subalgebra $\in \chi_1$.

Definition 3.9. Let H and J be two single-valued neutrosophic soft sets in a K-algebra $\mathcal{K}$ and $f : (H, E, \chi_H) \rightarrow (J, E, \chi_J)$. Then, f is called a *relatively single-valued neutrosophic soft open* function if for every single-valued neutrosophic soft open set V in χ_H, the image $f(V) \in \chi_J$.

Definition 3.10. If f is a mapping such that $f : (\mathcal{K}_1, E, \chi_1) \rightarrow (\mathcal{K}_2, E, \chi_2)$. Then f is a mapping from (H, E, χ_H) into (J, E, χ_J) if $f(H) \subset J$, where (H, E, χ_H) and (J, E, χ_J) are two single-valued neutrosophic soft subspaces of $(\mathcal{K}_1, E, \chi_1)$ and $(\mathcal{K}_2, E, \chi_2)$, respectively.

Definition 3.11. A mapping f such that $f : (H, E, \chi_H) \rightarrow (J, E, \chi_J)$ is called *relatively single-valued neutrosophic soft continuous* if for every single-valued neutrosophic soft open set $Y_J \in \chi_J$, $f^{-1}(y_J) \cap H \in \chi_H$.

Definition 3.12. Let $(\mathcal{K}_1, E, \chi_1)$ and $(\mathcal{K}_2, E, \chi_2)$ be two single-valued neutrosophic soft topological spaces. Then, a function $f : (\mathcal{K}_1, E, \chi_1) \rightarrow (\mathcal{K}_2, E, \chi_2)$ is called a *single-valued neutrosophic soft homomorphism* if it satisfies the following properties:

(i) f is a bijective function.

(ii) Both f and f^{-1} are single-valued neutrosophic soft continuous functions.

Proposition 3.13. Let $f : (\mathcal{K}_1, E, \chi_1) \rightarrow (\mathcal{K}_2, E, \chi_2)$ be a single-valued neutrosophic soft continues mapping and (H, E, χ_H) and (J, E, χ_J) two single-valued neutrosophic soft topological subspaces of $(\mathcal{K}_1, E, \chi_1)$ and $(\mathcal{K}_2, E, \chi_2)$, respectively. If $f(H) \subseteq J$, then f is a relatively single-valued neutrosophic soft continuous mapping from (H, E, χ_H) into (J, E, χ_J).

Proposition 3.14. Let $(\mathcal{K}_1, E, \chi_1)$ and $(\mathcal{K}_2, E, \chi_2)$ be two single-valued neutrosophic soft topological spaces, where χ_1 is a single-valued neutrosophic soft topology on $\mathcal{K}_1$ and χ_2 is an indiscrete single-valued neutrosophic soft topology on $\mathcal{K}_2$. Then for each $\theta \in E$, every function $f : (\mathcal{K}_1, E, \chi_1) \rightarrow (\mathcal{K}_2, E, \chi_2)$ is a single-valued neutrosophic soft continues function.

Proof. Let χ_1 be a single-valued neutrosophic soft topology on $\mathcal{K}_1$ and χ_2 an indiscrete single-valued neutrosophic soft topology on $\mathcal{K}_2$ such that $\chi_2 = \{\emptyset_E, 1_E\}$. Let $f : (\mathcal{K}_1, E, \chi_1) \rightarrow (\mathcal{K}_2, E, \chi_2)$ be any function. Now, to prove that f is a single-valued neutrosophic soft continues function for each $\theta \in E$, we show that f satisfies both conditions of Definition 3.8. Clearly, every member of χ_2 is a single-valued neutrosophic soft K-subalgebra of $\mathcal{K}_2$ for each $\theta \in E$. Now, there is only need to show that for all $H \in \chi_2$ and for each $\theta \in E$, $f^{-1}(H) \in \chi_1$. For this purpose, let us assume that $\emptyset_\theta \in \chi_2$, for any $u \in \mathcal{K}_1$ and $\theta \in E$, we have $f^{-1}(\emptyset_\theta)(u) = \emptyset_\theta(f(u)) = \emptyset_\theta(u) \Rightarrow \emptyset_\theta \in \chi_1$. Similarly, $f^{-1}(1_\theta)(u) = 1_\theta(f(u)) = 1_\theta(u) \Rightarrow 1_\theta \in \chi_1$. For an arbitrary choice of θ, result holds for each $\theta \in E$. This shows that f is a single-valued neutrosophic soft continues function. □

Proposition 3.15. Let χ_1 and χ_2 be any two discrete single-valued neutrosophic soft topological spaces on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively and $(\mathcal{K}_1, E, \chi_1)$ and $(\mathcal{K}_2, E, \chi_2)$ two discrete single-valued neutrosophic soft topological spaces. Then for each $\theta \in E$, every homomorphism $f : (\mathcal{K}_1, E, \chi_1) \rightarrow (\mathcal{K}_2, E, \chi_2)$ is a single-valued neutrosophic soft continuous function.

Proof. Let $H = \{(\mathcal{T}_{\zeta_H(\theta)}, \mathcal{I}_{\zeta_H(\theta)}, \mathcal{F}_{\zeta_H(\theta)}) : \theta \in E\}$ be a single-valued neutrosophic soft set in $\mathcal{K}_2$ defined by a set-valued function ζ_H. Let $f : (\mathcal{K}_1, E, \chi_1) \to (\mathcal{K}_2, E, \chi_2)$ be a homomorphism (not a usual inverse homomorphism). Since χ_1 and χ_2 be two discrete single-valued neutrosophic soft topologies, then for every $H \in \chi_2$, $f^{-1}(H) \in \chi_1$. Now, we show that for each $\theta \in E$, the mapping $f^{-1}(H)$ is a single-valued neutrosophic soft K-subalgebra of K-algebra $\mathcal{K}_1$. Then for any $s, t \in \mathcal{K}_1$ and $\theta \in E$, we have

$$\begin{aligned} f^{-1}(\mathcal{T}_{\zeta_H(\theta)})(s \odot t) &= \mathcal{T}_{\zeta_H(\theta)}\big(f(s \odot t)\big) \\ &= \mathcal{T}_{\zeta_H(\theta)}(f(s) \odot f(t)) \\ &\geq \min\{\mathcal{T}_{\zeta_H(\theta)}(f(s)) \odot \mathcal{T}_{\zeta_H(\theta)}(f(t))\} \\ &= \min\{f^{-1}(\mathcal{T}_{\zeta_H(\theta)})(s), f^{-1}(\mathcal{T}_{\zeta_H(\theta)})(t)\}, \end{aligned}$$

$$\begin{aligned} f^{-1}(\mathcal{I}_{\zeta_H(\theta)})(s \odot t) &= \mathcal{I}_{\zeta_H(\theta)}(f(s \odot t)) \\ &= \mathcal{I}_{\zeta_H(\theta)}(f(s) \odot f(t)) \\ &\geq \min\{\mathcal{I}_{\zeta_H(\theta)}(f(s)) \odot \mathcal{I}_{\zeta_H(\theta)}(f(t))\} \\ &= \min\{f^{-1}(\mathcal{I}_{\zeta_H(\theta)})(s), f^{-1}(\mathcal{I}_{\zeta_H(\theta)})(t)\}, \end{aligned}$$

$$\begin{aligned} f^{-1}(\mathcal{F}_{\zeta_H(\theta)})(s \odot t) &= \mathcal{F}_{\zeta_H(\theta)}(f(s \odot t)) \\ &= \mathcal{F}_{\zeta_H(\theta)}(f(s) \odot f(t)) \\ &\geq \min\{\mathcal{F}_{\zeta_H(\theta)}(f(s)) \odot \mathcal{F}_{\zeta_H(\theta)}(f(t))\} \\ &= \min\{f^{-1}(\mathcal{F}_{\zeta_H(\theta)})(s), f^{-1}(\mathcal{F}_{\zeta_H(\theta)})(t)\}, \end{aligned}$$

Therefore, $f^{-1}(H)$ is single-valued neutrosophic soft K-subalgebra of $\mathcal{K}_1$. Hence $f^{-1}(H) \in \chi_2$ which shows that f is a single-valued neutrosophic soft continuous function from $(\mathcal{K}_1, E, \chi_1)$ into $(\mathcal{K}_2, E, \chi_2)$. □

Proposition 3.16. Let χ_1 and χ_2 be any two single-valued neutrosophic soft topological spaces on $\mathcal{K}$ and $(\mathcal{K}, E, \chi_1)$ and $(\mathcal{K}, E, \chi_2)$ be two single-valued neutrosophic soft topological spaces. Then for each $\theta \in E$, every homomorphism $f : (\mathcal{K}_1, E, \chi_1) \to (\mathcal{K}_2, E, \chi_2)$ is a single-valued neutrosophic soft continuous function.

Definition 3.17. Let χ be a single-valued neutrosophic soft topology on K-algebra $\mathcal{K}$. Let $H = (\mathcal{T}_{\zeta_H}, \mathcal{I}_{\zeta_H}, \mathcal{F}_{\zeta_H})$ be a single-valued neutrosophic soft K-algebra (K-subalgebra) of $\mathcal{K}$ and χ_H a single-valued neutrosophic soft topology over H. Then H is called a *single-valued neutrosophic soft topological* K-algebra of $\mathcal{K}$ if the self mapping $\rho_a : (H, E, \chi_H) \to (H, E, \chi_H)$ defined as $\rho_a(u) = u \odot a$, $\forall\, a \in \mathcal{K}$, is a relatively single-valued neutrosophic soft continuous mapping.

Theorem 3.18. Let χ_1 and χ_2 be two single-valued neutrosophic soft topological spaces on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively. Let $f : \mathcal{K}_1 \to \mathcal{K}_2$ be a homomorphism of K-algebras such that $f^{-1}(\chi_2) = \chi_1$. If for each $\theta \in E$, $H = \{\mathcal{T}_{\zeta_H}, \mathcal{I}_{\zeta_H}, \mathcal{F}_{\zeta_H}\}$ is a single-valued neutrosophic soft topological K-algebra of $\mathcal{K}_2$, then for each $\theta \in E$, $f^{-1}(H)$ is a single-valued neutrosophic soft topological K-algebra of $\mathcal{K}_1$.

Proof. In order to prove that $f^{-1}(H)$ is a single-valued neutrosophic soft topological K-algebra of K-algebra $\mathcal{K}_1$. Firstly, we show that $f^{-1}(H)$ is a single-valued neutrosophic soft K-algebra of $\mathcal{K}_1$. One can easily show

that for all $s \neq e \in G$ and $\theta \in E$, $\mathcal{T}_{\zeta_\theta}(e) \geq \mathcal{T}_{\zeta_\theta}(s), \mathcal{I}_{\zeta_\theta}(e) \geq \mathcal{I}_{\zeta_\theta}(s), \mathcal{F}_{\zeta_\theta}(e) \leq \mathcal{F}_{\zeta_\theta}(s)$.
Let for any $s, t \in \mathcal{K}_1$ and $\theta \in E$,

$$\begin{aligned}\mathcal{T}_{f^{-1}(H)}(s \odot t) &= \mathcal{T}_H(f(s \odot t))\\ &\geq \min\{\mathcal{T}_H(f(s)), \mathcal{T}_H(f(t))\}\\ &= \min\{\mathcal{T}_{f^{-1}(H)}(s), \mathcal{T}_{f^{-1}(H)}(t)\},\end{aligned}$$

$$\begin{aligned}\mathcal{I}_{f^{-1}(H)}(s \odot t) &= \mathcal{I}_H(f(s \odot t))\\ &\geq \min\{\mathcal{I}_H(f(s)), \mathcal{I}_H(f(t))\}\\ &= \min\{\mathcal{I}_{f^{-1}(H)}(s), \mathcal{I}_{f^{-1}(H)}(t)\},\end{aligned}$$

$$\begin{aligned}\mathcal{F}_{f^{-1}(H)}(s \odot t) &= \mathcal{F}_H(f(s \odot t))\\ &\geq \min\{\mathcal{F}_H(f(s)), \mathcal{F}_H(f(t))\}\\ &= \min\{\mathcal{F}_{f^{-1}(H)}(s), \mathcal{F}_{f^{-1}(H)}(t)\}.\end{aligned}$$

This shows that $f^{-1}(H)$ is a single-valued neutrosophic soft K-algebra of $\mathcal{K}_1$.
Since f is a homomorphism and also a single-valued neutrosophic soft continuous mapping, then clearly, f is relatively single-valued neutrosophic soft continuous mapping from (H, E, χ_H) into $(f^{-1}(H), E, \chi_{f^{-1}(H)})$ such that for a single-valued neutrosophic soft set V in χ_H, and a single-valued neutrosophic soft set U in $\chi_{(f^{-1}(H)}$,

$$f^{-1}(V) = U. \tag{1}$$

Now, we prove that the self mapping $\rho_a : (f^{-1}(H), E, \chi_{f^{-1}(H)}) \to (f^{-1}(H), E, \chi_{f^{-1}(H)})$ is relatively single-valued neutrosophic soft continuous mapping. Now, for any $a \in \mathcal{K}_1$ and $\theta \in E$, we have

$$\begin{aligned}\mathcal{T}_{\rho_a^{-1}(U)}(s) &= \mathcal{T}_{(U)}(\rho_a(s)) = \mathcal{T}_{(U)}(s \odot a)\\ &= \mathcal{T}_{f^{-1}(V)}(s \odot a) = \mathcal{T}_{(V)}(f(s \odot a))\\ &= \mathcal{T}_{(V)}(f(s) \odot f(a)) = \mathcal{T}_{(V)}(\rho_{f(a)}(f(s)))\\ &= \mathcal{T}_{\rho^{-1}f(a)V}(f(s)) = \mathcal{T}_{f^{-1}}(\rho^{-1}_{f(a)}(V)(s)),\end{aligned}$$

$$\begin{aligned}\mathcal{I}_{\rho_a^{-1}(U)}(s) &= \mathcal{I}_{(U)}(\rho_a(s)) = \mathcal{I}_{(U)}(s \odot a)\\ &= \mathcal{I}_{f^{-1}(V)}(s \odot a) = \mathcal{I}_{(V)}(f(s \odot a))\\ &= \mathcal{I}_{(V)}(f(s) \odot f(a)) = \mathcal{I}_{(V)}(\rho_{f(a)}(f(s)))\\ &= \mathcal{I}_{\rho^{-1}f(a)V}(f(s)) = \mathcal{I}_{f^{-1}}(\rho^{-1}_{f(a)}(V)(s)),\end{aligned}$$

$$\begin{aligned}\mathcal{F}_{\rho_a^{-1}(U)}(s) &= \mathcal{F}_{(U)}(\rho_a(s)) = \mathcal{F}_{(U)}(s \odot a)\\ &= \mathcal{F}_{f^{-1}(V)}(s \odot a) = \mathcal{F}_{(V)}(f(s \odot a))\\ &= \mathcal{F}_{(V)}(f(s) \odot f(a)) = \mathcal{F}_{(V)}(\rho_{f(a)}(f(s)))\\ &= \mathcal{F}_{\rho^{-1}f(a)V}(f(s)) = \mathcal{F}_{f^{-1}}(\rho_{f(a)}^{-1}(V)(s)).\end{aligned}$$

This implies that $\rho_a^{-1}(U) = f^{-1}(\rho_{f(a)}^{-1}(V))$. Thus, $\rho_a^{-1}(U) \cap f^{-1}(H) = f^{-1}(\rho_{f(a)}^{-1}(V)) \cap f^{-1}(H)$ is a single-valued neutrosophic soft set in $f^{-1}(H)$ and a single-valued neutrosophic soft set in $\chi_{f^{-1}(H)}$. Hence $f^{-1}(H)$ is a single-valued neutrosophic soft topological K-algebra of $\mathcal{K}_1$. This completes the proof. □

Theorem 3.19. Let χ_1 and χ_2 be two single-valued neutrosophic soft topologies on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively and $f : \mathcal{K}_1 \to \mathcal{K}_2$ an isomorphism of K-algebras such that $f(\chi_1) = \chi_2$. If for each $\theta \in E$, $H = \{(\mathcal{T}_{\zeta_H(\theta)}, \mathcal{I}_{\zeta_H(\theta)}, \mathcal{F}_{\zeta_H(\theta)}) : \theta \in E\}$ is a single-valued neutrosophic soft topological K-algebra of K-algebra $\mathcal{K}_1$, then for each $\theta \in E$, $f(H)$ is a single-valued neutrosophic soft topological K-algebra of $\mathcal{K}_2$.

Proof. Let H be a single-valued neutrosophic soft topological K-algebra of $\mathcal{K}_1$. For $u, v \in \mathcal{K}_2$.

Let $t_o \in f^{-1}(u)$, $s_o \in f^{-1}(v)$ such that

$$\mathcal{T}_H(t_o) = \sup_{t\in f^{-1}(u)} \mathcal{T}_H(t), \mathcal{T}_H(y_o) = \sup_{t\in f^{-1}(v)} \mathcal{T}_H(t).$$

We now have,

$$\begin{aligned}\mathcal{T}_{f(H)}(u \odot v) = &\sup_{t\in f^{-1}(u\odot v)} \mathcal{T}_H(t)\\ \geq\ & \mathcal{T}_H(t_o, s_o)\\ \geq\ & \min\{\mathcal{T}_H(t_o), \mathcal{T}_H(s_o)\}\\ =\ & \min\{\sup_{t\in f^{-1}(u)} \mathcal{T}_H(t), \sup_{a\in f^{-1}(v)} \mathcal{T}_H(t)\}\\ =\ & \min\{\mathcal{T}_{f(H)}(u), \mathcal{T}_{f(H)}(v)\},\end{aligned}$$

$$\begin{aligned}\mathcal{I}_{f(H)}(u \odot v) = &\sup_{t\in f^{-1}(u\odot v)} \mathcal{I}_H(t)\\ \geq\ & \mathcal{I}_H(t_o, s_o)\\ \geq\ & \min\{\mathcal{I}_H(t_o), \mathcal{I}_H(s_o)\}\\ =\ & \min\{\sup_{t\in f^{-1}(u)} \mathcal{I}_H(t), \sup_{t\in f^{-1}(v)} \mathcal{I}_H(t)\}\\ =\ & \min\{\mathcal{I}_{f(H)}(u), \mathcal{I}_{f(H)}(v)\},\end{aligned}$$

$$\begin{aligned}\mathcal{F}_{f(H)}(u \odot v) =& \inf_{t \in f^{-1}(u \odot v)} \mathcal{F}_H(t)\\ \leq& \ \mathcal{F}_H(t_o, s_o)\\ \leq& \ \max\{\mathcal{F}_H(t_o), \mathcal{F}_H(s_o)\}\\ =& \ \max\{\inf_{t \in f^{-1}(u)} \mathcal{F}_H(t), \inf_{t \in f^{-1}(v)} \mathcal{F}_H(t)\}\\ =& \ \max\{\mathcal{F}_{f(H)}(u), \mathcal{F}_{f(H)}(v)\}.\end{aligned}$$

Hence $f(H)$ is a single-valued neutrosophic soft K-subalgebra of $\mathcal{K}_2$. To show that $f(H)$ is a single-valued neutrosophic soft topological K-algebra of $\mathcal{K}_2$, i.e., the self map $\rho_b : (f(H), \chi_{f(H)}) \to (f(H), \chi_{f(H)})$, defined as $\rho_b(v) = v \odot b$, $\forall\ b \in \mathcal{K}_2$ is a relatively single-valued neutrosophic soft continuous mapping. Let Y_H be a single-valued neutrosophic soft set in χ_H, then there exists a single-valued neutrosophic soft set Y in χ_1 be such that $Y_H = Y \cap H$.

$$\rho^{-1}{}_b(Y_{f(H)}) \cap f(H) \in \chi_{f(H)}$$

Then $f(Y_H) = f(Y \cap H) = f(Y) \cap f(H)$ is a single-valued neutrosophic soft set in $\chi_{f(H)}$ since f is an injective function. Thus, f is relatively single-valued neutrosophic soft open. Since f is also an onto function, then for all $b \in \mathcal{K}_2$ and $a \in \mathcal{K}_1$, $a = f(b)$, we have

$$\begin{aligned}\mathcal{T}_{f^{-1}(\rho^{-1}{}_b(Y_{f(H)}))}(u) &= \mathcal{T}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(H)}))}(u)\\ &= \mathcal{T}_{\rho^{-1}{}_{f(a)}(Y_{f(H)})}\big(f(u)\big)\\ &= \mathcal{T}_{(Y_{f(H)})}\big(\rho_{f(a)}\big(f(u)\big)\big)\\ &= \mathcal{T}_{(Y_{f(H)})}\big(f(u) \odot f(a)\big)\\ &= \mathcal{T}_{f^{-1}(Y_{f(H)})}(u \odot a)\\ &= \mathcal{T}_{f^{-1}(Y_{f(H)})}(\rho_a(u))\\ &= \mathcal{T}_{\rho^{-1}{}_{(a)}}(f^{-1}(Y_{f(H)}))(u),\end{aligned}$$

$$\begin{aligned}\mathcal{I}_{f^{-1}(\rho^{-1}{}_b(Y_{f(H)}))}(u) &= \mathcal{I}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(H)}))}(u)\\ &= \mathcal{I}_{\rho^{-1}{}_{f(a)}(Y_{f(H)})}\big(f(u)\big)\\ &= \mathcal{I}_{(Y_{f(H)})}\big(\rho_{f(a)}\big(f(u)\big)\big)\\ &= \mathcal{I}_{(Y_{f(H)})}\big(f(u) \odot f(a)\big)\\ &= \mathcal{I}_{f^{-1}(Y_{f(H)})}(u \odot a)\\ &= \mathcal{I}_{f^{-1}(Y_{f(H)})}(\rho_a(u))\\ &= \mathcal{I}_{\rho^{-1}{}_{(a)}}(f^{-1}(Y_{f(H)}))(u),\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{f^{-1}(\rho^{-1}{}_{b}(Y_{f(H)}))}(u) &= \mathcal{F}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(H)}))}(u) \\
&= \mathcal{F}_{\rho^{-1}{}_{f(a)}(Y_{f(H)})}\big(f(u)\big) \\
&= \mathcal{F}_{(Y_{f(H)})}\big(\rho_{f(a)}\big(f(u)\big)\big) \\
&= \mathcal{F}_{(Y_{f(H)})}\big(f(u) \odot f(a)\big) \\
&= \mathcal{F}_{f^{-1}(Y_{f(H)})}(u \odot a) \\
&= \mathcal{F}_{f^{-1}(Y_{f(H)})}(\rho_a(u)) \\
&= \mathcal{F}_{\rho^{-1}{}_{(a)}}(f^{-1}(Y_{f(H)}))(u).
\end{aligned}$$

This shows that $f^{-1}(\rho^{-1}_{(b)}((Y_{f(H)}))) = \rho^{-1}_{(a)}(f^{-1}(Y_{(H)}))$. Since $\rho_a : (H, \chi_H) \to (H, \chi_H)$ is relatively single-valued neutrosophic soft continuous mapping and f is also relatively single-valued neutrosophic soft continues function. Therefore, $f^{-1}(\rho^{-1}_{(b)}((Y_{f(H)}))) \cap H = \rho^{-1}_{(a)}(f^{-1}(Y_{(H)})) \cap H$ is a single-valued neutrosophic soft set in χ_H. Thus, $f(f^{-1}(\rho_{(b)}((Y_{f(H)}))) \cap \mathcal{A}) = \rho^{-1}_{(b)}(Y_{f(\mathcal{A})}) \cap f(\mathcal{A})$ is a single-valued neutrosophic soft set in $\chi_{\mathcal{A}}$. □

Example 3.20. Consider a K-algebra $\mathcal{K}$ on a cyclic group of order 8 and Cayley's table for $\odot$ is given Example 3.3, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7\}$. Consider a set of parameters $E = \{l_1, l_2\}$ and single-valued neutrosophic soft sets H, J, L defined as:

$$\zeta_H(l_1) = \{(e, 0.8, 0.7, 0.2), (h, 0.6, 0.5, 0.4)\},$$
$$\zeta_H(l_2) = \{(e, 0.7, 0.7, 0.2), (h, 0.6, 0.6, 0.5)\},$$

$$\zeta_J(l_1) = \{(e, 0.7, 0.7, 0.2), (h, 0.4, 0.1, 0.5)\},$$
$$\zeta_J(l_2) = \{(e, 0.4, 0.6, 0.6), (h, 0.3, 0.5, 0.7)\},$$

$$\zeta_L(l_1) = \{(e, 0.9, 0.8, 0.1), (h, 0.7, 0.6, 0.4)\},$$
$$\zeta_L(l_2) = \{(e, 0.9, 0.7, 0.1), (h, 0.7, 0.6, 0.4)\}$$

for all $h \neq e \in G$. Then the family $\chi_{\mathcal{K}} = \{\emptyset_E, 1_E, H, J, L\}$ is a single-valued neutrosophic soft topology on $\mathcal{K}$ and $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is a single-valued neutrosophic soft topological space over $\mathcal{K}$. We define another single-valued neutrosophic soft set Q in $\mathcal{K}$ as:

$$\zeta_Q(l_1) = \{(e, 0.8, 0.5, 0.1), (h, 0.6, 0.4, 0.3)\},$$
$$\zeta_Q(l_2) = \{(e, 0.5, 0.6, 0.5), (h, 0.3, 0.4, 0.6)\}.$$

It is obvious that Q is a single-valued neutrosophic soft K-algebra of $\mathcal{K}$.
Now, we prove that the self map $\rho_a : (Q, E, \chi_Q) \to (Q, E, \chi_Q)$, defined as $\rho_a(s) = s \odot a$ for all $a \in \mathcal{K}$, is a relatively single-valued neutrosophic soft continuous mapping.
We get $Q \cap \emptyset_E = \emptyset_E, Q \cap 1_E = 1_E, Q \cap H = R_1, Q \cap J = R_2, Q \cap L = R_3$, where R_1, R_2, R_3 are as follows:

$$\zeta_{R_1}(l_1) = \{(e, 0.8, 0.5, 0.2), (h, 0.6, 0.4, 0.4)\},$$
$$\zeta_{R_1}(l_2) = \{(e, 0.5, 0.6, 0.5), (h, 0.3, 0.4, 0.6)\},$$

$$\zeta_{R_2}(l_1) = \{(e, 0.7, 0.5, 0.2), (h, 0.4, 0.1, 0.5)\},$$
$$\zeta_{R_2}(l_2) = \{(e, 0.4, 0.6, 0.6), (h, 0.3, 0.4, 0.7)\},$$

$$\zeta_{R_3}(l_1) = \{(e, 0.8, 0.5, 0.1), (h, 0.4, 0.1, 0.5)\},$$
$$\zeta_{R_3}(l_2) = \{(e, 0.5, 0.6, 0.5), (h, 0.3, 0.4, 0.7)\}.$$

Thus, $\chi_Q = \{\emptyset_E, 1_E, R_1, R_2, R_3\}$ is a relatively topology of Q and (Q, E, χ_Q) is a single-valued neutrosophic soft subspace of $(\mathcal{K}, E, \chi_{\mathcal{K}})$. Since ρ_a is a homomorphism, then for a single-valued neutrosophic soft set $R \in \chi_Q$, $\rho_a^{-1}(R) \cap Q \in \chi_Q$. Which shows that $\rho_a : (Q, E, \chi_Q) \to (Q, E, \chi_Q)$ is relatively single-valued neutrosophic soft continuous mapping. Therefore, Q is a single-valued neutrosophic soft topological K-algebra.

4 Single-Valued Neutrosophic Soft C_5-connected K-Algebras

In this section, we discuss single-valued neutrosophic soft C_5-connected K-algebras.

Definition 4.1. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space over $\mathcal{K}$. A *single-valued neutrosophic soft separation* of $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is a pair of nonempty single-valued neutrosophic soft open sets H, J if the following conditions hold:

(i) $H \cup J = 1_E$.

(ii) $H \cap J = \emptyset_E$.

Definition 4.2. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space over $\mathcal{K}$. Then $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is called a *single-valued neutrosophic soft C_5-disconnected* if there exists a single-valued neutrosophic soft separation of $(\mathcal{K}, E, \chi_{\mathcal{K}})$, otherwise C_5-connected.

Definition 4.2 can be written as:

Definition 4.3. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space over $\mathcal{K}$. If there exists a single-valued neutrosophic soft open set and single-valued neutrosophic soft closed set L such that $L \neq 1_E$ and $L \neq \emptyset_E$, then $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is called a *single-valued neutrosophic soft C_5-disconnected*, otherwise $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is called a single-valued neutrosophic soft C_5-connected.

Example 4.4. By considering Example 3.3, we consider a single-valued neutrosophic soft topological space $\chi_{\mathcal{K}} = \{\emptyset_E, 1_E, H, J, L\}$. Since $H \cap J \neq \emptyset_E$, $H \cap L \neq \emptyset_E$, $J \cap L \neq \emptyset_E$ and $H \cup J \neq 1_E$, $H \cup L \neq 1_E$, $J \cup L \neq 1_E$. Thus, $\chi_{\mathcal{K}}$ is a single-valued neutrosophic soft C_5-connected.

Example 4.5. Every indiscrete single-valued neutrosophic soft space is C_5-connected since the only single-valued neutrosophic soft sets in single-valued neutrosophic soft indiscrete space that are both single-valued neutrosophic soft open and single-valued neutrosophic soft closed are $\emptyset_E$ and 1_E.

Theorem 4.6. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space on K-algebra $\mathcal{K}$. Then $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is a single-valued neutrosophic soft C_5-connected if and only if $\chi_{\mathcal{K}}$ contains only $\emptyset_E$ and 1_E which are both single-valued neutrosophic soft open and single-valued neutrosophic soft closed.

Proof. Straightforward. □

Proposition 4.7. Let $\mathcal{K}_1$ and $\mathcal{K}_2$ be two K-algebras and $(\mathcal{K}_1, E, \chi_{\mathcal{K}_1})$, $(\mathcal{K}_2, E, \chi_{\mathcal{K}_2})$ two single-valued neutrosophic soft topological spaces on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively. Let $f : \mathcal{K}_1 \to \mathcal{K}_2$ be a single-valued neutrosophic soft continuous surjective function. If $(\mathcal{K}_1, E, \chi_{\mathcal{K}_1})$ is a single-valued neutrosophic soft C_5-connected space, then $(\mathcal{K}_2, E, \chi_{\mathcal{K}_2})$ is also single-valued neutrosophic soft C_5-connected.

Proof. Let $(\mathcal{K}_1, E, \chi_{\mathcal{K}_1})$ and $(\mathcal{K}_2, E, \chi_{\mathcal{K}_2})$ be two single-valued neutrosophic soft topological spaces and $(\mathcal{K}_1, E, \chi_{\mathcal{K}_1})$ be a single-valued neutrosophic soft C_5-connected space. We prove that $(\mathcal{K}_2, E, \chi_{\mathcal{K}_2})$ is also single-valued neutrosophic soft C_5-connected. Let us suppose on contrary that $(\mathcal{K}_2, \chi_2)$ be a single-valued neutrosophic soft C_5-disconnected space. According to Definition 4.3, we have both single-valued neutrosophic soft open set and single-valued neutrosophic soft closed set L such that $L \neq 1_{SN}$ and $L \neq \emptyset_{SN}$. Then $f^{-1}(L) = 1_{SN}$ or $f^{-1}(L) = \emptyset_{SN}$ since f is a single-valued neutrosophic soft continuous surjective mapping , where $f^{-1}(L)$ is both single-valued neutrosophic soft open set and single-valued neutrosophic soft closed set. Therefore, $L = f(f^{-1}(L)) = f(1_{SN}) = 1_{SN}$ and $L = f(f^{-1}(L)) = f(\emptyset_{SN}) = \emptyset_{SN}$, a contradiction. Hence $(\mathcal{K}_2, E, \chi_2)$ is a single-valued neutrosophic soft C_5-connected space. □

5 Single-Valued Neutrosophic Soft Super Connected K-Algebras

Definition 5.1. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space over $\mathcal{K}$ and $H = \{\mathcal{T}_{\zeta_H}, \mathcal{I}_{\zeta_H}, \mathcal{F}_{\zeta_H}\}$ a single-valued neutrosophic soft set in $\mathcal{K}$. Then the *interior* and *closure* of H in a K-algebra $\mathcal{K}$ is defines as:

$$H^{Int} = \bigcup\{O : O \text{ is a single-valued neutrosophic soft open set in } \mathcal{K} \text{ and } O \subseteq H\},$$
$$H^{Clo} = \bigcap\{C : C \text{ is a single-valued neutrosophic soft closed set in } \mathcal{K} \text{ and } H \subseteq C\}.$$

It is interesting to note that H^{Int}, being union of single-valued neutrosophic soft open sets is single-valued neutrosophic soft open and H^{Clo}, being intersection of single-valued neutrosophic soft closed set is single-valued neutrosophic soft closed.

Theorem 5.2. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space on $\mathcal{K}$. Let $H = \{\mathcal{T}_{\zeta_H}, \mathcal{I}_{\zeta_H}, \mathcal{F}_{\zeta_H}\}$ be a single-valued neutrosophic soft set in $\chi_{\mathcal{K}}$. Then H^{Int} is the largest single-valued neutrosophic soft open set contained in H.

Proof. Obvious. □

Proposition 5.3. Let H be a single-valued neutrosophic soft set in $\mathcal{K}$. Then the following properties hold:

(i) $(1_E)^{Int} = 1_E$.

(ii) $(\emptyset_E)^{Clo} = \emptyset_E$.

(iii) $\overline{(H)}^{Int} = \overline{(H)^{Clo}}$.

(iv) $\overline{(H)}^{Clo} = \overline{(H)^{Int}}$.

Corollary 5.4. If H is a single-valued neutrosophic soft set in $\mathcal{K}$, then H is single-valued neutrosophic soft open if and only if $H^{Int} = H$ and H is a single-valued neutrosophic soft closed if and only if $H^{Clo} = H$.

Definition 5.5. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space on $\mathcal{K}$ and $\chi_{\mathcal{K}}$ be a single-valued neutrosophic soft topology on $\mathcal{K}$. Let $H = \{\mathcal{T}_{\zeta_H}, \mathcal{I}_{\zeta_H}, \mathcal{F}_{\zeta_H}\}$ be a single-valued neutrosophic soft open set in $\mathcal{K}$. Then H is called a *single-valued neutrosophic soft regular open* if

$$H = (H^{Clo})^{Int}.$$

Remark 5.6. (1) Every single-valued neutrosophic soft regular is single-valued neutrosophic soft open.
(2) Every single-valued neutrosophic soft clopen set is single-valued neutrosophic soft regular open.

Definition 5.7. Let $\chi_{\mathcal{K}}$ be a single-valued neutrosophic soft topology on $\mathcal{K}$. Then $\mathcal{K}$ is called a *single-valued neutrosophic soft super disconnected* if there exists a single-valued neutrosophic soft regular open set $H = \{\mathcal{T}_{\zeta_H}, \mathcal{I}_{\zeta_H}, \mathcal{F}_{\zeta_H}\}$ such that $1_E \neq H$ and $\emptyset_E \neq H$. But if there does not exist such a single-valued neutrosophic soft regular open set H such that $1_E \neq H$ and $\emptyset_E \neq H$, then $\mathcal{K}$ is called *single-valued neutrosophic soft super connected.*

Example 5.8. Consider a K-algebra on a cyclic group of order 8 and Cayley's table for $\odot$ is given in Example 3.3, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7\}$. We have a single-valued neutrosophic soft topology $\chi_{\mathcal{K}} = \{\emptyset_E, 1_E, H, J\}$, where H, J with a parametric set $E = \{l_1, l_2\}$ are given as:

$$\zeta_H(l_1) = \{(e, 0.8, 0.7, 0.2), (h, 0.6, 0.5, 0.4)\},$$
$$\zeta_H(l_2) = \{(e, 0.7, 0.7, 0.2), (h, 0.6, 0.6, 0.5)\},$$

$$\zeta_J(l_1) = \{(e, 0.7, 0.7, 0.2), (h, 0.4, 0.1, 0.5)\},$$
$$\zeta_J(l_2) = \{(e, 0.4, 0.6, 0.6), (h, 0.3, 0.5, 0.7)\},$$

for all $h \neq e \in G$.
Let L be a single-valued neutrosophic soft set in $\mathcal{K}$, defined by:

$$\zeta_L(l_1) = \{(e, 0.9, 0.8, 0.1), (h, 0.7, 0.6, 0.4)\},$$
$$\zeta_L(l_2) = \{(e, 0.9, 0.7, 0.1), (h, 0.7, 0.6, 0.4)\}.$$

Now, we have single-valued neutrosophic soft open sets : $\emptyset_E, 1_E, H, J$.
single-valued neutrosophic soft closed sets : $(\emptyset_E)^c = 1_E,\ (1_E)^c = \emptyset_E,\ (H)^c = H^{'},\ (J)^c = J^{'}$, where $H^{'}, J^{'}$ are obtained as:

$$\zeta_{H'}(l_1) = \{(e, 0.2, 0.7, 0.8), (h, 0.4, 0.5, 0.6)\},$$
$$\zeta_{H'}(l_2) = \{(e, 0.2, 0.7, 0.7), (h, 0.5, 0.6, 0.6)\},$$

$$\zeta_{J'}(l_1) = \{(e, 0.2, 0.7, 0.7), (h, 0.5, 0.1, 0.4)\},$$
$$\zeta_{J'}(l_2) = \{(e, 0.6, 0.6, 0.4), (h, 0.7, 0.5, 0.3)\},$$

for all $h \neq e \in G$. Then, interior and closure of a single-valued neutrosophic soft set L is obtained as:

$$L^{Int} = H,$$
$$L^{Clo} = 1_E.$$

For L to be a single-valued neutrosophic soft regular open, then $L = (L^{Clo})^{Int}$. But since $L = (1_E)^{Int} = 1_E \neq L$. This shows that $1_E \neq L \neq \emptyset_E$ is not a single-valued neutrosophic soft regular open set. By Definition 5.7, defined K-algebra is a single-valued neutrosophic soft super connected K-algebra.

6 Single-Valued Neutrosophic Soft Compactness K-Algebras

Definition 6.1. Let $\chi_{\mathcal{K}}$ be a single-valued neutrosophic soft topology on $\mathcal{K}$. Let H be a single-valued neutrosophic soft set in $\mathcal{K}$. A collection $\Omega = \{(\mathcal{T}_{\zeta_{H_i}}, \mathcal{I}_{\zeta_{H_i}}, \mathcal{F}_{\zeta_{H_i}}) : i \in I\}$ of single-valued neutrosophic soft sets in $\mathcal{K}$ is called a *single-valued neutrosophic soft open covering* of H if $H \subseteq \bigcup \Omega$. A finite sub-collection of Ω say $(\Omega^{'})$ is also a single-valued neutrosophic soft open covering of H, called a *finite subcovering* of H.

Definition 6.2. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space of $\mathcal{K}$. Let H be a single-valued neutrosophic soft set in $\mathcal{K}$. Then H is called a *single-valued neutrosophic soft compact* if every single-valued neutrosophic soft open covering Ω of H has a finite sub-covering $(\Omega^{'})$.

Example 6.3. A single-valued neutrosophic soft topological space $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is single-valued neutrosophic soft compact if either $\mathcal{K}$ is finite or $\chi_{\mathcal{K}}$ is a finite single-valued neutrosophic soft topology on $\mathcal{K}$.

Proposition 6.4. Let $f : (\mathcal{K}_1, E, \chi_{\mathcal{K}_1}) \to (\mathcal{K}_2, E, \chi_{\mathcal{K}_2})$ be a single-valued neutrosophic soft continuous mapping, where $(\mathcal{K}_1, E, \chi_{\mathcal{K}_1})$ and $(\mathcal{K}_2, E, \chi_{\mathcal{K}_2})$ are two single-valued neutrosophic soft topological spaces of $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively. If H is a single-valued neutrosophic soft compact in $(\mathcal{K}_1, E, \chi_{\mathcal{K}_1})$, then $f(H)$ is single-valued neutrosophic soft compact in $(\mathcal{K}_2, E, \chi_{\mathcal{K}_2})$.

Proof. Let f be a single-valued neutrosophic soft continuous map from $\mathcal{K}_1$ into $\mathcal{K}_2$. Let $\Omega = \{f^{-1}(H_i : i \in I)\}$ be a single-valued neutrosophic soft open covering of H and $\Delta = \{H_i : i \in I\}$ a single-valued neutrosophic soft open covering of $f(H)$. Then there exists a single-valued neutrosophic soft finite sub-covering $\bigcup\limits_{I=1}^{n} f^{-1}(H_i)$ such that

$$H \subseteq \bigcup_{i=1}^{n} f^{-1}(H_i).$$

Thus,

$$f(H) \subseteq \bigcup_{i=1}^{n} (H_i)$$

$$
\begin{aligned}
H &\subseteq \bigcup_{i=1}^{n} f^{-1}(H_i)\\
f(H) &\subseteq f\Big(\bigcup_{i=1}^{n} f^{-1}(H_i)\Big)\\
f(H) &\subseteq \bigcup_{i=1}^{n}\big(f(f^{-1}(H_i))\big)\\
f(H) &\subseteq \bigcup_{i=1}^{n}(H_i).
\end{aligned}
$$

This shows that there exists a single-valued neutrosophic soft finite sub-covering of $f(H)$. Therefore, $f(H)$ is single-valued neutrosophic soft compact in $(\mathcal{K}_2, E, \chi_{\mathcal{K}_2})$. □

7 Single-Valued Neutrosophic Soft Hausdorff K-Algebras

Definition 7.1. Let $H = \{\mathcal{T}_{\zeta_H}, \mathcal{I}_{\zeta_H}, \mathcal{F}_{\zeta_H}\}$ be a single-valued neutrosophic soft set in a $\mathcal{K}$. Then H is called a *single-valued neutrosophic soft point* if, for $\theta \in E$

$$\zeta_H(\theta) \neq \emptyset_E,$$

and

$$\zeta_H(\theta') = \emptyset_E,$$

for all $\theta' \in E - \{\theta\}$. A single-valued neutrosophic soft point in H is denoted by θ_H.

Definition 7.2. A single-valued neutrosophic soft point θ_H is said to *belong* to a single-valued neutrosophic soft set J, i.e., $\theta_H \in J$ if, for $\theta \in E$

$$\zeta_H(\theta) \leq \zeta_J(\theta).$$

Definition 7.3. Let $(\mathcal{K}, E, \chi_{\mathcal{K}})$ be a single-valued neutrosophic soft topological space over $\mathcal{K}$ and θ_L, θ_Q be two single-valued neutrosophic soft points in $\mathcal{K}$. If for these two single-valued neutrosophic soft points, there exist two disjoint single-valued neutrosophic soft open sets H, J such that $\theta_L \in H$ and $\theta_Q \in J$. Then $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is called a *single-valued neutrosophic soft Hausdorff topological space* over $\mathcal{K}$ and $\mathcal{K}$ is called a single-valued neutrosophic soft Hausdorff K-algebra.

Example 7.4. Consider a K-algebra $\mathcal{K}$ on a cyclic group of order 8 and Cayley's table for $\odot$ is given in Example 3.3, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7\}$. Let $E = \{l\}$ and $\chi_{\mathcal{K}} = \{\emptyset_E, 1_E, H, J\}$ be a single-valued neutrosophic soft topological space over $\mathcal{K}$. We define two single-valued neutrosophic soft points l_L, l_Q such that

$$
\begin{aligned}
l_L &= \{(e, 1, 0, 1), (h, 0, 0, 1)\},\\
l_Q &= \{(e, 0, 0, 1), (h, 0, 1, 0)\}.
\end{aligned}
$$

Since for $l \in E, \zeta_L(l) \neq \emptyset_E, \zeta_Q(l) \neq \emptyset_E$, and $l_L \neq l_Q$, then clearly l_L and l_Q are two single-valued neutrosophic soft points. Now, consider two single-valued neutrosophic soft open sets H and J defined as:

$$\zeta_H(l) = \{(e, 1, 1, 0), (h, 0, 0, 1)\},$$
$$\zeta_J(l) = \{(e, 0, 0, 1), (h, 1, 1, 0)\},$$

for all $h \neq e \in G$. Since $\zeta_L(l) \leq \zeta_H(l)$ and $\zeta_Q(l) \leq \zeta_J(l)$, i.e., $l_L \in H$ and $l_Q \in J$ and $H \cap J = \emptyset_E$. Thus, $(\mathcal{K}, E, \chi_{\mathcal{K}})$ is a single-valued neutrosophic soft Hausdorff space and $\mathcal{K}$ is a single-valued neutrosophic soft Hausdorff K-algebra.

Theorem 7.5. Let $f : (\mathcal{K}_1, E, \chi_1) \to (\mathcal{K}_2, E, \chi_2)$ be a single-valued neutrosophic soft homomorphism. Then $\mathcal{K}_1$ is a single-valued neutrosophic soft Hausdorff space if and only if $\mathcal{K}_2$ is a single-valued neutrosophic soft Hausdorff K-algebra.

Proof. Let $f : (\mathcal{K}_1, E, \chi_1) \to (\mathcal{K}_2, E, \chi_2)$ be a single-valued neutrosophic soft homomorphism and χ_1, χ_2 be two single-valued neutrosophic soft topologies on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively. Suppose that $\mathcal{K}_1$ is a single-valued neutrosophic soft Hausdorff space. To prove that $\mathcal{K}_2$ is a single-valued neutrosophic soft Hausdorff K-algebra, Let for $l \in E$, l_L and l_Q be two single-valued neutrosophic soft points in χ_2 such that $l_L \neq l_Q$ with $u, v \in \mathcal{K}_1$, $u \neq v$. Then for these two distinct single-valued neutrosophic soft points, there exist two single-valued neutrosophic soft open sets H and J such that $l_L \in H$, $l_Q \in J$ with $H \bigcap J = \emptyset_E$. For $x \in \mathcal{K}_1$, we consider

$$(f^{-1}(l_L))(x) = l_L(f^{-1}(x)) = \begin{cases} s \in (0,1] & if x = f^{-1}(u), \\ 0 & otherwise. \end{cases}$$
$$= ((f^{-1}(l))_L(x))$$

Therefore, $f^{-1}(l_L) = (f^{-1}(l))_L$. Likewise, $f^{-1}(l_Q) = (f^{-1}(l))_Q$. Since f is a single-valued neutrosophic soft continuous function from $\mathcal{K}_1$ into $\mathcal{K}_2$ and also f^{-1} is a single-valued neutrosophic soft continuous function from $\mathcal{K}_2$ into $\mathcal{K}_1$, then there exist two disjoint single-valued neutrosophic soft open sets $f(H)$ and $f(J)$ of single-valued neutrosophic soft points l_L and l_Q, respectively be such that $f(H) \bigcap f(J) = f(\emptyset_E) = \emptyset_E$. This shows that $\mathcal{K}_2$ is a single-valued neutrosophic soft Hausdorff K-algebra. The proof of converse part is straightforward. □

Theorem 7.6. let $f : \mathcal{K}_1 \to \mathcal{K}_2$ be a bijective single-valued neutrosophic soft continuous function, where $\mathcal{K}_1$ is a single-valued neutrosophic soft compact K-algebra and $\mathcal{K}_2$ is a single-valued neutrosophic soft Hausdorff K-algebra. Then mapping f is a $\mathcal{K}_1$ is a single-valued neutrosophic soft homomorphism.

Proof. Let f be a bijective single-valued neutrosophic soft mapping from a single-valued neutrosophic soft compact K-algebra into a single-valued neutrosophic soft Hausdorff K-algebra. Then clearly, f is a single-valued neutrosophic soft homomorphism. We only prove that f is single-valued neutrosophic soft closed since f is a bijective mapping. Let a single-valued neutrosophic soft set $Q = \{\mathcal{T}_{\zeta_Q}, \mathcal{I}_{\zeta_Q}, \mathcal{F}_{\zeta_Q}\}$ be closed in K-algebra $\mathcal{K}_1$. Now if $Q = \emptyset_E$, then $f(Q) = \emptyset_E$ is single-valued neutrosophic soft closed in $\mathcal{K}_2$. But if $Q \neq \emptyset_E$, then being a subset of a single-valued neutrosophic soft compact K-algebra, Q is single-valued neutrosophic soft compact. Also $f(Q)$ is single-valued neutrosophic soft compact, being a single-valued neutrosophic soft continuous image of a single-valued neutrosophic soft compact K-algebra. Hence f is closed thus, f is a single-valued neutrosophic soft homomorphism. □

8 Conclusions

In 1998, Smarandache originally considered the concept of neutrosophic set from philosophical point of view. The notion of a single-valued neutrosophic set is a subclass of the neutrosophic set from a scientific and engineering point of view, and an extension of intuitionistic fuzzy sets [32]. In 1999, Molodtsov introduced the idea of soft set theory as another powerful mathematical tool to handle indeterminate and inconsistent data. This theory fixes the problem of establishing the membership function for each specific case by giving a parameterized outlook to indeterminacy. By using a hybrid model of these two mathematical techniques with a topological structure, we have developed the concept of single-valued neutrosophic soft topological K-algebras to analyze the element of indeterminacy in K-algebras. We have defined some certain concepts such as the interior, closure, C_5-connected, super connected, compactness and Hausdorff of single-valued neutrosophic soft topological K-algebras. In future, we aim to extend our notions to (1) Rough neutrosophic K-algebras, (2) Soft rough neutrosophic K-algebras, (3) Bipolar neutrosophic soft K-algebras, and (4) Rough neutrosophic K-algebras.

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