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Neutrosophic Travelling Salesman Problem in Trapezoidal Fuzzy number using Branch and Bound Technique

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ABSTRACT--Travelling salesman problem is a well-known studied problem and intensely used in combinatorial optimization. In this article, we discuss a Neutrosophic fuzzy travelling salesman problem in which each element is considered as a Neutrosophic trapezoidal fuzzy numbers. Here, we provide the Branch and Bound technique is to find the optimal solution. The efficiency of this method is proved by solving a numerical example.
Keywords: Neutrosophic fuzzy number, Trapezoidal Neutrosophic fuzzy number, Branch and Bound technique.

1. INTRODUCTION

The travelling salesman problem was first introduced by Irish Mathematician W.R. Hamilton. Travelling salesman problem is a well-known popular and extensively studied problem in the field of combinatorial optimization. In the general form of travelling salesman problem, a salesman has to visits the entire cities only once and return to the home town with minimum cost. The design of the problem is simple. Fuzzy sets were proposed by Prof. L.A. Zadeh in 1965, to handle data and information feature of non-statistical ambiguity. One of the main applications of fuzzy arithmetic is accommodated the parameters and is represented by a fuzzy number.

The Neutrosophic set was first proposed by F. Smarandache in 1995. The concept of Neutrosophic components are characterized by three truth, indeterminacy and falsity membership values respectively and it is non-standard unit interval. Here, we generate the Neutrosophic trapezoidal fuzzy number to Neutrosophic number by using the graded mean ranking. To find the optimal solution of Neutrosophic trapezoidal fuzzy travelling salesman problem by the method called Branch and Bound technique. Numerical example also included to clear the optimization.

2. II. PRELIMINARIES

A. *Definition [11]*

A **fuzzy set** is characterized by its membership function taking values from the domain, space or the universe of discourse mapped into the unit interval $[0,1]$. A fuzzy set A in the universal set X is defined as $A = \{(x, \mu_A(x))/x \epsilon X\}$. Here $\mu_A(x): A \to [0,1]$ is the grade of the membership function and $\mu_A(x)$ is the grade value of $x \in X$ in the fuzzy set A.

B. *2.2 Definition [11]*

A fuzzy set A is called **Normal** if there exists an element $x \in X$ whose membership value is one, i.e., $\mu_A(x) = 1$.

C. Definition [9]

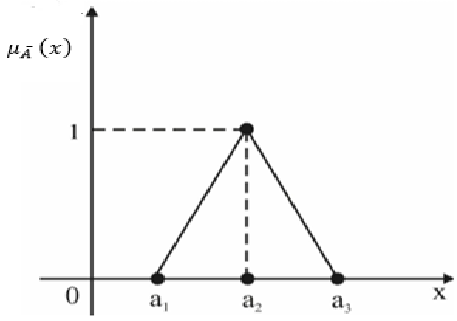
A fuzzy set A of real line R with membership function $\mu_A(x): R \to [0,1]$ is called **fuzzy number** if

(i) A is normal and convexity.
(ii) A must be bounded.
(iii) $\mu_A(x)$ is piecewise continuous.

D. Definition [9]

A fuzzy number $A = (a_0, a_1, a_2)$ is said to be **triangular fuzzy number** if its membership function is given by

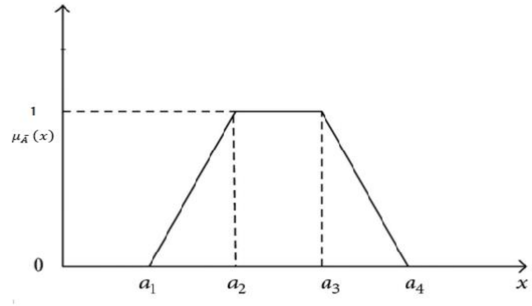
$$\mu_A(x)=\begin{cases} \frac{x-a_0}{a_1-a_0}, a_0 \leq x \leq a_1 \\ 1, x = a_1 \\ \frac{a_2-x}{a_2-a_1}, a_1 \leq x \leq a_2 \\ 0, otherwise \end{cases}$$



E. Definition [9]

A fuzzy number $A = (a_1, a_2, a_3, a_4)$ is a **Trapezoidal fuzzy number** where a_1, a_2, a_3, a_4 are real numbers and its membership function is given below,

$$\mu_A(x)=\begin{cases} \frac{x-a_1}{a_2-a_1}\ if\ a_1 \leq x \leq a_2 \\ 1\quad if\ a_2 \leq x \leq a_3 \\ \frac{a_3-x}{a_4-a_3}\ if a_3 \leq x \leq a_4 \end{cases}$$



F. Operations on trapezoidal fuzzy numbers

Addition:

$$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

Subtraction:

$$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 + c_2, c_1 + b_2, d_1 + a_2)$$

Element wise subtraction:

$$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2)$$

G. Ranking of Trapezoidal Fuzzy Number [2]

Suppose the trapezoidal fuzzy number, the ranking function R: F(R)→ R by graded mean is defined as

$$R(A) = \left[\frac{a_1 + 2a_2 + a_3 + 3a_4}{7}\right]$$

H. Definition [4]

Let X be a universe. A **Neutrosophic set** A over X is defined by

$A^N = \{\langle x: T_{a^N}(x), I_{a^N}(x), F_{a^N}(x)\rangle : x\epsilon X\}$where $T_{a^N}, I_{a^N}, F_{a^N}: X \to]0^-, 3^+[$ are called the truth, indeterminacy and falsity membership function of the element $x \in X$ to the set A^N with $0^- \leq T_{a^N}(x) + I_{a^N}(x) + F_{a^N}(x) \leq 3^+$

I. Definition [4]

Let X be the finite universe of discourse and $F^N[0,1]$ denoted by the set of all triangular fuzzy numbers on [0,1]. A **Neutrosophic triangular fuzzy set** A in X is represented by $A^N = \{\langle x: T_{a^N}(x), I_{a^N}(x), F_{a^N}(x)\rangle / x\epsilon X\}$

Where the Neutrosophic triangular fuzzy numbers $T_{a^N}(x) = \left(T_{a_1}^N(x), T_{a_2}^N(x), T_{a_3}^N(x)\right), I_{a^N}(x) = \left(I_{a_1}^N(x), I_{a_2}^N(x), I_{a_3}^N(x)\right), F_{a^N}(x) = \left(F_{a_1}^N(x), F_{a_2}^N(x), F_{a_3}^N(x)\right)$ be the truth, indeterminacy and falsity membership degree of x in A and for every $x \in X$ such that $0^- \leq T_{a^N}(x) + I_{a^N}(x) + F_{a^N}(x) \leq 3^+$.

J. Definition [4]

Let X be a universe of discourse, a **Neutrosophic trapezoidal fuzzy set** A in X is defined as the following form:

$A^N = \{\langle x: T_{a^N}(x), I_{a^N}(x), F_{a^N}(x)\rangle / x\epsilon X\}$

Where $\{T_{a^N}(x), I_{a^N}(x), F_{a^N}(x)\} \subset [0,1]$ are the three trapezoidal fuzzy number $T_{a^N}(x) = \left(T_{a_1}^N(x), T_{a_2}^N(x), T_{a_3}^N(x), T_{a_4}^N(x)\right): X \to [0,1]$, $I_{a^N}(x) = \left(I_{a_1}^N(x), I_{a_2}^N(x), I_{a_3}^N(x), I_{a_4}^N(x)\right): X \to [0,1]$, $F_{a^N}(x) = \left(F_{a_1}^N(x), F_{a_2}^N(x), F_{a_3}^N(x), F_{a_3}^N(x)\right): X \to [0,1]$ with the condition $0^- \le T_{a^N}(x) + I_{a^N}(x) + F_{a^N}(x) \le 3^+$.

3. Neutrosophic Fuzzy Travelling Salesman Problem

Suppose a salesman has to visits n cities. He visits one particular city and return to the home town within a short period of time. The fuzzy Neutrosophic travelling salesman problem in the following matrix may be formulated as

City → ↓	1	2	⋯	J	⋯	n
1	∞	$T_{12}^N, I_{12}^N, F_{12}^N$	⋯	$T_{1j}^N, I_{1j}^N, F_{1j}^N$	⋯	$T_{1n}^N, I_{1n}^N, F_{1n}^N$
2	$T_{21}^N, I_{21}^N, F_{21}^N$	∞	⋯	$T_{2j}^N, I_{2j}^N, F_{2j}^N$	⋯	$T_{2n}^N, I_{2n}^N, F_{2n}^N$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
J	$T_{j1}^N, I_{j1}^N, F_{j1}^N$	$T_{j2}^N, I_{j2}^N, F_{j2}^N$	⋯	∞	⋯	$T_{jn}^N, I_{jn}^N, F_{jn}^N$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
N	$T_{n1}^N, I_{n1}^N, F_{n1}^N$	$T_{n2}^N, I_{n2}^N, F_{n2}^N$	⋯	$T_{nj}^N, I_{nj}^N, F_{nj}^N$	⋯	∞

4. Procedure for solving the Neutrosophic fuzzy travelling salesman problem

Step 1: Find the least cost tour starting at A, travelling through the other cities exactly once and returning to A

Step 2: Compute the given problem is to Neutrosophic travelling salesman problem using graded mean ranking.

Step 3: A row (column) is to be reduced iff it contains at least one zero and all the remaining entries are non-zero.

Step 4: Consider upper bound as ∞.

Step 5: Initial node is to split the other branches and compute the cost of each node.

Step 6: The reduced cost for every is $C^N(i,j) + R^N + \widehat{R^N}$ where R^N is the reduced cost of initial node and $\widehat{R^N}$ is the reduced cost of current node.

Step 7: Compute the least cost branch and terminate the other.

Step 8: Stop when only one branch survives.

Step 9: Finally find the optimum cost.

5. NUMERICAL EXAMPLE

Consider the following Neutrosophic Fuzzy travelling salesman problem

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{array} \\ \begin{pmatrix} \infty & [(1,3,4,8);(5,8,11,15);(6,10,14,17)] & [(3,6,7,9);(7,9,11,16);(10,11,16,19)] & [(5,7,8,12);(7,11,15,18);(8,14,16,20)] \\ [(0,2,3,7);(3,6,7,9);(6,9,12,16)] & \infty & [(1,4,6,9);(5,6,9,10);(8,11,14,18)] & [(4,7,10,14);(9,13,16,18);(11,15,19,22)] \\ [(4,5,9,11);(6,7,10,11);(9,12,14,17)] & [(6,9,10,12);(10,11,15,17);(12,16,19,21)] & \infty & [(1,4,6,9);(4,5,8,9);(3,7,10,12)] \\ [(7,9,14,15);(9,11,16,17);(9,10,16,20)] & [(3,6,8,11);(4,7,10,14);(6,9,13,18)] & [(9,14,15,20);(11,15,19,22);(12,16,20,23)] & \infty \end{pmatrix} \end{array}$$

Solution:

By using the graded mean ranking formula, the given problem is as follows:

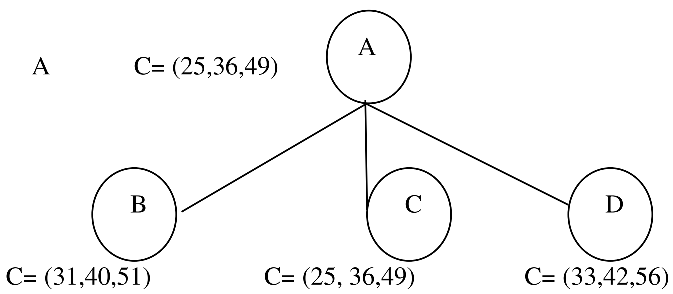
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & (5,11,13) & (7,12,15) & (9,14,16) \\ (4,7,12) & \infty & (6,8,14) & (10,15,18) \\ (8,9,14) & (10,14,18) & \infty & (6,7,9) \\ (12,14,15) & (8,10,13) & (16,18,19) & \infty \end{bmatrix} \end{array}$$

Applying row reduction and column reduction using the element wise subtraction, the above matrix becomes

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & (0,0,0) & (0,0,0) & (4,3,3) \\ (0,0,0) & \infty & (0,0,0) & (6,8,6) \\ (2,2,5) & (4,7,9) & \infty & (0,0,0) \\ (6,4,2) & (0,0,0) & (6,7,4) & \infty \end{bmatrix} \begin{array}{c} (5,11,13) \\ (4,7,12) \\ (6,7,9) \\ (8,10,13) \end{array} \\ (0,0,0) \quad (0,0,0) \quad (2,1,2) \quad (0,0,0) = (25,36,49) \end{array}$$

The above matrix is taken as **A** and the Reduced cost $R^N = (25,36,49)$

Initially, Upper = ∞



Finding $C^N(A,B)$. Set A,B as ∞.

$$\begin{array}{c} \\ A\\B\\C\\D\end{array}\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & (0,0,0) & (6,8,6) \\ (2,2,5) & \infty & \infty & (0,0,0) \\ (6,4,2) & \infty & (6,7,4) & \infty \end{bmatrix}\end{array}$$

The reduced matrix is as follows:

$$\begin{array}{c} \\ A\\B\\C\\D\end{array}\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & (0,0,0) & (6,8,6) \\ (2,2,5) & \infty & \infty & (0,0,0) \\ (0,0,0) & \infty & (0,3,2) & \infty \end{bmatrix}\end{array}\begin{array}{c} \\ (0,0,0)\\(0,0,0)\\(0,0,0)\\(6,4,2)\end{array}$$

$$C^N(A,B)+R^N+\widehat{R^N}=(31,40,51)$$

Next finding $C^N(A,C)$. Set A,C as ∞.

$$\begin{array}{c} \\ A\\B\\C\\D\end{array}\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ (0,0,0) & \infty & \infty & (6,8,6) \\ \infty & (4,7,9) & \infty & (0,0,0) \\ (6,4,2) & (0,0,0) & \infty & \infty \end{bmatrix}\end{array}$$

The above matrix is already reduced. So, the reduced cost is

$$C^N(A,C)+R^N+\widehat{R^N}=(25,36,49)$$

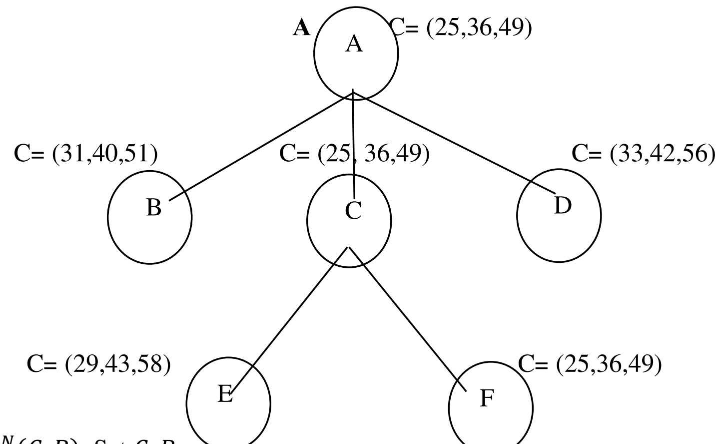
Next finding $C^N(A,D)$. Set A,D as ∞.

$$\begin{array}{c} \\ A\\B\\C\\D\end{array}\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ (0,0,0) & \infty & (0,0,0) & \infty \\ (2,2,5) & (4,7,9) & \infty & \infty \\ \infty & (0,0,0) & (6,7,4) & \infty \end{bmatrix}\end{array}$$

The reduced matrix is as follows:

$$\begin{array}{c} \\ A\\B\\C\\D\end{array}\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ (0,0,0) & \infty & (0,0,0) & \infty \\ (0,0,0) & (4,7,9) & \infty & \infty \\ \infty & (0,0,0) & (6,7,4) & \infty \end{bmatrix}\end{array}\begin{array}{c} \\ (0,0,0)\\(0,0,0)\\(2,2,5)\\(0,0,0)\end{array}$$

$$C^N(A,D)+R^N+\widehat{R^N}=(33,42,56)$$



Here, finding $C^N(C,B)$. Set C,B as ∞.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ (0,0,0) & \infty & \infty & (6,8,6) \\ \infty & \infty & \infty & \infty \\ (6,4,2) & \infty & \infty & \infty \end{bmatrix} \end{array}$$

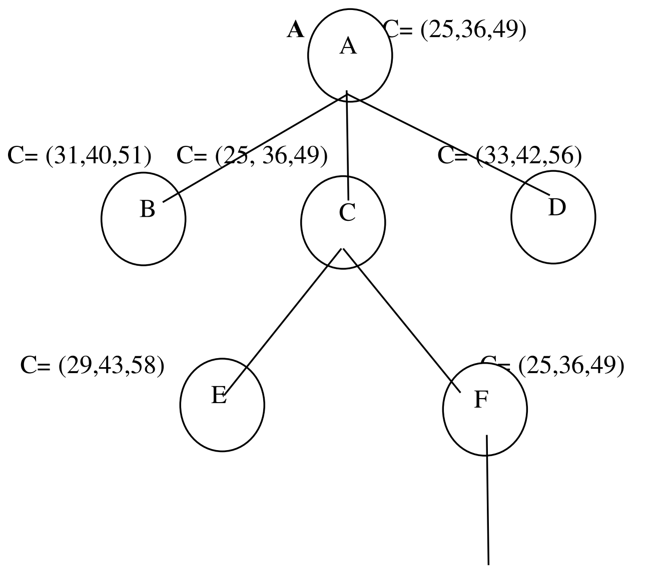
The above matrix is already reduced. So, the reduced cost is

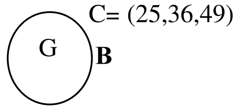
$$C^N(C,B)+R^N+\widehat{R^N}=(29,43,58)$$

Next finding $C^N(C,D)$. Set C,D as ∞.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ (0,0,0) & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ (6,4,2) & (0,0,0) & \infty & \infty \end{bmatrix} \end{array}$$

The above matrix is already reduced. So, the reduced cost is $C^N(C,B)+R^N+\widehat{R^N}=(25,36,49)$





Here, finding $C^N(D,B)$. Set D,B as ∞.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty \\ (0,0,0) & \infty & (0,0,0) & \infty \\ (0,0,0) & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix} \end{array}$$

The above matrix is already reduced. So, the reduced cost is

$$C^N(D,B)+R^N+\widehat{R^N}=(25,36,49)$$

The upper bound becomes $(25,36,49)$ and the branches are terminated. The optimum schedule is to be $A \rightarrow C \rightarrow D \rightarrow B \rightarrow A$ and the optimum cost is $(25,36,49)$.

6. CONCLUSION

In this paper, we discussed a Neutrosophic travelling salesman problem in real life situations, and it is represented by a trapezoidal fuzzy number. We develop a problem to Neutrosophic concept by using a graded mean ranking and applying Branch and Bound technique to find the optimal cost of the travelling salesman problem.

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