Neutrosophic Treatment of the Modified Simplex Algorithm to find the Optimal Solution for Linear Models

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Abstract

Science is the basis for managing the affairs of life and human activities, and living without knowledge is a form of wandering and a kind of loss. Using scientific methods helps us understand the foundations of choice, decision-making, and adopting the right solutions when solutions abound and options are numerous. Operational research is considered the best that scientific development has provided because its methods depend on the application of scientific methods in solving complex issues and the optimal use of available resources in various fields, private and governmental work in peace and war, in politics and economics, in planning and implementation, and in various aspects of life. Its basic essence is to use the data provided for the issue under study to build a mathematical model that is the optimal solution. It is the basis on which decision makers rely in managing institutions and companies, and when operations research methods meet with the neutrosophic teacher, we get ideal solutions that take into account all the circumstances and fluctuations that may occur in the work environment over time. One of the most important operations research methods is the linear programming method. Which prompted us to reformulate the linear models, the graphical method, and the simplex method, which are used to obtain the optimal solution for linear models using the concepts of neutrosophic science. In this research, and as a continuation of what we presented previously, we will reformulate the modified simplex algorithm that was presented to address the difficulty that we were facing when applying the direct simplex algorithm. It is the large number of calculations required to be performed in each step of the solution, which requires a lot of time and effort.

keywords: Linear programming; neutrosophic logic; neutrosophic simplex algorithm; modified neutrosophic simplex algorithm.

1. Introduction:

Operational research is the application of scientific methods to complex issues. It is the art of finding the best solutions and arriving at them through scientific methods so that nothing is left to chance or luck. With the computer, without which it would not have been possible to achieve solutions to many of these issues raised, the use of operations research methods helps us understand the foundations of selection and decision-making and reach agreement on the best ones in order to adopt the correct solutions when solutions abound and options are numerous [1,2,3,4,5], and even the studies presented complete the scientific and practical issues. It is preferable to use the concepts of neutrosophic science, the science that has proven its ability to provide appropriate solutions for all conditions that may occur in the case under study and helps decision makers to adopt ideal decisions that achieve the greatest profit and the lowest cost. Therefore, we have reformulated many research methods. Processes using neutrosophic concepts, see [6-19]. We continue to build neutrosophic processes research by reformulating the modified simplex algorithm and the synthetic algorithm used to find the optimal solution for linear models using neutrosophic concepts.

2.   Discussion:

The neutrosophic linear model is given by the following general formula: [20]

The neutrosophic general form is given the linear mathematical model in the abbreviated form as follows:

$$\text{Max } or \text{ Min } f(x) = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i + \delta_i \quad ; i = 1,2, \dots, m$$

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i + \delta_i \quad ; i = 1,2, \dots, m$$

$$x_j \geq 0 \quad ; j = 1,2, \dots, n$$

Where $Nc_j = c_j \pm \varepsilon_j$, $Nb_i = b_i \pm \delta_i$, a_{ij}, $j = 1,2, \dots, n$, $i = 1,2, \dots, m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables.

It is worthy to mention that the coefficients subscribed by the index N are of neutrosophic values.

The objective function coefficients $Nc_1, Nc_2, \dots, Nc_n$ have neutrosophic meaning are intervals of possible values:

That is, $Nc_j \in [\lambda_{j1}, \lambda_{j2}]$, where $\lambda_{j1}, \lambda_{j2}$ are the upper and the lower bounds of the objective variables x_j respectively, $j = 1,2, \dots, n$. Also, we have the values of the right-hand side of the inequality constraints $Nb_1, Nb_2, \dots, Nb_m$ are regarded as neutrosophic interval values:

$Nb_i \in [\mu_{i1}, \mu_{i2}]$, here, μ_{i1}, μ_{i2} are the upper and the lower bounds of the constraint

$i = 1,2, \dots, m$.

The optimal solution is obtained using the direct simplex method, which is summarized in three basic stages, as we presented: [21]

The first stage: converting the imposed model into the formal form

The second stage: converting the regular form into a basic form to obtain the non-negative basic solutions

The third stage: searching for the ideal solution required among the non-negative basic solutions.

In this research, we will reformulate the modified Simplex algorithm using the concepts of neutrosophic science. The algorithm was presented to address the difficulty that we faced when applying the direct Simplex algorithm, which is the large number of calculations required to be performed in each step of the solution, which requires a lot of time and effort. We explain the steps of the modified simplex algorithm through the following neutrosophic linear mathematical model:

$$Max\ Z = Nc_1 x_1 + Nc_2 x_2 + \dots + Nc_n x_n$$

$$subject\ to\ \begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq Nb_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq Nb_2 \\ a_{31}x_1 + a_{32}x_2 + \dots + a_{3n}x_n \leq Nb_3 \\ \qquad\qquad\qquad\quad . \\ \qquad\qquad\qquad\quad . \\ \qquad\qquad\qquad\quad . \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq Nb_m \end{cases}$$

With the non-negativity conditions $x_1, x_2, \dots, x_n \geq 0$.

To find the optimal solution for this linear neutrosophic model using the modified simplex algorithm.

1-   We write the neutrosophic linear model in standard form, we get the following model:

$$\max Z = c_{1N} x_1 + c_{2N} x_2 + \dots + c_{nN} x_n + 0y_1 + 0y_2 + \dots + 0y_m$$

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$$subject\ to \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + y_1 = b_{1N} \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + y_2 = b_{2N} \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + y_3 = b_{3N} \\ \qquad \qquad \cdot \\ \qquad \qquad \cdot \\ \qquad \qquad \cdot \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + y_m = b_{mN} \end{cases}$$

With the non-negativity conditions $x_1, x_2, \ldots, x_n, y_1, y_2, y_3, \ldots, y_m \geq 0$. The below tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function.

We convert the regular linear model to the basal form and place the coefficients in a short table whose first column includes the basic variables and whose top row includes the non- basic variables only.

- We define the pivot column, which is the column corresponding to the largest positive value in the objective function row because the objective function is a maximization function (but if the objective function is a minimization function, it is the column corresponding to the most negative values). Let this column be the column of the variable x_s.
- We define the pivot row; The pivot row is determined by the following indicator:

$$\theta = min\left[\frac{b_{iN}}{a_{is}}\right] = \frac{b_{tN}}{a_{ts}} > 0; \quad a_{is} > 0, b_{iN} > 0$$

Let this row be the line of the base variable y_t

- Then the fulcrum element is the element resulting from the intersection of the fulcrum column and the fulcrum row, i.e., We explain the second step in the following table:

Table No. 1: Anchor element table   -

Non-Basic Variables / Basic Variables	x_1	x_2	…..	x_s	….	x_n	Nb_i
y_1	a_{11}	a_{12}	……	a_{1s}	…..	a_{1n}	Nb_1
y_2	a_{21}	a_{22}	…….	a_{1s}	….	a_{2n}	Nb_2
……..	…….	……..	…….	……	…..	……	…….
y_t	a_{t1}	a_{t2}	……	a_{ts}	……	a_{tn}	Nb_t
……	……	……	….	…..	…..	……	…….
y_m	a_{m1}	a_{m2}	…….	a_{1s}	….	a_{mn}	Nb_m
NZ	Nc_1	Nc_2	……	Nc_s	…..	Nc_n	$NZ - Nc_o$

We calculate the new elements corresponding to the pivot row and the pivot column according to the following steps:

1- We put opposite the pivot element a_{ts} the reciprocal of $\frac{1}{a_{ts}}$
2- We calculate the elements of the row corresponding to the pivot row (except the pivot row element) by dividing the elements of the pivot row by the anchor element a_{ts}
3- We calculate all the elements of the column opposite the fulcrum (except the fulcrum element) by dividing the elements of the fulcrum column by the fulcrum element a_{ts} and then multiplying them by(-1)
4- We calculate the other elements from the following relationships:

$$a'_{ij} = a_{ij} - a_{tj}\frac{a_{is}}{a_{ts}} = \frac{a_{ij}a_{ts} - a_{tj}a_{is}}{a_{ts}} \qquad (1)$$

$$Nb'_i = Nb_i - Nb_t\frac{a_{is}}{a_{ts}} = \frac{Nb_i a_{ts} - Nb_t a_{is}}{a_{ts}} \qquad (2)$$

$$Nc'_j = Nc_j - Nc_s\frac{a_{tj}}{a_{ts}} = \frac{Nc_j a_{ts} - Nc_s a_{tj}}{a_{ts}} \qquad (3)$$

We get the following table:

Table No. 2: The first stage in searching for the optimal solution

Non-Basic Variables / Basic Variables	x_1	x_2	…..	y_t	….	x_n	Nb'_i

y_1	a'_{11}	a'_{12}	......	$\dfrac{-a_{1s}}{a_{ts}}$	.....	a'_{1n}	Nb'_1
y_2	a'_{21}	a'_{22}	....	$\dfrac{-a_{2s}}{a_{ts}}$	....	a'_{21}	Nb'_2
....	....	....	....	......	.....	......	....
x_s	$\dfrac{a_{t1}}{a_{ts}}$	$\dfrac{a_{t2}}{a_{ts}}$	......	$\dfrac{1}{a_{ts}}$	......	$\dfrac{a_{tn}}{a_{ts}}$	$\dfrac{Nb_t}{a_{ts}}$
......	......	......	....	.....	.....	......	....
y_m	a'_{m1}	a'_{m1}	....	$\dfrac{-a_{ms}}{a_{ts}}$	....	a'_{mn}	Nb'_m
NZ	Nc'_1	Nc'_2	......	$\dfrac{-Nc_s}{a_{ts}}$	.....	Nc'_n	$NZ - Nc'_0$

We apply the stopping criterion of the Simplex algorithm to the objective function row in Table (2) below:
Since the objective function is of the maximize type, the objective function row in the table must not contain any positive value, (but if the objective function is of the minimize type, the objective function row in the new table must not contain any negative value), in the event that the Criterion: We return to step No. (3) and repeat the same steps until the stopping criterion is met and we obtain the desired ideal solution.
We explain the above through the following example: (This example was presented in research [21])

Example:

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Table No. 3: Issue dat

Products / Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	B	
F_1	2	3	[14,20]
F_2	2	1	[10,16]
F_3	0	3	[12,18]
F_4	3	0	[15,21]
Profit Returned per unit	[5,8]	[3,6]	

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible.

Symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2 , the problem will be reformulated from the neutrosophic perspective as follow:

$$\max Z \in [5,8]\, x_1 + [3,6]\, x_2$$

$$subject\ to \begin{cases} 2x_1 + 3x_2 \leq [14,20] \\ 2x_1 + x_2 \leq [10,16] \\ 3x_2 \leq [12,18] \\ 3x_1 \leq [15,21] \end{cases}$$

$$x_1 \geq 0, x_2 \geq 0$$

We apply the modified simplex algorithm:

1- The standard form of the previous linear model is:

$$\max Z \in [5,8]\, x_1 + [3,6]\, x_2 + 0y_1 + 0y_2 + 0y_3 + 0y_4$$

$$subject\ to \begin{cases} 2x_1 + 3x_2 + y_1 = [14,20] \\ 2x_1 + x_2 + y_2 = [10,16] \\ 3x_2 + y_3 = [12,18] \\ 3x_1 + y_4 = [15,21] \end{cases}$$

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$$x_1 \geq 0, x_2 \geq 0, y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$$

2- We organize the previous information in the following modified simplex table:
   Table No. 2: Simplex table according to the modified Neutrosophic simplex algorithm.   -3

Non-basic var. / Basic var.	x_1	x_1	b_i
y_1	2	3	[14,20]
y_2	2	1	[10,16]
y_3	0	3	[12,18]
y_4	3	0	[15,21]
Objective fun.	[5,8]	[3,6]	$Z - 0$

We know $[a, b] \leq [c, d]$ if $a \leq c$ and $b \leq d$,Therefore.

It is clear that $\max([5,8], [3,6]) = [5,8]$ versus to the column of x_1 , meaning that the variable x_1 should be placed instead of one of the basic variables.

Now to demonstrate which basic variables should be ejected, the following calculation has been done:

$$\theta \in min \left[\frac{[14,20]}{2}, \frac{[10,16]}{2}, \frac{[15,21]}{3} \right] = \frac{[15,21]}{3} = [5,7]$$

The value of θ indicates that the row versus to the variable y_4, and the element positioned in the cross row/column is 3 which is the pivot element, divide the elements of the row versus to y_4 yields.

4- We calculate the elements of the new table using relationships (1), (2), (3), we obtain the following table

Table No. 3: Table of the first step in searching for the optimal solution

Non-basic var. / Basic var.	x_1	x_2	b_i'
y_1	$\frac{-2}{3}$	3	[4,6]
y_2	$\frac{-2}{3}$	1	[0,4]
y_3	0	3	[12,18]
x_1	$\frac{1}{3}$	0	[5,7]
Object. Fun.	$\left[\frac{-8}{3}, \frac{-5}{3} \right]$	[3,6]	$Z - [25,56]$

We apply a stopping criterion in the algorithm. We find:

It still there is a non-negative value in the row of the objective function (i.e., [3,6])

This means that we have not yet reached the optimal solution, so we repeat the previous steps as follows:

which is versus to the x_2 column, this leads to the fact that the variable x_2 should be entered into the basic variables. Now, the question is: which basic variable should be ejected?

Track the following calculation to answer this question,

And here too because $[a,b] \le [c,d]$ if $a \le c$ and $b \le d$,Therefore.

$$\theta \in \min\left[\frac{[4,6]}{3}, \frac{[0,4]}{1}, \frac{[12,18]}{3}\right] = \frac{[4,6]}{3} = [\tfrac{4}{3}, 2]$$

which is versus to the slack variable y_1, the pivot element equal 3, hence the row versus to y_1 should be divided by 3 , the required calculations yield the following tableau:

Table No. 4: Final solution table

Non-basic var. / basic var.	x_1	x_2	b_i'
x_2	$\frac{-2}{9}$	$\frac{1}{3}$	$[\tfrac{4}{3}, 2]$
y_2	$\frac{-4}{9}$	$\frac{-1}{3}$	$[\frac{-4}{3}, 2]$
y_3	$\frac{2}{3}$	-1	$[8,12]$
x_1	1	0	$[5,7]$
Object. Fun.	$[-6,-1]$	$[-2,-1]$	$Z - [29,68]$

We apply the algorithm stopping criterion. We find that the criterion has been met and thus we have reached the optimal solution.

The optimal solution for the linear model is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3}, 2\right], y_2^* \in \left[\frac{-4}{3}, 2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

The value of the objective function corresponds to:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$$

It is clear from the row of the objective function that all the elements are neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3}, 2\right], y_2^* \in \left[\frac{-4}{3}, 2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function, yields:

This means that the company must produce quantity $x_1^* \in [5,7]$,of product A and quantity $x_2^* \in \left[\frac{4}{3}, 2\right]$ of product B, thereby achieving a maximum profit of:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$$

It should be noted that the previous problem was solved in the research [21] using the direct simplex algorithm, and the solution tables are as follows:

Non-basic var. / Basic var.	x_1	x_1	y_1	y_2	y_3	y_4	b_i
y_1	2	3	1	0	0	0	$[14,20]$
y_2	2	1	0	1	0	0	$[10,16]$
y_3	0	3	0	0	1	0	$[12,18]$

y_4	3	0	0	0	0	1	[15,21]
Objective fun.	[5,8]	[3,6]	0	0	0	0	$Z-0$

Table No. 5: Simplex table according to the direct neutrosophic simplex algorithm

Table No. 6: Table of the first step in searching for the optimal solution

Non-basic var. / Basic var.	x_1	x_2	y_1	y_2	y_3	y_4	b_i
y_1	0	3	1	0	0	$\dfrac{-2}{3}$	[4,6]
y_2	0	1	0	1	0	$\dfrac{-2}{3}$	[0,4]
y_3	0	3	0	0	1	0	[12,18]
x_1	1	0	0	0	0	$\dfrac{1}{3}$	[5,7]
Object. Fun.	0	[3,6]	0	0	0	$[\dfrac{-8}{3},\dfrac{-5}{3}]$	$Z-[25,56]$

Table No. 7: Final solution table

Non-basic var. / basic var.	x_1	x_2	y_1	y_2	y_3	y_4	b_i
x_2	0	1	$\dfrac{1}{3}$	0	0	$\dfrac{-2}{9}$	$[\dfrac{4}{3},2]$
y_2	0	0	$\dfrac{1}{3}$	1	0	$\dfrac{-4}{9}$	$[\dfrac{-4}{3},2]$
y_3	0	0	-1	0	1	$\dfrac{2}{3}$	[8,12]
x_1	1	0	0	0	0	$\dfrac{1}{3}$	[5,7]
Object. Fun.	0	0	[-2, -1]	0	0	$[\dfrac{-6}{9},-1]$	$Z-[29,68]$

It is clear from the row of the objective function that all the elements are either zero or neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3},2\right], y_2^* \in \left[\frac{-4}{3},2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3},2\right] = [25,56] + [4,12] = [29,68]$$

3. Conclusion and results:

From the previous study, we note that we obtained the same optimal solution that was obtained when we used the direct simplex method, but with a much smaller number of calculations than the number we did in the direct simplex algorithm, as is clear by comparing the solution tables using the modified simplex method, Tables No. (2)., No. (3), No. (4), with solution tables using the direct simplex method, Tables No. (5), No. (6), No. (7). Therefore, in order to shorten time and effort, we focus on the necessity of using the modified Simplex method to find the optimal solution for linear models. Especially when the model contains a large number of variables and constraints.

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