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New multiparametric similarity measure and distance measure for interval neutrosophic set with IoT industry evaluation

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ABSTRACT In the epoch of Internet of Things (IoT), we are confronted five challenges (Connectivity, Value, Security, Telepresence and Intelligence) with complex structures. IoT industry decision making is critically important for countries or societies to enhance the effectiveness and validity of leadership, which can greatly accelerate industrialized and large-scale development. In the case of IoT industry decision evaluation, the essential problem arises serious incompleteness, impreciseness, subjectivity and incertitude. Interval neutrosophic set (INS), disposing the indeterminacy portrayed by truth membership T, indeterminacy membership I, and falsity membership F with interval form, is a more viable and effective means to seize indeterminacy. The main purpose of this paper is to investigate the multiparametric distance measure and similarity measure. Meanwhile, some interesting properties of distance measure and similarity measure are proved. Then, the objective weights of diverse attributes are ascertained by deviation-based method. Also, we explore the combination weight, which reveal both the objective preference and the subjective preference. The validity of algorithm is illustrated by an IoT industry decision making issue, along with the effect of diverse parameters on the ranking. Finally, a comparison of the developed with the existing interval neutrosophic decision making methods has been executed in the light of the counter-intuitive phenomena and unauthentic issue for displaying their effectiveness.

INDEX TERMS Interval neutrosophic set, Similarity measure, Combination weight, Decision making

I. INTRODUCTION

THE Internet of Things (IoT) is deemed to an economic and technology wave of global information industry after the Computer and Internet. It refers to the intelligent wireless network which connects all things to the Internet for the goal of information communicating and data exchanging by sensing devices in keeping with data authentication, and has branched out into the scope of technological capability to data collection, monitoring, automation, sharing, control and collaboration. IoT has revolutionized scientific developments, affected our daily performances, even influenced the planning and policies of the countries. Nevertheless, the influential factors of "IoT" centers on not only the connectivity and intelligence but also on their telepresence and security [1]. In addition, other trait, such as value, is also continually considered [2, 3]. A brief description of these traits is shown in Figure 1.

Presently, IoT has already been applied to numerous fields, such as smart cities [4, 5], healthcare [6, 7], energy [8, 9], agriculture [10, 11], building automation [12, 13], industry [14, 15], transportation [16, 17], military [18, 19] and so on. The Microsoft founder Bill Gates once said, IoT is the future of technology. The tech behemoths want to seek their own business opportunities in the blue sea of the IoT market. Whereas, if they want their business opportunities become market monopoly share, it is not enough to depend on themselves alone. Hence, they had better select a befitting leading company to collaborate with. As the top of the

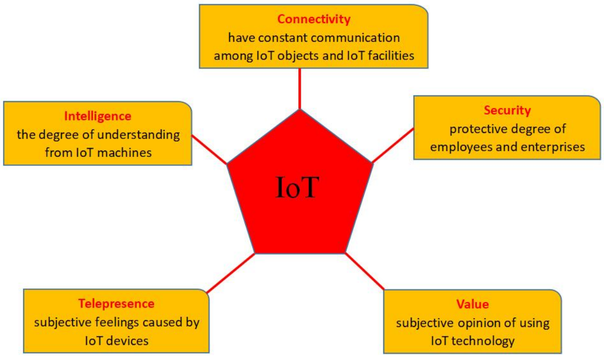

FIGURE 1: The five traits at IoT.

pyramid of the IoT market, Amazon Web Services, ARM, Huawei, Cisco, IBM, Microsoft and Intel are certainly good partners to be considered. Therefore, the idea that regards the process to select the applicable company to collaborate with to the dominating multiple attribute decision making (MADM) problem comes to my mind. However, the increasingly complex decision-making environments and willy-nilly decision makers (DMs) have made it difficult to express decision information with uncertainty in the process of solving the above MADM problems [20–22].

Neutrosophic set (NS), initiatively introduced by Smarandache [23], has regarded as a more efficient tool to describe incongruous and uncertain information in a philosophical point compared with the intuitionistic fuzzy set (IFS) [24]. From scientific point, the neutrosophic set and set-theoretic operators should be prescriptive, or it will have difficult in employing real environments. As a consequence, Wang et al. [25] presented the concept of single valued neutrosophic set (SVNS) as well as some interesting and ground-breaking properties of SVNSs. Up to now, SVNS has drawn much attention and achieved some influential achievements [26–30].

As a matter of fact, the truth membership T, indeterminacy membership I, and falsity membership F of a sure statement cannot be denoted precisely in real environment but are defined by the possible interval form. Therefore, Wang et al. [31] presented the significant theory of interval neutrosophic sets (INSs) and defined the set-theoretic operators of INSs. Pramanik and Mondal [32] stated the interval neutrosophic MADM method based on grey relational analysis (GRA). Peng and Dai [33] presented three MADM methods based on MABAC, similarity measure and EDAS. Broumi and Smarandache [34] investigated the correlation coefficient for INS. Broumi et al. [35] solved the shortest path problem under interval valued neutrosophic setting. Yang et al. [36] introduced the novel MADM method based on linear assignment method under interval neutrosophic environment. Karaşan and Kahraman [37] initiated the interval-valued neutrosophic EDAS method for the prioritization of the UN national sustainable development goals. Bolturk and Kahraman [38] developed the interval-valued neutrosophic method based on AHP with cosine similarity measure. Wang et al. [39] presented the fuzzy stochastic MADM methods with interval neutrosophic probability based on regret theory. Ye [40] presented the subtraction and division operations for INS. Moreover, some applications [41] are focused on the aggregation operators [42–53] and information measures [54–67].

Due to the counterintuitive phenomena [42, 44, 47, 49, 51, 67] and unauthentic issue [62] of the existing interval neutrosophic decision making methods, they may be hard for DMs to choose convincible or optimal alternatives. As a consequence, the goal of this paper is to deal the above issue by proposing a new similarity measure method for INS, which can have without above problems.

For counting the distance measure and similarity measure of two INSs, we introduce a novel way to build the distance measure and similarity measure which depend on three parameters, namely, t_1, t_2 and p, where p is the L_p norm, t_1, t_2 identify the level of uncertainty. Meanwhile, their relation with the similarity measures for INSs are discussed in detail. Moreover, the effect of the different parameters on the ordering of the alternatives are presented.

Considering that diverse attributes' weights would possess influence in ordering results of given alternatives, enlightened by Peng and Dai [33] and Zhang et al. [59], we also propose a fire-new approach to calculate the attributes' weight by uniting the subjective ones with the objective ones. The proposed model is different from the existing interval neutrosophic weight determining approaches, which can be classed as two sides: (1) subjective weighting determine approaches and (2) the objective weighting determining approaches, which can be calculated by bran-new deviation-based method. Some subjective weighting approaches place emphasis on the preference of the decision makers [42, 44–52], but they neglect the objective evaluation information. Meanwhile, the objective weighting determining approaches don't take the information of experts into account, that is to say, these approaches cannot take the preference of the experts into consideration [68]. The characteristic of the proposed weighting model can reveal the subjective preference information and the objective preference information simultaneously. For this reason, uniting objective weight and subjective weight, a combination model for achieving attributes' weights is developed.

To obtain these goals, the major research contributions are listed in the following:

(i) The novel similarity measure and distance measure for INSs is presented. The effect of the three parameters (t_1, t_2, p) on the ordering of the alternatives are presented.

(ii) A comparison between the initiated and the existing similarity measures [54, 55, 57, 58, 60–66]. has been executed in the light of counter-intuitive examples for showing its viability and effectiveness.

(iii) The novel weight model is presented for averting effect of objective aspect and subjective aspect.

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(iv) The proposed algorithm with some existing decision making algorithms [42, 44, 47, 49, 51, 62, 67] are compared by some examples.

The rest of this paper is listed as follows: In Section 2, the basic notions of NS, SVNS and INSs are briefly retrospected, which will be employed in the analysis throughout this paper. In Section 3, some new distance measures and similarity measures are proposed and proved. Meanwhile, a comparison between the initiated and the existing measures is executed in the light of counter-intuitive examples. In Section 4, an algorithm for INS based decision making are shown and the effect of the different parameters on the ordering of the objects are discussed in detail. In Section 5, a comparison with some existing algorithms are examined. The paper is concluded in Section 6.

II. PRELIMINARIES

In this section, we first recall some basic ideas of NS, SVNS, INS and their properties.

A. NEUTROSOPHIC SET AND SINGLE-VALUED NEUTROSOPHIC SET

Definition 1: [23] Let X be the universe of discourse, with a mass of elements in X presented by x. A neutrosophic set B in X is summarized as a truth membership function $T_B(x)$, an indeterminacy membership function $I_B(x)$, and a falsity membership function $F_B(x)$. The functions $T_B(x), I_B(x)$, and $F_B(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_B(x) : X \to]0^-, 1^+[$, $I_B(x) : X \to]0^-, 1^+[$, and $F_B(x) : X \to]0^-, 1^+[$.

There is limiting condition on the sum of $T_B(x), I_B(x)$, and $F_B(x)$, so $0^- \leq \sup T_B(x) + \sup I_B(x) + \sup F_B(x) \leq 3^+$.

Definition 2: [25] Let X be the universe of discourse, with a mass of elements in X presented by x. A SVNS N in X is summarized as a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$. Then, the SVNS N can be presented as follows:

$$N = \{< x, T_N(x), I_N(x), F_N(x) >|\ x \in X\}, \quad (1)$$

where $T_N(x), I_N(x), F_N(x) \in [0, 1]$ for $\forall x \in X$. Meanwhile, the sum of $T_N(x), I_N(x)$, and $F_N(x)$ fulfills the condition $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$. For a SVNS N in X, the triplet $(T_N(x), I_N(x), F_N(x))$ is called single-valued neutrosophic number (SVNN). As a matter of convenience, we can simply use $x = (T_x, I_x, F_x)$ to denote a SVNN as an element in the SVNS N.

B. INTERVAL NEUTROSOPHIC SET

Interval neutrosophic set (INS), initiatively introduced by Wang et al. [31], has regarded as a more efficient tool to describe incongruous information in a philosophical point compared with the interval-valued intuitionistic fuzzy set (IVIFS) and SVNS. Wang et al. [31] introduced the definition of INS as follows:

Definition 3: [31] Let X be a universe of discourse, with a class of elements in X denoted by x. An INS A in X is summarized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then an INS A can be denoted as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >|\ x \in X\}. \quad (2)$$

For each point x in X, $T_A(x) = [T_A^L(x), T_A^U(x)], I_A(x) = [I_A^L(x), I_A^U(x)], F_A(x) = [F_A^L(x), F_A^U(x)] \subseteq [0, 1]$, and $0 \leq T_A^U(x) + I_A^U(x) + F_A^U(x) \leq 3$. For convenience, Peng and Dai [33] can simply use $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ to represent an INN as an element in the INS A.

Definition 4: [31] An INS N is contained in other INS M, $N \subseteq M$ if and only if $T_N^L(x) \leq T_M^L(x), T_N^U(x) \leq T_M^U(x), I_N^L(x) \geq I_M^L(x), I_N^U(x) \geq I_M^U(x), F_N^L(x) \geq F_M^L(x), F_N^U(x) \geq F_M^U(x)$ for $\forall x$.

Definition 5: [40, 42] Let $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two INNs, and $\lambda > 0$, then the operations for the INNs are defined as follows:

(1) $\lambda x_1 = ([1-(1-T_1^L)^\lambda, 1-(1-T_1^U)^\lambda], [(I_1^L)^\lambda, (I_1^U)^\lambda], [(F_1^L)^\lambda, (F_1^U)^\lambda]);$

(2) $x_1^\lambda = ([(T_1^L)^\lambda, (T_1^U)^\lambda], [1 - (1 - I_1^L)^\lambda, 1 - (1 - I_1^U)^\lambda], [1 - (1 - F_1^L)^\lambda, 1 - (1 - F_1^U)^\lambda]);$

(3) $x_1 \bigoplus x_2 = ([T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L * I_2^L, I_1^U * I_2^U], [F_1^L * F_2^L, F_1^U * F_2^U]);$

(4) $x_1 \bigotimes x_2 = ([T_1^L * T_2^L, T_1^U * T_2^U], [I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U]);$

(5) $x_1^c = ([F_1^L, F_1^U], [1 - I_1^U, 1 - I_1^L], [T_1^L, T_1^U]).$

Theorem 1: [42] Let $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two INNs, and $\lambda, \lambda_1, \lambda_2 > 0$, then we have

(1) $x_1 \bigoplus x_2 = x_2 \bigoplus x_1;$

(2) $x_1 \bigotimes x_2 = x_2 \bigotimes x_1;$

(3) $\lambda(x_1 \bigoplus x_2) = \lambda x_1 \bigoplus \lambda x_2;$

(4) $(x_1 \bigotimes x_2)^\lambda = x_1^\lambda \bigotimes x_2^\lambda;$

(5) $\lambda_1 x_1 \bigoplus \lambda_2 x_1 = (\lambda_1 + \lambda_2) x_1;$

(6) $x_1^{\lambda_1} \bigotimes x_1^{\lambda_2} = x_1^{\lambda_1 + \lambda_2}.$

Definition 6: [57] Let $x_1 = ([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])$ and $x_2 = ([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])$ be two INNs, then the Hamming distance between x_1 and x_2 can be defined as follows:

$$d_h(x_1, x_2) = \frac{1}{6} \begin{pmatrix} |\ T_1^L - T_2^L\ | + |\ T_1^U - T_2^U\ | + \\ |\ I_1^L - I_2^L\ | + |\ I_1^U - I_2^U\ | + \\ |\ F_1^L - F_2^L\ | + |\ F_1^U - F_2^U\ | \end{pmatrix}. \quad (3)$$

Definition 7: [33] Let $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ be an INN, then the proposed score function $s(x)$ is defined as follows:

$$s(x) = \frac{2}{3} + \frac{T^L + T^U}{6} - \frac{I^L + I^U}{6} - \frac{F^L + F^U}{6}. \tag{4}$$

It measures the Hamming similarity $(1 - d_h(x, x^*))$ between $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ and the ideal solution $x^* = ([1, 1], [0, 0], [0, 0])$ for the comparison of INNs.

Example 1: Suppose that $x_1 = ([0, 0], [0, 0], [1, 1])$ and $x_2 = ([0, 0], [1, 1], [0, 0])$. If we utilize the score function of Peng and Dai [33] to compare, we can have $s(x_1) = s(x_2) = \frac{1}{3}$. As a matter of fact, $x_1 \neq x_2$.

Notice the deficiencies of the score function s of Peng and Dai [33], we add an accuracy function in the following.

Definition 8: Let $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ be an INN, then the accuracy function $a(x)$ is defined as follows:

$$a(x) = T^L + T^U - F^L - F^U. \tag{5}$$

For any two INNs x, y,
(1) if $s_(x) > s(y)$, then $x \succ y$;
(2) if $s(x) = s(y)$ and $a(x) > a(y)$, then $x \succ y$.

Definition 9: [55] Let M, N and O be three INSs on X. A distance measure $D(M, N)$ is a mapping D : INS$(X) \times$ INS$(X) \to [0, 1]$, possessing the following properties:

$(D1)$ $0 \leq D(M, N) \leq 1$;
$(D2)$ $D(M, N) = 0$, iff $M = N$;
$(D3)$ $D(M, N) = D(N, M)$;
$(D4)$ If $M \subseteq N \subseteq O$, then $D(M, N) \leq D(M, O)$ and $D(N, O) \leq D(M, O)$.

Definition 10: [55] Let M, N and O be three INSs on X. A similarity measure $S(M, N)$ is a mapping S : INS$(X) \times$ INS$(X) \to [0, 1]$, possessing the following properties:
$(S1)$ $0 \leq S(M, N) \leq 1$;
$(S2)$ $S(M, N) = 1$, iff $M = N$;
$(S3)$ $S(M, N) = S(N, M)$;
$(S4)$ If $M \subseteq N \subseteq O$, then $S(M, N) \geq S(M, O)$ and $S(N, O) \geq S(M, O)$.

III. SOME NEW TYPES OF INFORMATION MEASURE BETWEEN INSS

Theorem 2: Let M and N be two INSs in X where $X = \{x_1, x_2, \cdots, x_n\}$, then $D(M, N)$ is the distance measure between two INSs M and N in X.

$$D(M,N) = \sqrt[p]{\begin{array}{l} \frac{1}{6n(t_1+2)^p} \sum_{i=1}^{n} \begin{pmatrix} |-t_1(T_M^L(x_i) - T_N^L(x_i)) + \\ (I_M^L(x_i) - I_N^L(x_i)) + \\ (F_M^L(x_i) - F_N^L(x_i))|^p + \\ |-t_1(T_M^U(x_i) - T_N^U(x_i)) + \\ (I_M^U(x_i) - I_N^U(x_i)) + \\ (F_M^U(x_i) - F_N^U(x_i))|^p \end{pmatrix} + \\ \frac{1}{6n(t_2+2)^p} \sum_{i=1}^{n} \begin{pmatrix} |-t_2(I_M^L(x_i) - I_N^L(x_i)) - \\ (F_M^L(x_i) - F_N^L(x_i)) + \\ (T_M^L(x_i) - T_N^L(x_i))|^p + \\ |-t_2(I_M^U(x_i) - I_N^U(x_i)) - \\ (F_M^U(x_i) - F_N^U(x_i)) + \\ (T_M^U(x_i) - T_N^U(x_i))|^p + \\ |-t_2(F_M^L(x_i) - F_N^L(x_i)) - \\ (I_M^L(x_i) - I_N^L(x_i)) + \\ (T_M^L(x_i) - T_N^L(x_i))|^p + \\ |-t_2(F_M^U(x_i) - F_N^U(x_i)) - \\ (I_M^U(x_i) - I_N^U(x_i)) + \\ (T_M^U(x_i) - T_N^U(x_i))|^p \end{pmatrix} \end{array}} \tag{6}$$

where p is the L_p norm, and t_1 and t_2 denote the level of uncertainty with the condition $t_1, t_2 \geq 0$.

Proof:

$(D1)$ For two INSs M and N, we have $0 \leq T_M^L(x_i), T_N^L(x_i), I_M^L(x_i), I_N^L(x_i), F_M^L(x_i), F_N^L(x_i), T_M^U(x_i), T_N^U(x_i), I_M^U(x_i), I_N^U(x_i), F_M^U(x_i), F_N^U(x_i) \leq 1$.

Therefore, $|T_M^L(x_i) - T_N^L(x_i)| \leq 1, |I_M^L(x_i) - I_N^L(x_i)| \leq 1, |F_M^L(x_i) - F_N^L(x_i)| \leq 1, |T_M^U(x_i) - T_N^U(x_i)| \leq 1, |I_M^U(x_i) - I_N^U(x_i)| \leq 1, |F_M^U(x_i) - F_N^U(x_i)| \leq 1$ and $|t_1(T_M^L(x_i) - T_N^L(x_i))| \leq t_1, |t_2(I_M^L(x_i) - I_N^L(x_i))| \leq t_2, |t_2(F_M^L(x_i) - F_N^L(x_i))| \leq t_2, |t_1(T_M^U(x_i) - T_N^U(x_i))| \leq t_1, |t_2(I_M^U(x_i) - I_N^U(x_i))| \leq t_2, |t_2(F_M^U(x_i) - F_N^U(x_i))| \leq t_2$.

Thus, we can achieve

$-(t_1 + 2) \leq -t_1(T_M^L(x_i) - T_N^L(x_i)) + (I_M^L(x_i) - I_N^L(x_i)) + (F_M^L(x_i) - F_N^L(x_i)) \leq t_1 + 2$,

$-(t_1 + 2) \leq -t_1(T_M^U(x_i) - T_N^U(x_i)) + (I_M^U(x_i) - I_N^U(x_i)) + (F_M^U(x_i) - F_N^U(x_i)) \leq t_1 + 2$,

$-(t_2 + 2) \leq -t_2(I_M^L(x_i) - I_N^L(x_i)) - (F_M^L(x_i) - F_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i)) \leq t_2 + 2$,

$-(t_2 + 2) \leq -t_2(I_M^U(x_i) - I_N^U(x_i)) - (F_M^U(x_i) - F_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i)) \leq t_2 + 2$,

$-(t_2 + 2) \leq -t_2(F_M^L(x_i) - F_N^L(x_i)) - (I_M^L(x_i) - I_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i)) \leq t_2 + 2$,

$-(t_2 + 2) \leq -t_2(F_M^U(x_i) - F_N^U(x_i)) - (I_M^U(x_i) - I_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i)) \leq t_2 + 2$.

It means that

$0 \leq |-t_1(T_M^L(x_i) - T_N^L(x_i)) + (I_M^L(x_i) - I_N^L(x_i)) + (F_M^L(x_i) - F_N^L(x_i))|^p \leq (t_1 + 2)^p$,

$0 \leq |-t_1(T_M^U(x_i) - T_N^U(x_i)) + (I_M^U(x_i) - I_N^U(x_i)) + (F_M^U(x_i) - F_N^U(x_i))|^p \leq (t_1 + 2)^p$,

$0 \leq |-(t_2 + 2) \leq -t_2(I_M^L(x_i) - I_N^L(x_i)) - (F_M^L(x_i) - F_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq (t_2 + 2)^p$,

$0 \leq |-(t_2 + 2) \leq -t_2(I_M^U(x_i) - I_N^U(x_i)) - (F_M^U(x_i) - F_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq (t_2 + 2)^p$,

$0 \leq |-(t_2 + 2) \leq -t_2(I_M^L(x_i) - I_N^L(x_i)) - (F_M^L(x_i) - F_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq (t_2 + 2)^p$,

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$0 \leq |-(t_2+2) \leq -t_2(I_M^U(x_i) - I_N^U(x_i)) - (F_M^U(x_i) - F_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq (t_2+2)^p$.

Consequently, by adding above inequalities, we can achieve $0 \leq D(M,N) \leq 1$.

$(D2)$ Let M and N be two INSs. If $M = N$, then $T_M^L(x_i) = T_N^L(x_i), T_M^U(x_i) = T_N^U(x_i), I_M^L(x_i) = I_N^L(x_i), I_M^U(x_i) = I_N^U(x_i), F_M^L(x_i) = F_N^L(x_i), F_M^U(x_i) = F_N^U(x_i)$.

Thus, the distance measure $D(M,N) = 0$.

Conversely, if $D(M,N) = 0$, then

$|-t_1(T_M^L(x_i)-T_N^L(x_i))+(I_M^L(x_i)-I_N^L(x_i))+(F_M^L(x_i)-F_N^L(x_i))| = 0$,

$|-t_1(T_M^U(x_i)-T_N^U(x_i))+(I_M^U(x_i)-I_N^U(x_i))+(F_M^U(x_i)-F_N^U(x_i))| = 0$,

$|-t_2(I_M^L(x_i)-I_N^L(x_i))-(F_M^L(x_i)-F_N^L(x_i))+(T_M^L(x_i)-T_N^L(x_i))| = 0$,

$|-t_2(I_M^U(x_i)-I_N^U(x_i))-(F_M^U(x_i)-F_N^U(x_i))+(T_M^U(x_i)-T_N^U(x_i))| = 0$,

$|-t_2(F_M^L(x_i)-F_N^L(x_i))-(I_M^L(x_i)-I_N^L(x_i))+(T_M^L(x_i)-T_N^L(x_i))| = 0$,

$|-t_2(F_M^U(x_i)-F_N^U(x_i))-(I_M^U(x_i)-I_N^U(x_i))+(T_M^U(x_i)-T_N^U(x_i))| = 0$.

Hence, after solving, we can have $T_M^L(x_i) = T_N^L(x_i), T_M^U(x_i) = T_N^U(x_i)$, $I_M^L(x_i) = I_N^L(x_i), I_M^U(x_i) = I_N^U(x_i), F_M^L(x_i) = F_N^L(x_i), F_M^U(x_i) = F_N^U(x_i)$.

Consequently, $M = N$.

$(D3)$ Let M and N be two INSs. We can rewrite the following equations:

$|-t_1(T_M^L(x_i)-T_N^L(x_i))+(I_M^L(x_i)-I_N^L(x_i))+(F_M^L(x_i)-F_N^L(x_i))|^p$
$= |(-1)\{-t_1(T_N^L(x_i) - T_M^L(x_i)) + (I_N^L(x_i) - I_M^L(x_i)) + (F_N^L(x_i) - F_M^L(x_i))\}|^p$
$= |-t_1(T_N^L(x_i)-T_M^L(x_i))+(I_N^L(x_i)-I_M^L(x_i))+(F_N^L(x_i)-F_M^L(x_i))|^p$,

$|-t_1(T_M^U(x_i)-T_N^U(x_i))+(I_M^U(x_i)-I_N^U(x_i))+(F_M^U(x_i)-F_N^U(x_i))|^p$
$= |(-1)\{-t_1(T_N^U(x_i) - T_M^U(x_i)) + (I_N^U(x_i) - I_M^U(x_i)) + (F_N^U(x_i) - F_M^U(x_i))\}|^p$
$= |-t_1(T_N^U(x_i)-T_M^U(x_i))+(I_N^U(x_i)-I_M^U(x_i))+(F_N^U(x_i)-F_M^U(x_i))|^p$,

$|-t_2(I_M^L(x_i)-I_N^L(x_i))-(F_M^L(x_i)-F_N^L(x_i))+(T_M^L(x_i)-T_N^L(x_i))|^p$
$= |(-1)\{-t_2(I_M^L(x_i) - I_N^L(x_i)) - (F_M^L(x_i) - F_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))\}|^p$
$= |-t_2(I_N^L(x_i)-I_M^L(x_i))-(F_N^L(x_i)-F_M^L(x_i))+(T_N^L(x_i)-T_M^L(x_i))|^p$,

$|-t_2(I_M^U(x_i)-I_N^U(x_i))-(F_M^U(x_i)-F_N^U(x_i))+(T_M^U(x_i)-T_N^U(x_i))|^p$
$= |(-1)\{-t_2(I_M^U(x_i) - I_N^U(x_i)) - (F_M^U(x_i) - F_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))\}|^p$
$= |-t_2(I_N^U(x_i)-I_M^U(x_i))-(F_N^U(x_i)-F_M^U(x_i))+(T_N^U(x_i)-T_M^U(x_i))|^p$,

$|-t_2(F_M^L(x_i)-F_N^L(x_i))-(I_M^L(x_i)-I_N^L(x_i))+(T_M^L(x_i)-T_N^L(x_i))|^p$
$= |(-1)\{-t_2(F_M^L(x_i) - F_N^L(x_i)) - (I_M^L(x_i) - I_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))\}|^p$
$= |-t_2(F_N^L(x_i)-F_M^L(x_i))-(I_N^L(x_i)-I_M^L(x_i))+(T_N^L(x_i)-T_M^L(x_i))|^p$,

$|-t_2(F_M^U(x_i)-F_N^U(x_i))-(I_M^U(x_i)-I_N^U(x_i))+(T_M^U(x_i)-T_N^U(x_i))|^p$
$= |(-1)\{-t_2(F_M^U(x_i) - F_N^U(x_i)) - (I_M^U(x_i) - I_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))\}|^p$
$= |-t_2(F_N^U(x_i)-F_M^U(x_i))-(I_N^U(x_i)-I_M^U(x_i))+(T_N^U(x_i)-T_M^U(x_i))|^p$.

There, $D(M,N) = D(N,M)$.

$(D4)$ Let M, N, and O be three INSs. The distance measures between M and N, and M and O are the followings:

$$D(M,N) = \sqrt[p]{\begin{array}{l} \frac{1}{6n(t_1+2)^p}\sum_{i=1}^{n}\left(\begin{array}{r} |-t_1(T_M^L(x_i)-T_N^L(x_i))+ \\ (I_M^L(x_i)-I_N^L(x_i))+ \\ (F_M^L(x_i)-F_N^L(x_i))|^p+ \\ |-t_1(T_M^U(x_i)-T_N^U(x_i))+ \\ (I_M^U(x_i)-I_N^U(x_i))+ \\ (F_M^U(x_i)-F_N^U(x_i))|^p \end{array}\right)+ \\ \frac{1}{6n(t_2+2)^p}\sum_{i=1}^{n}\left(\begin{array}{r} |-t_2(I_M^L(x_i)-I_N^L(x_i))- \\ (F_M^L(x_i)-F_N^L(x_i))+ \\ (T_M^L(x_i)-T_N^L(x_i))|^p+ \\ |-t_2(I_M^U(x_i)-I_N^U(x_i))- \\ (F_M^U(x_i)-F_N^U(x_i))+ \\ (T_M^U(x_i)-T_N^U(x_i))|^p+ \\ |-t_2(F_M^L(x_i)-F_N^L(x_i))- \\ (I_M^L(x_i)-I_N^L(x_i))+ \\ (T_M^L(x_i)-T_N^L(x_i))|^p+ \\ |-t_2(F_M^U(x_i)-F_N^U(x_i))- \\ (I_M^U(x_i)-I_N^U(x_i))+ \\ (T_M^U(x_i)-T_N^U(x_i))|^p \end{array}\right) \end{array}}$$

$$D(M,O) = \sqrt[p]{\begin{array}{l} \frac{1}{6n(t_1+2)^p}\sum_{i=1}^{n}\left(\begin{array}{r} |-t_1(T_M^L(x_i)-T_O^L(x_i))+ \\ (I_M^L(x_i)-I_O^L(x_i))+ \\ (F_M^L(x_i)-F_O^L(x_i))|^p+ \\ |-t_1(T_M^U(x_i)-T_O^U(x_i))+ \\ (I_M^U(x_i)-I_O^U(x_i))+ \\ (F_M^U(x_i)-F_O^U(x_i))|^p \end{array}\right)+ \\ \frac{1}{6n(t_2+2)^p}\sum_{i=1}^{n}\left(\begin{array}{r} |-t_2(I_M^L(x_i)-I_O^L(x_i))- \\ (F_M^L(x_i)-F_O^L(x_i))+ \\ (T_M^L(x_i)-T_O^L(x_i))|^p+ \\ |-t_2(I_M^U(x_i)-I_O^U(x_i))- \\ (F_M^U(x_i)-F_O^U(x_i))+ \\ (T_M^U(x_i)-T_O^U(x_i))|^p+ \\ |-t_2(F_M^L(x_i)-F_O^L(x_i))- \\ (I_M^L(x_i)-I_O^L(x_i))+ \\ (T_M^L(x_i)-T_O^L(x_i))|^p+ \\ |-t_2(F_M^U(x_i)-F_O^U(x_i))- \\ (I_M^U(x_i)-I_O^U(x_i))+ \\ (T_M^U(x_i)-T_O^U(x_i))|^p \end{array}\right) \end{array}}$$

If $M \subseteq N \subseteq O$, then $T_M^L(x_i) \leq T_N^L(x_i) \leq T_O^L(x_i), I_M^L(x_i) \geq I_N^L(x_i) \geq I_O^L(x_i), T_M^U(x_i) \leq T_N^U(x_i) \leq T_O^U(x_i), I_M^U(x_i) \geq I_N^U(x_i) \geq I_O^U(x_i), F_M^L(x_i) \geq F_N^L(x_i) \geq F_O^L(x_i)$, and $F_M^U(x_i) \geq F_N^U(x_i) \geq F_O^U(x_i)$.

Therefore,

$T_M^L(x_i) - T_N^L(x_i) \geq T_M^L(x_i) - T_O^L(x_i), T_M^U(x_i) - T_N^U(x_i) \geq T_M^U(x_i) - T_O^U(x_i)$,

$I_M^L(x_i) - I_N^L(x_i) \leq I_M^L(x_i) - I_O^L(x_i), I_M^U(x_i) - I_N^U(x_i) \leq I_M^U(x_i) - I_O^U(x_i)$,

$F_M^L(x_i) - F_N^L(x_i) \leq F_M^L(x_i) - F_O^L(x_i), F_M^U(x_i) - F_N^U(x_i) \leq F_M^U(x_i) - F_O^U(x_i)$.

Further, we can obtain

$|-t_1(T_M^L(x_i) - T_N^L(x_i)) + (I_M^L(x_i) - I_N^L(x_i)) + (F_M^L(x_i) - F_N^L(x_i))|^p \leq |-t_1(T_M^L(x_i) - T_O^L(x_i)) + (I_M^L(x_i) - I_O^L(x_i)) + (F_M^L(x_i) - F_O^L(x_i))|^p$,

$|-t_1(T_M^U(x_i) - T_N^U(x_i)) + (I_M^U(x_i) - I_N^U(x_i)) + (F_M^U(x_i) - F_N^U(x_i))|^p \leq |-t_1(T_M^U(x_i) - T_O^U(x_i)) + (I_M^U(x_i) - I_O^U(x_i)) + (F_M^U(x_i) - F_O^U(x_i))|^p$,

$|-t_2(I_M^L(x_i) - I_N^L(x_i)) - (F_M^L(x_i) - F_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq |-t_2(I_M^L(x_i) - I_O^L(x_i)) - (F_M^L(x_i) - F_O^L(x_i)) + (T_M^L(x_i) - T_O^L(x_i))|^p$,

$|-t_2(I_M^U(x_i) - I_N^U(x_i)) - (F_M^U(x_i) - F_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq | - t_2(I_M^U(x_i) - I_O^U(x_i)) - (F_M^U(x_i) - F_O^U(x_i)) + (T_M^U(x_i) - T_O^U(x_i))|^p$,

$|-t_2(F_M^L(x_i) - F_N^L(x_i)) - (I_M^L(x_i) - I_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq |-t_2(F_M^L(x_i) - F_O^L(x_i)) - (I_M^L(x_i) - I_O^L(x_i)) + (T_M^L(x_i) - T_O^L(x_i))|^p$,

$|-t_2(F_M^U(x_i) - F_N^U(x_i)) - (I_M^U(x_i) - I_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq | - t_2(F_M^U(x_i) - F_O^U(x_i)) - (I_M^U(x_i) - I_O^U(x_i)) + (T_M^U(x_i) - T_O^U(x_i))|^p$.

Thus, we can achieve $D(M,N) \leq D(M,O)$.

Similarly, $D(N,O) \leq D(M,O)$.

However, in most real environment, the diverse sets may possess diverse weights. Therefore, the weight $w_i (i = 1, 2, \cdots, n)$ of the element $x_i \in X$ should be taken into account. We present a weighted distance measure $D^w(M,N)$ between INSs in the following.

$$D^w(M,N) = \sqrt[p]{\begin{array}{l} \frac{1}{6n(t_1+2)^p} \sum_{i=1}^{n} w_i \begin{pmatrix} |-t_1(T_M^L(x_i) - T_N^L(x_i)) + \\ (I_M^L(x_i) - I_N^L(x_i)) + \\ (F_M^L(x_i) - F_N^L(x_i))|^p + \\ |-t_1(T_M^U(x_i) - T_N^U(x_i)) + \\ (I_M^U(x_i) - I_N^U(x_i)) + \\ (F_M^U(x_i) - F_N^U(x_i))|^p \end{pmatrix} + \\ \frac{1}{6n(t_2+2)^p} \sum_{i=1}^{n} w_i \begin{pmatrix} |-t_2(I_M^L(x_i) - I_N^L(x_i)) - \\ (F_M^L(x_i) - F_N^L(x_i)) + \\ (T_M^L(x_i) - T_N^L(x_i))|^p + \\ |-t_2(I_M^U(x_i) - I_N^U(x_i)) - \\ (F_M^U(x_i) - F_N^U(x_i)) + \\ (T_M^U(x_i) - T_N^U(x_i))|^p + \\ |-t_2(F_M^L(x_i) - F_N^L(x_i)) - \\ (I_M^L(x_i) - I_N^L(x_i)) + \\ (T_M^L(x_i) - T_N^L(x_i))|^p + \\ |-t_2(F_M^U(x_i) - F_N^U(x_i)) - \\ (I_M^U(x_i) - I_N^U(x_i)) + \\ (T_M^U(x_i) - T_N^U(x_i))|^p \end{pmatrix} \end{array}} \quad (7)$$

where $p > 0, t_1, t_2 \geq 0$, and w_i is the weights of the element x_i with $\sum_{i=1}^n w_i = 1$.

Theorem 3: Let M and N be two INSs in X where $X = \{x_1, x_2, \cdots, x_n\}$, then $D^w(M,N)$ is the distance measure between two INSs M and N in X.

Proof:

$(D1)$ If we product the inequality defined above with w_i, then we can easily have

$0 \leq w_i| - t_1(T_M^L(x_i) - T_N^L(x_i)) + (I_M^L(x_i) - I_N^L(x_i)) + (F_M^L(x_i) - F_N^L(x_i))|^p \leq w_i(t_1+2)^p$,

$0 \leq w_i| - t_1(T_M^U(x_i) - T_N^U(x_i)) + (I_M^U(x_i) - I_N^U(x_i)) + (F_M^U(x_i) - F_N^U(x_i))|^p \leq w_i(t_1+2)^p$,

$0 \leq w_i| - t_2(I_M^L(x_i) - I_N^L(x_i)) - (F_M^L(x_i) - F_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq w_i(t_2+2)^p$,

$0 \leq w_i| - t_2(I_M^U(x_i) - I_N^U(x_i)) - (F_M^U(x_i) - F_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq w_i(t_2+2)^p$,

$0 \leq w_i| - t_2(F_M^L(x_i) - F_N^L(x_i)) - (I_M^L(x_i) - I_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq w_i(t_2+2)^p$,

$0 \leq w_i| - t_2(F_M^U(x_i) - F_N^U(x_i)) - (I_M^U(x_i) - I_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq w_i(t_2+2)^p$.

Furthermore, we can write the following inequality

$0 \leq \sum_{i=1}^{n} w_i|-t_1(T_M^L(x_i) - T_N^L(x_i)) + (I_M^L(x_i) - I_N^L(x_i)) + (F_M^L(x_i) - F_N^L(x_i))|^p \leq (t_1+2)^p$,

$0 \leq \sum_{i=1}^{n} w_i|-t_1(T_M^U(x_i) - T_N^U(x_i)) + (I_M^U(x_i) - I_N^U(x_i)) + (F_M^U(x_i) - F_N^U(x_i))|^p \leq (t_1+2)^p$,

$0 \leq \sum_{i=1}^{n} w_i|-t_2(I_M^L(x_i) - I_N^L(x_i)) - (F_M^L(x_i) - F_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq (t_2+2)^p$,

$0 \leq \sum_{i=1}^{n} w_i|-t_2(I_M^U(x_i) - I_N^U(x_i)) - (F_M^U(x_i) - F_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq (t_2+2)^p$,

$0 \leq \sum_{i=1}^{n} w_i|-t_2(F_M^L(x_i) - F_N^L(x_i)) - (I_M^L(x_i) - I_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq (t_2+2)^p$.

$0 \leq \sum_{i=1}^{n} w_i|-t_2(F_M^U(x_i) - F_N^U(x_i)) - (I_M^U(x_i) - I_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq (t_2+2)^p$.

It is easy to know that $\sum_{i=1}^{n} w_i(t_1+2)^p$ or $\sum_{i=1}^{n} w_i(t_2+2)^p$ is equal to $(t_1+2)^p$ or $(t_2+2)^p$ since $\sum_{i=1}^{n} w_i = 1$.

Hence,

$0 \leq \sum_{i=1}^{n} w_i|-t_1(T_M^L(x_i) - T_N^L(x_i)) + (I_M^L(x_i) - I_N^L(x_i)) + (F_M^L(x_i) - F_N^L(x_i))|^p \leq (t_1+2)^p, 0 \leq \sum_{i=1}^{n} w_i|-t_1(T_M^U(x_i) - T_N^U(x_i)) + (I_M^U(x_i) - I_N^U(x_i)) + (F_M^U(x_i) - F_N^U(x_i))|^p \leq (t_1+2)^p, 0 \leq \sum_{i=1}^{n} w_i| - t_2(I_M^L(x_i) - I_N^L(x_i)) - (F_M^L(x_i) - F_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq (t_2+2)^p, 0 \leq \sum_{i=1}^{n} w_i| - t_2(I_M^U(x_i) - I_N^U(x_i)) - (F_M^U(x_i) - F_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq (t_2+2)^p, 0 \leq \sum_{i=1}^{n} w_i|-t_2(F_M^L(x_i) - F_N^L(x_i)) - (I_M^L(x_i) - I_N^L(x_i)) + (T_M^L(x_i) - T_N^L(x_i))|^p \leq (t_2+2)^p$,
$0 \leq \sum_{i=1}^{n} w_i| - t_2(F_M^U(x_i) - F_N^U(x_i)) - (I_M^U(x_i) - I_N^U(x_i)) + (T_M^U(x_i) - T_N^U(x_i))|^p \leq (t_2+2)^p$.

Consequently, by the above Eq (7), we can obtain $0 \leq D^w(M,N) \leq 1$.

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$(D2)-(D4)$ It is straightforward.

Theorem 4: If $D(M,N)$ and $D^w(M,N)$ are distance measures between INSs M and N, then $S(M,N)=1-D(M,N)$ and $S^w(M,N)=1-D^w(M,N)$ are similarity measures between M and N, respectively.

$$S(M,N)=1-\sqrt[p]{\begin{array}{l}\dfrac{1}{6n(t_1+2)^p}\displaystyle\sum_{i=1}^{n}\left(\begin{array}{r}|-t_1(T_M^L(x_i)-T_N^L(x_i))+\\(I_M^L(x_i)-I_N^L(x_i))+\\(F_M^L(x_i)-F_N^L(x_i))|^p+\\|-t_1(T_M^U(x_i)-T_N^U(x_i))+\\(I_M^U(x_i)-I_N^U(x_i))+\\(F_M^U(x_i)-F_N^U(x_i))|^p\end{array}\right)+\\ \dfrac{1}{6n(t_2+2)^p}\displaystyle\sum_{i=1}^{n}\left(\begin{array}{r}|-t_2(I_M^L(x_i)-I_N^L(x_i))-\\(F_M^L(x_i)-F_N^L(x_i))+\\(T_M^L(x_i)-T_N^L(x_i))|^p+\\|-t_2(I_M^U(x_i)-I_N^U(x_i))-\\(F_M^U(x_i)-F_N^U(x_i))+\\(T_M^U(x_i)-T_N^U(x_i))|^p+\\|-t_2(F_M^L(x_i)-F_N^L(x_i))-\\(I_M^L(x_i)-I_N^L(x_i))+\\(T_M^L(x_i)-T_N^L(x_i))|^p+\\|-t_2(F_M^U(x_i)-F_N^U(x_i))-\\(I_M^U(x_i)-I_N^U(x_i))+\\(T_M^U(x_i)-T_N^U(x_i))|^p\end{array}\right)\end{array}} \tag{8}$$

$$S^w(M,N)=1-\sqrt[p]{\begin{array}{l}\dfrac{1}{6(t_1+2)^p}\displaystyle\sum_{i=1}^{n}w_i\left(\begin{array}{r}|-t_1(T_M^L(x_i)-T_N^L(x_i))+\\(I_M^L(x_i)-I_N^L(x_i))+\\(F_M^L(x_i)-F_N^L(x_i))|^p+\\|-t_1(T_M^U(x_i)-T_N^U(x_i))+\\(I_M^U(x_i)-I_N^U(x_i))+\\(F_M^U(x_i)-F_N^U(x_i))|^p\end{array}\right)+\\ \dfrac{1}{6(t_2+2)^p}\displaystyle\sum_{i=1}^{n}w_i\left(\begin{array}{r}|-t_2(I_M^L(x_i)-I_N^L(x_i))-\\(F_M^L(x_i)-F_N^L(x_i))+\\(T_M^L(x_i)-T_N^L(x_i))|^p+\\|-t_2(I_M^U(x_i)-I_N^U(x_i))-\\(F_M^U(x_i)-F_N^U(x_i))+\\(T_M^U(x_i)-T_N^U(x_i))|^p+\\|-t_2(F_M^L(x_i)-F_N^L(x_i))-\\(I_M^L(x_i)-I_N^L(x_i))+\\(T_M^L(x_i)-T_N^L(x_i))|^p+\\|-t_2(F_M^U(x_i)-F_N^U(x_i))-\\(I_M^U(x_i)-I_N^U(x_i))+\\(T_M^U(x_i)-T_N^U(x_i))|^p\end{array}\right)\end{array}} \tag{9}$$

Theorem 5: Let M and N be two INSs, then we have

(1) $D(M,M\otimes N)=D(N,M\oplus N)$;
(2) $D(M,M\oplus N)=D(N,M\otimes N)$;
(3) $S(M,M\otimes N)=S(N,M\oplus N)$;
(4) $S(M,M\oplus N)=S(N,M\otimes N)$.

Proof: We only prove the (1), and the (2)-(4) can be proved in the similar way.

(1) According to Definition 5 and Eq. (6), and for $D(M,M\bigotimes N)$ with $\forall x_i\in X$, we can have

$|-t_1(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))+(I_M^L(x_i)-(I_M^L(x_i)+I_N^L(x_i)-I_M^L(x_i)I_N^L(x_i)))+(F_M^L(x_i)-(F_M^L(x_i)+F_N^L(x_i)-F_M^L(x_i)F_N^L(x_i)))|^p$
$=|-t_1(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))+(-I_N^L(x_i)+I_M^L(x_i)I_N^L(x_i))+(-F_N^L(x_i)+F_M^L(x_i)F_N^L(x_i))|^p$,

$|-t_1(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))+(I_M^U(x_i)-(I_M^U(x_i)+I_N^U(x_i)-I_M^U(x_i)I_N^U(x_i)))+(F_M^U(x_i)-(F_M^U(x_i)+F_N^U(x_i)-F_M^U(x_i)F_N^U(x_i)))|^p$
$=|-t_1(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))+(-I_N^U(x_i)+I_M^U(x_i)I_N^U(x_i))+(-F_N^U(x_i)+F_M^U(x_i)F_N^U(x_i))|^p$,

$|-t_2(I_M^L(x_i)-(I_M^L(x_i)+I_N^L(x_i)-I_M^L(x_i)I_N^L(x_i)))-(F_M^L(x_i)-(F_M^L(x_i)+F_N^L(x_i)-F_M^L(x_i)F_N^L(x_i)))+(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))|^p$
$=|-t_2(-I_N^L(x_i)+I_M^L(x_i)I_N^L(x_i))-(-F_N^L(x_i)+F_M^L(x_i)F_N^L(x_i))+(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))|^p$,

$|-t_2(I_M^U(x_i)-(I_M^U(x_i)+I_N^U(x_i)-I_M^U(x_i)I_N^U(x_i)))-(F_M^U(x_i)-(F_M^U(x_i)+F_N^U(x_i)-F_M^U(x_i)F_N^U(x_i)))+(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))|^p$
$=|-t_2(-I_N^U(x_i)+I_M^U(x_i)I_N^U(x_i))-(-F_N^U(x_i)+F_M^U(x_i)F_N^U(x_i))+(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))|^p$,

$|-t_2(F_M^L(x_i)-(F_M^L(x_i)+F_N^L(x_i)-F_M^L(x_i)F_N^L(x_i)))-(I_M^L(x_i)-(I_M^L(x_i)+I_N^L(x_i)-I_M^L(x_i)I_N^L(x_i)))+(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))|^p$
$=|-t_2(-F_N^L(x_i)+F_M^L(x_i)F_N^L(x_i))-(-I_N^L(x_i)+I_M^L(x_i)I_N^L(x_i))+(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))|^p$,

$|-t_2(F_M^U(x_i)-(F_M^U(x_i)+F_N^U(x_i)-F_M^U(x_i)F_N^U(x_i)))-(I_M^U(x_i)-(I_M^U(x_i)+I_N^U(x_i)-I_M^U(x_i)I_N^U(x_i)))+(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))|^p$
$=|-t_2(-F_N^U(x_i)+F_M^U(x_i)F_N^U(x_i))-(-I_N^U(x_i)+I_M^U(x_i)I_N^U(x_i))+(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))|^p$.

For $D(N,M\bigoplus N)$ with $\forall x_i\in X$, we can have

$|-t_1(T_N^L(x_i)-(T_M^L(x_i)+T_N^L(x_i)-T_M^L(x_i)T_N^L(x_i)))+(I_N^L(x_i)-I_M^L(x_i)I_N^L(x_i))+(F_N^L(x_i)-F_M^L(x_i)F_N^L(x_i))|^p$
$=|-t_1(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))+(-I_N^L(x_i)+I_M^L(x_i)I_N^L(x_i))+(-F_N^L(x_i)+F_M^L(x_i)F_N^L(x_i))|^p$,

$|-t_1(T_N^U(x_i)-(T_M^U(x_i)+T_N^U(x_i)-T_M^U(x_i)T_N^U(x_i)))+(I_N^U(x_i)-I_M^U(x_i)I_N^U(x_i))+(F_N^U(x_i)-F_M^U(x_i)F_N^U(x_i))|^p$
$=|-t_1(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))+(-I_N^U(x_i)+I_M^U(x_i)I_N^U(x_i))+(-F_N^U(x_i)+F_M^U(x_i)F_N^U(x_i))|^p$,

$|-t_2(I_N^L(x_i)-I_M^L(x_i)I_N^L(x_i))-(F_N^L(x_i)-F_M^L(x_i)F_N^L(x_i))+(T_N^L(x_i)-(T_M^L(x_i)+T_N^L(x_i)-T_M^L(x_i)T_N^L(x_i)))|^p$
$=|-t_2(-I_N^L(x_i)+I_M^L(x_i)I_N^L(x_i))-(-F_N^L(x_i)+F_M^L(x_i)F_N^L(x_i))+(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))|^p$,

$|-t_2(I_N^U(x_i)-I_M^U(x_i)I_N^U(x_i))-(F_N^U(x_i)-F_M^U(x_i)F_N^U(x_i))+(T_N^U(x_i)-(T_M^U(x_i)+T_N^U(x_i)-T_M^U(x_i)T_N^U(x_i)))|^p$
$=|-t_2(-I_N^U(x_i)+I_M^U(x_i)I_N^U(x_i))-(-F_N^U(x_i)+F_M^U(x_i)F_N^U(x_i))+(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))|^p$,

$|-t_2(F_N^L(x_i)-F_M^L(x_i)F_N^L(x_i))-(I_N^L(x_i)-I_M^L(x_i)I_N^L(x_i))+(T_N^L(x_i)-(T_M^L(x_i)+T_N^L(x_i)-T_M^L(x_i)T_N^L(x_i)))|^p$
$=|-t_2(-F_N^L(x_i)+F_M^L(x_i)F_N^L(x_i))-(-I_N^L(x_i)+I_M^L(x_i)I_N^L(x_i))+(T_M^L(x_i)-T_M^L(x_i)T_N^L(x_i))|^p$,

$|-t_2(F_N^U(x_i)-F_M^U(x_i)F_N^U(x_i))-(I_N^U(x_i)-I_M^U(x_i)I_N^U(x_i))+(T_N^U(x_i)-(T_M^U(x_i)+T_N^U(x_i)-T_M^U(x_i)T_N^U(x_i)))|^p$
$=|-t_2(-F_N^U(x_i)+F_M^U(x_i)F_N^U(x_i))-(-I_N^U(x_i)+I_M^U(x_i)I_N^U(x_i))+(T_M^U(x_i)-T_M^U(x_i)T_N^U(x_i))|^p$.

Consequently, we can obtain $D(M,M\otimes N)=D(N,M\oplus N)$.

For stating the advantage of the explored similarity measure S, a comparison between the initiated similarity measure and the existing similarity measures is established. Some existing similarity measures are presented in Table 1.

In the following, we utilize seven sets of INSs to compare the experimental results of the initiated similarity measure S with the existing similarity measures [54, 55, 57, 58, 60–66] , as shown in Table 2. From Table 2, it is clear that the proposed similarity measure S can overcome the shortcomings of getting the unreasonable results of the existing similarity measures [54, 55, 57, 58, 60–66]. We will state the four major drawbacks in detail as follows:

(1) It is easily seen that the first axiom of similarity measure ($S1$) is not satisfied by S_A since $S_A(M,N) = -0.05$ when $M = ([0.3,0.4],[0.2,0.3],[0.4,0.5])$ and $N = ([0.6,0.8],[0.4,0.6],[0.8,1.0])$ (Case 2) which are indeed $0 \leq S_A(M,N) \leq 1$. The similar cases are shown in Cases 3, 4, 6 and 7. Moreover, the second axiom of similarity measure ($S2$) is not satisfied by S_{B4} and S_{Y4} when $M = ([0.3,0.4],[0.2,0.3],[0.4,0.5]), N = ([0.6,0.8],[0.4,0.6],[0.8,1.0])$ (Case 2). In fact, they are indeed not equal to each other. In addition, we can see that $S_{B1}(M,N) = S_{B2}(M,N) = S_{B3}(M,N) = S_{Y2}(M,N) = S_{Y3}(M,N) = S_{Y5}(M,N) = S_J(M,N) = S_{Y7}(M,N) = S_L(M,N) = S_A(M,N) = 0$ when $M = ([1,1],[0,0],[0,0]), N = ([0,0],[0,0],[0,0])$ (Case 1), $S_{Y5}(M,N) = S_J(M,N) = S_L(M,N) = 0$ when $M = ([1,1],[0,1],[1,1]), N = ([0.6,0.7],[0.5,0.5],[0,0])$ (Case 6) and $M = ([1,1],[0,1],[1,1]), N = ([0.8,0.9],[0.5,0.5],[0,0])$ (Case 7). Actually, they are indeed not opposite to each other.

(2) Some similarity measures (S_{Y1}, S_{B1}, S_{B2}, S_{Y2}, S_{Y3}, S_{Y5}, S_{Y6}, S_J, S_{Y7}, S_F, S_M, S_L) have no capabilities to distinguish positive difference from negative difference. For example, $S_{Y1}(M,N) = S_{Y1}(M1,N1) = 0.65$ when $M = ([0.3,0.4],[0.2,0.3],[0.4,0.5]), N = ([0.6,0.8],[0.4,0.6],[0.8,1.0])$ (Case 2), $M1 = ([0.3,0.4],[0.2,0.3],[0.8,1.0])$ and $N1 = ([0.6,0.8],[0.4,0.6],[0.4,0.5])$ (Case 3). The same counter-intuitive example exists for S_{B1}, S_{B2}, S_{Y2}, S_{Y3}, S_{Y5}, S_{Y6}, S_J, S_{Y7}, S_F, S_M, S_L.

(3) Some similarity measures have no capabilities to deal the division by zero problem. For example, S_{B4} and S_{Y4} when $M = ([1,1],[0,0],[0,0]), N = ([0,0],[0,0],[0,0])$ (Case 1). Another example exists for S_{B1}, S_{B2}, S_{B3}, S_{B4}, S_{Y2}, S_{Y3}, S_{Y4}, S_{Y7} and S_{Y4} when $M = ([0,0],[0,0],[0,0]), N = ([0,0],[0,0],[0,0])$ (Case 5).

(4) Another charming counter-intuitive case occurs when $M1 = ([1,1],[0,1],[1,1]), N1 = ([0.6,0.7],[0.5,0.5],[0,0])$ (Case 6) and $M1 = ([1,1],[0,1],[1,1]), N2 = ([0.8,0.9],[0.5,0.5],[0,0])$ (Case 7). In this case, it is expected that the similarity degree between $M1$ and $N1$ is equal or greater than the similarity degree between $M1$ and $N2$ since they are ordered as $M1 \prec N1 \prec N2$ by means of comparison method shown in Definition 7 . However, the similarity degree between $M1$ and $N2$ is greater than the similarity degree between $M1$ and $N1$ when S_{Y1}, S_{B1}, S_{B2}, S_{B3}, S_{B4}, S_{Y2}, S_{Y3}, S_{Y4}, S_{Y6}, S_{Y7}, S_F, S_M are used, which does not seem to be reasonable (It violates the last axiom of similarity measure ($S4$)). On the other hand, our proposed similarity measure $S(M1,N1) = 0.67$ and $S(M1,N2) = 0.6483$. Therefore, the developed similarity measure is in agreement with real case. The presented similarity measure S is the similarity measure that have no the counter-intuitive cases as illustrated in Table 2.

IV. AN INTERVAL NEUTROSOPHIC DECISION MAKING ALGORITHM BASED SIMILARITY MEASURE

A. THE DESCRIPTION OF INTERVAL NEUTROSOPHIC MADM PROBLEM

The core of the MADM issue with interval neutrosophic information is to verify the ideal choice from a series of alternatives which are assessed by a set of attributes, where assessed values are INNs presented by the experts. Afterwards, this kind of issue can be depicted by mathematical symbols in the following.

Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, \cdots, C_n\}$ be a series of n attributes, and $W = \{w_1, w_2, \cdots, w_n\}$ be weight vector assigned for the attributes by the decision makers with $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$. Assume that the evaluation of the alternative A_i with respect to attribute C_j is represented by interval neutrosophic matrix $P = (p_{ij})_{m\times n} = ([T_{ij}^L, T_{ij}^U],[I_{ij}^L, I_{ij}^U],[F_{ij}^L, F_{ij}^U])_{m\times n}(i = 1,2,\cdots,m; j = 1,2,\cdots,n)$. The values united with the alternatives for MADM problems can be shown in Table 3.

B. THE METHOD OF DETERMINING THE COMBINED WEIGHTS

DMs or experts frequently give the attribute weights in a subjective manner. This subjective evaluation of attribute weights from diverse DMs or experts often result in diverse weights for one attribute. To achieve more realistic attribute weights of the issue, a decision procedure is proposed by combining the subjective weights given by DMs or experts and objective weights computed based on the deviation-based method. The use of a combination of subjective and objective weights can help us reduce the sensitivity of the decision process and thus changing the weights of the DMs or experts. The framework for using the proposed method is shown in Fig. 2.

1) Determining the objective weights: deviation-based method

The deviation-based method is a resultful means for the calculation of objective weight, which is used in management decision practices [69]. It can be employed in calculating the attribute weight by means of decision matrix and it is responsible for specifically detail data. The attribute weight can be revealed and decided by intrinsic willingness and specific information. The objective weight achieved through deviation-based method is shown in Eq. (10)-(14).

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TABLE 1: Existing similarity measures.

Authors	Similarity measure
Ye [57]	$S_{Y1}(M,N) = 1 - \frac{1}{6n}\sum_{i=1}^{n}\left\{\begin{array}{c} \lvert T_M^L(x_i)-T_N^L(x_i)\rvert + \lvert T_M^U(x_i)-T_N^U(x_i)\rvert + \lvert I_M^L(x_i)-I_N^L(x_i)\rvert + \\ \lvert I_M^U(x_i)-I_N^U(x_i)\rvert + \lvert F_M^L(x_i)-F_N^L(x_i)\rvert + \lvert F_M^U(x_i)-F_N^U(x_i)\rvert \end{array}\right\}$
Broumi and Smarandache [55]	$S_{B1}(M,N) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n}\left\{\begin{array}{c} \lvert T_M^L(x_i)-T_N^L(x_i)\rvert^p + \lvert T_M^U(x_i)-T_N^U(x_i)\rvert^p + \lvert I_M^L(x_i)-I_N^L(x_i)\rvert^p + \\ \lvert I_M^U(x_i)-I_N^U(x_i)\rvert^p + \lvert F_M^L(x_i)-F_N^L(x_i)\rvert^p + \lvert F_M^U(x_i)-F_N^U(x_i)\rvert^p \end{array}\right\}}{\sum_{i=1}^{n}\left\{\begin{array}{c} \lvert T_M^L(x_i)+T_N^L(x_i)\rvert^p + \lvert T_M^U(x_i)+T_N^U(x_i)\rvert^p + \lvert I_M^L(x_i)+I_N^L(x_i)\rvert^p + \\ \lvert I_M^U(x_i)+I_N^U(x_i)\rvert^p + \lvert F_M^L(x_i)+F_N^L(x_i)\rvert^p + \lvert F_M^U(x_i)+F_N^U(x_i)\rvert^p \end{array}\right\}}}$
Broumi and Smarandache [55]	$S_{B2}(M,N) = \frac{\sum_{i=1}^{n}\left\{\begin{array}{c} \min\{T_M^L(x_i),T_N^L(x_i)\} + \min\{T_M^U(x_i),T_N^U(x_i)\} + \min\{I_M^L(x_i),I_N^L(x_i)\} + \\ \min\{I_M^U(x_i),I_N^U(x_i)\} + \min\{F_M^L(x_i),F_N^L(x_i)\} + \min\{F_M^U(x_i),F_N^U(x_i)\} \end{array}\right\}}{\sum_{i=1}^{n}\left\{\begin{array}{c} \max\{T_M^L(x_i),T_N^L(x_i)\} + \max\{T_M^U(x_i),T_N^U(x_i)\} + \max\{I_M^L(x_i),I_N^L(x_i)\} \\ +\max\{I_M^U(x_i),I_N^U(x_i)\} + \max\{F_M^L(x_i),F_N^L(x_i)\} + \max\{F_M^U(x_i),F_N^U(x_i)\} \end{array}\right\}}$
Broumi and Smarandache [55]	$S_{B3}(M,N) = \frac{\sum_{i=1}^{n}\left\{\begin{array}{c} T_M^L(x_i)T_N^L(x_i) + T_M^U(x_i)T_N^U(x_i) + I_M^L(x_i)I_N^L(x_i) + \\ I_M^U(x_i)I_N^U(x_i) + F_M^L(x_i)F_N^L(x_i) + F_M^U(x_i)F_N^U(x_i) \end{array}\right\}}{\max\left\{\begin{array}{c} \sum_{i=1}^{n}\{T_M^L(x_i)^2 + T_M^U(x_i)^2 + I_M^L(x_i)^2 + I_M^U(x_i)^2 + F_M^L(x_i)^2 + F_M^U(x_i)^2\}, \\ \sum_{i=1}^{n}\{T_N^L(x_i)^2 + T_N^U(x_i)^2 + I_N^L(x_i)^2 + I_N^U(x_i)^2 + F_N^L(x_i)^2 + F_N^U(x_i)^2\} \end{array}\right\}}$
Broumi and Smarandache [54, 55]	$S_{B4}(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{\left\{\begin{array}{r} (T_M^L(x_i)+T_M^U(x_i))(T_N^L(x_i)+T_N^U(x_i)) + (I_M^L(x_i)+I_M^U(x_i))(I_N^L(x_i)+I_N^U(x_i)) + \\ (F_M^L(x_i)+F_M^U(x_i))(F_N^L(x_i)+F_N^U(x_i)) \end{array}\right\}}{\left\{\begin{array}{l} \sqrt{(T_M^L(x_i)+T_M^U(x_i))^2 + (I_M^L(x_i)+I_M^U(x_i))^2 + (F_M^L(x_i)+F_M^U(x_i))^2} \\ \times\sqrt{(T_N^L(x_i)+T_N^U(x_i))^2 + (I_N^L(x_i)+I_N^U(x_i))^2 + (F_N^L(x_i)+F_N^U(x_i))^2} \end{array}\right\}}$
Ye [58]	$S_{Y2}(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{\left\{\begin{array}{c} T_M^L(x_i)T_N^L(x_i) + T_M^U(x_i)T_N^U(x_i) + I_M^L(x_i)I_N^L(x_i) + \\ I_M^U(x_i)I_N^U(x_i) + F_M^L(x_i)F_N^L(x_i) + F_M^U(x_i)F_N^U(x_i) \end{array}\right\}}{\left\{\begin{array}{r} T_M^L(x_i)^2 + T_M^U(x_i)^2 + I_M^L(x_i)^2 + I_M^U(x_i)^2 + F_M^L(x_i)^2 + F_M^U(x_i)^2 + \\ T_N^L(x_i)^2 + T_N^U(x_i)^2 + I_N^L(x_i)^2 + I_N^U(x_i)^2 + F_N^L(x_i)^2 + F_N^U(x_i)^2 - \\ (T_M^L(x_i)T_N^L(x_i) + I_M^L(x_i)I_N^L(x_i) + F_M^L(x_i)F_N^L(x_i)) \\ -(T_M^U(x_i)T_N^U(x_i) + I_M^U(x_i)I_N^U(x_i) + F_M^U(x_i)F_N^U(x_i)) \end{array}\right\}}$
Ye [58]	$S_{Y3}(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{\left\{\begin{array}{c} 2(T_M^L(x_i)T_N^L(x_i) + T_M^U(x_i)T_N^U(x_i) + I_M^L(x_i)I_N^L(x_i) \\ +I_M^U(x_i)I_N^U(x_i) + F_M^L(x_i)F_N^L(x_i) + F_M^U(x_i)F_N^U(x_i)) \end{array}\right\}}{\left\{\begin{array}{c} T_M^L(x_i)^2 + T_M^U(x_i)^2 + I_M^L(x_i)^2 + I_M^U(x_i)^2 + F_M^L(x_i)^2 + F_M^U(x_i)^2 + \\ T_N^L(x_i)^2 + T_N^U(x_i)^2 + I_N^L(x_i)^2 + I_N^U(x_i)^2 + F_N^L(x_i)^2 + F_N^U(x_i)^2 \end{array}\right\}}$
Ye [58]	$S_{Y4}(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{\left\{\begin{array}{c} T_M^L(x_i)T_N^L(x_i) + T_M^U(x_i)T_N^U(x_i) + I_M^L(x_i)I_N^L(x_i) + \\ I_M^U(x_i)I_N^U(x_i) + F_M^L(x_i)F_N^L(x_i) + F_M^U(x_i)F_N^U(x_i) \end{array}\right\}}{\left\{\begin{array}{l} \sqrt{T_M^L(x_i)^2 + T_M^U(x_i)^2 + I_M^L(x_i)^2 + I_M^U(x_i)^2 + F_M^L(x_i)^2 + F_M^U(x_i)^2} \\ \times\sqrt{T_N^L(x_i)^2 + T_N^U(x_i)^2 + I_N^L(x_i)^2 + I_N^U(x_i)^2 + F_N^L(x_i)^2 + F_N^U(x_i)^2} \end{array}\right\}}$
Ye [60]	$S_{Y5}(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cos\left\{\begin{array}{c} \frac{\pi}{4}\Big(\max\{\lvert T_M^L(x_i)-T_N^L(x_i)\rvert, \lvert I_M^L(x_i)-I_N^L(x_i)\rvert, \lvert F_M^L(x_i)-F_N^L(x_i)\rvert\} + \\ \max\{\lvert T_M^U(x_i)-T_N^U(x_i)\rvert, \lvert I_M^U(x_i)-I_N^U(x_i)\rvert, \lvert F_M^U(x_i)-F_N^U(x_i)\rvert\}\Big) \end{array}\right\}$
Ye [60]	$S_{Y6}(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cos\left\{\begin{array}{c} \frac{\pi}{12}(\lvert T_M^L(x_i)-T_N^L(x_i)\rvert + \lvert I_M^L(x_i)-I_N^L(x_i)\rvert + \lvert F_M^L(x_i)-F_N^L(x_i)\rvert + \\ \lvert T_M^U(x_i)-T_N^U(x_i)\rvert + \lvert I_M^U(x_i)-I_N^U(x_i)\rvert + \lvert F_M^U(x_i)-F_N^U(x_i)\rvert) \end{array}\right\}$
Ji and Zhang [61]	$S_J(M,N) = 1 - \frac{1}{2n}\sum_{i=1}^{n}\max\left\{\begin{array}{c} \lvert T_M^L(x_i)-T_N^L(x_i)\rvert + \lvert T_M^U(x_i)-T_N^U(x_i)\rvert, \lvert I_M^L(x_i)-I_N^L(x_i)\rvert + \\ \lvert I_M^U(x_i)-I_N^U(x_i)\rvert, \lvert F_M^L(x_i)-F_N^L(x_i)\rvert + \lvert F_M^U(x_i)-F_N^U(x_i)\rvert \end{array}\right\}$
Ye [62]	$S_{Y7}(M,N) = \frac{2\sum_{i=1}^{n}\left\{\begin{array}{c} T_M^L(x_i)T_N^L(x_i) + T_M^U(x_i)T_N^U(x_i) + I_M^L(x_i)I_N^L(x_i) + \\ I_M^U(x_i)I_N^U(x_i) + F_M^L(x_i)F_N^L(x_i) + F_M^U(x_i)F_N^U(x_i) \end{array}\right\}}{\sum_{i=1}^{n}\left\{\begin{array}{c} T_M^L(x_i)^2 + T_M^U(x_i)^2 + I_M^L(x_i)^2 + I_M^U(x_i)^2 + F_M^L(x_i)^2 + F_M^U(x_i)^2 + \\ T_N^L(x_i)^2 + T_N^U(x_i)^2 + I_N^L(x_i)^2 + I_N^U(x_i)^2 + F_N^L(x_i)^2 + F_N^U(x_i)^2 \end{array}\right\}}$
Fu and Ye [63]	$S_F(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{e^{-\frac{1}{6}\left\{\begin{array}{c} \lvert T_M^L(x_i)-T_N^L(x_i)\rvert + \lvert I_M^L(x_i)-I_N^L(x_i)\rvert + \lvert F_M^L(x_i)-F_N^L(x_i)\rvert + \\ \lvert T_M^U(x_i)-T_N^U(x_i)\rvert + \lvert I_M^U(x_i)-I_N^U(x_i)\rvert + \lvert F_M^U(x_i)-F_N^U(x_i)\rvert \end{array}\right\}} - e^{-1}}{1-e^{-1}}$
Mondal et al. [64]	$S_M(M,N) = 1 - \frac{1}{n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{12}\left\{\begin{array}{r} \lvert\lambda(T_M^L(x_i)-T_N^L(x_i)) + (1-\lambda)(T_M^U(x_i)-T_N^U(x_i))\rvert + \lvert\lambda(I_M^L(x_i)-I_N^L(x_i)) + \\ (1-\lambda)(I_M^U(x_i)-I_N^U(x_i))\rvert + \lvert\lambda(F_M^L(x_i)-F_N^L(x_i)) + (1-\lambda)(F_M^U(x_i)-F_N^U(x_i))\rvert \end{array}\right\}\right)$
Liu [65]	$S_L(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cot\frac{\pi}{4}\left\{\begin{array}{r} 1+\max\{\lvert T_M^L(x_i)-T_N^L(x_i)\rvert, \lvert I_M^L(x_i)-I_N^L(x_i)\rvert, \lvert F_M^L(x_i)-F_N^L(x_i)\rvert, \\ \lvert T_M^U(x_i)-T_N^U(x_i)\rvert, \lvert I_M^U(x_i)-I_N^U(x_i)\rvert, \lvert F_M^U(x_i)-F_N^U(x_i)\rvert\} \end{array}\right\}$
Aydoğdu [66]	$S_A(M,N) = 1 - \frac{1}{2}\left\{\begin{array}{l} \max_i(\lvert T_M^L(x_i)-T_N^L(x_i)\rvert) + \max_i(\lvert T_M^U(x_i)-T_N^U(x_i)\rvert) + \\ \max_i(\lvert I_M^L(x_i)-I_N^L(x_i)\rvert) + \max_i(\lvert I_M^U(x_i)-I_N^U(x_i)\rvert) + \\ \max_i(\lvert F_M^L(x_i)-F_N^L(x_i)\rvert) + \max_i(\lvert F_M^U(x_i)-F_N^U(x_i)\rvert) \end{array}\right\}$

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TABLE 2: The comparison of similarity measures.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
M	$\{< x, [1,1], [0,0], [0,0] >\}$	$\{< x, [0.3,0.4], [0.2,0.3], [0.4,0.5] >\}$	$\{< x, [0.3,0.4], [0.2,0.3], [0.8,1.0] >\}$	$\{< x, [1,1], [0,0], [0,0] >\}$	$\{< x, [0,0], [0,0], [0,0] >\}$	$\{< x, [1,1], [0,1], [1,1] >\}$	$\{< x, [1,1], [0,1], [1,1] >\}$
N	$\{< x, [0,0], [0,0], [0,0] >\}$	$\{< x, [0.6,0.8], [0.4,0.6], [0.8,1.0] >\}$	$\{< x, [0.6,0.8], [0.4,0.6], [0.4,0.5] >\}$	$\{< x, [0,0], [1,1], [1,1] >\}$	$\{< x, [0,0], [0,0], [0,0] >\}$	$\{< x, [0.6,0.7], [0.5,0.5], [0,0] >\}$	$\{< x, [0.8,0.9], [0.5,0.5], [0,0] >\}$
S_{Y1} [57]	0.6667	0.6500	0.6500	0.0000	1.0000	0.3833	0.4500
S_{B1} [55]	0.0000	0.6667	0.6667	0.0000	N/A	0.5005	0.6056
S_{B2} [55]	0.0000	0.5000	0.5000	0.0000	N/A	0.3273	0.4000
S_{B3} [55]	0.0000	0.5000	0.7822	0.0000	N/A	0.3600	0.4400
S_{B4} [54, 55]	N/A	1.0000	0.8003	0.0000	N/A	0.7317	0.7436
S_{Y2} [58]	0.0000	0.6667	0.6667	0.0000	N/A	0.3956	0.4632
S_{Y3} [58]	0.0000	0.8000	0.8000	0.0000	N/A	0.5669	0.6331
S_{Y4} [58]	N/A	1.0000	0.8002	0.0000	N/A	0.6928	0.7046
S_{Y5} [60]	0.0000	0.7604	0.7604	0.0000	1.0000	0.0000	0.0000
S_{Y6} [60]	0.8660	0.8526	0.8526	0.0000	1.0000	0.5664	0.6494
S_J [61]	0.0000	0.5500	0.5500	0.0000	1.0000	0.0000	0.0000
S_{Y7} [62]	0.0000	0.8000	0.8000	0.0000	N/A	0.5669	0.6331
S_F [63]	0.6667	0.6500	0.6500	0.0000	1.0000	0.3833	0.4500
S_M [64]	0.7321	0.7180	0.7180	0.0000	1.0000	0.6311	0.6895
S_L [65]	0.0000	0.4142	0.4142	0.0000	1.0000	0.0000	0.0000
S_A [66]	0.0000	-0.0500	-0.0500	-2.0000	1.0000	-0.8500	-0.6500
S(proposed)	0.7000	0.8600	0.8250	0.0000	1.0000	0.6700	0.6483

The red background color denotes that it cannot make a decision due to the same results;
The pink background color denotes results do not satisfy similarity measure definition $(S1)$;
The green background color denotes results do not satisfy similarity measure definition $(S2)$;
The yellow background color denotes results do not satisfy similarity measure definition $(S4)$;
The blue background color denotes the counterintuitive phenomena;
"N/A" denotes it cannot compute the degree of similarity due to "the division by zero problem".
Note: ($p = 1$ in $S_{B1}, \lambda = 0.5$ in S_M, and $t_1 = 2, t_2 = 3, p = 1$ in S.)

TABLE 3: The interval neutrosophic MADM matrix.

	C_1	C_2	$\cdots$	C_n
A_1	$([T_{11}^L, T_{11}^U], [I_{11}^L, I_{11}^U], [F_{11}^L, F_{11}^U])$	$([T_{12}^L, T_{12}^U], [I_{12}^L, I_{12}^U], [F_{12}^L, F_{12}^U])$	$\cdots$	$([T_{1n}^L, T_{1n}^U], [I_{1n}^L, I_{1n}^U], [F_{1n}^L, F_{1n}^U])$
A_2	$([T_{21}^L, T_{21}^U], [I_{21}^L, I_{21}^U], [F_{21}^L, F_{21}^U])$	$([T_{22}^L, T_{22}^U], [I_{22}^L, I_{22}^U], [F_{22}^L, F_{22}^U])$	$\cdots$	$([T_{2n}^L, T_{2n}^U], [I_{2n}^L, I_{2n}^U], [F_{2n}^L, F_{2n}^U])$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$([T_{m1}^L, T_{m1}^U], [I_{m1}^L, I_{m1}^U], [F_{m1}^L, F_{m1}^U])$	$([T_{m2}^L, T_{m2}^U], [I_{m2}^L, I_{m2}^U], [F_{m2}^L, F_{m2}^U])$	$\cdots$	$([T_{mn}^L, T_{mn}^U], [I_{mn}^L, I_{mn}^U], [F_{mn}^L, F_{mn}^U])$

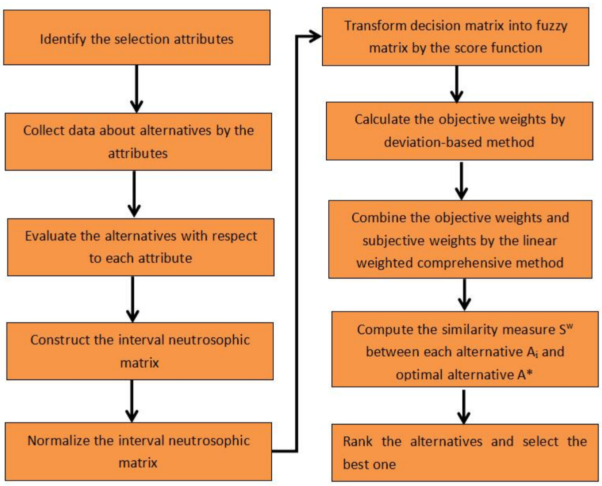

FIGURE 2: The framework for using the proposed method.

At first, we think in normalizing information since there exists some benefit attributes and cost attributes in decision making matrix. These two classes of attributes react opposite-ly. In other words, the bigger value means the better behavior of a benefit attribute but reveals the worse behavior of a cost attribute. As a consequence, for ensuring all attributes are simultaneous, we proceed to shift the cost attributes into benefit attributes by means of the following equation.

$$p'_{ij} = ([(T_{ij}^L)', (T_{ij}^U)'], [(I_{ij}^L)', (I_{ij}^U)'], [(F_{ij}^L)', (F_{ij}^U)']) = \begin{cases} ([T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U]), \ C_j \text{ is benefit attribute}, \\ ([F_{ij}^L, F_{ij}^U], [1 - I_{ij}^U, 1 - I_{ij}^L], [T_{ij}^L, T_{ij}^U]), \ C_j \text{ is cost attribute}. \end{cases} \tag{10}$$

According to this equation, we can achieve the normalized interval neutrosophic matrix $P' = (p'_{ij})_{m \times n}$.

Next, we calculate the score function $s_{ij} (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ of p'_{ij} by Definition 7.

$$s_{ij} = \frac{2}{3} + \frac{(T_{ij}^L)' + (T_{ij}^U)'}{6} - \frac{(I_{ij}^L)' + (I_{ij}^U)'}{6} - \frac{(F_{ij}^L)' + (F_{ij}^U)'}{6} \tag{11}$$

Later, normalize the score decision matrix $N = (N_{ij})_{m \times n}$ as follow:

$$N_{ij} = \frac{s_{ij}}{\sum_{i=1}^m s_{ij}}. \tag{12}$$

Calculate the average score decision matrix $N = (N_j)_{n \times 1}$ of each attribute as follows:

$$N_j = \sum_{i=1}^m N_{ij}/m. \tag{13}$$

Finally, calculate an objective weight (ω_j) for each attribute using a deviation-based method as follows:

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$$\delta_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(N_{ij} - N_j)^2}. \tag{14}$$

$$\omega_j = \frac{\delta_j}{\sum_{j=1}^{n}\delta_j}. \tag{15}$$

2) Determining the combined weights: the linear weighted comprehensive method

Assume that the subjective weight, provided by the experts directly, is $w = \{w_1, w_2, \cdots, w_n\}$, where $\sum_{j=1}^{n} w_j = 1, 0 \leq w_j \leq 1$. The vector of the objective weight, calculated by Eq.(15) directly, is $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$, where $\sum_{j=1}^{n} \omega_j = 1, 0 \leq \omega_j \leq 1$.

Consequently, the vector of the combined weight $\varpi = \{\varpi_1, \varpi_2, \cdots, \varpi_n\}$ can be denoted as follows:

$$\varpi_j = \frac{w_j * \omega_j}{\sum_{j=1}^{n} w_j * \omega_j}, \tag{16}$$

where $\sum_{j=1}^{n} \varpi_j = 1, 0 \leq \varpi_j \leq 1$.

Subjective weights and objective weights are combined using a nonlinear weighted synthesis approach. According to the multiplier effect, the bigger the value of subjective weight information and objective weight information, the bigger the combination weight, and vice versa. In addition, It is easily seen that Eq. (16) pushes the limitations of considering only objective or subjective influences. The advantage of Eq. (16) lies in that the rank and attribute weights of alternatives can simultaneously display subjective information and objective information.

C. THE INTERVAL NEUTROSOPHIC SIMILARITY MEASURE METHOD

For this subsection, we present an algorithm for MADM issue by means of the explored similarity measure S^w among INSs. The notion of ideal point has been successfully applied to settle the optimal alternative in MADM problem. Although ideal alternative doesn't exist in real world, it does provide a priceless theoretical framework against which to evaluate alternatives. Therefore, we denote the ideal alternative A^* as the INN $A_j = ([1, 1], [0, 0], [0, 0])$ for $\forall j$.

As a consequence, based on Eq. (17), the developed similarity measure S^w between alternative A_i and ideal alternative A^* denoted by the INSs is shown in the following.

$$S^w(A_i, A^*) = 1 - \sqrt[p]{\begin{aligned}&\frac{1}{6(t_1+2)^p}\sum_{i=1}^{n} w_i \begin{pmatrix} |-t_1(T_{ij}^L - 1) + I_{ij}^L + F_{ij}^L|^p + \\ |-t_1(T_{ij}^U - 1) + I_{ij}^U + F_{ij}^U|^p \end{pmatrix} + \\ &\frac{1}{6(t_2+2)^p}\sum_{i=1}^{n} w_i \begin{pmatrix} |-t_2 I_{ij}^L - F_{ij}^L + (T_{ij}^L - 1)|^p + \\ |-t_2 I_{ij}^U - F_{ij}^U + (T_{ij}^U - 1)|^p + \\ |-t_2 F_{ij}^L - I_{ij}^L + (T_{ij}^L - 1)|^p + \\ |-t_2 F_{ij}^U - I_{ij}^U + (T_{ij}^U - 1)|^p \end{pmatrix}\end{aligned}} \tag{17}$$

Generally speaking, interval neutrosophic similarity measure method involves the following steps:

Algorithm 1 :Similarity measure

1: Input the interval neutrosophic decision matrix $P = (P_{ij})_{m\times n}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$.
2: Transform the matrix $P = (P_{ij})_{m\times n}$ into a normalized matrix $P' = (P'_{ij})_{m\times n}$ by Eq.(10).
3: Compute the score matrix $S = (s_{ij})_{m\times n}$ of $P' = (P'_{ij})_{m\times n}$ by Eq. (11).
4: Normalize the score decision matrix $N = (N_{ij})_{m\times n}$ by Eq. (12).
5: Calculate the objective weight $\omega = (\omega_j)_{n\times 1}$ by Eq. (15).
6: Compute combined weight $\varpi = (\varpi_j)_{n\times 1}$ by Eq. (16).
7: Compute the similarity measure $S^w(A_i, A^*)(i = 1, 2, \cdots, m)$ by Eq. (17)
8: Rank the alternatives according to the decreasing values of similarity measure $S^w(A_i, A^*)$.

D. A CASE STUDY IN IOT INDUSTRY EVALUATION

The IoT is the next communications industry dividend after 4G. The inter-library connection exceeded the interpersonal connection in 2017, and the connection scale is expected to reach 100 billion in 2025. Communication companies such as Huawei are paying close attention to the IoT, mainly due to the rapid growth of their connectivity and the business opportunities behind massive connections. Connections are only part of the IoT industry's ecosystem and can support a market scale of up to trillions. Machina divides the industrial ecology of the IoT into five parts: module, connection, equipment, service and application. From the perspective of market value, the market value of the connection is only about 10%, and the market value of the application is the highest, up to 35%. It is believed that the total investment of China's operators in the construction of the IoT network will exceed 100 billion yuan in 2017-2018. From this calculation, only the scale of China's IoT industry will exceed 100 billion yuan. Ly et al. [3] analyzed the five crucial factors (Connectivity, Value, Security, Telepresence and Intelligence) in building triumphant IoT systems for IoT-related companies which is shown in Figure 1.

Example 2: Assume that there are five companies $A = \{A_1, A_2, A_3, A_4, A_5\}$ to be considered for the assessment of IoT industry. The expert chooses the decision attribute set $C = \{C_1, C_2, C_3, C_4, C_5\}$ to be C_1 (denoted as Connectivity), C_2 (denoted as Value), C_3 (denoted as Security), C_4 (denoted as Telepresence) and C_5 (denoted as Intelligence). Based on the general evolving principle and the characteristics of the IoT industry, we can determine that all attributes are benefit attributes. Suppose that the expert has the following prior weight set given by his/her prior experience or preference: $w = (w_1, w_2, w_3, w_4, w_5) =$

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$(0.3, 0.2, 0.14, 0.16, 0.2)$. The assessments for teachers arising from questionnaire investigation to the expert and constructing an interval neutrosophic matrix with its tabular form given by Table 4.

In what follows, we utilize the algorithm $(p = 1, t_1 = 2, t_2 = 3)$ proposed above to select excellent IoT company under interval neutrosophic information.

Step 1: Input the interval neutrosophic decision matrix $P = (P_{ij})_{5\times5}$ shown in Table 4.

Step 2: Because all attributes are benefit attributes, hence there is no need to transform.

Step 3: Compute the score matrix $S = (s_{ij})_{5\times5}$ of $P' = (P'_{ij})_{5\times5}$ by Eq. (11) as follows:

$$S = \begin{pmatrix} 0.8500 & 0.8167 & 0.8167 & 0.8500 & 0.8167 \\ 0.8333 & 0.7667 & 0.7667 & 0.8167 & 0.7833 \\ 0.8167 & 0.7167 & 0.7500 & 0.7500 & 0.7167 \\ 0.7000 & 0.6833 & 0.7333 & 0.7167 & 0.6833 \\ 0.6167 & 0.5833 & 0.6000 & 0.6667 & 0.5500 \end{pmatrix}$$

Step 4: Normalize the score decision matrix $N = (N_{ij})_{5\times5}$ by Eq. (12) as follows:

$$N = \begin{pmatrix} 0.2227 & 0.2290 & 0.2227 & 0.2237 & 0.2300 \\ 0.2183 & 0.2150 & 0.2091 & 0.2149 & 0.2207 \\ 0.2140 & 0.2009 & 0.2045 & 0.1974 & 0.2019 \\ 0.1834 & 0.1916 & 0.2000 & 0.1886 & 0.1925 \\ 0.1616 & 0.1636 & 0.1636 & 0.1754 & 0.1549 \end{pmatrix}$$

Step 5: Calculate the objective weight $\omega = (\omega_j)_{5\times1}$ by Eq. (15) as follows:

$\omega_1 = 0.2167, \omega_2 = 0.2033, \omega_3 = 0.1805,$
$\omega_4 = 0.1599, \omega_5 = 0.2396.$

Step 6: Compute combined weight $\varpi = (\varpi_j)_{5\times1}$ by Eq. (16).

$\varpi_1 = 0.3180, \varpi_2 = 0.1989, \varpi_3 = 0.1236,$
$\varpi_4 = 0.1251, \varpi_5 = 0.2344.$

Step 7: Compute the similarity measure S^w by Eq. (17) as follows:

$S^w(A_1, A^*) = 0.8317, S^w(A_2, A^*) = 0.7980,$
$S^w(A_3, A^*) = 0.7580, S^w(A_4, A^*) = 0.7033,$
$S^w(A_5, A^*) = 0.6039.$

Step 8: Rank the companies according to the similarity measure, the ranking order is $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

In Figure 3, we can know that the presented weight model can efficaciously reveal the objective preference information and subjective preference information. However, the combination weight developed by Peng and Dai [33] can't reflect the discrepancy compared with weight given by experts. In other words, the proposed weight determining model can state the laws between the data given by decision maker but not limit to the given subjective weight. Meanwhile, Zhang et al.'s [59] weight determining model also can deliver the objective and subjective information.

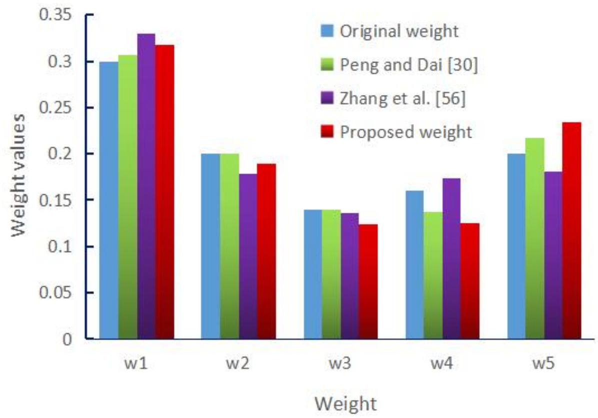

FIGURE 3: The comparison of the combined weight information.

E. EFFECT OF THE PARAMETERS P, T_1 AND T_2 ON THE ORDERING IN PROPOSED ALGORITHM

However, in order to analyze the effect of the parameters p, t_1 and t_2 on the measure values, an experiment (Example 2) was performed by taking different values of $p (p = 1, 2, \cdots 9)$ corresponding to a different value of the uncertainty parameters $t_1 (t_1 = 1, 2, \cdots, 9)$ and $t_2 (t_2 = 1, 2, \cdots, 9)$.

On the basis of these different pairs of parameters, similarity measure S^w was computed, and its results are summarized in Figs. 4-6. From these, the important points have been concluded in the following.

(1) For a fixed value of t_1 and t_2, it has been observed that the decision values corresponding to each alternative decrease with the increase in the value of p (Fig. 7). The decision values of five alternatives have a clear distinction from (a) to (i). From (a) to (i), the final results all keep as $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

(2) For a fixed value of p and t_2, as t_1 increases, the decision values corresponding to most of alternatives monotonically decreases (Fig. 8). Moreover, it can be easily seen that the decision values of all alternatives become decreasing slowly when t_1 is increasing. From (a) to (i), the five alternatives have a gradually increasing gap from [0.5731,0.8314] to [0.5193,0.8203]. For alternative A_1, it monotonically decreases when (a) to (d) while it first monotonically increases to $t_1 = 2$ and monotonically decreases when (e) to (i). For alternative A_2, it first monotonically increases to $t_1 = 2$ and keep stationary to $t_1 = 3$, later monotonically decreases when (a) to (c). Meanwhile, it first monotonically increases to $t_1 = 2$ and monotonically decreases when (d) to (i). For alternatives A_3, A_4 and A_5, they all keep an decreasing trend. The final results all keep as $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

(3) For a fixed value of p and t_1, as t_2 increases, the decision values corresponding to each alternative most monotonically decreases (Fig. 9). Moreover, it can be easily seen that the decision values of all alternatives become decreasing or increasing slowly when t_2 is increasing. It can be easily seen that the decision values of five alternatives are monotonically increases when (a) to (b). For alternative A_1, it monotonically increases when (a) while it monotonically decreases when (b) to (i). For alternative A_2, it first keep stationary to

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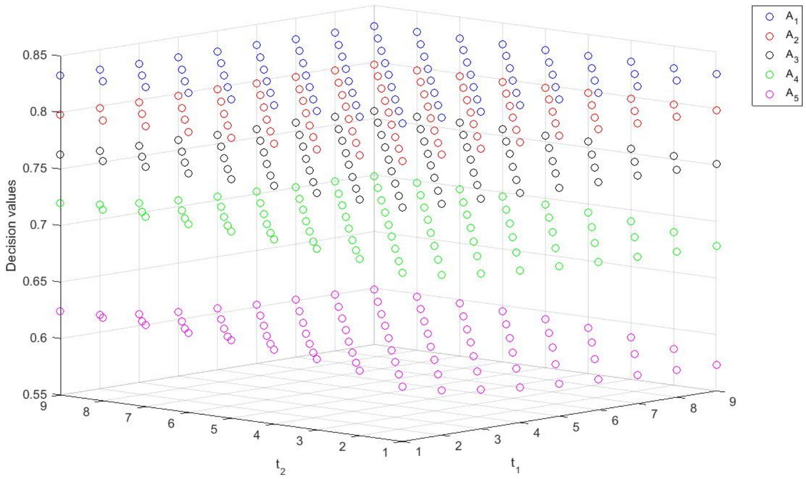
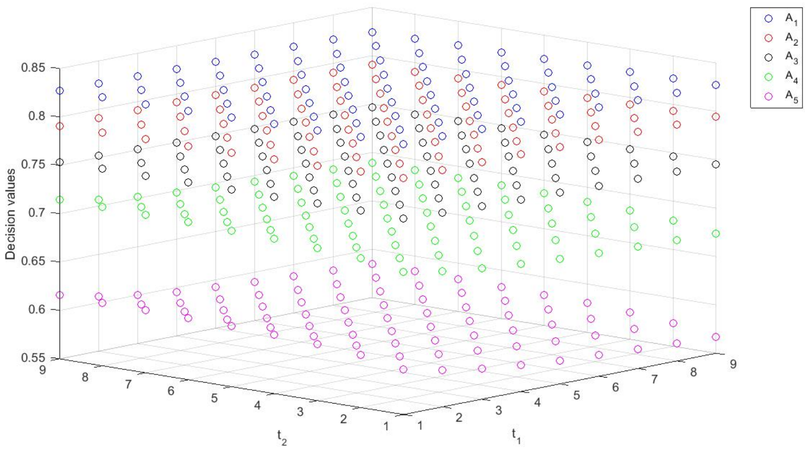
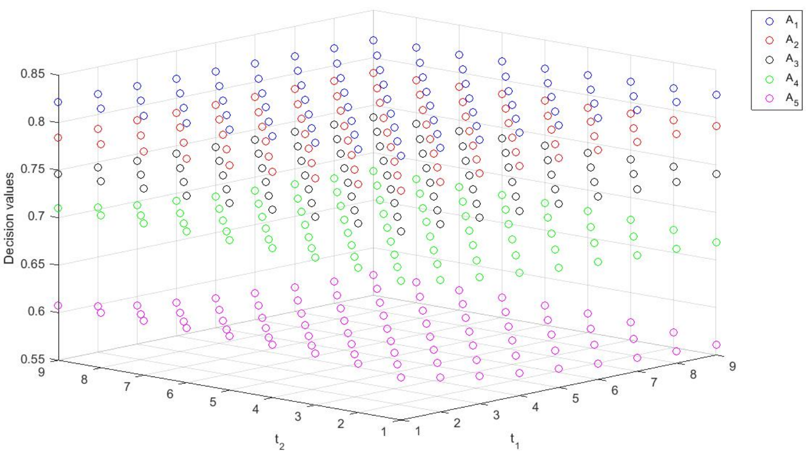


(a) p=1



(b) p=2



(c) p=3

FIGURE 4: The total changing trend of parameters t_1 and t_2 in proposed algorithm when p=1,2,3.

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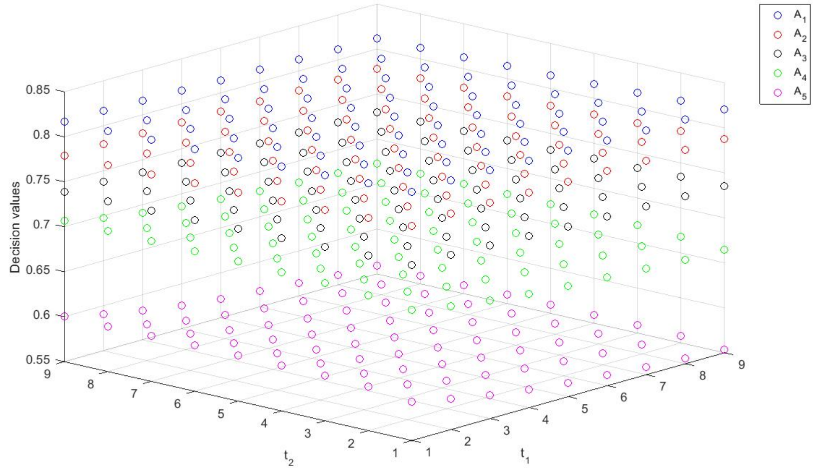
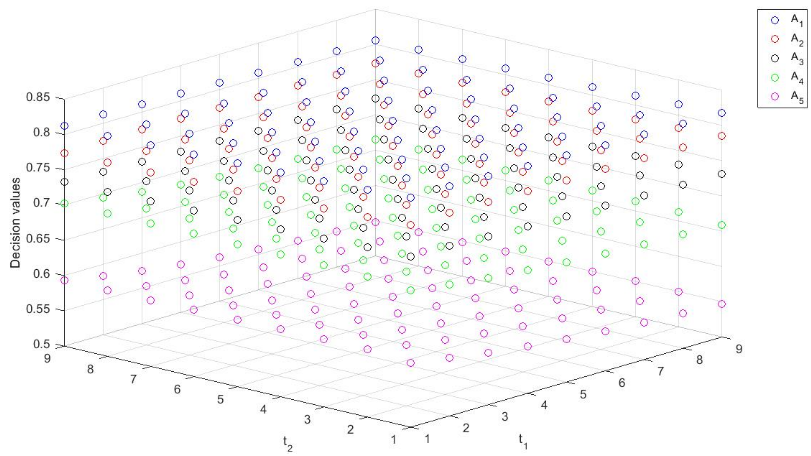
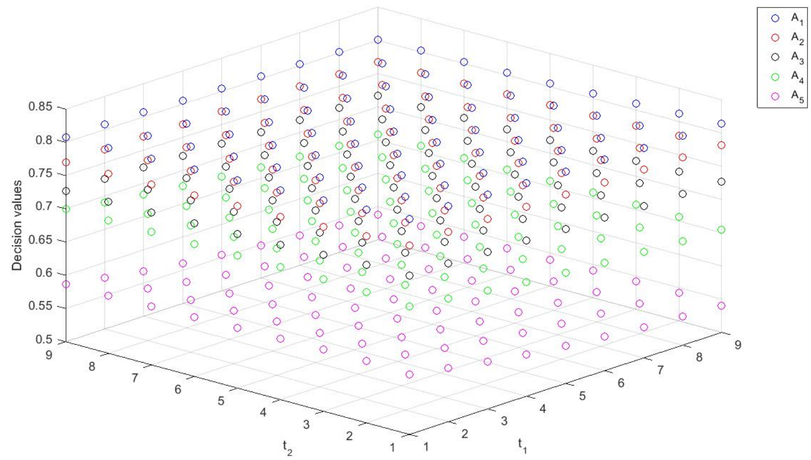


(a) p=4



(b) p=5



(c) p=6

FIGURE 5: The total changing trend of parameters t_1 and t_2 in proposed algorithm when p=4,5,6.

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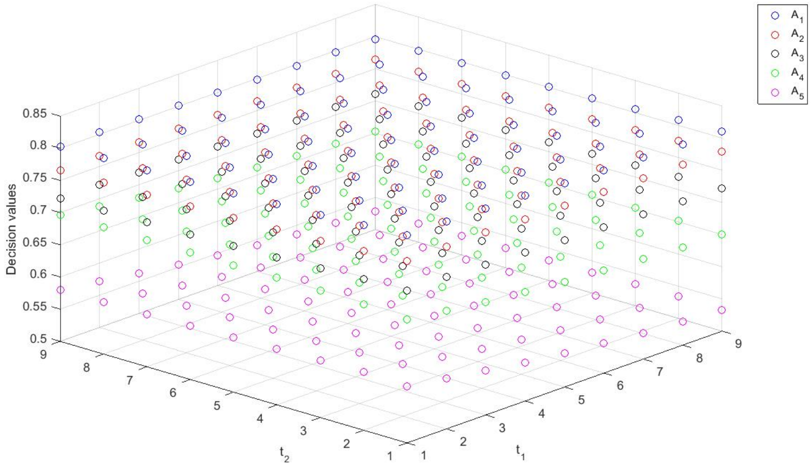
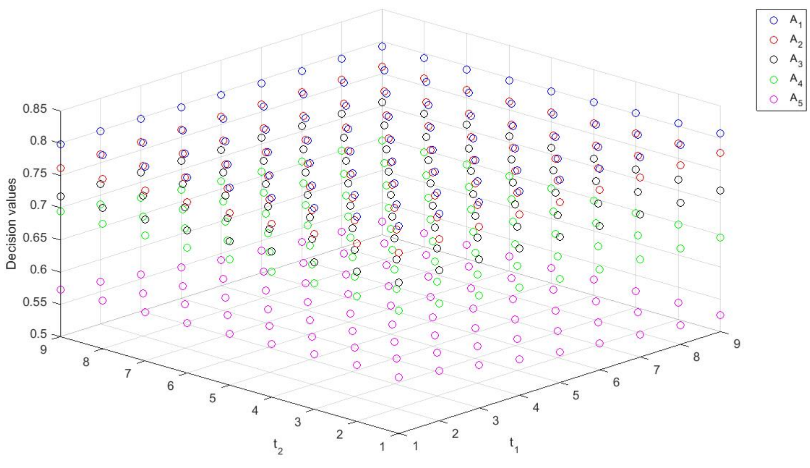
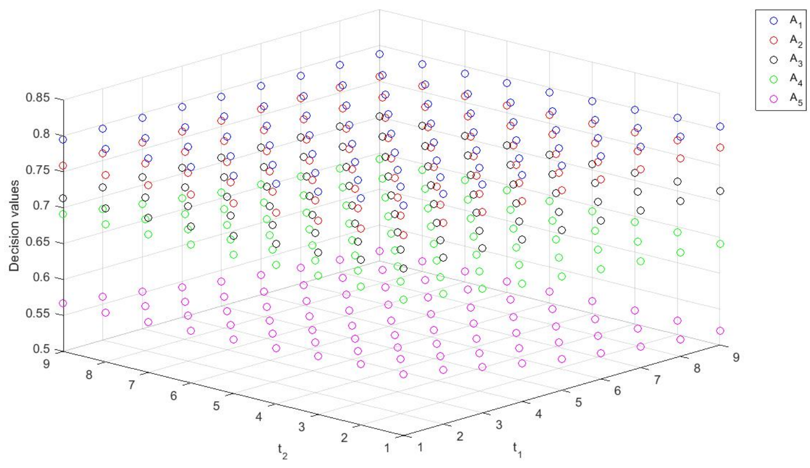


(a) p=7



(b) p=8



(c) p=9

FIGURE 6: The total changing trend of parameters t_1 and t_2 in proposed algorithm when p=7,8,9.

TABLE 4: The interval neutrosophic matrix in Example 2.

	C_1	C_2	C_3
A_1	$([0.8,0.9],[0.1,0.2],[0.1,0.2])$	$([0.8,0.9],[0.1,0.2],[0.2,0.3])$	$([0.8,0.9],[0.2,0.3],[0.1,0.2])$
A_2	$([0.8,0.8],[0.1,0.2],[0.1,0.2])$	$([0.8,0.9],[0.2,0.3],[0.3,0.3])$	$([0.7,0.8],[0.2,0.3],[0.1,0.3])$
A_3	$([0.7,0.8],[0.1,0.2],[0.1,0.2])$	$([0.7,0.8],[0.2,0.3],[0.3,0.4])$	$([0.6,0.8],[0.2,0.3],[0.1,0.3])$
A_4	$([0.5,0.6],[0.1,0.3],[0.2,0.3])$	$([0.6,0.7],[0.2,0.3],[0.3,0.4])$	$([0.6,0.8],[0.2,0.3],[0.2,0.3])$
A_5	$([0.3,0.4],[0.2,0.3],[0.2,0.3])$	$([0.5,0.6],[0.3,0.4],[0.4,0.5])$	$([0.5,0.7],[0.4,0.5],[0.3,0.4])$
	C_4	C_5	
A_1	$([0.8,0.9],[0.1,0.2],[0.1,0.2])$	$([0.7,0.8],[0.1,0.2],[0.1,0.2])$	
A_2	$([0.8,0.9],[0.2,0.2],[0.2,0.2])$	$([0.7,0.8],[0.2,0.3],[0.1,0.2])$	
A_3	$([0.8,0.9],[0.2,0.3],[0.3,0.4])$	$([0.6,0.7],[0.2,0.3],[0.2,0.3])$	
A_4	$([0.7,0.8],[0.2,0.3],[0.3,0.4])$	$([0.5,0.6],[0.2,0.3],[0.2,0.3])$	
A_5	$([0.6,0.7],[0.2,0.3],[0.4,0.4])$	$([0.4,0.6],[0.5,0.6],[0.2,0.4])$	

$t_2 = 4$ and drop to $t_2 = 5$, later keep stationary to $t_2 = 9$ when (a) and monotonically decreases when (b) to (i). For alternative A_3, it monotonically increases when (a) while it monotonically decreases when (d) to (i). Meanwhile, it first keep stationary to $t_2 = 2$ and monotonically decreases when (c). For alternative A_4, most of them keep an increasing trend when (a) to (e). For alternative A_5, it monotonically increases when (a) to (c) while it first monotonically increases and monotonically decreases when (e) to (i). The final results all keep as $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$.

F. EFFECTIVENESS TEST

For stating the availability of the developed algorithm, the algorithm has been tested on certain test criteria which has proposed by Wang and Triantaphyllou [70] in the following.

Test criterion 1: If we instead of the decision values of non-optimal alternative with worse alternative, then the optimal alternative should not alter.

Test criterion 2: MADM method should comply with transitive property.

Test criterion 3: When a appointed issue is disassembled into smaller ones and the same MADM method has been employed, then the assorted ordering of the alternatives should be same as the ranking of original one.

In the following, we have affirmed these test criteria on our developed MADM method based on interval neutrosophic similarity measure.

1) Effectiveness test by criterion 1

For this test, if we exchange the truth degrees and falsity degrees with opposite indeterminacy degrees of alternatives A_2, A_3, A_4 (non-optimal) and A_5 (worse) in the matrix P, then the switched decision matrix turns into P' which is shown in Table 5.

According to above information, the presented similarity measure S^w has been applied, the optimal alternatives is A_1 which is same as that of the original ranking. Therefore, the proposed algorithm is feasible under the test criterion 1.

2) Effectiveness test by criteria 2 and 3

According to these tests, if we resolved the appointed problem into the sub-issues $\{A_1, A_2, A_3\}$, $\{A_2, A_3, A_4\}$, $\{A_3, A_4, A_5\}$ and $\{A_1, A_4, A_5\}$ and the procedure steps of the algorithm has been employed, then we obtain the ranking of these smaller issues is $A_1 \succ A_2 \succ A_3$, $A_2 \succ A_3 \succ A_4$, $A_3 \succ A_4 \succ A_5$ and $A_1 \succ A_4 \succ A_5$, respectively. Hence, by uniting above criteria 2 and 3, we obtain the overall ranking order of the alternatives is $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ which is equal that of the original ranking order. Therefore, the developed algorithm is feasible under the test criteria 2 and 3.

V. COMPARATIVE ANALYSIS AND DISCUSSION

In the following, some existing decision making methods [42, 44, 47, 49, 51, 62, 67] and their limitations are discussed in detail. Two examples are given to show the advantages of our proposed algorithm. For better comparisons, we take the unified weight (proposed combined weight) in this paper.

Example 3: Continue to Example 2. Suppose that the assessments for IoT companies arising from another group of experts are presented which is shown in Table 6.

Remark 1: From the Table 7, we can see that the red background color denote some unreasonable results due to the counterintuitive phenomena which is discussed in Peng and Dai [33]. For G_{WINN3} and G_{WINN4} in [62], it would cause the unauthentic situation due to the the selection of λ. In Ye's definition [62], the preference value of current alternative and preference value of ideal alternative in corresponding attribute are assigned the λ and $1-\lambda$, respectively. As a matter of fact, it will cause unexpected change unless the $\lambda = 0$. That is to say, the $\lambda = 0$ will make the current preference value not influence the final decision value in the whole decision process under the ideal environment. It is easily known that the optimal alternative and corresponding the ordering are same as the results of INNWG [42], INNEWG [42], IVNSGWA ($\lambda \to 0$) [44], INPGWA ($\lambda \to 0$) [49], INDWMMP$^{(1,0,0,0,0)}$ [51], Cross-Entropy [67], G_{WINN3}, G_{WINN4} ($\lambda = 0$).

Example 4: Continue to Example 2. Suppose that the assessments for IoT companies arising from another group of experts are presented which is shown in Table 8.

TABLE 5: The switched interval neutrosophic matrix P' in Example 2.

	C_1	C_2	C_3
A_1	$([0.8, 0.9], [0.1, 0.2], [0.1, 0.2])$	$([0.8, 0.9], [0.1, 0.2], [0.2, 0.3])$	$([0.8, 0.9], [0.2, 0.3], [0.1, 0.2])$
A_2	$([0.1, 0.2], [0.8, 0.9], [0.8, 0.8])$	$([0.3, 0.3], [0.7, 0.8], [0.8, 0.9])$	$([0.1, 0.3], [0.7, 0.8], [0.7, 0.8])$
A_3	$([0.1, 0.2], [0.8, 0.9], [0.7, 0.8])$	$([0.3, 0.4], [0.7, 0.8], [0.7, 0.8])$	$([0.1, 0.3], [0.7, 0.8], [0.6, 0.8])$
A_4	$([0.2, 0.3], [0.7, 0.9], [0.5, 0.6])$	$([0.3, 0.4], [0.7, 0.8], [0.6, 0.7])$	$([0.2, 0.3], [0.7, 0.8], [0.6, 0.8])$
A_5	$([0.2, 0.3], [0.7, 0.8], [0.3, 0.4])$	$([0.4, 0.5], [0.6, 0.7], [0.5, 0.6])$	$([0.3, 0.4], [0.5, 0.6], [0.5, 0.7])$
	C_4	C_5	
A_1	$([0.8, 0.9], [0.1, 0.2], [0.1, 0.2])$	$([0.7, 0.8], [0.1, 0.2], [0.1, 0.2])$	
A_2	$([0.2, 0.2], [0.8, 0.8], [0.8, 0.9])$	$([0.1, 0.2], [0.7, 0.8], [0.7, 0.8])$	
A_3	$([0.3, 0.4], [0.7, 0.8], [0.8, 0.9])$	$([0.2, 0.3], [0.7, 0.8], [0.6, 0.7])$	
A_4	$([0.3, 0.4], [0.7, 0.8], [0.7, 0.8])$	$([0.2, 0.3], [0.7, 0.8], [0.5, 0.6])$	
A_5	$([0.4, 0.4], [0.7, 0.8], [0.6, 0.7])$	$([0.2, 0.4], [0.4, 0.5], [0.4, 0.6])$	

TABLE 6: The interval neutrosophic matrix in Example 3.

	C_1	C_2	C_3
A_1	$([1, 1], [0.1, 0.1], [0.1, 0.1])$	$([0.1, 0.2], [0.2, 0.3], [0.3, 0.4])$	$([0.1, 0.3], [0.3, 0.4], [0.2, 0.3])$
A_2	$([0.8, 0.9], [0.1, 0.2], [0.1, 0.1])$	$([0.8, 0.9], [0.2, 0.3], [0.3, 0.4])$	$([0.7, 0.8], [0.3, 0.4], [0.2, 0.3])$
A_3	$([0.7, 0.8], [0.1, 0.2], [0.1, 0.1])$	$([0.7, 0.8], [0.2, 0.3], [0.3, 0.4])$	$([0.6, 0.7], [0.3, 0.4], [0.2, 0.3])$
A_4	$([0.6, 0.7], [0.1, 0.2], [0.1, 0.1])$	$([0.6, 0.7], [0.2, 0.3], [0.3, 0.4])$	$([0.6, 0.7], [0.3, 0.4], [0.2, 0.3])$
A_5	$([0.6, 0.7], [0.1, 0.2], [0.1, 0.1])$	$([0.6, 0.7], [0.2, 0.3], [0.3, 0.4])$	$([0.5, 0.6], [0.3, 0.4], [0.2, 0.3])$
	C_4	C_5	
A_1	$([0.1, 0.3], [0.2, 0.3], [0.2, 0.3])$	$([0.2, 0.3], [0.2, 0.3], [0.3, 0.4])$	
A_2	$([0.8, 0.9], [0.2, 0.3], [0.2, 0.3])$	$([0.7, 0.8], [0.2, 0.3], [0.3, 0.4])$	
A_3	$([0.8, 0.9], [0.2, 0.3], [0.2, 0.3])$	$([0.6, 0.7], [0.2, 0.3], [0.3, 0.4])$	
A_4	$([0.7, 0.8], [0.2, 0.3], [0.2, 0.3])$	$([0.5, 0.6], [0.2, 0.3], [0.3, 0.4])$	
A_5	$([0.6, 0.8], [0.2, 0.3], [0.2, 0.3])$	$([0.4, 0.5], [0.2, 0.3], [0.3, 0.4])$	

TABLE 7: A comparison study with some existing methods in Example 3.

Algorithms	Ranking	Optimal alternative
Algorithm 1: Similarity measure	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Zhang et al. [42]: INNWA	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhang et al. [42]: INNWG	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Zhang et al. [42]: INNEWG	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Zhao et al. [44]: IVNSGWA ($\lambda = 1$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhao et al. [44]: IVNSGWA ($\lambda \to 0$)	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Ye [47]: CIINWAA	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [47]: CIINWGA	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Liu and Tang [49]: INPGWA ($\lambda = 1$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Liu and Tang [49]: INPGWA ($\lambda \to 0$)	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Liu and You [51]: INWMMP$^{(1,0,0,0,0)}$	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Liu and You [51]: INDWMMP$^{(1,0,0,0,0)}$	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Ye [67]: Cross-Entropy	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Ye [62]:Dice measure G_{WINN3} ($\lambda = 0$)	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Ye [62]:Dice measure G_{WINN3} ($\lambda = 1$)	$A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$	A_1
Ye [62]:Dice measure G_{WINN4} ($\lambda = 0$)	$A_2 \succ A_3 \succ A_1 \succ A_4 \succ A_5$	A_2
Ye [62]:Dice measure G_{WINN4} ($\lambda = 1$)	$A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$	A_1

$p = 1, t_1 = 2, t_2 = 3$ in Algorithm 1;
The red background color denotes the counterintuitive phenomena;
The green background color denotes the unauthentic results.

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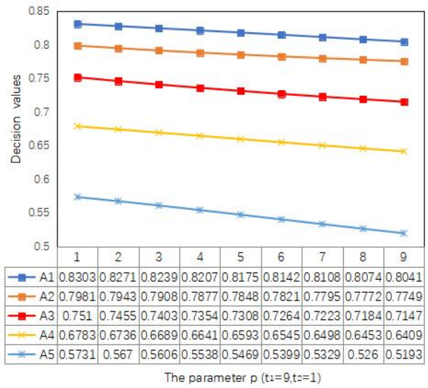
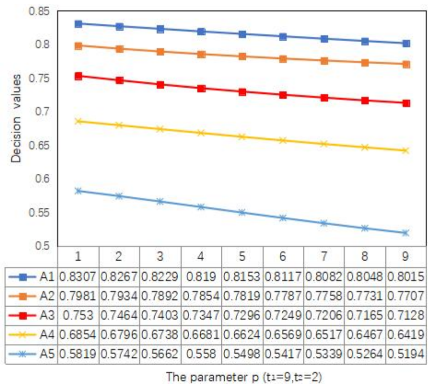
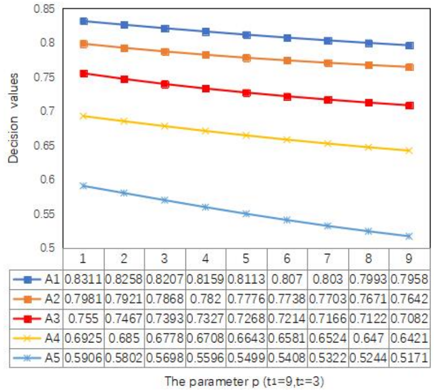


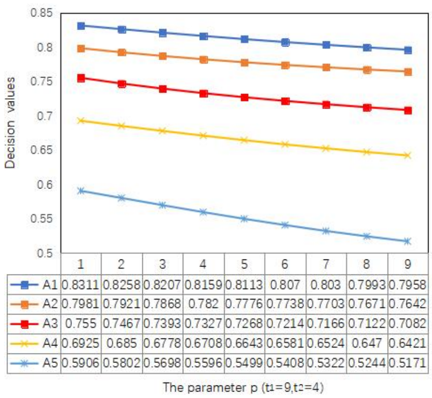
(a) $t_1 = 9, t_2 = 1$



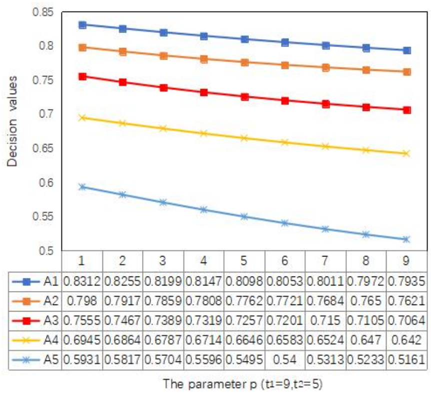
(b) $t_1 = 9, t_2 = 2$



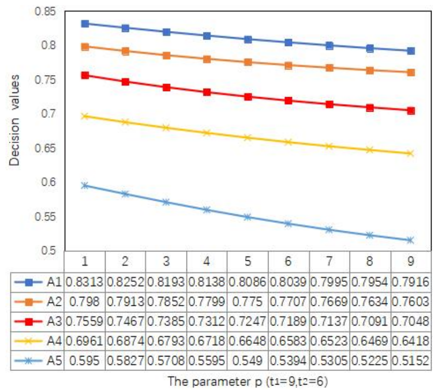
(c) $t_1 = 9, t_2 = 3$



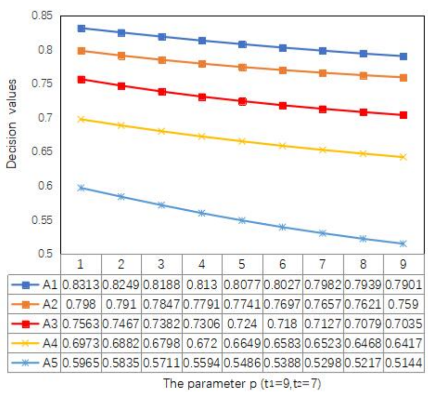
(d) $t_1 = 9, t_2 = 4$



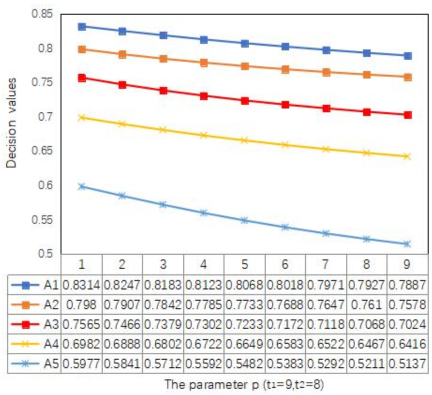
(e) $t_1 = 9, t_2 = 5$



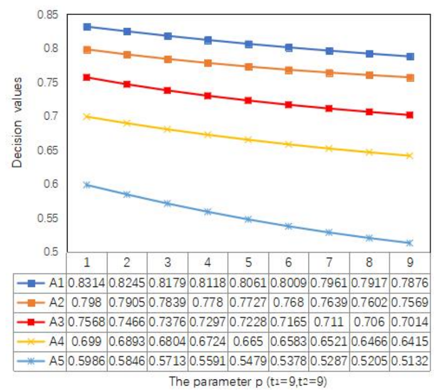
(f) $t_1 = 9, t_2 = 6$



(g) $t_1 = 9, t_2 = 7$



(h) $t_1 = 9, t_2 = 8$



(i) $t_1 = 9, t_2 = 9$

FIGURE 7: The changing trend of parameter p.

Remark 2: From the Table 9, we can see that the red background color denote some unreasonable results due to the counterintuitive phenomena which is discussed in Peng and Dai [33]. For G_{WINN3} and G_{WINN4} in [62], it would cause the unauthentic situation due to the the selection of λ. In other words, it will not obtain a convincing result. It is easily known that the optimal alternative and corresponding the ordering are same as the results of INNWA [42], IVNSGWA ($\lambda = 1$) [44], INPGWA ($\lambda = 1$) [49], INDWMMP$^{(1,0,0,0,0)}$ [51], Cross-Entropy [67], G_{WINN3}, G_{WINN4} ($\lambda = 0$).

VI. CONCLUSION

The main contributions can be illustrated and reviewed in the following.

(1) The formulae of interval neutrosophic similarity measures and distance measures are proposed, and their properties are proved. Meanwhile, the diverse desirable relations between the developed similarity measures and distance measures have also been elicited. Especially, a comparison

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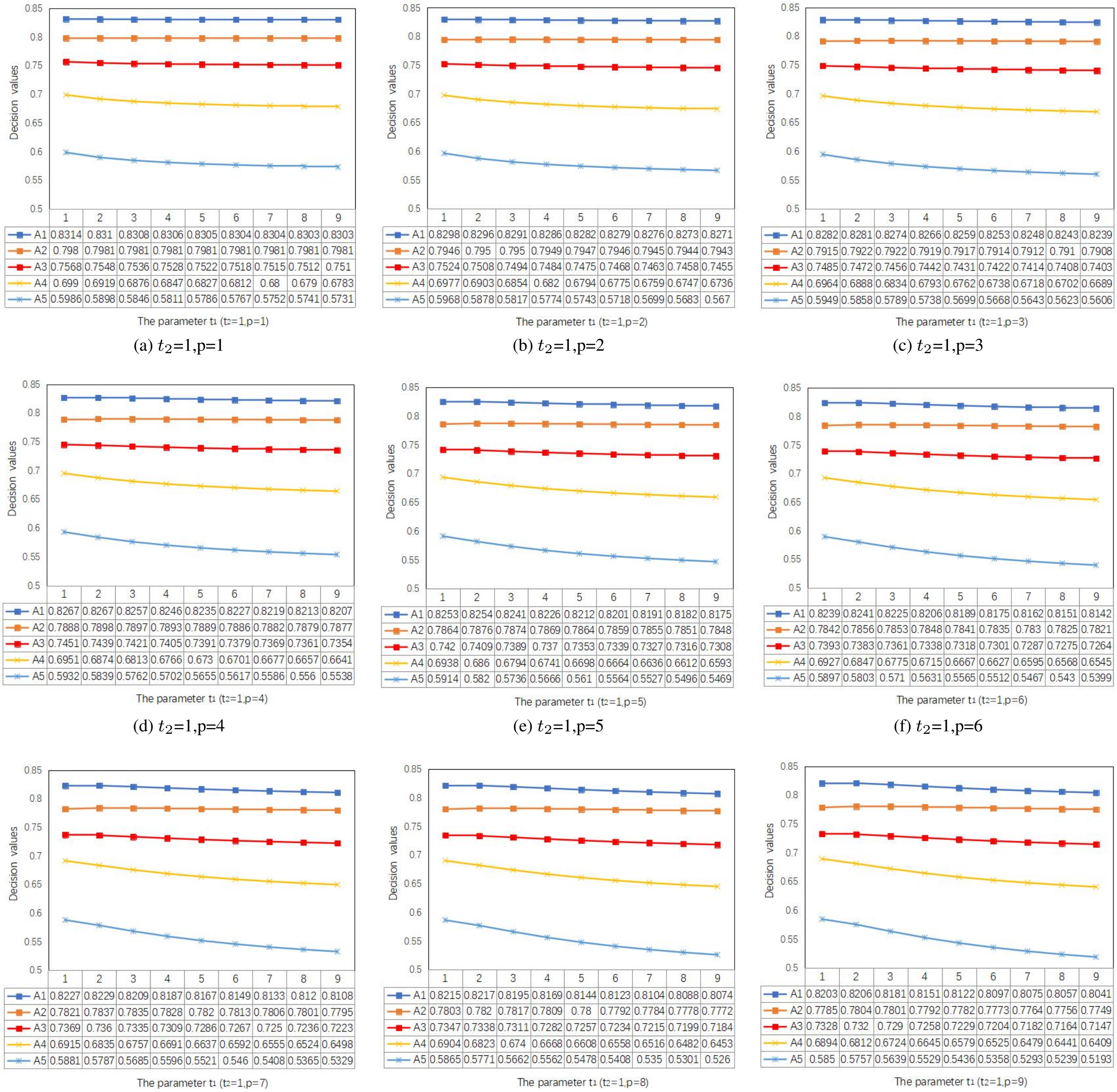

(a) t_2=1,p=1 (b) t_2=1,p=2 (c) t_2=1,p=3

(d) t_2=1,p=4 (e) t_2=1,p=5 (f) t_2=1,p=6

(g) t_2=1,p=7 (h) t_2=1,p=8 (i) t_2=1,p=9

FIGURE 8: The changing trend of parameter t_1.

with some existing literature [54, 55, 57, 58, 60–66] are constructed in Table 2 to state the effectiveness of proposed similarity measure. Compared with the existing similarity measure [74], it possesses more application scenarios of interval form.

(2) Deviation-based method for achieving objective weight is given, later, the combination weights are introduced, which can effectually reveal the subjective weight and objective weight, whereas the combination weight presented in Peng and Dai [33] can't reflect the discrepancy by comparing with the known weight (Fig. 3).

(3) An algorithm for solving interval neutrosophic decision making issue by multiparametric similarity measure is presented. The effect of the parameters p, t_1 and t_2 on the ranking in Algorithm 1 is discussed in detailed (Figs. 4, 5, 6, 7, 8, 9). Compared with the existing interval neutrosophic decision making algorithms (Table 7 and Table 9), are (i) it can achieve the best alternative out of counter-intuitive issues

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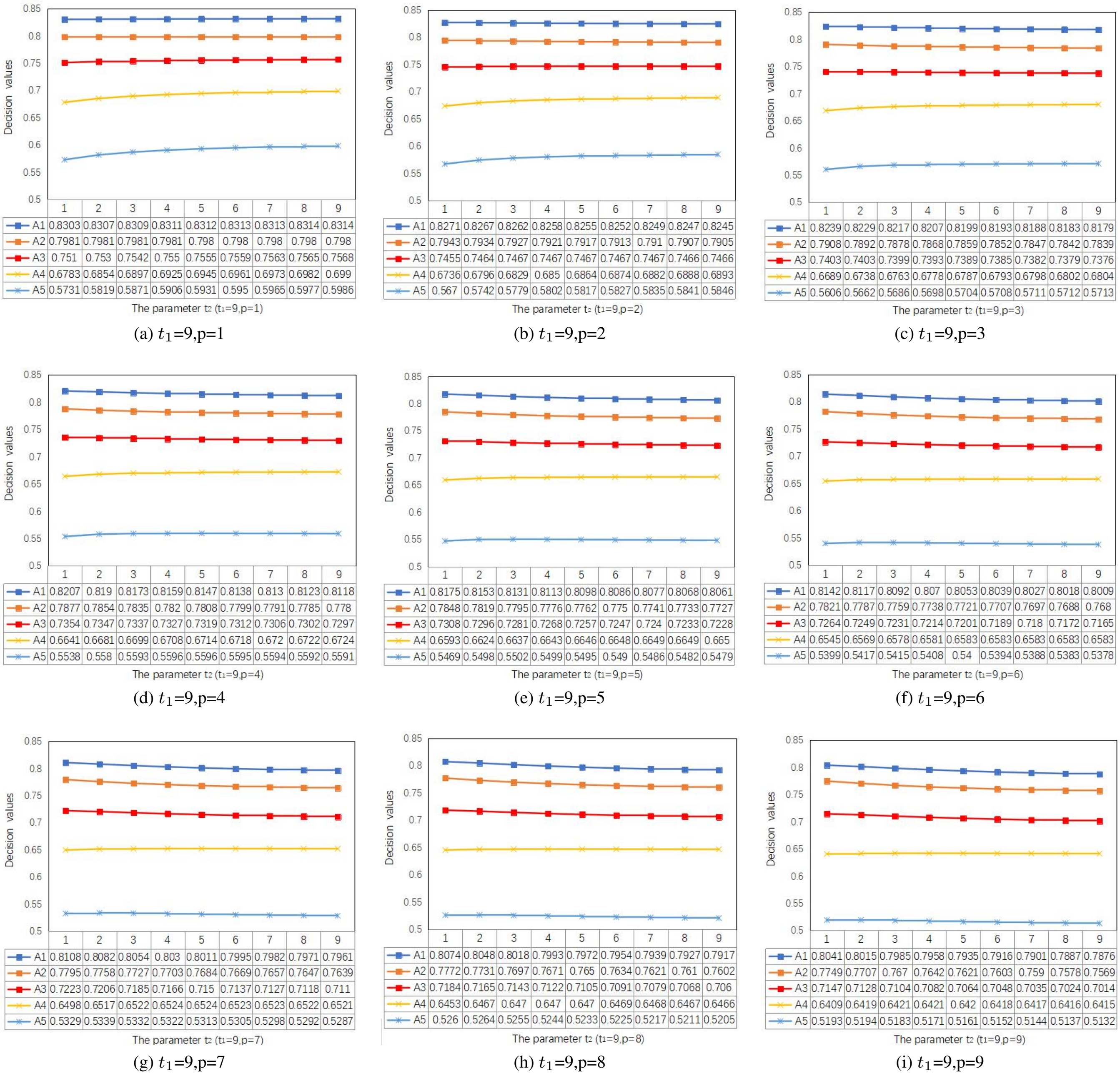

(a) t_1=9,p=1 (b) t_1=9,p=2 (c) t_1=9,p=3 (d) t_1=9,p=4 (e) t_1=9,p=5 (f) t_1=9,p=6 (g) t_1=9,p=7 (h) t_1=9,p=8 (i) t_1=9,p=9

FIGURE 9: The changing trend of parameter t_2.

[42, 44, 47, 49, 51, 67]; (ii) it has no unauthentic issues [62].

In the future, we will employ the similarity measures in other ways, such as gene selection [71]. Besides, as this paper is just an applied research focusing on the similarity measures of INSs, we shall attempt to design some softwares to preferably realize the initiated information measure in daily life. Meanwhile, we also will take them into and diverse fuzzy environment [72, 73, 75–79].

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TABLE 8: The interval neutrosophic matrix in Example 4.

	C_1	C_2	C_3
A_1	$([0.8,0.9],[0.1,0.2],[0.2,0.3])$	$([0.8,0.9],[0.1,0.3],[0.1,0.2])$	$([0,0],[0.1,0.2],[0.1,0.2])$
A_2	$([0.3,0.4],[0.1,0.2],[0.2,0.3])$	$([0.3,0.4],[0.1,0.3],[0.1,0.2])$	$([0.1,0.2],[0.1,0.2],[0.1,0.2])$
A_3	$([0.3,0.4],[0.1,0.2],[0.2,0.3])$	$([0.2,0.3],[0.1,0.3],[0.1,0.2])$	$([0.1,0.2],[0.1,0.2],[0.1,0.2])$
A_4	$([0.2,0.3],[0.1,0.2],[0.2,0.3])$	$([0.2,0.3],[0.1,0.3],[0.1,0.2])$	$([0.1,0.2],[0.1,0.2],[0.1,0.2])$
A_5	$([0.1,0.2],[0.1,0.2],[0.2,0.3])$	$([0.2,0.3],[0.1,0.3],[0.1,0.2])$	$([0.1,0.2],[0.1,0.2],[0.1,0.2])$
	C_4	C_5	
A_1	$([0.8,0.9],[0.1,0.3],[0.1,0.2])$	$([0.9,0.9],[0.1,0.3],[0.1,0.2])$	
A_2	$([0.3,0.4],[0.1,0.3],[0.1,0.2])$	$([0.2,0.3],[0.1,0.3],[0.1,0.2])$	
A_3	$([0.3,0.4],[0.1,0.3],[0.1,0.2])$	$([0.2,0.3],[0.1,0.3],[0.1,0.2])$	
A_4	$([0.2,0.3],[0.1,0.3],[0.1,0.2])$	$([0.1,0.2],[0.1,0.3],[0.1,0.2])$	
A_5	$([0.1,0.2],[0.1,0.3],[0.1,0.2])$	$([0.1,0.2],[0.1,0.3],[0.1,0.2])$	

TABLE 9: A comparison study with some existing methods in Example 4.

Algorithms	Ranking	Optimal alternative
Algorithm 1: Similarity measure	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhang et al. [42]: INNWA	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhang et al. [42]: INNWG	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Zhang et al. [42]: INNEWG	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Zhao et al. [44]: IVNSGWA ($\lambda = 1$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Zhao et al. [44]: IVNSGWA ($\lambda \to 0$)	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Ye [47]: CIINWAA	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [47]: CIINWGA	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Liu and Tang [49]: INPGWA ($\lambda = 1$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Liu and Tang [49]: INPGWA ($\lambda \to 0$)	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Liu and You [51]: INWMM$^{P(1,0,0,0,0)}$	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Liu and You [51]: INDWMM$^{P(1,0,0,0,0)}$	$A_2 \succ A_3 \succ A_4 \succ A_5 \succ A_1$	A_2
Ye [67]: Cross-Entropy	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [62]:Dice measure G_{WINN3} ($\lambda = 0$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [62]:Dice measure G_{WINN3} ($\lambda = 1$)	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$	A_5
Ye [62]:Dice measure G_{WINN4} ($\lambda = 0$)	$A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$	A_1
Ye [62]:Dice measure G_{WINN4} ($\lambda = 1$)	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$	A_5

$p = 1, t_1 = 2, t_2 = 3$ in Algorithm 1;
The red background color denotes the counterintuitive phenomena;
The green background color denotes the unauthentic results.

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