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METHODOLOGIES AND APPLICATION

Novel decision-making method based on bipolar neutrosophic information

Jianming Zhan[1] · Muhammad Akram[2] · Muzzamal Sitara[2]

Abstract

In 1998, Smarandache originally considered the concept of neutrosophic set from philosophical point of view. The notion of a single-valued neutrosophic set is a subclass of the neutrosophic set from a scientific and engineering point of view and an extension of intuitionistic fuzzy sets. A bipolar single-valued neutrosophic set is an extension of a bipolar fuzzy set, which provides us an additional possibility to represent uncertainty, imprecise, incomplete and inconsistent information existing in real situations. In this research article, we apply the concept of bipolar single-valued neutrosophic sets to graph structures and present a novel framework for handling bipolar neutrosophic information by combining bipolar neutrosophic sets with graph structures. Several basic notions concerning bipolar single-valued neutrosophic graph structures are introduced, and some related properties are investigated. We also consider the applications of bipolar single-valued neutrosophic graph structures in decision making. In particular, efficient algorithms are developed to solve decision-making problems regarding recognition of each country's participation in its conspicuous relationships, detection of psychological improvement of patients in a mental hospital and uncovering the undercover reasons for global terrorism.

Keywords Graph structure · Bipolar single-valued neutrosophic graph structure · φ-complement · Decision making · Algorithm

1 Introduction

Fuzzy set theory Zadeh (1965) is meticulous research field having abundant applications in diverse disciplines. Fuzzy set theory is a strong tool for dealing approximate reasoning problems. On those complex phenomenons while classical set theory does not represent reasonably, it is necessary to consider implicit counter property. Bipolar fuzzy sets Zhang (1994) are an extension of fuzzy sets whose membership degree range is [− 1,1]. Bosc and Pivert (2013) discussed the concept of bipolar fuzzy relations. According to him, "bipolarity refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative affects. Positive information states what is possible, satisfactory, permitted, desired, or considered as being acceptable. On the other hand, negative statements express what is impossible, rejected, or forbidden. Negative preferences correspond to constraints, since they specify which values or objects have to be rejected (i.e., those that do not satisfy the constraints), while positive preferences correspond to wishes, as they specify which objects are more desirable than others (i.e., satisfy user wishes) without rejecting those that do not meet the wishes."

Thus, bipolar fuzzy sets indeed have potential impacts on many fields, including artificial intelligence, computer science, information science, cognitive science, decision science, management science, economics, neural science, quantum computing, medical science and social science. In many phenomenons like information fusion, uncertainty and indeterminacy are doubtlessly quantified. Smarandache (1998) proposed the idea of neutrosophic sets, and he mingled tricomponent logic, nonstandard analysis and philosophy. It

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is a branch of philosophy which studies the origin, nature and scope of neutralities as well as their interactions with different ideational spectra. For convenient and advantageous usage of neutrosophic sets in science and engineering, Myithili et al. (2016) proposed the single-valued neutrosophic (SVNS) sets, whose three independent components have values in standard unit interval. Deli et al. (2015) narrated the notion of bipolar neutrosophic sets as an extension of the fuzzy sets, bipolar fuzzy sets and neutrosophic sets.

Fuzzy graph theory is finding an increasing number of applications in modeling real-time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. Kaufmann's initial definition of a fuzzy graph Kauffman (1973) was based on Zadeh's fuzzy relations Zadeh (1971). Rosenfeld Rosenfeld (1975) described the structure of fuzzy graphs obtaining analogs of several graph theoretical concepts. Bhattacharya (1987) gave some remarks on fuzzy graphs. Mordeson and Peng (1994) discussed some operations on fuzzy graphs. The complement of a fuzzy graph was defined by Mordeson and Peng (1994) and further studied by Sunitha and Vijayakumar (2002). Akram (2011, 2013) worked on certain notions of bipolar fuzzy graphs. Further theory of bipolar fuzzy graphs was developed in Akram and Anam (2017), Akram and Sitara (2017), Akram and Sarwar (2017), Sarwar and Akram (2017), Singh and Kumar (2014a), Singh and Kumar (2014b) and Yang et al. (2013). On the other hand, Dinesh and Ramakrishnan (2011) introduced the concept of the fuzzy graph structure. Akram and Akmal (2016) considered the notion of bipolar fuzzy graph structures.

Akram and Shahzadi (2017) first dealt with neutrosophic soft graphs. Single-valued neutrosophic minimum spanning tree and its clustering method were studied by Ye (2014). The same author gave many clustering algorithms based on different methods, including similarity measures, netting method and distance-based similarity measures (Ye 2013, 2014a, b). In this research article, we apply the concept of bipolar single-valued neutrosophic sets to graph structures and present a novel framework for handling bipolar neutrosophic information by combining bipolar neutrosophic sets with graph structures. We introduce several basic notions concerning bipolar single-valued neutrosophic graph structures and investigate some related properties. We also consider the applications of bipolar single-valued neutrosophic graph structures in decision making. In particular, efficient algorithms are developed to solve decision-making problems regarding recognition of each country's participation in its conspicuous relationships, detection of psychological improvement of patients in a mental hospital and uncovering the undercover reasons for global terrorism. Bipolar single-valued neutrosophic graphs, extension of bipolar fuzzy graph, handle the phenomenons concerning bipolar information in which indeterminacy is explicitly quantified. We deal with multiple relations in graph structure model; a bipolar neutrosophic graph structure is more advantageous and utilitarian. Obviously, a bipolar neutrosophic graph structure can deal with more than one factors involved for particular type of relationship between any two vertices, which is properly explained in this research paper with some interesting real-life applications. For other notations, terminologies and applications are not mentioned in this research article; the readers are referred to Alcantud (2016), Ali et al. (2016), Dinesh (2011), Greco and Kadzinski (2018), Luo et al. (2018), Majumdar and Samanta (2014), Mordeson and Nair (2001), Myithili et al. (2016), Pramanik et al. (2016), Sayed et al. (2018), Smarandache (1999), Turksen (1986), Wu et al. (2017) and Zhan et al. (2018).

2 Bipolar single-valued neutrosophic graph structures

Sampathkumar (2006) introduced the graph structure which is a generalization of undirected graph and is quite useful in studying some structures like graphs, signed graphs, labeled graphs and edge-colored graphs.

Definition 2.1 Sampathkumar (2006) A *graph structure* $\check{G}_s = (V, R_1, \ldots, R_m)$ consists of a non-empty set V together with relations $R_1, R_2, \ldots, R_m$ on V which are mutually disjoint such that each R_k, $1 \le k \le m$, is symmetric and irreflexive.

One can represent a graph structure $G = (V, R_1, ..., R_m)$ in the plane just like a graph where each edge is labeled as R_k, $1 \le k \le m$.

Smarandache (1998) introduced neutrosophic sets as a generalization of fuzzy sets and intuitionistic fuzzy sets. A neutrosophic set is identified by three functions called truth membership (T), indeterminacy membership (I) and falsity membership (F) whose values are real standard or nonstandard subset of unit interval $]^-0, 1^+[$, where $^-0 = 0-\epsilon, 1^+ = 1+\epsilon$, ϵ is an infinitesimal number. To apply neutrosophic set in real-life problems more conveniently, Smarandache (1998) and Myithili et al. (2016) defined single-valued neutrosophic sets which take the value from the subset of [0, 1]. Thus, a single-valued neutrosophic set is an instance of neutrosophic set and can be used expediently to deal with real-world problems, especially in decision support.

Definition 2.2 Smarandache (1998) A *neutrosophic set* N on a non-empty set V is an object of the form

$$N = \{(v, T_N(v), I_N(v), F_N(v)) : v \in V\}$$

where $T_N, I_N, F_N : V \to]0^-, 1^+[$ and there is no restriction on the sum of $T_N(v)$, $I_N(v)$ and $F_N(v)$ for all $v \in V$.

Definition 2.3 Myithili et al. (2016) A *single-valued neutrosophic set* N on a non-empty set V is an object of the form

$$N = \{(v, T_N(v), I_N(v), F_N(v)) : v \in V\}$$

where $T_N, I_N, F_N : V \to [0, 1]$ and sum of $T_N(v)$, $I_N(v)$ and $F_N(v)$ is confined between 0 and 3 for all $v \in V$.

Definition 2.4 Deli et al. (2015) A *bipolar single-valued neutrosophic set* B on a non-empty set V is an object of the form

$$B = \{(v, T_B^P(v), I_B^P(v), F_B^P(v), T_B^N(v), I_B^N(v), F_B^N(v)) : v \in V\}$$

where $T_B^P, I_B^P, F_B^P : V \to [0, 1]$ and $T_B^N, I_B^N, F_B^N : V \to [-1, 0]$. The positive values $T_B^P(v), I_B^P(v), F_B^P(v)$ denote the truth, indeterminacy and falsity membership values of an element $v \in V$, whereas negative values $T_B^N(v), I_B^N(v), F_B^N(v)$ indicate the implicit counterproperty of truth, indeterminacy and falsity membership values of an element $v \in V$.

Definition 2.5 A *bipolar single-valued neutrosophic graph* on a non-empty set V is a pair $G = (B, R)$, where B is a bipolar single-valued neutrosophic set on V and R is a bipolar single-valued neutrosophic relation in V such that

- $T_R^P(bd) \le T_B^P(b) \wedge T_B^P(d)$, $I_R^P(bd) \le I_B^P(b) \wedge I_B^P(d)$, $F_R^P(bd) \le F_B^P(b) \vee F_B^P(d)$,
- $T_R^N(bd) \ge T_B^N(b) \vee T_B^N(d)$, $I_R^N(bd) \ge I_B^N(b) \vee I_B^N(d)$, $F_R^N(bd) \ge F_B^N(b) \wedge F_B^N(d)$,

for all $b, d \in V$.

We now define bipolar single-valued neutrosophic graph structure.

Definition 2.6 $\check{G}_{bn} = (B, B_1, B_2, \ldots, B_m)$ is called *bipolar single-valued neutrosophic graph structure (BSVNGS)* of graph structure $\check{G}_s = (V, R_1, R_2, \ldots, R_m)$ if $B =< b, T^P(b), I^P(b), F^P(b), T^N(b), I^N(b), F^N(b) >$ and $B_k =< bd, T_k^P(bd), I_k^P(bd), F_k^P(bd), T_k^N(bd), I_k^N(bd), F_k^N(bd) >$ are bipolar single-valued neutrosophic (BSVN) sets on V and R_k, respectively, such that

- $T_k^P(bd) \le T^P(b) \wedge T^P(d)$, $I_k^P(bd) \le I^P(b) \wedge I^P(d)$, $F_k^P(bd) \le F^P(b) \vee F^P(d)$,
- $T_k^N(bd) \ge T^N(b) \vee T^N(d)$, $I_k^N(bd) \ge I^N(b) \vee I^N(d)$, $F_k^N(bd) \ge F^N(b) \wedge F^N(d)\}$,

for all $b, d \in V$.

We note that $0 \le T_k^P(bd) + I_k^P(bd) + F_k^P(bd) \le 3$, $-3 \le T_k^N(bd) + I_k^N(bd) + F_k^N(bd) \le 0$, for all $bd \in R_k$, $k \in \{1, 2, \ldots, m\}$.

Table 1 BSVN set B on vertex set V

B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
T^P	0.4	0.3	0.3	0.4	0.2	0.3	0.4	0.2
I^P	0.3	0.2	0.3	0.3	0.1	0.3	0.4	0.3
F^P	0.4	0.3	0.2	0.4	0.2	0.5	0.2	0.3
T^N	− 0.4	− 0.3	− 0.3	− 0.4	− 0.2	− 0.3	− 0.4	− 0.2
I^N	− 0.3	− 0.2	− 0.3	− 0.3	− 0.1	− 0.3	− 0.4	− 0.3
F^N	− 0.4	− 0.3	− 0.2	− 0.4	− 0.2	− 0.5	− 0.2	− 0.3

We illustrate the concept of BSVNGS with an example.

Example 2.7 Consider a graph structure $\check{G}_s = (V, R_1, R_2, R_3)$ such that $V = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$ and $R_1 = \{b_1b_2, b_2b_7, b_4b_8, b_6b_8, b_5b_6, b_3b_4\}$, $R_2 = \{b_1b_5, b_5b_7, b_3 b_6, b_7b_8\}$, $R_3 = \{b_1b_3, b_2b_4\}$. Let B be a BSVN set on V given in Table. 1 and B_1, B_2, B_3 be BSVN sets on R_1, R_2, R_3, respectively given in Table. 2.

Routine calculations show that $\check{G}_{bn} = (B, B_1, B_2, B_3)$ is a bipolar single-valued neutrosophic graph structure, as shown in Fig. 1.

Definition 2.8 A BSVNGS $\check{G}_{b1} = (B_1, B_{11}, B_{12}, \ldots, B_{1m})$ of graph structure $\check{G}_{s1} = (V_1, R_{11}, R_{12}, \ldots, R_{1m})$ is isomorphic to BSVNGS $\check{G}_{b2} = (B_2, B_{21}, B_{22}, \ldots, B_{2m})$ of graph structure $\check{G}_{s2} = (V_2, R_{21}, R_{22}, ..., R_{2m})$ if there exists a pair (h, φ), where $h : V_1 \to V_2$ is bijection and φ is a permutation on $\{1, 2, \ldots, m\}$ such that:

(i) $T_{B_1}^P(b) = T_{B_2}^P(h(b))$, $I_{B_1}^P(b) = I_{B_2}^P(h(b))$, $F_{B_1}^P(b) = F_{B_2}^P(h(b))$,
(ii) $T_{B_1}^N(b) = T_{B_2}^N(h(b))$, $I_{B_1}^N(b) = I_{B_2}^N(h(b))$, $F_{B_1}^N(b) = F_{B_2}^N(h(b))$,
(iii) $T_{B_{1k}}^P(bd) = T_{B_{2\varphi(k)}}^P(h(b)h(d))$, $I_{B_{1k}}^P(bd) = I_{B_{2\varphi(k)}}^P(h(b)h(d))$, $F_{B_{1k}}^P(bd) = F_{B_{2\varphi(k)}}^P(h(b)h(d))$,
(iv) $T_{B_{1k}}^N(bd) = T_{B_{2\varphi(k)}}^N(h(b)h(d))$, $I_{B_{1k}}^N(bd) = I_{B_{2\varphi(k)}}^N(h(b)h(d))$, $F_{B_{1k}}^N(bd) = F_{B_{2\varphi(k)}}^N(h(b)h(d))$,

for all $b \in V_1$, $bd \in B_{1k}$, $k \in \{1, 2, \ldots, m\}$.

Example 2.9 Let $\check{G}_{b1} = (B, B_1, B_2)$ and $\check{G}_{b2} = (B', B_1', B_2')$ be two BSVNGSs of two GSs.

$\check{G}_{s1} = (V, R_1, R_2)$ and $\check{G}_{s2} = (V', R_1', R_2')$ as shown in Figs. 2 and 3, respectively.

These two BSVNGSs $\check{G}_{b1}$ and $\check{G}_{b2}$ are isomorphic under (h, φ), where $h : V \to V'$ is a bijection and φ is a permutation on the set $\{1, 2\}$ defined as $\varphi(1) = 2$, $\varphi(2) = 1$, and satisfy following conditions:

- $T_B^P(b_i) = T_{B'}^P(h(b_i))$, $I_B^P(b_i) = I_{B'}^P(h(b_i))$, $F_B^P(b_i) = F_{B'}^P(h(b_i))$,

Table 2 BSVN sets B_1, B_2 and B_3

B_1	b_1b_2	b_2b_7	b_4b_8	b_6b_8	b_5b_6	b_3b_4	B_2	b_1b_5	b_5b_7	b_3b_6	b_7b_8	B_3	b_1b_3	b_2b_4
T^P	0.3	0.3	0.2	0.2	0.2	0.3	T^P	0.2	0.2	0.3	0.2	T^P	0.3	0.3
I^P	0.2	0.2	0.3	0.3	0.1	0.3	I^P	0.1	0.1	0.3	0.3	I^P	0.3	0.2
F^P	0.4	0.3	0.4	0.5	0.5	0.4	F^P	0.4	0.2	0.5	0.3	F^P	0.4	0.4
T^N	− 0.3	− 0.3	− 0.2	− 0.2	− 0.2	− 0.3	T^N	− 0.2	− 0.2	− 0.3	− 0.2	T^N	− 0.3	− 0.3
I^N	− 0.2	− 0.2	− 0.3	− 0.3	− 0.1	− 0.3	I^N	− 0.1	− 0.1	− 0.3	− 0.3	I^N	− 0.3	− 0.2
F^N	− 0.4	− 0.3	− 0.4	− 0.5	− 0.5	− 0.4	F^N	− 0.4	− 0.2	− 0.5	− 0.3	F^N	− 0.4	− 0.4

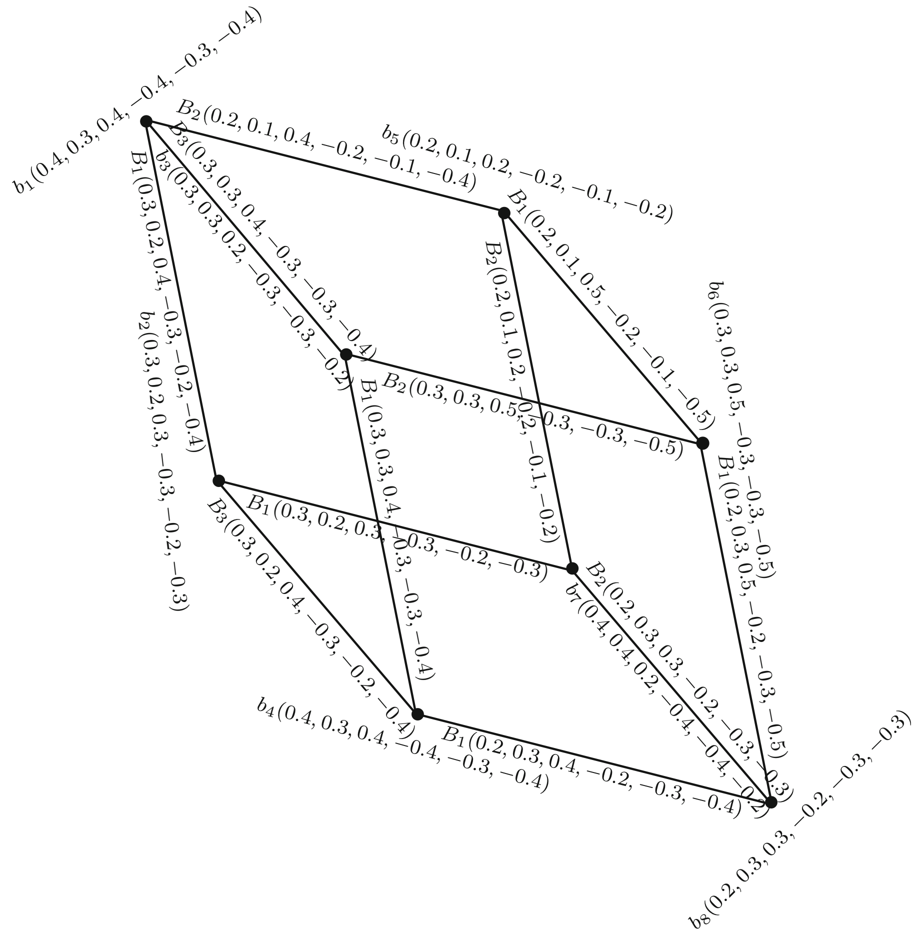


Fig. 1 Bipolar single-valued neutrosophic graph structure

- $T_B^N(b_i) = T_{B'}^N(h(b_i))$, $I_B^N(b_i) = I_{B'}^N(h(b_i))$, $F_B^N(b_i) = F_{B'}^N(h(b_i))$,
- $T_{B_k}^P(b_ib_j) = T_{B'_{\varphi(k)}}^P(h(b_i)h(b_j))$, $I_{B_k}^P(b_ib_j) = I_{B'_{\varphi(k)}}^P(h(b_i)h(b_j))$, $F_{B_k}^P(b_ib_j) = F_{B'_{\varphi(k)}}^P(h(b_i)h(b_j))$,
- $T_{B_k}^N(b_ib_j) = T_{B'_{\varphi(k)}}^N(h(b_i)h(b_j))$, $I_{B_k}^N(b_ib_j) = I_{B'_{\varphi(k)}}^N(h(b_i)h(b_j))$, $F_{B_k}^N(b_ib_j) = F_{B'_{\varphi(k)}}^N(h(b_i)h(b_j))$,

for all $b_i \in V$, $b_ib_j \in R_k$, $k = 1, 2$.

Definition 2.10 A BSVNGS $\check{G}_{b1} = (B_1, B_{11}, B_{12}, \ldots, B_{1m})$ of graph structure $\check{G_{s1}} = (V_1, R_{11}, R_{12}, \ldots, R_{1m})$ is said to be identical to BSVNGS $\check{G}_{b2} = (B_2, B_{21}, B_{22}, ..., B_{2m})$ of graph structure $\check{G_{s2}} = (V_2, R_{21}, R_{22}, \ldots, R_{2m})$ if $h : V_1 \to V_2$ is a bijection such that:

(i) $T_{B_1}^P(b) = T_{B_2}^P(h(b))$, $I_{B_1}^P(b) = I_{B_2}^P(h(b))$, $F_{B_1}^P(b) = F_{B_2}^P(h(b))$,
(ii) $T_{B_1}^N(b) = T_{B_2}^N(h(b))$, $I_{B_1}^N(b) = I_{B_2}^N(h(b))$, $F_{B_1}^N(b) = F_{B_2}^N(h(b))$,
(iii) $T_{B_{1k}}^P(bd) = T_{B_{2k}}^P(h(b)h(d))$, $I_{B_{1k}}^P(bd) = I_{B_{2k}}^P(h(b)h(d))$, $F_{B_{1k}}^P(bd) = F_{B_{2k}}^P(h(b)h(d))$,

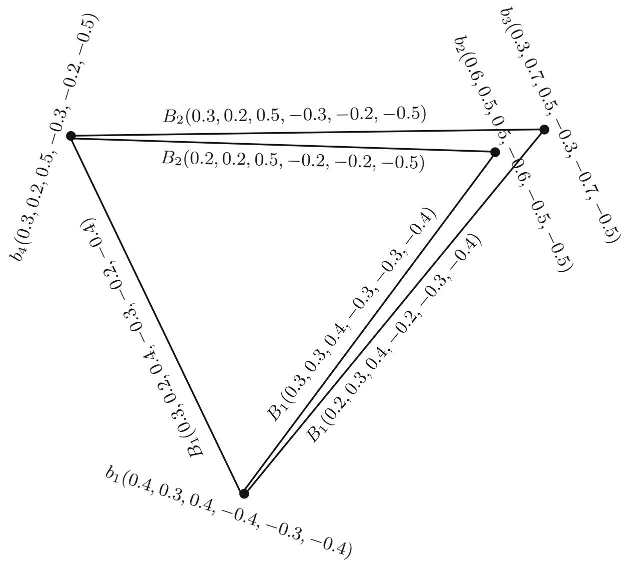


Fig. 2 A BSVNGS $\check{G}_{b1}$

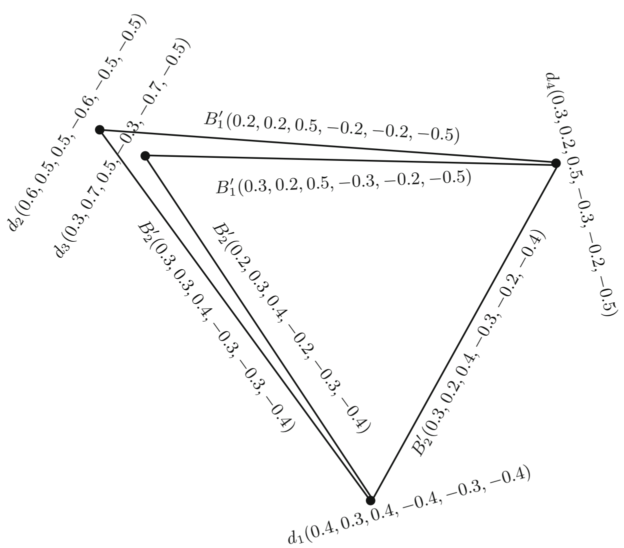


Fig. 3 A BSVNGS $\check{G}_{b2}$

(iv) $T^N_{B_{1k}}(bd) = T^N_{B_{2k}}(h(b)h(d)), I^N_{B_{1k}}(bd) = I^N_{B_{2k}}(h(b)h(d)),$ $F^N_{B_{1k}}(bd) = F^N_{B_{2k}}(h(b)h(d)),$

for all $b \in V_1, bd \in R_{1k}, k \in \{1, 2, \ldots, m\}$.

Example 2.11 Let $\check{G}_{b1} = (B, B_1, B_2)$ and $\check{G}_{b2} = (B', B_1', B_2')$ be two BSVNGSs of the GSs. $\check{G_{s1}} = (V, R_1, R_2)$ and $\check{G_{s2}} = (V', R_1', R_2')$, respectively, as depicted in Figs. 4 and 5.

BSVNGS $\check{G}_{b1}$ is identical to $\check{G}_{b2}$ under $h : V \to V'$ defined as:
$h(b_1) = d_2, h(b_2) = d_1, h(b_3) = d_4, h(b_4) = d_3, h(b_5) = d_5, h(b_6) = d_8, h(b_7) = d_7, h(b_8) = d_6$. Moreover,

- $T^P_B(b_i) = T^P_{B'}(h(b_i)),\ I^P_B(b_i) = I^P_{B'}(h(b_i)),\ F^P_B(b_i) = F^P_{B'}(h(b_i)),$
- $T^N_B(b_i) = T^N_{B'}(h(b_i)),\ I^N_B(b_i) = I^N_{B'}(h(b_i)),\ F^N_B(b_i) = F^N_{B'}(h(b_i)),$
- $T^P_{B_k}(b_ib_j) = T^P_{B_k'}(h(b_i)h(b_j)),\ I^P_{B_k}(b_ib_j) = I^P_{B_k'}(h(b_i)h(b_j)),\ F^P_{B_k}(b_ib_j) = F^P_{B_k'}(h(b_i)h(b_j)),$
- $T^N_{B_k}(b_ib_j) = T^N_{B_k'}(h(b_i)h(b_j)),\ I^N_{B_k}(b_ib_j) = I^N_{B_k'}(h(b_i)h(b_j)),\ F^N_{B_k}(b_ib_j) = F^N_{B_k'}(h(b_i)h(b_j)),$

for all $b_i \in V, b_ib_j \in R_k, k = 1, 2$.

Definition 2.12 Let $\check{G}_{bn} = (B, B_1, B_2, \ldots, B_m)$ be a BSVNGS and φ be a permutation on $\{B_1, B_2, \ldots, B_m\}$ and also on set $\{1, 2, \ldots, m\}$, that is, $\varphi(B_k) = B_l$ if and only if $\varphi(k) = l$ for all k. If $bd \in B_k$ and

(i) $T^P_{B^\varphi_k}(bd) = T^P_B(b) \wedge T^P_B(d) - \bigvee_{l \neq k} T^P_{\varphi(B_l)}(bd),\quad T^N_{B^\varphi_k}(bd) = T^N_B(b) \vee T^N_B(d) - \bigwedge_{l \neq k} T^N_{\varphi(B_l)}(bd),$

(ii) $I^P_{B^\varphi_k}(bd) = I^P_B(b) \wedge I^P_B(d) - \bigvee_{l \neq k} I^P_{\varphi(B_l)}(bd),\quad I^N_{B^\varphi_k}(bd) = I^N_B(b) \vee I^N_B(d) - \bigwedge_{l \neq k} I^N_{\varphi(B_l)}(bd),$

(iii) $F^P_{B^\varphi_k}(bd) = F^P_B(b) \vee F^P_B(d) - \bigwedge_{l \neq k} T^P_{\varphi(B_l)}(bd),\quad F^N_{B^\varphi_k}(bd) = F^N_B(b) \wedge F^N_B(d) - \bigvee_{l \neq k} T^N_{\varphi(B_l)}(bd),$

$k \in \{1, 2, \ldots, m\}$, then $bd \in B^\varphi_e$, where e is selected such that

- $T^P_{B^\varphi_e}(bd) \geq T^P_{B^\varphi_k}(bd),\quad I^P_{B^\varphi_e}(bd) \geq I^P_{B^\varphi_k}(bd),\quad F^P_{B^\varphi_e}(bd) \geq F^P_{B^\varphi_k}(bd),$
- $T^N_{B^\varphi_e}(bd) \leq T^N_{B^\varphi_k}(bd),\quad I^N_{B^\varphi_e}(bd) \leq I^N_{B^\varphi_k}(bd),\quad F^N_{B^\varphi_e}(bd) \leq F^N_{B^\varphi_k}(bd),$

for all k. Then, BSVNGS $(B, B^\varphi_1, B^\varphi_2, \ldots, B^\varphi_m)$ is called φ-complement of BSVNGS $\check{G}_{bn}$ and denoted by $\check{G}^{\varphi c}_{bn}$.

Example 2.13 Let $B = \{(b_1, 0.4, 0.4, 0.7, -0.4, -0.4, -0.7), (b_2, 0.6, 0.6, 0.4, -0.6, -0.6, -0.4), (b_3, 0.8, 0.5, 0.3, -0.8, -0.5, -0.3)\}$, $B_1 = \{(b_1b_3, 0.4, 0.4, 0.3, -0.4, -0.4, -0.3)\}$, $B_2 = \{(b_2b_3, 0.6, 0.4, 0.3, -0.6, -0.4, -0.3)\}$, $B_3 = \{(b_1b_2, 0.4, 0.3, 0.4, -0.4, -0.3, -0.4)\}$ be BSVN sets on $V = \{b_1, b_2, b_3\}$, $R_1 = \{b_1b_3\}$, $R_2 = \{b_2b_3\}$, $R_3 = \{b_1b_2\}$, respectively.

Consider $\check{G}_{bn} = (B, B_1, B_2, B_3)$ is a BSVNGS of GS $\check{G} = (V, R_1, R_2, R_3)$. Let $\varphi(B_1) = B_2, \varphi(B_2) = B_3, \varphi(B_3) = B_1$, where φ is permutation on $\{B_1, B_2, B_3\}$.
Through simple calculations for edges $b_1b_3, b_2b_3, b_1b_2 \in B_1, B_2, B_3$, respectively, we see that $b_1b_3 \in B^\varphi_3, b_2b_3 \in B^\varphi_1,$

Fig. 4 A BSVNGS $\check{G}_{b1}$

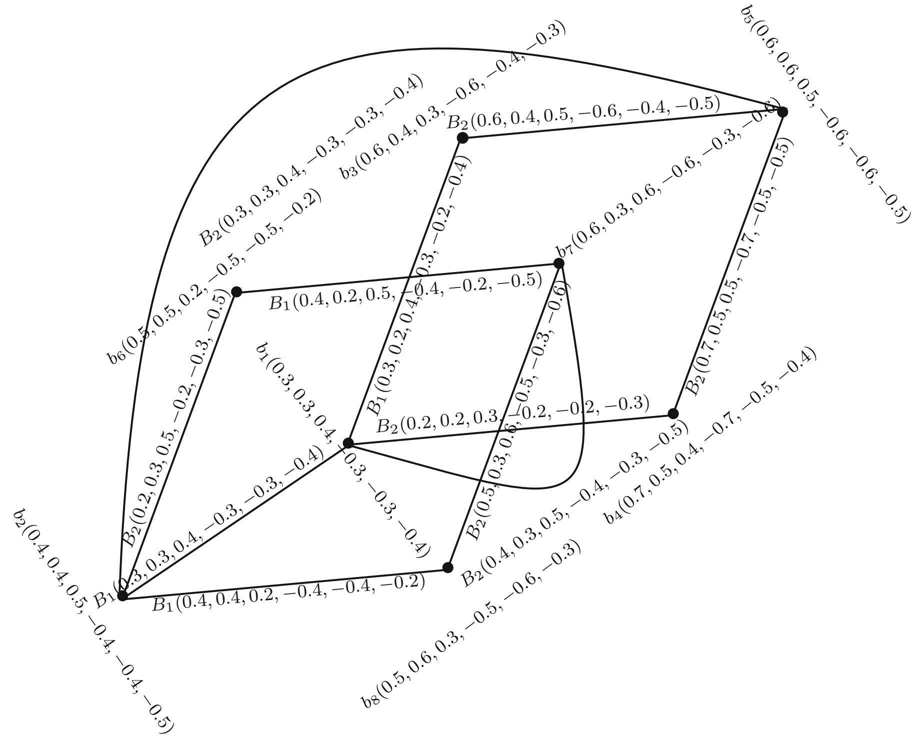
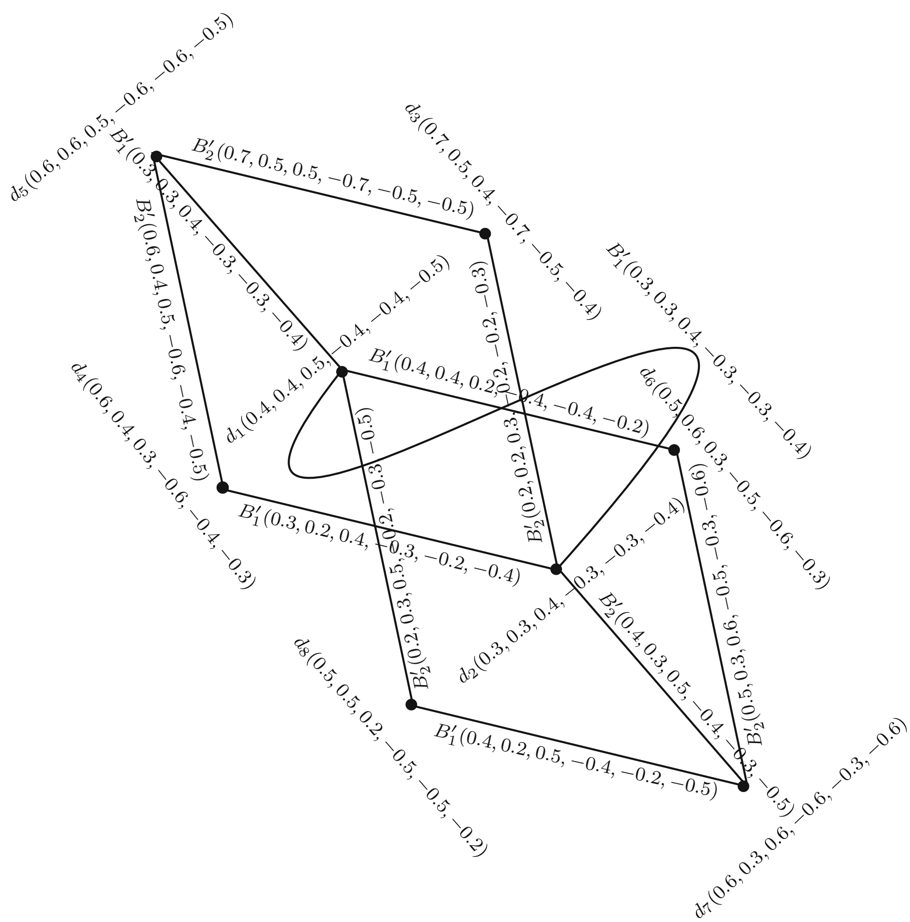


Fig. 5 A BSVNGS $\check{G}_{b2}$



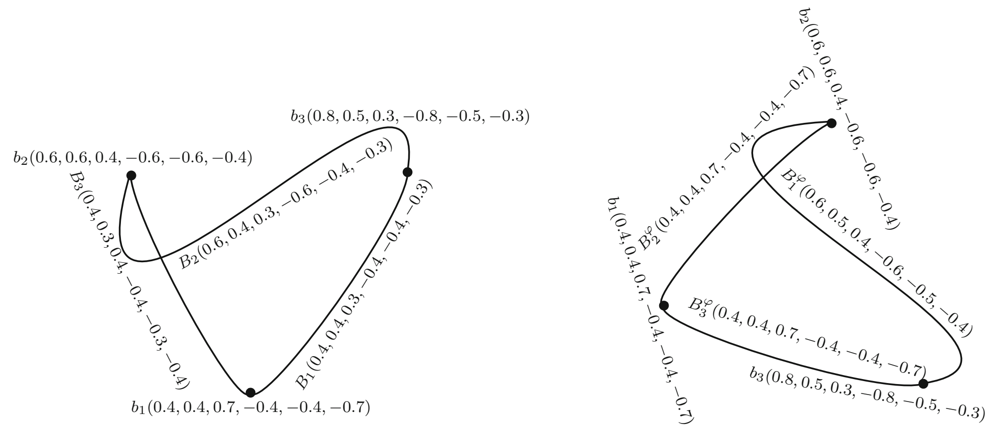


Fig. 6 Two BSVNGSs $\check{G}_{bn}$, $\check{G}_{bn}^{\psi c}$

$b_1 b_2 \in B_2^{\varphi}$. So, $\check{G}_{bn}^{\varphi c}$ =$(B, B_1^{\varphi}, B_2^{\varphi}, B_3^{\varphi})$ is φ-complement of BSVNGS $\check{G}_{bn}$ as shown in Fig. 6.

Proposition 2.14 *The φ-complement of a BSVNGS $\check{G}_{bn} = (B, B_1, B_2, \ldots, B_m)$ is a strong BSVNGS. Moreover, if $\varphi(k) = e$, where $k, e \in \{1, 2, \ldots, m\}$, then all B_e-edges in BSVNGS $(B, B_1, B_2, \ldots, B_m)$ become B_k^{φ}-edges in $(B, B_1^{\varphi}, B_2^{\varphi}, \ldots, B_m^{\varphi})$.*

Proof By definition of φ-complement, we have

$$T_{B_k^{\varphi}}^{P}(bd) = T_B^P(b) \wedge T_B^P(d) - \bigvee_{l \neq k} T_{\varphi(B_l)}^{P}(bd),$$
$$T_{B_k^{\varphi}}^{N}(bd) = T_B^N(b) \vee T_B^N(d) - \bigwedge_{l \neq k} T_{\varphi(B_l)}^{N}(bd),$$
$$I_{B_k^{\varphi}}^{P}(bd) = I_B^P(b) \wedge I_B^P(d) - \bigvee_{l \neq k} I_{\varphi(B_l)}^{P}(bd),$$
$$I_{B_k^{\varphi}}^{N}(bd) = I_B^N(b) \vee I_B^N(d) - \bigwedge_{l \neq k} I_{\varphi(B_l)}^{N}(bd),$$
$$F_{B_k^{\varphi}}^{P}(bd) = F_B^P(b) \vee F_B^P(d) - \bigwedge_{l \neq k} F_{\varphi(B_l)}^{P}(bd),$$
$$F_{B_k^{\varphi}}^{N}(bd) = F_B^N(b) \wedge F_B^N(d) - \bigvee_{l \neq k} F_{\varphi(B_l)}^{N}(bd),$$

where, $k \in \{1, 2, \ldots, m\}$. For expression of $T_{B_k^{\varphi}}^{P}$:

As $T_B^P(b) \wedge T_B^P(d) \geq 0$, $\bigvee_{l \neq k} T_{\varphi(B_l)}^{P}(bd) \geq 0$ and $T_{B_k}^{P}(bd) \leq T_B^P(b) \wedge T_B^P(d)$ for all B_k. This implies $\bigvee_{l \neq k} T_{\varphi(B_l)}^{P}(bd) \leq T_B^P(b) \wedge T_B^P(d)$. This shows that $T_B^P(b) \wedge T_B^P(b) - \bigvee_{l \neq k} T_{\varphi(B_l)}^{P}(bd) \geq 0$. Hence, $T_{B_k^{\varphi}}^{P}(bd) \geq 0$ for all k. Furthermore, $T_{B_k^{\varphi}}^{P}(bd)$ obtains maximum value when $\bigvee_{l \neq k} T_{\varphi(B_l)}^{P}(bd)$ is zero. Undoubtedly, when $\varphi(B_k) = B_e$ and bd is a B_e-edge, then $\bigvee_{l \neq k} T_{\varphi(B_l)}^{P}(bd)$ acquires zero value. Hence

$$T_{B_k^{\varphi}}^{P}(bd) = T_B^P(b)$$
$$\wedge T_B^P(d), \; for \; bd \in B_e, \; \varphi(B_k) = B_e.$$

For expression of $T_{B_k^{\varphi}}^{N}$:

Since $T_B^N(b) \vee T_B^N(d) \leq 0$, $\bigwedge_{l \neq k} T_{\varphi(B_l)}^{N}(bd) \leq 0$ and $T_{B_k}^{N}(bd) \geq T_B^N(b) \vee T_B^N(d)$ for all B_k. This implies $\bigwedge_{l \neq k} T_{\varphi(B_l)}^{N}(bd) \geq T_B^N(b) \vee T_B^N(d)$. It indicates $T_B^N(b) \vee T_B^N(d) - \bigwedge_{l \neq k} T_{\varphi(B_l)}(bd) \leq 0$. Hence, $T_{B_k^{\varphi}}^{N}(bd) \leq 0$ for all k. Furthermore, $T_{B_k^{\varphi}}^{N}(bd)$ is minimum when $\bigwedge_{l \neq k} T_{\varphi(B_l)}^{N}(bd)$ is zero. Certainly, when $\varphi(B_k) = B_e$ and bd is a B_e-edge, then $\bigwedge_{l \neq k} T_{\varphi(B_l)}^{N}(bd)$ is zero. Hence,

$$T_{B_k^{\varphi}}^{N}(bd) = T_B^N(b)$$
$$\vee T_B^N(d), \; for \; bd \in B_e, \; \varphi(B_k) = B_e.$$

Similarly, for expression of $I_{B_k^{\varphi}}^{P}$ and $I_{B_k^{\varphi}}^{N}$, results are:
Since $I_B^P(b) \wedge I_B^P(d) \geq 0$, $\bigvee_{l \neq k} I_{\varphi(B_l)}^{P}(bd) \geq 0$ and $I_{B_k}^{P}(bd) \leq I_B^P(b) \wedge I_B^P(d) \, \forall B_k$. This implies $\bigvee_{l \neq k} I_{\varphi(B_l)}^{P}(bd) \leq I_B^P(b) \wedge I_B^P(d)$, which shows that $I_B^P(b) \wedge I_B^P(d) - \bigvee_{l \neq k} I_{\varphi(B_l)}^{P}(bd) \geq 0$. Hence, $I_{B_k^{\varphi}}^{P}(bd) \geq 0$ for all k. Furthermore, $I_{B_k^{\varphi}}^{P}(bd)$ obtains maximum value when $\bigvee_{l \neq k} I_{\varphi(B_l)}^{P}(bd)$ is zero. Undoubtedly, when $\varphi(B_k) = B_e$ and bd is a B_e-edge, then $\bigvee_{l \neq k} I_{\varphi(B_l)}^{P}(bd)$ acquires zero value. Hence,

$$I_{B_k^{\varphi}}^{P}(bd) = I_B^P(b)$$
$$\wedge I_B^P(d), \quad for \; bd \in B_e, \; \varphi(B_k) = B_e.$$

Moreover, as $I_B^N(b) \vee I_B^N(d) \leq 0$, $\bigwedge_{l \neq k} I_{\varphi(B_l)}^{N}(bd) \leq 0$ and $I_{B_k}^{N}(bd) \geq I_B^N(b) \vee I_B^N(d)$ for all B_k. This implies $\bigwedge_{l \neq k} I_{\varphi(B_l)}^{N}(bd) \geq I_B^N(b) \vee I_B^N(d)$. This indicates $I_B^N(b) \vee I_B^N(d) - \bigwedge_{l \neq k} I_{\varphi(B_l)}(bd) \leq 0$. Hence, $I_{B_k^{\varphi}}^{N}(bd) \leq 0$ for all

k. Furthermore, $I^N_{B^\varphi_k}(bd)$ is minimum when $\bigwedge_{l\neq k} I^N_{\varphi(B_l)}(bd)$ is zero. Certainly, when $\varphi(B_k) = B_e$ and bd is a B_e-edge, then $\bigwedge_{l\neq k} I^N_{\varphi(B_l)}(bd)$ is zero. Hence,

$$I^N_{B^\varphi_k}(bd) = I^N_B(b) \vee I^N_B(d)\ for\ bd \in B_e,\ \varphi(B_k) = B_e.$$

On similar basis, we derive expressions for $F^P_{B^\varphi_k}$ and $F^N_{B^\varphi_k}$ as:

Since $F^P_B(b) \vee F^P_B(d) \geq 0$, $\bigwedge_{l\neq k} F^P_{\varphi(B_l)}(bd) \geq 0$ and $F^P_{B_k}(bd) \leq F^P_B(b) \vee F^P_B(d)$ for all B_k. This implies $\bigwedge_{l\neq k} F^P_{\varphi(B_l)}(bd) \leq F^P_B(b) \vee F^P_B(d)$. It indicates $F^P_B(b) \vee F^P_B(d) - \bigwedge_{l\neq k} FP_{\varphi(B_l)}(bd) \geq 0$. Hence, $F^P_{B^\varphi_k}(bd) \geq 0$ for all k. Furthermore, $F^P_{B^\varphi_k}(bd)$ is maximum when $\bigwedge_{l\neq k} F^P_{\varphi(B_l)}(bd)$ is zero. When $\varphi(B_k) = B_e$ and bd is a B_e-edge, then $\bigwedge_{l\neq k} F^P_{\varphi(B_l)}(bd)$ is zero. Hence,

$$F^P_{B^\varphi_k}(bd) = F^P_B(b) \vee F^P_B(d)$$

$for\ bd \in B_e,\ \varphi(B_k) = B_e.$

Moreover, as $F^N_B(b) \wedge F^N_B(d) \leq 0$, $\bigvee_{l\neq k} F^N_{\varphi(B_l)}(bd) \leq 0$ and $F^N_{B_k}(bd) \geq F^N_B(b) \wedge F^N_B(d)$ $\forall B_k$. This implies $\bigvee_{l\neq k} F^N_{\varphi(B_l)}(bd) \geq F^N_B(b) \wedge F^N_B(d)$. This shows that $F^N_B(b) \wedge F^N_B(b) - \bigvee_{l\neq k} F^N_{\varphi(B_l)}(bd) \leq 0$. Hence, $F^N_{B^\varphi_k}(bd) \leq 0\ \forall\ k$. Furthermore, $F^N_{B^\varphi_k}(bd)$ obtains minimum value when $\bigvee_{l\neq k} F^N_{\varphi(B_l)}(bd)$ is zero. Undoubtedly, when $\varphi(B_k) = B_e$ and bd is a B_e-edge, then $\bigvee_{l\neq k} F^N_{\varphi(B_l)}(bd)$ acquires zero value. Hence,

$$F^N_{B^\varphi_k}(bd) = F^N_B(b) \wedge F^N_B(d),\ for\ bd \in B_e,\ \varphi(B_k) = B_e.$$

This completes the proof. □

Definition 2.15 Let $\check{G}_{bn} = (B, B_1, B_2, \ldots, B_m)$ be a BSV NGS and φ be a permutation on $\{1, 2, \ldots, m\}$. Then

(i) $\check{G}_{bn}$ is self-complementary BSVNGS if $\check{G}_{bn}$ is isomorphic to $\check{G}^{\varphi c}_{bn}$.
(ii) $\check{G}_{bn}$ is strong self-complementary BSVNGS if $\check{G}_{bn}$ is identical to $\check{G}^{\varphi c}_{bn}$.

Let $\check{G}_{bn} = (B, B_1, B_2, \ldots, B_m)$ be a BSVNGS. Then,

Definition 2.16 (i) $\check{G}_{bn}$ is totally self-complementary BSV NGS if $\check{G}_{bn}$ is isomorphic to $\check{G}^{\varphi c}_{bn}$ for all permutations φ on $\{1, 2, \ldots, m\}$.
(ii) $\check{G}_{bn}$ is totally strong self-complementary BSVNGS if $\check{G}_{bn}$ is identical to $\check{G}^{\varphi c}_{bn}$ for all permutations φ on $\{1, 2, \ldots, m\}$.

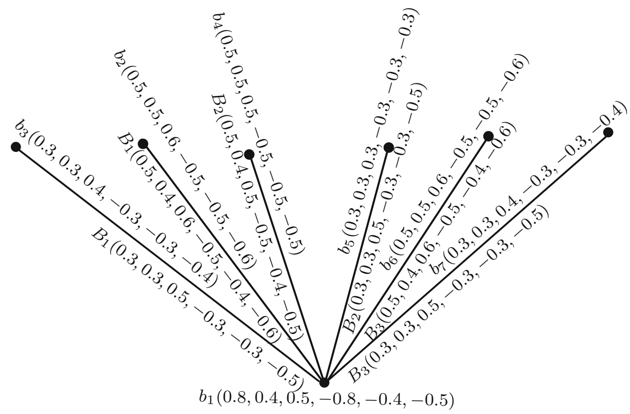


Fig. 7 Totally strong self-complementary BSVNGS

Example 2.17 A BSVNGS $\check{G}_{bn} = (B, B_1, B_2, B_3)$ shown in Fig. 7 is totally strong self-complementary BSVNGS since it is identical to its $\varphi-$complement for all permutations φ on $\{1, 2, 3\}$.

Theorem 2.18 *A BSVNGS is totally self-complementary if and only if it is a strong BSVNGS.*

Proof Consider a strong BSVNGS $\check{G}_{bn}$ and a permutation φ on {1,2, ..., m}. By proposition 2.14, φ-complement of BSVNGS $\check{G}_{bn} = (B, B_1, B_2, \ldots, B_m)$ is a strong BSVNGS. Moreover, if $\varphi^{-1}(e) = k$, where $k, e \in \{1, 2, \ldots, m\}$, then all B_e-edges in BSVNGS $(B, B_1, B_2, \ldots, B_m)$ become B^φ_k-edges in $(B, B^\varphi_1, B^\varphi_2, \ldots, B^\varphi_m)$. This leads

$$T^P_{B_e}(bd) = T^P_B(b) \wedge T^P_B(d) = T^P_{B^\varphi_k}(bd),$$
$$I^P_{B_e}(bd) = I^P_B(b) \wedge I^P_B(d) = I^P_{B^\varphi_k}(bd),$$
$$F^P_{B_e}(bd) = F^P_B(b) \vee F^P_B(d) = F^P_{B^\varphi_k}(bd),$$
$$T^N_{B_e}(bd) = T^N_B(b) \vee T^N_B(d) = T^N_{B^\varphi_k}(bd),$$
$$I^N_{B_e}(bd) = I^N_B(b) \vee I^N_B(d) = I^N_{B^\varphi_k}(bd),$$
$$F^N_{B_e}(bd) = F^N_B(b) \wedge F^N_B(d) = F^P_{B^\varphi_k}(bd).$$

Therefore, under $h : V \to V$ (identity mapping), $\check{G}_{bn}$ and $\check{G}^{\varphi c}_{bn}$ are isomorphic such that $T^P_B(b) = T^P_B(h(b))$, $I^P_B(b) = I^P_B(h(b))$, $F^P_B(b) = F^P_B(h(b))$, $T^N_B(b) = T^N_B(h(b))$, $I^N_B(b) = I^N_B(h(b))$, $F^N_B(b) = F^N_B(h(b))$. Further

$$T^P_{B_e}(bd) = T^P_{B^\varphi_k}(h(b)h(d)) = T^P_{B^\varphi_k}(bd),$$
$$T^N_{B_e}(bd) = T^N_{B^\varphi_k}(h(b)h(d)) = T^N_{B^\varphi_k}(bd),$$
$$I^P_{B_e}(bd) = I^P_{B^\varphi_k}(h(b)h(d)) = I^P_{B^\varphi_k}(bd),$$
$$I^N_{B_e}(bd) = I^N_{B^\varphi_k}(h(b)h(d)) = I^N_{B^\varphi_k}(bd),$$

$$F^{P}_{B_e}(bd) = F^{P}_{B^{\varphi}_k}(h(b)h(d)) = F^{P}_{B^{\varphi}_k}(bd),$$
$$F^{N}_{B_e}(bd) = F^{N}_{B^{\varphi}_k}(h(b)h(d)) = F^{N}_{B^{\varphi}_k}(bd),$$

for all $bd \in R_e$, for $\varphi^{-1}(e) = k$; $e, k = 1, 2, \ldots, m$.

These relations hold for each permutation φ on $\{1, 2, \ldots, m\}$. Hence, $\check{G}_{bn}$ is totally self-complementary BSVNGS. Conversely, let $\check{G}_{bn}$ be isomorphic to $\check{G}^{\varphi c}_{bn}$ for each permutation φ on $\{1, 2, \ldots, m\}$. Moreover, according to the definitions of φ-complement and isomorphism of BSVNGS

$$\begin{aligned}
T^{P}_{B_e}(bd) &= T^{P}_{B^{\varphi}_k}(h(b)h(d)) = T^{P}_{B}(h(b)) \wedge T^{P}_{B}(h(d))\\
&= T^{P}_{B}(b) \wedge T^{P}_{B}(d),\\
T^{N}_{B_e}(bd) &= T^{N}_{B^{\varphi}_k}(h(b)h(d))\\
&= T^{N}_{B}(h(b)) \vee T^{N}_{B}(h(d))\\
&= T^{N}_{B}(b) \vee T^{N}_{B}(d),\\
I^{P}_{B_e}(bd) &= I^{P}_{B^{\varphi}_k}(h(b)h(d)) = I^{P}_{B}(h(b)) \wedge I^{P}_{B}(h(d))\\
&= I^{P}_{B}(b) \wedge I^{P}_{B}(d),\\
I^{N}_{B_e}(bd) &= I^{N}_{B^{\varphi}_k}(h(b)h(d)) = I^{N}_{B}(h(b)) \vee I^{N}_{B}(h(d))\\
&= I^{N}_{B}(b) \vee I^{N}_{B}(d),\\
F^{P}_{B_e}(bd) &= F^{P}_{B^{\varphi}_k}(h(b)h(d)) = F^{P}_{B}(h(b)) \vee F^{P}_{B}(h(d))\\
&= F^{P}_{B}(b) \vee F^{P}_{B}(d),\\
F^{N}_{B_e}(bd) &= F^{N}_{B^{\varphi}_k}(h(b)h(d)) = F^{N}_{B}(h(b)) \wedge F^{N}_{B}(h(d))\\
&= F^{N}_{B}(b) \wedge F^{N}_{B}(d).
\end{aligned}$$

for all $bd \in R_e$, where, $e = 1, 2, \ldots, m$. Hence $\check{G}_{bn}$ is a strong BSVNGS. □

Remark 2.19 Every self-complementary BSVNGS is always a totally self-complementary BSVNGS.

Theorem 2.20 *If $\check{G}_s = (V, R_1, R_2, \ldots, R_m)$ is a totally strong self-complementary GS and $B = (T^P_B, I^P_B, F^P_B, T^N_B, I^N_B, F^N_B)$ is a BSVN subset of V where $T^P_B, I^P_B, F^P_B, T^N_B, I^N_B, F^N_B$ are constant functions, then every strong BSVNGS of $\check{G}_s$ with BSVN vertex set B is necessarily totally strong self-complementary BSVNGS.*

Proof Let $f, f' \in [0, 1]$, $g, g' \in [0, 1]$ and $i, i' \in [0, 1]$ be six constants and

$$T^P_B(b) = f,\ I^P_B(b) = g,\ F^P_B(b) = i,$$
$$T^N_B(b) = f',\ I^N_B(b) = g',\ F^N_B(b) = i',\ for all b \in V.$$

Since $\check{G}_s$ is a totally strong self-complementary GS, so for every permutation φ^{-1} on $\{1, 2, \ldots, m\}$ there exists a bijection $h : V \to V$ such that for every B_e-edge bd, h(b)h(d) [a B_k-edge in $\check{G}_s$] is a B_e-edge in $\check{G}_s^{\varphi^{-1}c}$. Thus, for every B_e-edge (bd), (h(b)h(d)) [a B_k-edge in $\check{G}_{bn}$] is a B^{φ}_e-edge in $\check{G}_{bn}^{\varphi^{-1}c}$.

Moreover, $\check{G}_{bn}$ is a strong BSVNGS. So

$$T^P_B(b) = f = T^P_B(h(b)),\ I^P_B(b) = g = I^P_B(h(b)),$$
$$F^P_B(b) = i = F^P_B(h(b)),$$
$$T^N_B(b) = f' = T^N_B(h(b)),\ I^N_B(b) = g' = I^N_B(h(b)),$$
$$F^N_B(b) = i' = F^N_B(h(b)), \forall b \in V.$$

And

$$\begin{aligned}
T^{P}_{B_e}(bd) &= T^P_B(b) \wedge T^P_B(d) = T^P_B(h(b)) \wedge T^P_B(h(d))\\
&= T^{P}_{B^{\varphi}_k}(h(b)h(d)),\\
I^{P}_{B_e}(bd) &= I^P_B(b) \wedge I^P_B(d) = I^P_B(h(b)) \wedge I^P_B(h(d))\\
&= I^{P}_{B^{\varphi}_k}(h(b)h(d)),\\
F^{P}_{B_e}(bd) &= F^P_B(b) \vee I^P_B(d) = F^P_B(h(b)) \vee F^P_B(h(d))\\
&= F^{P}_{B^{\varphi}_k}(h(b)h(d)),\\
T^{N}_{B_e}(bd) &= T^N_B(b) \vee T^N_B(d) = T^N_B(h(b)) \vee T^N_B(h(d))\\
&= T^{N}_{B^{\varphi}_k}(h(b)h(d)),\\
I^{N}_{B_e}(bd) &= I^N_B(b) \vee I^N_B(d) = I^N_B(h(b)) \vee I^N_B(h(d))\\
&= I^{N}_{B^{\varphi}_k}(h(b)h(d)),\\
F^{N}_{B_e}(bd) &= F^N_B(b) \wedge I^N_B(d) = F^N_B(h(b)) \wedge F^N_B(h(d))\\
&= F^{N}_{B^{\varphi}_k}(h(b)h(d)),
\end{aligned}$$

for all $bd \in B_k$, $k = 1, 2, \ldots, m$.

This shows that $\check{G}_{bn}$ is a strong self-complementary BSVNGS. This satisfies for every permutation φ and φ^{-1} on set $\{1, 2, \ldots, m\}$. Thus, $\check{G}_{bn}$ is a totally strong self-complementary BSVNGS. □

Remark 2.21 Converse of theorem 2.20 may not be true; for example, a BSVNSGS depicted in Fig. 7 is totally strong self-complementary BSVNGS; it is also strong BSVNGS with a totally strong self-complementary underlying GS but $T^P_B, I^P_B, F^P_B, T^N_B, I^N_B, F^N_B$ are not the constant functions.

3 Applications in decision making

1. Recognition of each country's participation in its conspicuous relationships

Naturally, any living organism cannot be 100% self-reliant, especially man who is the noblest of all creatures but he relies

Table 3 BSVN set B of eight countries in the universe

Country	T^P	I^P	F^P	T^N	I^N	F^N
China	0.9	0.4	0.2	− 0.9	− 0.2	− 0.2
Turkey	0.8	0.6	0.5	− 0.6	− 0.5	− 0.5
Bangladesh	0.7	0.6	0.5	− 0.4	− 0.6	− 0.5
America	0.8	0.4	0.4	− 0.8	− 0.2	− 0.4
India	0.6	0.5	0.5	− 0.5	− 0.6	− 0.6
Russia	0.8	0.4	0.3	− 0.9	− 0.3	− 0.4
Pakistan	0.8	0.5	0.4	− 0.5	− 0.6	− 0.6
Afghanistan	0.5	0.7	0.6	− 0.4	− 0.5	− 0.8

on many things for his survival including, plants, animals and other human beings. This dependence causes relationships among human beings. On the Earth, human population is divided into 195 countries. That's why interdependence of human beings results in many relationships among countries. In a relationship, both countries play their role so that these relations can be represented using BSVNGS, which will show that between two particular countries which type of relationship is the strongest and what are the degrees of its strength from both countries. There are some relationships which are not good for the universe in some aspects. BSVNGS will highlight that which one country of those two countries is strongest participant. To make that relationship weak, we will hit strong participant for that relationship. We can see that which relationship is prevailing and which are strong participants of it. We can also observe that which country is the most favorite for a particular relationship.
Let V be the set of eight countries in the universe:

$$V = \{\text{China, Turkey, Bangladesh, America, India, Russia, Pakistan, Afghanistan}\}.$$

Consider a BSVN set B on V as shown in Table 3.

In Table 3, T^P, F^P of a country indicate its positive and negative approach for the universe and I^P shows indeterminacy/ambiguity of its approach, whereas T^N and F^N denote its positive and negative reputation in the universe and I^N stands to represent the percentage of its uncertain reputation. We will use the following abbreviations for country names: Ch = China, TU = Turkey, Ba = Bangladesh, Am = America, In = India, Ru = Russia, Pak = Pakistan, Afg = Afghanistan.

Each pair of countries of set V has T^P, I^P, F^P, T^N, I^N and F^N values of different relationships in Tables 4, 5, 6, 7, 8, 9, 10, 11 and 12.

For each pair of countries, T^P, T^N of any particular relationship indicate strength of that relationship from first country and second country, respectively. Similarly, F^P, F^N represent percentage of weakness and I^P, I^N demonstrate percentage of ambiguity of that relationship from first country and second country, respectively.

Since many relations exist on set V, we define six relations on set V as:

R_1 = Project investment,
R_2 = Political support,
R_3 = Warfare activities,
R_4 = Nuclear weapons,
R_5 = Friendship,
R_6 = Respecting religious beliefs.

We will define the relations with those elements such that $(V, R_1, R_2, R_3, R_4, R_5, R_6)$ is a GS. Every element of a relation denotes a highly worthwhile relationship in that pair of countries. Since $(V, R_1, R_2, R_3, R_4, R_5, R_6)$ is a GS, every relation has unique elements; that is, they are present in that relation only. Hence, any pair of countries will be an element of that relation for which its value of T^P, I^N and F^N is high, and T^N, I^P and F^P values are comparatively low, according to the data of Tables 4, 5, 6, 7, 8, 9, 10, 11 and 12. Write down the T^P, I^P, F^P, T^N, I^N and F^N values of all elements in the relations, using the data of Tables 24, 25, 26, 27, 28, 29, 30, 31 and 32, such that $B_1, B_2, B_3, B_4, B_5, B_6$ are BSVN sets on $R_1, R_2, R_3, R_4, R_5, R_6$, respectively. Let

R_1 = {(Turkey, Pakistan), (Russia, India)},
R_2 = {(India, Afghanistan), (America, Pakistan), (America, Bangladesh)},
R_3 = {(India, Pakistan), (China, India), (Afghanistan, Pakistan), (America, Afghanistan), (Russia, Afg)},
R_4 = {(Russia, Pakistan), (America, India)},
R_5= {(Pakistan, China), (Turkey, China)},
R_6 ={(Pakistan, Bangladesh), (America, Russia), (Bangladesh, Turkey), (Turkey, Afghanistan)}.

Corresponding BSVN sets $B_1, B_2, B_3, B_4, B_5, B_6$, respectively are:

B_1 = {((Tu, Pak), 0.5, 0.1, 0.2, − 0.7, − 0.1, − 0.1), ((Ru, Ind), 0.4, 0.1, 0.3, − 0.5, − 0.1, − 0.1)},
B_2 = {((Ind, Afg), 0.4, 0.1, 0.1, − 0.5, − 0.1, − 0.1), ((Am, Pak),0.6, 0.1, 0.1, − 0.5, − 0.1, 0.0), ((Am, Bang), 0.1, 0.1, 0.2, − 0.3, − 0.3, − 0.4)},
B_3 = {((Ind, Pak), 0.4, 0.2, 0.3, − 0.6, − 0.1, 0.0), ((Ch, Ind), 0.5, 0.1, 0.2, − 0.5, − 0.1, − 0.1), ((Afg, Pak), 0.3, 0.4, 0.6, − 0.4, − 0.3, − 0.3), ((Am, Afg), 0.2, 0.2, 0.3, − 0.4, − 0.1, 0.0), ((Ru, Afg), 0.4, 0.2, 0.3, − 0.5, 0.0, 0.0)},
B_4 = {((Ru, Pak), 0.4, 0.2, 0.4, − 0.7, − 0.2, − 0.1), ((Am, Ind), 0.6, 0.1, 0.0, − 0.3, − 0.2, − 0.3)},
B_5 = {((Pak, Ch), 0.8, 0.1, 0.1, − 0.5, − 0.1, 0.0), ((Tu, Ch), 0.5, 0.1, 0.2, − 0.5, − 0.3, − 0.3)},

Table 4 BSVN set indicating relationships of China with some other countries in V

Type of relation	(China, Turkey)	(China, Bangladesh)	(China, America)
Project investment	(0.1,0.4,0.3,− 0.1,− 0.2, − 0.3)	(0.2,0.2,0.2,− 0.1,− 0.2, − 0.3)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.4)
Political support	(0.3,0.3,0.4, − 0.3,− 0.1,− 0.3)	(0.4,0.2,0.1,− 0.4,− 0.2,− 0.3)	(0.4,0.4,0.4,− 0.3,− 0.2,− 0.4)
Warfare activities	(0.1,0.4,0.5,− 0.1,− 0.2,− 0.2)	(0.1,0.3,0.5,− 0.1,− 0.2,− 0.3)	(0.6,0.1,0.1,− 0.7,− 0.2,− 0.1)
Nuclear weapons	(0.4,0.2,0.3,− 0.1,− 0.2,− 0.3)	(0.1,0.2,0.5,− 0.1,− 0.2,− 0.3)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.4)
Friendship	(0.5,0.3,0.3,− 0.5,− 0.1,− 0.2)	(0.4,0.2,0.1,− 0.4,− 0.1,− 0.3)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.3)
Respecting religious beliefs	(0.1,0.4,0.4,− 0.4,− 0.2,− 0.3)	(0.4,0.3,0.1,− 0.4,− 0.2,− 0.2)	(0.7,0.1,0.4,− 0.6,− 0.2,− 0.3)

Table 5 BSVN set indicating relationships of Turkey with some other countries in V

Type of relation	(Turkey, Bangladesh)	(Turkey, America)	(Turkey, India)
Project investment	(0.4,0.3,0.3,− 0.1,− 0.2,− 0.3)	(0.2,0.2,0.5,− 0.4,− 0.2,− 0.5)	(0.4,0.1,0.4,− 0.1,− 0.2,− 0.5)
Political support	(0.5,0.4,0.3,− 0.4,− 0.3,− 0.3)	(0.6,0.4,0.4,− 0.5,− 0.2,− 0.4)	(0.2,0.4,0.4,− 0.1,− 0.4,− 0.5)
Warfare activities	(0.1,0.4,0.4,− 0.1,− 0.3,− 0.5)	(0.1,0.2,0.4,− 0.3,− 0.2,− 0.4)	(0.2,0.4,0.5,− 0.3,− 0.2,− 0.4)
Nuclear weapons	(0.1,0.4,0.3,− 0.1,− 0.2,− 0.3)	(0.1,0.1,0.5,− 0.4,− 0.1,− 0.4)	(0.2,0.1,0.4,− 0.1,− 0.4,− 0.5)
Friendship	(0.6,0.4,0.4,− 0.3,− 0.2,− 0.3)	(0.4,0.4,0.4,− 0.5,− 0.1,− 0.5)	(0.2,0.3,0.5,− 0.3,− 0.2,− 0.3)
Respecting religious beliefs	(0.7,0.3,0.2,− 0.4,− 0.2,− 0.1)	(0.8,0.2,0.1,− 0.3,− 0.2,− 0.4)	(0.2,0.1,0.4,− 0.2,− 0.2,− 0.5)

Table 6 BSVN set indicating relationships of Bangladesh with some other countries in V

Type of relation	(Bangladesh, America)	(Bangladesh, India)	(Bangladesh, Russia)
Project investment	(0.1,0.4,0.4,− 0.1,− 0.2,− 0.3)	(0.1,0.5,0.5,− 0.1,− 0.3,− 0.4)	(0.2,0.1,0.5,− 0.3,− 0.2,− 0.3)
Political support	(0.3,0.3,0.4,− 0.1,− 0.1,− 0.2)	(0.4,0.2,0.4,− 0.2,− 0.4,− 0.5)	(0.3,0.2,0.4,− 0.2,− 0.3,− 0.3)
Warfare activities	(0.1,0.3,0.4,− 0.1,− 0.2,− 0.3)	(0.3,0.3,0.4,− 0.1,− 0.3,− 0.4)	(0.4,0.3,0.3,− 0.4,− 0.5,− 0.5)
Nuclear weapons	(0.1,0.4,0.5,− 0.1,− 0.2,− 0.5)	(0.2,0.2,0.4,− 0.1,− 0.5,− 0.6)	(0.5,0.4,0.4,− 0.3,− 0.2,− 0.4)
Friendship	(0.2,0.3,0.5,− 0.1,− 0.2,− 0.5)	(0.3,0.2,0.4,− 0.1,− 0.3,− 0.5)	(0.2,0.1,0.4,− 0.3,− 0.3,− 0.3)
Respecting religious beliefs	(0.4,0.4,0.4,− 0.1,− 0.2,− 0.4)	(0.4,0.3,0.4,− 0.1,− 0.4,− 0.3)	(0.5,0.2,0.4,− 0.2,− 0.3,− 0.3)

Table 7 BSVN set indicating relationships of America with some other countries in V

Type of relation	(America, India)	(America, Russia)	(America, Pakistan)
Project investment	(0.5,0.2,0.3,− 0.1,− 0.2,− 0.5)	(0.1,0.2,0.4,− 0.1,− 0.1,− 0.4)	(0.4,0.3,0.4,− 0.1,− 0.2,− 0.6)
Political support	(0.6,0.2,0.3,− 0.5,− 0.2,− 0.3)	(0.3,0.1,0.3,− 0.2,− 0.2,− 0.4)	(0.6,0.1,0.1,− 0.5,− 0.1,− 0.0)
Warfare activities	(0.1,0.4,0.1,− 0.1,− 0.2,− 0.5)	(0.4,0.2,0.4,− 0.5,− 0.2,− 0.3)	(0.5,0.1,0.1,− 0.1,− 0.2,− 0.6)
Nuclear weapons	(0.6,0.1,0.0,− 0.1,− 0.2,− 0.5)	(0.2,0.2,0.4,− 0.1,− 0.1,− 0.4)	(0.4,0.1,0.4,− 0.1,− 0.2,− 0.5)
Friendship	(0.4,0.2,0.3,− 0.5,− 0.2,− 0.3)	(0.3,0.2,0.4,− 0.1,− 0.2,− 0.4)	(0.4,0.1,0.3,− 0.5,− 0.1,− 0.1)
Respecting religious beliefs	(0.5,0.2,0.2,− 0.5,− 0.2,− 0.3)	(0.8,0.1,0.1,− 0.8,− 0.1,− 0.1)	(0.2,0.1,0.4,− 0.5,− 0.2,− 0.1)

Table 8 BSVN set indicating relationships of India with some other countries in V

Type of relation	(India, Russia)	(India, Pakistan)	(India, Afghanistan)
Project investment	(0.4,0.1,0.3,− 0.5,− 0.1,− 0.1)	(0.1,0.2,0.5,− 0.1,− 0.2,− 0.3)	(0.3,0.3,0.5,− 0.1,− 0.2,− 0.3)
Political support	(0.3,0.2,0.4,− 0.4,− 0.3, − 0.3)	(0.0,0.2,0.4,− 0.2,− 0.2,− 0.3)	(0.5,0.1,0.1,− 0.4,− 0.1,− 0.1)
Warfare activities	(0.1,0.3,0.5,− 0.1,− 0.2,− 0.5)	(0.6,0.1,0.0,− 0.4,− 0.2,− 0.3)	(0.4,0.4,0.1,− 0.1,− 0.2,− 0.2)
Nuclear weapons	(0.1,0.2,0.5,− 0.5,− 0.3,− 0.2)	(0.0,0.1,0.5,− 0.0,− 0.3,− 0.3)	(0.4,0.1,0.4,− 0.1,− 0.2,− 0.1)
Friendship	(0.4,0.3,0.2,− 0.3,− 0.2,− 0.4)	(0.1,0.2,0.5,− 0.3,− 0.2,− 0.3)	(0.4,0.4,0.2,− 0.4,− 0.2,− 0.2)
Respecting religious beliefs	(0.4,0.2,0.2,− 0.4,− 0.2,− 0.2)	(0.3,0.1,0.4,− 0.5,− 0.3,− 0.3)	(0.2,0.1,0.4,− 0.4,− 0.2,− 0.2)

Table 9 BSVN set indicating relationships of Russia with some other countries in V

Type of relation	(Russia, Pakistan)	(Russia, Afghanistan)	(Russia, China)
Project investment	(0.5,0.2,0.4,− 0.1,− 0.2,− 0.6)	(0.4,0.2,0.1,− 0.0,− 0.3,− 0.8)	(0.2,0.1,0.3,− 0.1,− 0.2,− 0.4)
Political support	(0.4,0.2,0.4,− 0.4,− 0.2,− 0.4)	(0.1,0.2,0.4,− 0.3,− 0.2,− 0.3)	(0.3,0.3,0.3,− 0.1,− 0.2,− 0.4)
Warfare activities	(0.1,0.2,0.4,− 0.1,− 0.1,− 0.6)	(0.5,0.0,0.0,− 0.4,− 0.2,− 0.3)	(0.2,0.1,0.3,− 0.1,− 0.2,− 0.3)
Nuclear weapons	(0.7,0.2,0.1,− 0.4,− 0.2,− 0.4)	(0.4,0.2,0.1,− 0.1,− 0.2,− 0.3)	(0.1,0.1,0.3,− 0.1,− 0.1,− 0.4)
Friendship	(0.4,0.2,0.3,− 0.1,− 0.2,− 0.3)	(0.4,0.2,0.1,− 0.1,− 0.2,− 0.3)	(0.2,0.4,0.3,− 0.1,− 0.2,− 0.3)
Respecting religious beliefs	(0.4,0.2,0.4,− 0.4,− 0.2,− 0.4)	(0.4,0.2,0.1,− 0.1,− 0.2,− 0.3)	(0.6,0.1,0.2,− 0.7,− 0.2,− 0.1)

Table 10 BSVN set indicating relationships of Pakistan with some other countries in V

Type of relation	(Pakistan, China)	(Turkey, Pakistan)	(Pakistan, Bangladesh)
Project investment	(0.4,0.2,0.4,− 0.5,− 0.1,− 0.1)	(0.7,0.1,0.2,− 0.5,− 0.1,− 0.1)	(0.1,0.1,0.4,− 0.1,− 0.2,− 0.3)
Political support	(0.8,0.2,0.1,− 0.5,− 0.0,− 0.0)	(0.4,0.2,0.4,− 0.5,− 0.2,− 0.3)	(0.6,0.1,0.4,− 0.4,− 0.2,− 0.3)
Warfare activities	(0.0,0.0,0.4,− 0.0,− 0.0,− 0.6)	(0.1,0.2,0.5,− 0.0,− 0.1,− 0.5)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.3)
Nuclear weapons	(0.4,0.2,0.3,− 0.1,− 0.2,− 0.2)	(0.1,0.1,0.5,− 0.1,− 0.2,− 0.5)	(0.1,0.1,0.5,− 0.1,− 0.1,− 0.6)
Friendship	(0.8,0.1,0.1,− 0.5,− 0.1,− 0.0)	(0.6,0.2,0.1,− 0.5,− 0.2,− 0.3)	(0.5,0.1,0.3,− 0.3,− 0.2,− 0.3)
Respecting religious beliefs	(0.8,0.2,0.2,− 0.4,− 0.2,− 0.1)	(0.5,0.1,0.1,− 0.1,− 0.1,− 0.1)	(0.7,0.1,0.1,− 0.4,− 0.1,− 0.0)

Table 11 BSVN set indicating relationships of Afghanistan with some other countries in V

Type of relation	(Afghanistan, China)	(Afghanistan, America)	(Afghanistan, Turkey)
Project investment	(0.1,0.2,0.6,− 0.1,− 0.2,− 0.7)	(0.0,0.1,0.6,− 0.0,− 0.0,− 0.6)	(0.0,0.1,0.6,− 0.1,− 0.2,− 0.6)
Political support	(0.3,0.3,0.5,− 0.1,− 0.2,− 0.8)	(0.0,0.1,0.6,− 0.0,− 0.0,− 0.8)	(0.2,0.1,0.4,− 0.1,− 0.1,− 0.5)
Warfare activities	(0.1,0.2,0.6,− 0.1,− 0.2,− 0.5)	(0.2,0.2,0.3,− 0.4,− 0.1,− 0.0)	(0.1,0.1,0.4,− 0.1,− 0.2,− 0.7)
Nuclear weapons	(0.1,0.4,0.6,− 0.1,− 0.2,− 0.8)	(0.0,0.0,0.6,− 0.0,− 0.0,− 0.7)	(0.0,0.1,0.5,− 0.1,− 0.1,− 0.8)
Friendship	(0.4,0.2,0.6,− 0.1,− 0.2,− 0.4)	(0.0,0.0,0.6,− 0.0,− 0.2,− 0.6)	(0.2,0.1,0.6,− 0.1,− 0.2,− 0.7)
Respecting religious beliefs	(0.4,0.2,0.5,− 0.1,− 0.2,− 0.3)	(0.0,0.2,0.4,− 0.1,− 0.2,− 0.5)	(0.5,0.1,0.2,− 0.4,− 0.1,− 0.1)

$B_6 = \{((\text{Pak}, \text{Bang}), 0.4, 0.1, 0.0, -0.7, -0.1, -0.1), ((\text{Am}, \text{Ru}), 0.8, 0.1, 0.1, -0.8, -0.1, -0.1), ((\text{Bang}, \text{Tu}), 0.4, 0.2, 0.1, -0.7, -0.3, -0.2), ((\text{Tu}, \text{Afg}), 0.4, 0.1, 0.1, -0.5, -0.1, -0.2)\}$.

Obviously, $(B, B_1, B_2, B_3, B_4, B_5, B_6)$ is a BSVNGS as shown in Fig. 8.

In BSVNGS, represented in Fig. 8, an edge depicts most dominating and attention-getting relationship of contiguous countries. For instance, between Turkey and Pakistan, most dominating and attention-getting relationship is project investment, participation of Turkey in this relationship's strength, weakness and indeterminacy is 70%, 10% and 10%, respectively, and participation of Pakistan in this relationship's strength, weakness and indeterminacy is 50%, 10% and 20%, respectively. This BSVNGS represents complete status of a relationship from its both participants. It can be very utilitarian for some particular relationships, like war-

Table 12 BSVN set indicating relationships of some countries in V

Type of relation	(China, India)	(Turkey, Russia)	(Bangladesh, Afghanistan)
Project investment	(0.2,0.1,0.5,− 0.1,− 0.2,− 0.6)	(0.4,0.2,0.5,− 0.1,− 0.2,− 0.6)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.8)
Political support	(0.1,0.1,0.4,− 0.1,− 0.2,− 0.6)	(0.4,0.3,0.4,− 0.3,− 0.2,− 0.5)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.3)
Warfare activities	(0.5,0.1,0.2,− 0.5,− 0.1,− 0.1)	(0.5,0.2,0.5,− 0.1,− 0.2,− 0.3)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.6)
Nuclear weapons	(0.1,0.2,0.5,− 0.1,− 0.1,− 0.6)	(0.4,0.1,0.5,− 0.1,− 0.2,− 0.5)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.8)
Friendship	(0.1,0.1,0.5,− 0.1,− 0.1,− 0.5)	(0.6,0.2,0.1,− 0.4,− 0.2,− 0.4)	(0.3,0.1,0.4,− 0.3,− 0.2,− 0.3)
Respecting religious beliefs	(0.4,0.3,0.1,− 0.5,− 0.1,− 0.1)	(0.4,0.2,0.1,− 0.6,− 0.2,− 0.1)	(0.5,0.1,0.1,− 0.5,− 0.2,− 0.1)

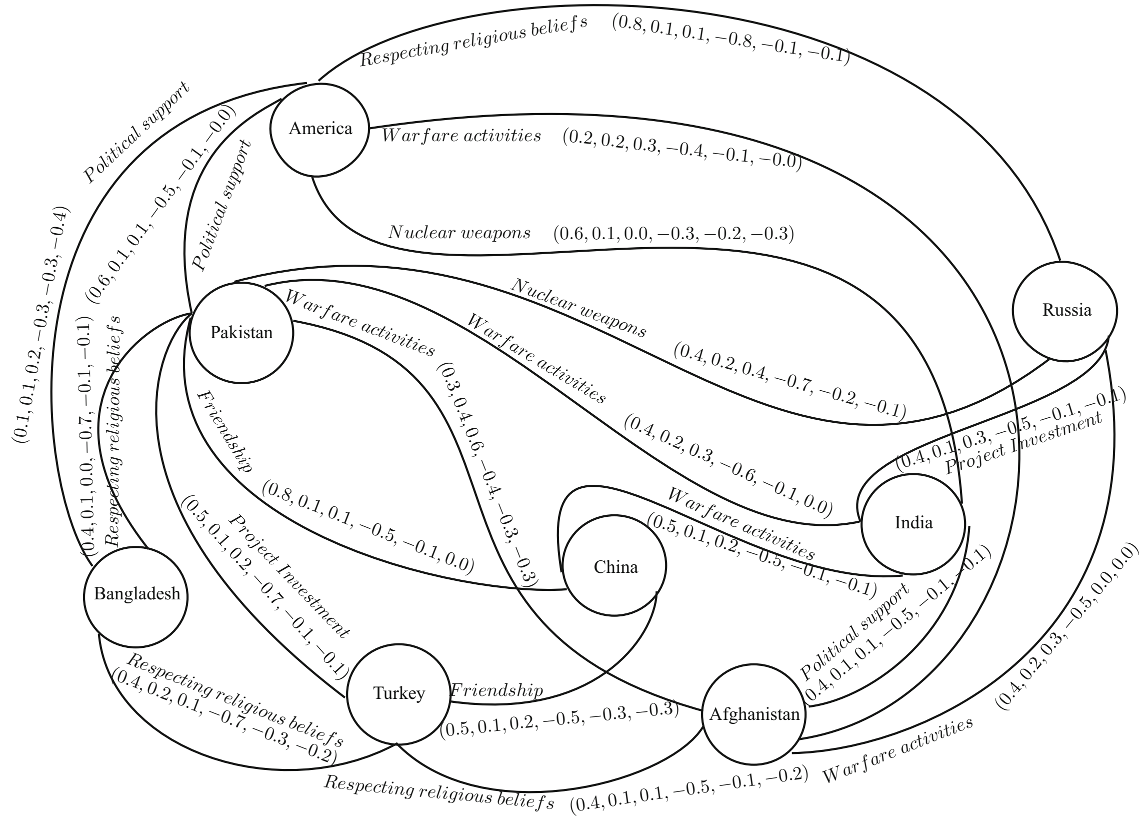


Fig. 8 A BSVNGS pointing out each country's participation in its conspicuous relationships

fare activities. It tells a country that in what percentage its warfare activities are answered from corresponding country and warns it that in case of war which kind of reaction can appear. This can guide United Nations and other peace-loving organizations that which country is most actively involved in peace-destroying relationship, for instance, warfare activities and nuclear weapons, and what is its percentage. For instance, India and Pakistan, America and Afghanistan are involved in warfare activities and percentage of these edges shows that India and America are more actively involved in it. So it guides United Nations Security Council that in what percentage it should pressurize India, America and other countries involved in it. It also highlights those countries whose attention-getting relationship is nuclear weapons and spotlights those countries which are its strong participants than their contiguous countries. For example, America and India have nuclear weapons as their dominating relationship and America is strong participant, as its participation for its strength is high. So, this BSVNGS indicates that in order to reduce nuclear proliferation, United Nations should pressurize America and Russia.

We now present the general procedure of our method which is used in our application in the following algorithm.

Algorithm 1

1. Begin
2. Input membership values $B(u_i)$ of n number of countries $u_1, u_2, \ldots, u_n$.
3. Input the adjacency matrix of countries with respect to $R_1, R_2, \ldots, R_m$ mutually disjoint, irreflexive and symmetric relations.
4. **do** i from $1 \to m$
5. **do** j from $1 \to m$
6. **do** k from $1 \to n$
7. **do** l from $k \to n$
8. **if** $(i \neq j, T^P_{R_i}(u_k u_l) > T^P_{R_j}(u_k u_l), I^P_{R_i}(u_k u_l) < I^P_{R_j}(u_k u_l), F^P_{R_i}(u_k u_l) < F^P_{R_j}(u_k u_l))$**then**
9. **if** $T^N_{R_i}(u_k u_l) < T^N_{R_j}(u_k u_l), I^N_{R_i}(u_k u_l) > I^N_{R_j}(u_k u_l), F^N_{R_i}(u_k u_l) > F^N_{R_j}(u_k u_l))$**then**
10. Label $u_k u_l$ as R_i
11. **end if**
12. **end if**
13. **end do**
14. **end do**

15. **end do**
16. **end do**
17. $T^P_{R_i}$, $F^P_{R_i}$, $I^P_{R_i}$ values of an edge between two different vertices (countries) a and b show participation of vertex (country) a in strength, weakness and indeterminacy of most conspicuous relationship R_i between them, whereas $T^N_{R_i}$, $F^N_{R_i}$, $I^N_{R_i}$ are corresponding values of vertex(country) b.
18. End

2. Detection of Psychological Improvement of Patients in a Mental Hospital

A society is an enduring and cooperating social group whose members have developed organized patterns of relationships through interaction with one another. In a society, interaction of its members is very important; this compactness causes all types of relationships among society members. As Homo sapiens have complex nature, for grouping of human beings we have to focus on many psychological aspects and behaviors. We use BSVNGS to judge behavior and other social aspects of any group of society. Using BSVNGS, we may get aware of conspicuous behavioral property of any two group members with each other. Most frequent occurrence of a particular behavioral property reveals that this group is good or bad in that perspective. This BSVNGS is very helpful for subgrouping and assigning of tasks. It may be very utilitarian in mental hospitals, where in a group of patients we estimate what is recent stage of their psychological behavior and psychiatrist can decide which patient is having high violence-based relationship with others. He may decrease his interaction with others and can decide either mental stage of his patients is getting better or worse. It helps a psychiatrist in taking precautionary measures and providing suitable environment for his patients. He can decide looking at the BSVNGS that which patient can properly survive in the society with better behavior and which one is still not capable of showing moderate behavior. A psychiatrist may use many types of psychological behaviors to estimate improvement in mental state of his patients including violence, friendliness, jealousy, helping, forgiving, communication and problem sharing, etc. Consider set V of eight patients in a mental hospital:

V = {Albert, Charles, Burton, Calvert, Christopher, David, Chapman, Joseph}.

Consider a BSVN set B on V as shown in Table 13.

In Table 13, T^P, F^P of a patient indicate his positive and negative thinking for the society and I^P shows indeterminacy/ambiguity of his thinking for society, whereas T^N, F^N denote his positive and negative reputation in the society and I^N represents the percentage of his uncertain reputation.

Table 13 BSVN set B of eight patients in a mental hospital

Patients	T^P	I^P	F^P	T^N	I^N	F^N
Albert	0.4	0.9	0.9	− 0.4	− 0.6	− 0.9
Charles	0.5	0.8	0.8	− 0.6	− 0.7	− 0.8
Burton	0.6	0.7	0.7	− 0.4	− 0.6	− 0.7
Calvert	0.7	0.8	0.7	− 0.5	− 0.6	− 0.6
Christopher	0.5	0.7	0.6	− 0.4	− 0.5	− 0.7
David	0.5	0.8	0.7	− 0.6	− 0.5	− 0.8
Chapman	0.4	0.7	0.7	− 0.4	− 0.6	− 0.8
Joseph	0.7	0.6	0.6	− 0.5	− 0.5	− 0.6

Each pair of patients of set V has T^P, I^P, F^P, T^N, I^N and F^N values of different psychological behaviors in Tables 14, 15, 16, 17, 18, 19, 20, 21 and 22.
For each pair of patients, T^P, T^N of any particular psychological behavior indicate percentage of strength of that behavior of first patient and second patient, respectively. Similarly, F^P, F^N represent percentage of weakness and I^P, I^N demonstrate percentage of ambiguity of that behavior of first patient and second patient, respectively. We consider seven relations on set V as:

R_1 = Violence,
R_2 = Friendliness,
R_3 = Jealousy,
R_4 = Helping,
R_5 = Forgiving,
R_6 = Problem sharing,
R_7 = Communication.

We will define the relations with those elements such that $(V, R_1, R_2, R_3, R_4, R_5, R_6, R_7)$ is a GS. Every element of relation denotes a conspicuous psychological behavior of that pair of patients. Since $(V, R_1, R_2, R_3, R_4, R_5, R_6, R_7)$ is a GS, every relation has unique elements; that is, they are present in that relation only. Hence, any pair of patients will be an element of that particular relation, for which its values of T^P, I^N and F^N are high, and T^N, I^P and F^P values are comparatively low, according to the data of Tables 14, 15, 16, 17, 18, 19, 20, 21 and 22. Write down the T^P, I^P, F^P, T^N, I^N and F^N values of all elements in the relations, using Tables 14, 15, 16, 17, 18, 19, 20, 21 and 22, such that B_1, B_2, B_3, B_4, B_5, B_6, B_7 are BSVN sets on the relations R_1, R_2, R_3, R_4, R_5, R_6, R_7, respectively.

Let

R_1 = {(Albert, Chapman), (Albert, Christopher), (Calvert, Albert), (David, Albert)},
R_2 = {(Calvert, Christopher)},
R_3 = {(Charles, Chapman), (Burton, Albert)},

Table 14 BSVN set indicating psychological behaviors of Albert with some other patients in V

Psychological behaviors	(Albert, Charles)	(Albert, Burton)	(Albert, Calvert)
Violence	(0.4,0.4,0.7,– 0.2,– 0.6,– 0.8)	(0.2,0.7,0.8,– 0.1,– 0.6,– 0.9)	(0.2,0.3,0.4,– 0.1,– 0.2,– 0.3)
Friendliness	(0.3,0.5,0.6,– 0.1,– 0.5,– 0.5)	(0.3,0.6,0.7,– 0.2,– 0.5,– 0.8)	(0.3,0.4,0.5,– 0.2,– 0.3,– 0.4)
Jealousy	(0.3,0.6,0.7,– 0.3,– 0.4,– 0.6)	(0.4,0.5,0.5,– 0.3,– 0.4,– 0.6)	(0.4,0.5,0.6,– 0.3,– 0.4,– 0.5)
Helping	(0.1,0.5,0.8,– 0.2,– 0.6,– 0.7)	(0.3,0.2,0.6,– 0.1,– 0.6,– 0.7)	(0.2,0.6,0.7,– 0.4,– 0.5,– 0.6)
Forgiving	(0.4,0.3,0.4,– 0.4,– 0.4,– 0.5)	(0.4,0.6,0.8,– 0.2,– 0.5,– 0.6)	(0.3,0.7,0.8,– 0.1,– 0.6,– 0.7)
Problem sharing	(0.3,0.6,0.7,– 0.3,– 0.5,– 0.7)	(0.3,0.7,0.7,– 0.3,– 0.4,– 0.9)	(0.4,0.8,0.9,– 0.2,– 0.2,– 0.8)
Communication	(0.2,0.7,0.6,– 0.4,– 0.4,– 0.8)	(0.2,0.6,0.6,– 0.1,– 0.6,– 0.8)	(0.1,0.1,0.4,– 0.3,– 0.3,– 0.8)

Table 15 BSVN set indicating psychological behaviors of Charles with some other patients in V

Psychological behaviors	(Charles, Burton)	(Charles, Calvert)	(Charles, Christopher)
Violence	(0.3,0.7,0.8,– 0.1,– 0.6,– 0.8)	(0.4,0.8,0.7,– 0.1,– 0.6,– 0.8)	(0.2,0.7,0.8,– 0.4,– 0.5,– 0.3)
Friendliness	(0.2,0.6,0.7,– 0.2,– 0.5,– 0.7)	(0.3,0.7,0.8,– 0.2,– 0.5,– 0.7)	(0.3,0.6,0.4,– 0.3,– 0.4,– 0.4)
Jealousy	(0.1,0.5,0.6,– 0.3,– 0.4,– 0.6)	(0.2,0.2,0.6,– 0.3,– 0.4,– 0.6)	(0.4,0.5,0.5,– 0.2,– 0.3,– 0.5)
Helping	(0.5,0.4,0.5,– 0.4,– 0.4,– 0.6)	(0.1,0.2,0.5,– 0.4,– 0.3,– 0.5)	(0.5,0.4,0.6,– 0.1,– 0.2,– 0.6)
Forgiving	(0.4,0.4,0.5,– 0.1,– 0.6,– 0.8)	(0.4,0.2,0.4,– 0.1,– 0.2,– 0.4)	(0.2,0.3,0.7,– 0.4,– 0.4,– 0.7)
Problem sharing	(0.5,0.6,0.6,– 0.3,– 0.5,– 0.7)	(0.5,0.3,0.4,– 0.5,– 0.2,– 0.4)	(0.3,0.5,0.4,– 0.1,– 0.3,– 0.4)
Communication	(0.2,0.6,0.7,– 0.2,– 0.4,– 0.6)	(0.4,0.4,0.5,– 0.3,– 0.3,– 0.5)	(0.4,0.4,0.4,– 0.1,– 0.2,– 0.3)

Table 16 BSVN set indicating psychological behaviors of Burton with some other patients in V

Psychological behaviors	(Burton, Calvert)	(Burton, Christopher)	(Burton, David)
Violence	(0.1,0.6,0.7,– 0.1,– 0.4,– 0.7)	(0.4,0.7,0.2,– 0.1,– 0.5,– 0.4)	(0.3,0.7,0.3,– 0.1,– 0.4,– 0.8)
Friendliness	(0.4,0.4,0.3,– 0.4,– 0.2,– 0.3)	(0.2,0.6,0.3,– 0.2,– 0.4,– 0.6)	(0.4,0.6,0.4,– 0.2,– 0.5,– 0.7)
Jealousy	(0.2,0.2,0.6,– 0.2,– 0.5,– 0.6)	(0.3,0.5,0.4,– 0.1,– 0.3,– 0.7)	(0.3,0.5,0.5,– 0.3,– 0.3,– 0.6)
Helping	(0.3,0.2,0.5,– 0.3,– 0.2,– 0.5)	(0.5,0.2,0.2,– 0.4,– 0.3,– 0.3)	(0.4,0.4,0.6,– 0.1,– 0.4,– 0.8)
Forgiving	(0.4,0.2,0.6,– 0.4,– 0.3,– 0.4)	(0.4,0.4,0.5,– 0.3,– 0.5,– 0.4)	(0.4,0.3,0.7,– 0.3,– 0.5,– 0.7)
Problem sharing	(0.1,0.5,0.4,– 0.1,– 0.4,– 0.3)	(0.2,0.3,0.6,– 0.1,– 0.4,– 0.6)	(0.5,0.1,0.3,– 0.4,– 0.3,– 0.6)
Communication	(0.2,0.6,0.7,– 0.2,– 0.5,– 0.4)	(0.3,0.3,0.7,– 0.3,– 0.3,– 0.7)	(0.4,0.2,0.4,– 0.3,– 0.4,– 0.7)

Table 17 BSVN set indicating psychological behaviors of Calvert with some other patients in V

Psychological behaviors	(Calvert, Christopher)	(Calvert, David)	(Calvert, Chapman)
Violence	(0.1,0.7,0.4,– 0.1,– 0.2,– 0.7)	(0.4,0.8,0.5,– 0.1,– 0.4,– 0.8)	(0.2,0.7,0.4,– 0.1,– 0.6,– 0.8)
Friendliness	(0.4,0.4,0.3,– 0.4,– 0.2,– 0.3)	(0.3,0.6,0.4,– 0.4,– 0.3,– 0.7)	(0.4,0.6,0.4,– 0.2,– 0.5,– 0.7)
Jealousy	(0.2,0.6,0.3,– 0.2,– 0.4,– 0.6)	(0.4,0.7,0.3,– 0.5,– 0.2,– 0.6)	(0.2,0.5,0.5,– 0.3,– 0.4,– 0.6)
Helping	(0.3,0.5,0.4,– 0.1,– 0.2,– 0.3)	(0.5,0.5,0.2,– 0.5,– 0.5,– 0.5)	(0.4,0.4,0.6,– 0.3,– 0.3,– 0.5)
Forgiving	(0.4,0.4,0.5,– 0.3,– 0.4,– 0.5)	(0.4,0.4,0.1,– 0.4,– 0.4,– 0.4)	(0.4,0.2,0.3,– 0.4,– 0.1,– 0.1)
Problem sharing	(0.1,0.7,0.6,– 0.1,– 0.5,– 0.3)	(0.3,0.6,0.6,– 0.3,– 0.3,– 0.2)	(0.3,0.3,0.6,– 0.1,– 0.2,– 0.4)
Communication	(0.2,0.6,0.7,– 0.2,– 0.2,– 0.4)	(0.2,0.7,0.7,– 0.1,– 0.2,– 0.3)	(0.2,0.2,0.7,– 0.2,– 0.3,– 0.3)

$R_4 = \{(\text{Charles, Burton}), (\text{Burton, Christopher})\}$,
$R_5 = \{(\text{Charles, Albert}), (\text{Calvert, Chapman})\}$,
$R_6 = \{(\text{Burton, David}), (\text{Chapman, Joseph}), (\text{Calvert, Charles})\}$,
$R_7 = \{(\text{Joseph, Burton}), (\text{Chapman, David})\}$.

Corresponding BSVN sets B_1, B_2, B_3, B_4, B_5, B_6, B_7, respectively, are:

B_1 $=\{((\text{Albert, Chapman}), 0.4, 0.1, 0.1, -0.2, -0.3, -0.5), ((\text{Albert, Christopher}), 0.4, 0.1, 0.0, -0.3, -0.2, -0.3), ((\text{Calvert, Albert}), 0.5, 0.1, 0.1, -0.4, -0.2,$

Table 18 BSVN set indicating psychological behaviors of Christopher with some other patients in V

Psychological behaviors	(Christopher, David)	(Christopher, Chapman)	(Christopher, Joseph)
Violence	(0.1,0.2,0.4,− 0.1,− 0.2,− 0.3)	(0.4,0.2,0.1,− 0.1,− 0.2,− 0.3)	(0.2,0.1,0.4,− 0.1,− 0.2,− 0.7)
Friendliness	(0.2,0.2,0.3,− 0.2,− 0.3,− 0.4)	(0.2,0.7,0.2,− 0.2,− 0.4,− 0.4)	(0.5,0.6,0.3,− 0.2,− 0.5,− 0.6)
Jealousy	(0.1,0.4,0.4,− 0.3,− 0.4,− 0.3)	(0.4,0.6,0.3,− 0.1,− 0.3,− 0.5)	(0.2,0.1,0.5,− 0.1,− 0.2,− 0.5)
Helping	(0.3,0.5,0.6,− 0.4,− 0.2,− 0.5)	(0.3,0.5,0.4,− 0.3,− 0.2,− 0.6)	(0.3,0.5,0.4,− 0.3,− 0.3,− 0.3)
Forgiving	(0.1,0.6,0.5,− 0.3,− 0.2,− 0.6)	(0.4,0.4,0.5,− 0.1,− 0.4,− 0.5)	(0.2,0.1,0.4,− 0.1,− 0.4,− 0.4)
Problem sharing	(0.4,0.7,0.4,− 0.4,− 0.3,− 0.7)	(0.3,0.3,0.6,− 0.2,− 0.3,− 0.5)	(0.4,0.4,0.4,− 0.2,− 0.5,− 0.5)
Communication	(0.5,0.2,0.3,− 0.1,− 0.2,− 0.3)	(0.4,0.2,0.7,− 0.3,− 0.2,− 0.3)	(0.2,0.1,0.6,− 0.4,− 0.2,− 0.3)

Table 19 BSVN set indicating psychological behaviors of David with some other patients in V

Psychological behaviors	(David, Chapman)	(David, Joseph)	(David, Albert)
Violence	(0.1,0.7,0.6,− 0.2,− 0.3,− 0.6)	(0.4,0.6,0.1,− 0.1,− 0.2,− 0.2)	(0.4,0.2,0.1,− 0.3,− 0.1,− 0.3)
Friendliness	(0.3,0.6,0.7,− 0.1,− 0.4,− 0.7)	(0.3,0.2,0.2,− 0.2,− 0.3,− 0.3)	(0.2,0.2,0.9,− 0.1,− 0.5,− 0.8)
Jealousy	(0.1,0.5,0.6,− 0.3,− 0.5,− 0.8)	(0.4,0.5,0.3,− 0.3,− 0.4,− 0.4)	(0.3,0.3,0.4,− 0.1,− 0.4,− 0.7)
Helping	(0.2,0.7,0.7,− 0.1,− 0.3,− 0.6)	(0.5,0.4,0.4,− 0.4,− 0.5,− 0.5)	(0.2,0.4,0.5,− 0.1,− 0.3,− 0.6)
Forgiving	(0.1,0.6,0.6,− 0.2,− 0.4,− 0.7)	(0.4,0.2,0.5,− 0.5,− 0.2,− 0.6)	(0.1,0.5,0.6,− 0.1,− 0.2,− 0.5)
Problem sharing	(0.1,0.7,0.3,− 0.1,− 0.5,− 0.8)	(0.3,0.6,0.1,− 0.1,− 0.3,− 0.7)	(0.2,0.6,0.7,− 0.2,− 0.4,− 0.4)
Communication	(0.3,0.5,0.6,− 0.4,− 0.3,− 0.6)	(0.2,0.7,0.1,− 0.2,− 0.4,− 0.8)	(0.3,0.7,0.8,− 0.1,− 0.2,− 0.3)

Table 20 BSVN set indicating psychological behaviors of Chapman with some other patients in V

Psychological behaviors	(Albert, Chapman)	(Chapman, Charles)	(Chapman, Burton)
Violence	(0.4,0.1,0.1,− 0.2,− 0.3,− 0.5)	(0.2,0.7,0.5,− 0.1,− 0.4,− 0.7)	(0.2,0.1,0.7,− 0.1,− 0.2,− 0.4)
Friendliness	(0.2,0.7,0.4,− 0.3,− 0.5,− 0.9)	(0.1,0.6,0.4,− 0.2,− 0.3,− 0.8)	(0.3,0.2,0.6,− 0.3,− 0.3,− 0.5)
Jealousy	(0.1,0.6,0.5,− 0.4,− 0.6,− 0.8)	(0.3,0.1,0.2,− 0.3,− 0.2,− 0.4)	(0.4,0.3,0.5,− 0.1,− 0.2,− 0.6)
Helping	(0.4,0.5,0.6,− 0.2,− 0.3,− 0.7)	(0.2,0.5,0.5,− 0.1,− 0.4,− 0.5)	(0.2,0.4,0.4,− 0.4,− 0.4,− 0.3)
Forgiving	(0.3,0.4,0.7,− 0.3,− 0.4,− 0.6)	(0.1,0.4,0.6,− 0.2,− 0.5,− 0.6)	(0.3,0.5,0.3,− 0.1,− 0.2,− 0.7)
Problem sharing	(0.2,0.3,0.8,− 0.4,− 0.5,− 0.5)	(0.2,0.3,0.7,− 0.3,− 0.6,− 0.7)	(0.4,0.2,0.4,− 0.3,− 0.6,− 0.3)
Communication	(0.1,0.2,0.9,− 0.2,− 0.6,− 0.6)	(0.1,0.1,0.8,− 0.1,− 0.2,− 0.8)	(0.2,0.7,0.4,− 0.1,− 0.2,− 0.8)

Table 21 BSVN set indicating psychological behaviors of Joseph with some other patients in V

Psychological behaviors	(Joseph, Albert)	(Joseph, Charles)	(Joseph, Calvert)
Violence	(0.1,0.5,0.9,− 0.4,− 0.2,− 0.8)	(0.4,0.3,0.8,− 0.3,− 0.2,− 0.3)	(0.7,0.3,0.4,− 0.3,− 0.2,− 0.1)
Friendliness	(0.4,0.2,0.8,− 0.2,− 0.4,− 0.7)	(0.3,0.2,0.7,− 0.2,− 0.3,− 0.5)	(0.2,0.1,0.5,− 0.2,− 0.3,− 0.3)
Jealousy	(0.1,0.6,0.7,− 0.1,− 0.2,− 0.3)	(0.4,0.6,0.6,− 0.1,− 0.4,− 0.6)	(0.5,0.5,0.6,− 0.1,− 0.2,− 0.2)
Helping	(0.2,0.2,0.3,− 0.4,− 0.3,− 0.6)	(0.2,0.5,0.5,− 0.4,− 0.2,− 0.3)	(0.2,0.4,0.4,− 0.4,− 0.4,− 0.3)
Forgiving	(0.1,0.5,0.6,− 0.3,− 0.2,− 0.5)	(0.5,0.4,0.4,− 0.3,− 0.3,− 0.7)	(0.4,0.3,0.5,− 0.3,− 0.2,− 0.4)
Problem sharing	(0.3,0.4,0.3,− 0.2,− 0.5,− 0.4)	(0.3,0.3,0.1,− 0.2,− 0.2,− 0.3)	(0.6,0.2,0.6,− 0.2,− 0.4,− 0.5)
Communication	(0.1,0.2,0.5,− 0.1,− 0.2,− 0.3)	(0.4,0.2,0.3,− 0.1,− 0.4,− 0.8)	(0.7,0.1,0.7,− 0.1,− 0.5,− 0.6)

$-0.1)$, ((David, Albert), $0.4, 0.2, 0.1, -0.3, -0.1, -0.3)\}$,
$B_2 = \{$((Calvert, Christopher),$0.4, 0.4, 0.3, -0.4, -0.2, -0.3)\}$,
$B_3 = \{$((Charles, Chapman),$0.3, 0.1, 0.2, -0.3, -0.2, -0.4)$, ((Burton, Albert),$0.4, 0.5, 0.5, -0.3, -0.4, -0.6)\}$,
$B_4 = \{$((Charles, Burton),$0.5, 0.4, 0.5, -0.4, -0.4, -0.6)$, ((Burton, Christopher),$0.5, 0.2, 0.2, -0.4, -0.3, -0.3)\}$,

Table 22 BSVN set indicating psychological behaviors of some patients in V

Psychological behaviors	(Albert, Christopher)	(Charles, David)	(Joseph, Burton)
Violence	(0.4,0.1,0.0,– 0.3,– 0.2,– 0.3)	(0.5,0.1,0.1,– 0.6,– 0.1,– 0.0)	(0.2,0.6,0.4,– 0.1,– 0.4,– 0.7)
Friendliness	(0.1,0.2,0.3,– 0.2,– 0.3,– 0.4)	(0.4,0.2,0.2,– 0.5,– 0.2,– 0.6)	(0.4,0.1,0.7,– 0.2,– 0.2,– 0.3)
Jealousy	(0.3,0.4,0.4,– 0.3,– 0.2,– 0.3)	(0.5,0.5,0.5,– 0.4,– 0.3,– 0.3)	(0.5,0.5,0.4,– 0.1,– 0.5,– 0.4)
Helping	(0.2,0.2,0.5,– 0.2,– 0.5,– 0.5)	(0.4,0.4,0.4,– 0.3,– 0.4,– 0.5)	(0.4,0.1,0.6,– 0.3,– 0.2,– 0.3)
Forgiving	(0.1,0.5,0.6,– 0.1,– 0.4,– 0.4)	(0.3,0.6,0.3,– 0.2,– 0.2,– 0.3)	(0.3,0.4,0.5,– 0.1,– 0.4,– 0.5)
Problem sharing	(0.1,0.2,0.7,– 0.2,– 0.3,– 0.5)	(0.5,0.7,0.2,– 0.1,– 0.5,– 0.4)	(0.2,0.6,0.4,– 0.3,– 0.2,– 0.3)
Communication	(0.1,0.6,0.3,– 0.1,– 0.2,– 0.4)	(0.4,0.2,0.1,– 0.1,– 0.2,– 0.3)	(0.6,0.2,0.3,– 0.4,– 0.1,– 0.2)

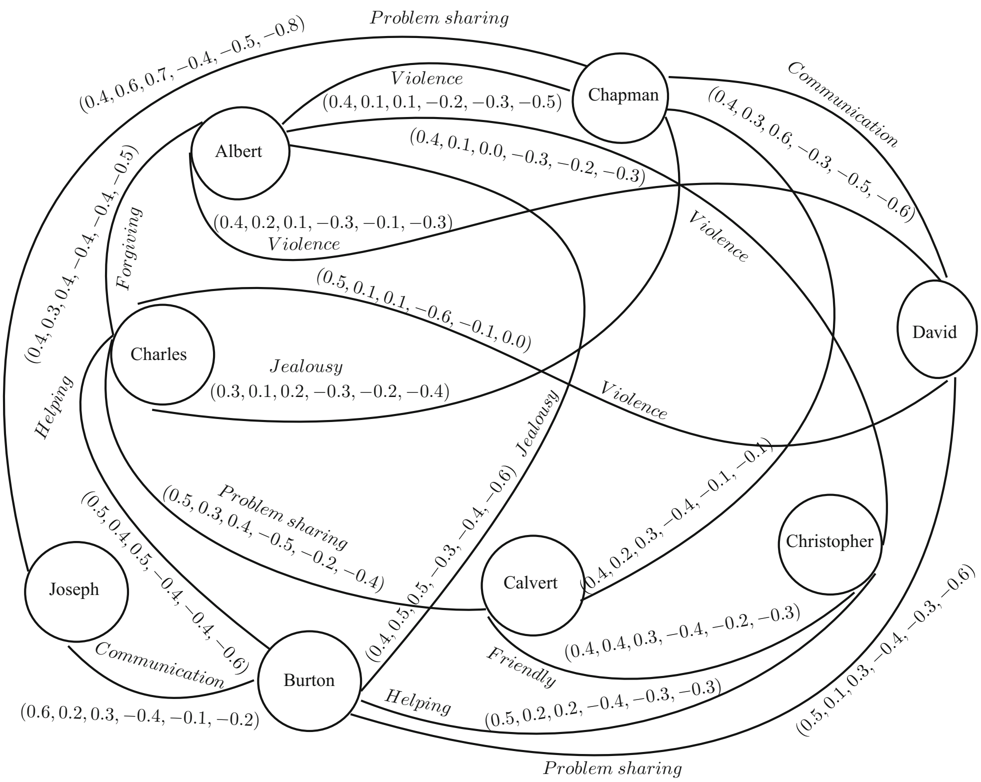


Fig. 9 BSVNGS showing psychological behaviors of patients in a mental hospital

B_5 ={((Charles, Albert),0.4, 0.3, 0.4, − 0.4, − 0.4, − 0.5), ((Calvert, Chapman),0.4, 0.2, 0.3, − 0.4, − 0.1, − 0.1)},
B_6 = {((Burton, David),0.5, 0.1, 0.3, − 0.4, − 0.3, − 0.6) ((Chapman, Joseph),0.4, 0.6, 0.7, − 0.4, − 0.5, − 0.8), ((Calvert, Charles),0.5, 0.3, 0.4, − 0.5, − 0.2, − 0.4)}.

Obviously, $(B, B_1, B_2, B_3, B_4, B_5, B_6)$ is a BSVNGS as depicted in Fig. 9.

In BSVNGS, depicted in Fig. 9, every edge depicts most conspicuous behavioral property of adjacent vertices (patients). For instance, most conspicuous behavioral property between Charles and Burton is helping, Charles' participation in its strength, weakness and indeterminacy is 50%, 40% and 50%, respectively, and Burton's participation in its strength, weakness and indeterminacy is 40%, 40% and 60%, respectively. This BSVNGS also accentuates that vertex Albert has highest degree for violence. This points out that Albert is most violent mental patient among these eight patients. So, his psychiatrist must focus on Albert's violent behavior, as Albert's most violent behavior is with Chapman and his positive behavior is with just one person, that is, Charles. So psychiatrist should try to minimize his interaction with Chapman to avoid his most violent behavior and should maximize his interaction with Charles to improve his good psychological behavior. Similarly, a psychiatrist can jude the behavior of all of his patients and can take better decisions for their well-being. He can make two BSVNGSs with a gap of some time period and can estimate their behavioral improvement by comparing those two BSVNGSs; it also helps him in continuation or cancelation of his decisions for his patients. General procedure regarding this application is deciphered through following algorithm.

Algorithm 2

1. Begin
2. Input membership values $B(u_i)$ of n number of patients $u_1, u_2, \ldots, u_n$ in a hospital.
3. Input the adjacency matrix of countries with respect to $R_1, R_2, \ldots, R_m$
   mutually disjoint, irreflexive and symmetric psychological behaviors.
4. Consider two patients u_k and u_l, $1 \le k \le n, k \le l \le n$.
5. If $T^P_{R_i}(u_k u_l) > T^P_{R_j}(u_k u_l)$, $I^P_{R_i}(u_k u_l) < I^P_{R_j}(u_k u_l)$, $F^P_{R_i}(u_k u_l) < F^P_{R_j}(u_k u_l)$, $T^N_{R_i}(u_k u_l) < T^N_{R_j}(u_k u_l)$, $I^N_{R_i}(u_k u_l) > I^N_{R_j}(u_k u_l)$, $F^N_{R_i}(u_k u_l) > F^N_{R_j}(u_k u_l)$, $i \neq j, 1 \le i, j \le m$
   then label $u_k u_l$ as R_i.
6. $T^P_{R_i}$, $F^P_{R_i}$, $I^P_{R_i}$ values of an edge between two different vertices (patients) u_k and u_l show participation of vertex(patient) u_k in strength, weakness and indeterminacy of most conspicuous psychological behavior R_i between them, whereas $T^N_{R_i}$, $F^N_{R_i}$, $I^N_{R_i}$ are corresponding values of vertex (patient) u_l.
7. End

3. Uncovering the undercover reasons for global terrorism

Terrorism is the process of using violence or threat of violence to purport a religious, ideological or political change. It is committed by some non-state actors and may undercover personnel servings on the behalf of respective governments of non-state actors. Terrorism achieves more than immediate target victims and also directed at targets covering a large spectrum of society. It is considered as fourth-generation warfare and violent crime. Nowadays, terrorism is major threat to the society and is illegal under anti-terrorism laws in all jurisdictions. It is considered as a war crime under laws of war, when it is used to target the noncombatants, including neutral military personnel, civilians or enemy prisoners of the war. Many political organizations have practiced the terrorism to achieve their objectives. It is being practiced by both left-wing and right-wing political organizations, religious groups, nationalist groups, revolutionaries and ruling governments. Terrorism can be used to exploit human fear to achieve these goals. According to Global Terrorism Database, non-state terrorism incidents are more than 61,000, claiming more than 140,000 lives from 2000 to 2014.

Table 23 Bipolar single-valued neutrosophic set B of set of countries V

Country	T^P	I^P	F^P	T^N	I^N	F^N
Pakistan	0.7	0.6	0.5	− 0.8	− 0.4	− 0.3
India	0.7	0.6	0.5	− 0.6	− 0.5	− 0.6
Afghanistan	0.8	0.4	0.3	− 0.9	− 0.3	− 0.2
America	0.4	0.4	0.6	− 0.3	− 0.2	− 0.7
Iraq	0.5	0.4	0.5	− 0.6	− 0.3	− 0.4
Israel	0.8	0.4	0.3	− 0.7	− 0.5	− 0.5
Palestine	0.4	0.5	0.7	− 0.5	− 0.6	− 0.6
Kashmir	0.6	0.4	0.8	− 0.7	− 0.6	− 0.6

Nowadays, many countries are blamed to be responsible for global terrorism. That's why many countries are doing war against terror. When a country blames other country to be responsible for terrorism in its territory, many factors are responsible for it. But there are some reasons or factors which are strongly responsible for that particular incident and cannot be ignored. We can use bipolar neutrosophic graph structure to highlight those factors which are highly responsible for terrorism between any two countries or global terrorism due to those countries. There are many undercover reasons highly responsible for global terrorism, including religious factors, military interference/drone attacks, political benefits, to pressurize other government, to occupy other country, benefits of powerful countries, to revenge past political occupation. Let V be the set of eight countries/estates on the globe which are either victims of terrorism or said to be responsible for global terrorism:

$$V = \{\text{Pakistan, India, Afghanistan, America, Iraq,} \\ \text{Israel, Palestine, Kashmir}\}.$$

Consider a BSVN set B on V as shown in Table 23. In Table 23, T^P, T^N of a country depict the percentage of its actual involvement in terrorism and the percentage of its involvement on the basis of other countries' point of view, respectively, whereas F^P, F^N of a country represent the percentage of its actual noninvolvement and of reputation of its noninvolvement, respectively. Similarly, I^P, I^N indicate ambiguity in its involvement and reputation with respect to terrorism.

Every pair of countries in set V has T^P, I^P, F^P, T^N, I^N and F^N values of different undercover reasons in Tables 24, 25, 26, 27, 28, 29, 30, 31 and 32.
For each pair of countries, T^P, T^N of any terrorism reason show the percentage of responsibility from first country and second country, respectively. Similarly, F^P, F^N indicate percentage of being not responsible and I^P, I^N denote the percentage of ambiguity of being responsible for that

Table 24 BSVN set indicating reasons for terrorism due to relationships of Pakistan and other countries

Terrorism Reasons	(India, Pakistan)	(Pakistan, Afghanistan)	(America, Pakistan)
Religious factors	(0.5,0.3,0.3,– 0.4,– 0.4,– 0.4)	(0.3,0.2,0.2,– 0.4,– 0.2,– 0.3)	(0.2,0.2,0.4,– 0.2,– 0.2,– 0.6)
Political benefits	(0.6,0.3,0.3,– 0.4,– 0.3,– 0.4)	(0.3,0.3,0.3,– 0.4,– 0.3,– 0.3)	(0.3,0.3,0.3,– 0.1,– 0.1,– 0.6)
To pressurize other government	(0.7,0.2,0.3,– 0.5,– 0.1,– 0.4)	(0.1,0.3,0.5,– 0.3,– 0.2,– 0.2)	(0.3,0.3,0.2,0.0,– 0.2,– 0.7)
To occupy other country	(0.5,0.2,0.3,– 0.2,– 0.2,– 0.6)	(0.1,0.2,0.4,– 0.1,– 0.3,– 0.3)	(0.3,0.4,0.4,0.0,0.0,– 0.7)
Benefits of powerful countries	(0.4,0.4,0.4,– 0.4,– 0.4,– 0.4)	(0.6,0.2,0.3,– 0.5,– 0.2,– 0.2)	(0.2,0.2,0.3,– 0.1,– 0.2,– 0.6)
Military interference/Drones	(0.3,0.4,0.3,– 0.1,– 0.1,– 0.5)	(0.4,0.1,0.5,– 0.4,– 0.2,– 0.3)	(0.4,0.2,0.2,– 0.2,– 0.1,– 0.6)

Table 25 BSVN set indicating reasons for terrorism due to relationships of India and other countries

Terrorism Reasons	(India, Afghanistan)	(India, America)	(India, Iraq)
Religious factors	(0.4,0.3,0.4,– 0.3,– 0.2,– 0.3)	(0.2,0.2,0.5,– 0.4,– 0.2,– 0.5)	(0.4,0.1,0.4,– 0.5,– 0.2,– 0.5)
Political benefits	(0.7,0.3,0.2,– 0.4,– 0.2,– 0.2)	(0.3,0.4,0.3,– 0.5,– 0.2,– 0.4)	(0.2,0.4,0.3,– 0.1,– 0.3,– 0.5)
To pressurize other government	(0.3,0.4,0.4,– 0.1,– 0.3,– 0.3)	(0.1,0.2,0.4,– 0.3,– 0.2,– 0.4)	(0.2,0.4,0.5,– 0.3,– 0.2,– 0.4)
To occupy other country	(0.4,0.3,0.4,– 0.2,– 0.2,– 0.3)	(0.1,0.1,0.5,– 0.3,– 0.1,– 0.4)	(0.2,0.1,0.4,– 0.1,– 0.3,– 0.5)
Benefits of powerful countries	(0.5,0.4,0.4,– 0.3,– 0.2,– 0.3)	(0.4,0.3,0.4,– 0.2,– 0.1,– 0.5)	(0.2,0.3,0.5,– 0.3,– 0.2,– 0.3)
Military interference/Drones	(0.3,0.3,0.5,– 0.3,– 0.2,– 0.2)	(0.2,0.2,0.1,– 0.3,– 0.2,– 0.4)	(0.2,0.1,0.4,– 0.2,– 0.3,– 0.5)

Table 26 BSVN set indicating reasons for terrorism due to relationships of Afghanistan and other countries

Terrorism Reasons	(America, Afghanistan)	(Iraq, Afghanistan)	(Afghanistan, Israel)
Religious factors	(0.1,0.4,0.6,– 0.2,– 0.2,– 0.5)	(0.3,0.4,0.5,– 0.1,– 0.3,– 0.4)	(0.5,0.1,0.5,– 0.3,– 0.2,– 0.3)
Political benefits	(0.3,0.3,0.3,– 0.1,– 0.2,– 0.6)	(0.4,0.2,0.4,– 0.2,– 0.2,– 0.4)	(0.3,0.2,0.4,– 0.2,– 0.3,– 0.3)
To pressurize other government	(0.2,0.3,0.3,– 0.1,– 0.2,– 0.6)	(0.3,0.3,0.4,– 0.1,– 0.3,– 0.3)	(0.4,0.3,0.3,– 0.4,– 0.3,– 0.5)
To occupy other country	(0.3,0.4,0.2,0.0,– 0.2,– 0.5)	(0.2,0.2,0.4,– 0.1,– 0.3,– 0.4)	(0.3,0.4,0.4,– 0.3,– 0.2,– 0.4)
Benefits of powerful countries	(0.2,0.3,0.4,– 0.1,– 0.2,– 0.5)	(0.5,0.2,0.3,– 0.6,– 0.2,– 0.2)	(0.2,0.1,0.4,– 0.3,– 0.3,– 0.3)
Military interference/Drones	(0.4,0.2,0.1,– 0.3,– 0.2,– 0.5)	(0.1,0.3,0.4,– 0.1,– 0.3,– 0.4)	(0.4,0.2,0.4,– 0.2,– 0.3,– 0.3)

Table 27 BSVN set indicating reasons for terrorism due to relationships of America and other countries

Terrorism Reasons	(America, Kashmir)	(America, Iraq)	(America, Palestine)
Religious factors	(0.3,0.2,0.5,– 0.3,– 0.2,– 0.5)	(0.1,0.4,0.6,– 0.1,– 0.1,– 0.5)	(0.4,0.3,0.5,– 0.2,– 0.2,– 0.6)
Political benefits	(0.4,0.2,0.4,– 0.3,– 0.1,– 0.5)	(0.4,0.3,0.3,– 0.2,– 0.1,– 0.5)	(0.4,0.1,0.3,– 0.3,– 0.1,– 0.5)
To pressurize other government	(0.1,0.3,0.5,– 0.1,– 0.2,– 0.6)	(0.4,0.4,0.4,– 0.2,– 0.2,– 0.6)	(0.3,0.1,0.3,– 0.3,– 0.2,– 0.6)
To occupy other country	(0.2,0.3,0.5,– 0.1,– 0.2,– 0.6)	(0.2,0.3,0.4,– 0.1,– 0.1,– 0.7)	(0.4,0.1,0.4,– 0.1,– 0.2,– 0.5)
Benefits of powerful countries	(0.3,0.2,0.4,– 0.2,– 0.2,– 0.5)	(0.3,0.4,0.4,– 0.1,– 0.2,– 0.5)	(0.4,0.1,0.5,– 0.3,– 0.1,– 0.1)
Military interference/Drones	(0.1,0.2,0.5,– 0.2,– 0.2,– 0.7)	(0.2,0.3,0.3,– 0.1,– 0.1,– 0.6)	(0.2,0.1,0.4,– 0.3,– 0.2,– 0.7)

Table 28 BSVN set indicating reasons for terrorism due to relationships of Iraq and other countries

Terrorism Reasons	(Iraq, Israel)	(Iraq, Palestine)	(Iraq, Kashmir)
Religious factors	(0.4,0.1,0.3,– 0.5,– 0.1,– 0.1)	(0.4,0.2,0.5,– 0.1,– 0.2,– 0.3)	(0.3,0.3,0.3,– 0.1,– 0.2,– 0.3)
Political benefits	(0.3,0.2,0.4,– 0.4,– 0.3,– 0.3)	(0.3,0.2,0.4,– 0.2,– 0.2,– 0.3)	(0.2,0.1,0.1,– 0.4,– 0.1,– 0.1)
To pressurize other government	(0.1,0.3,0.5,– 0.1,– 0.2,– 0.5)	(0.2,0.1,0.6,– 0.2,– 0.2,– 0.6)	(0.4,0.4,0.5,– 0.1,– 0.2,– 0.6)
To occupy other country	(0.1,0.2,0.5,– 0.5,– 0.3,– 0.2)	(0.1,0.1,0.5,0.0,– 0.3,– 0.5)	(0.4,0.1,0.6,– 0.1,– 0.2,– 0.5)
Benefits of powerful countries	(0.4,0.3,0.2,– 0.3,– 0.2,– 0.4)	(0.1,0.2,0.5,– 0.3,– 0.2,– 0.3)	(0.4,0.4,0.2,– 0.4,– 0.2,– 0.2)
Military interference/Drones	(0.4,0.2,0.2,– 0.4,– 0.2,– 0.2)	(0.3,0.1,0.7,– 0.3,– 0.3,– 0.6)	(0.2,0.1,0.7,– 0.1,– 0.2,– 0.6)

Table 29 BSVN set indicating reasons for terrorism due to relationships of Israel and other countries

Terrorism Reasons	(Israel, Palestine)	(Israel, Kashmir)	(Israel, Pakistan)
Religious factors	(0.3,0.2,0.5,– 0.1,– 0.2,– 0.6)	(0.4,0.2,0.1,– 0.4,– 0.3,– 0.3)	(0.2,0.1,0.3,– 0.1,– 0.2,– 0.4)
Political benefits	(0.4,0.2,0.2,0.0,– 0.2,– 0.5)	(0.3,0.2,0.4,– 0.3,– 0.2,– 0.3)	(0.3,0.3,0.3,– 0.1,– 0.2,– 0.4)
To pressurize other government	(0.3,0.2,0.4,0.0,– 0.1,– 0.6)	(0.5,0.0,0.3,– 0.3,– 0.2,– 0.3)	(0.2,0.1,0.3,– 0.1,– 0.2,– 0.5)
To occupy other country	(0.4,0.0,0.1,– 0.1,0.0,– 0.5)	(0.4,0.2,0.4,– 0.1,– 0.2,– 0.6)	(0.1,0.1,0.3,– 0.1,– 0.1,– 0.4)
Benefits of powerful countries	(0.3,0.2,0.3,– 0.1,– 0.2,– 0.6)	(0.4,0.2,0.1,– 0.3,– 0.2,– 0.3)	(0.2,0.4,0.3,– 0.1,– 0.2,– 0.3)
Military interference/Drones	(0.3,0.2,0.1,– 0.1,– 0.2,– 0.6)	(0.4,0.2,0.5,– 0.1,– 0.2,– 0.6)	(0.2,0.1,0.2,– 0.1,– 0.2,– 0.5)

Table 30 BSVN set indicating reasons for terrorism due to relationships of Palestine and other countries

Terrorism Reasons	(Palestine, India)	(Palestine, Pakistan)	(Palestine, Afghanistan)
Religious factors	(0.4,0.2,0.4,– 0.4,– 0.1,– 0.6)	(0.4,0.1,0.2,– 0.5,– 0.1,– 0.1)	(0.4,0.1,0.4,– 0.5,– 0.2,– 0.3)
Political benefits	(0.4,0.2,0.5,– 0.3,0.0,– 0.6)	(0.4,0.2,0.4,– 0.2,– 0.2,– 0.3)	(0.3,0.1,0.4,– 0.2,– 0.2,– 0.3)
To pressurize other government	(0.0,0.0,0.4,0.0,0.0,– 0.6)	(0.1,0.2,0.6,0.0,– 0.1,– 0.5)	(0.2,0.1,0.4,– 0.1,– 0.2,– 0.3)
To occupy other country	(0.2,0.2,0.6,– 0.1,– 0.2,– 0.4)	(0.1,0.1,0.5,– 0.1,– 0.2,– 0.5)	(0.1,0.1,0.5,– 0.1,– 0.1,– 0.6)
Benefits of powerful countries	(0.3,0.1,0.4,– 0.5,– 0.1,– 0.5)	(0.3,0.2,0.4,– 0.5,– 0.2,– 0.3)	(0.4,0.1,0.3,– 0.3,– 0.2,– 0.3)
Military interference/Drones	(0.2,0.2,0.5,– 0.2,– 0.2,– 0.6)	(0.3,0.1,0.7,– 0.1,– 0.1,– 0.5)	(0.4,0.1,0.5,– 0.2,– 0.1,– 0.5)

Table 31 BSVN set indicating reasons for terrorism due to relationships of Kashmir and other countries

Terrorism Reasons	(Kashmir, India)	(Kashmir, Afghanistan)	(Kashmir, Pakistan)
Religious factors	(0.1,0.2,0.8,– 0.1,– 0.2,– 0.4)	(0.4,0.1,0.6,– 0.2,– 0.2,– 0.6)	(0.6,0.1,0.4,– 0.7,– 0.2,– 0.3)
Political benefits	(0.0,0.3,0.7,– 0.5,– 0.2,– 0.4)	(0.0,0.1,0.6,– 0.3,– 0.3,– 0.3)	(0.2,0.1,0.4,– 0.1,– 0.3,– 0.5)
To pressurize other government.	(0.1,0.2,0.7,– 0.1,– 0.2,– 0.5)	(0.2,0.2,0.5,– 0.4,– 0.2,– 0.5)	(0.1,0.1,0.4,– 0.1,– 0.2,– 0.6)
To occupy other country	(0.2,0.2,0.7,– 0.6,– 0.1,– 0.1)	(0.0,0.0,0.6,0.0,– 0.3,– 0.5)	(0.0,0.1,0.5,– 0.1,– 0.2,– 0.5)
Benefits of powerful countries	(0.1,0.4,0.8,– 0.3,– 0.2,– 0.3)	(0.5,0.3,0.4,– 0.6,– 0.2,– 0.3)	(0.2,0.1,0.6,– 0.1,– 0.2,– 0.4)
Military interference/Drones	(0.1,0.2,0.7,– 0.5,– 0.2,– 0.3)	(0.0,0.2,0.4,– 0.1,– 0.2,– 0.5)	(0.2,0.1,0.6,– 0.4,– 0.2,– 0.5)

terrorism reason from first country and second country, respectively.

There are many relations on the set V; here, we define six relations on set V as: R_1 = Religious factors, R_2 = Political benefits, R_3 = To pressurize other government, R_4 = To occupy other country, R_5 = Benefits of powerful countries, R_6 = Military interference/drone attacks. We will define the relations with those elements such that $(V, R_1, R_2, R_3, R_4, R_5, R_6)$ is a GS. Every element in a relation indicates most prominent undercover reason for global terrorism due to that pair of countries. Since $(V, R_1, R_2, R_3, R_4, R_5, R_6)$ is a graph structure, elements in every relation are unique; that is, they belong to just that relation. Hence, we will consider any pair of countries as an element of that particular relation, for which its value of T^P, I^N and F^N is high, and T^N, I^P and F^P values are low as compared to other relations, according to the data of Tables 24, 25, 26, 27, 28, 29, 30, 31 and 32. Write down the

Table 32 BSVN set indicating reasons for terrorism due to relationships of some countries

Terrorism Reasons	(America, Israel)	(India, Israel)	(Pakistan, Iraq)
Religious factors	(0.2,0.1,0.5,– 0.1,– 0.2,– 0.6)	(0.4,0.2,0.5,– 0.1,– 0.2,– 0.6)	(0.2,0.1,0.4,– 0.1,– 0.2,– 0.3)
Political benefits	(0.1,0.1,0.4,– 0.1,– 0.2,– 0.6)	(0.4,0.3,0.4,– 0.3,– 0.2,– 0.5)	(0.2,0.1,0.4,– 0.1,– 0.2,– 0.3)
To pressurize other government.	(0.4,0.1,0.2,– 0.3,– 0.1,– 0.5)	(0.5,0.2,0.5,– 0.1,– 0.2,– 0.3)	(0.2,0.1,0.4,– 0.1,– 0.2,– 0.3)
To occupy other country	(0.1,0.2,0.5,– 0.1,– 0.1,– 0.6)	(0.4,0.1,0.5,– 0.1,– 0.2,– 0.5)	(0.2,0.1,0.4,– 0.1,– 0.2,– 0.3)
Benefits of powerful countries	(0.3,0.1,0.5,– 0.1,– 0.1,– 0.5)	(0.6,0.2,0.4,– 0.4,– 0.2,– 0.4)	(0.5,0.1,0.2,– 0.6,– 0.2,– 0.3)
Military interference/Drones	(0.1,0.3,0.5,– 0.3,– 0.1,– 0.6)	(0.4,0.2,0.5,– 0.3,– 0.2,– 0.5)	(0.5,0.1,0.5,– 0.3,– 0.2,– 0.4)

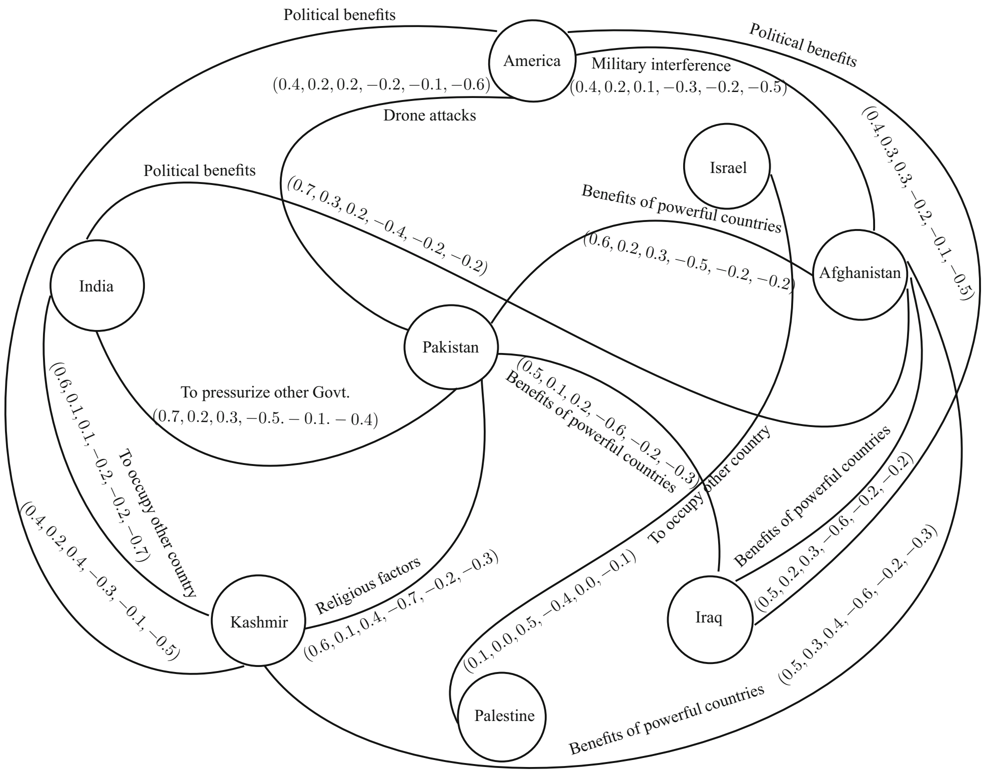


Fig. 10 A BSVNGS highlighting undercover reasons for global terrorism due to any pair of countries

T^P, I^P, F^P, T^N, I^N and F^N values of all elements in the relations, using the data of Tables 24, 25, 26, 27, 28, 29, 30, 31 and 32, such that B_1, B_2, B_3, B_4, B_5, B_6 are BSVN sets on R_1, R_2, R_3, R_4, R_5, R_6, respectively. Let

R_1 = {(Kashmir, Pakistan)},
R_2 = {(India, Afghanistan), (America, Iraq), (America, Kashmir)},
R_3 ={(India, Pakistan)},
R_4 = {(India, Kashmir), (Palestine, Israel)},
R_5 = {(Pakistan, Afghanistan), (Kashmir, Afghanistan), (Iraq, Pakistan), (Afghanistan, Iraq)},
R_6 ={(America, Pakistan), (America, Afghanistan)}.

Corresponding BSVN sets B_1, B_2, B_3, B_4, B_5, B_6 are:

B_1 = {((Kashmir, Pakistan), 0.6,0.1,0.4, − 0.7, − 0.2, − 0.3)},
B_2= {((India, Afghanistan),0.7, 0.3, 0.2, − 0.4, − 0.2, − 0.2), ((America, Iraq),0.4, 0.3, 0.3, − 0.2, − 0.1, − 0.5), ((America, Kashmir), 0.4,0.2,0.4, − 0.3, − 0.1, − 0.5) },
B_3 = {((India, Pakistan),0.7,0.2,0.3, − 0.5, − 0.1, − 0.4)},
B_4 = {((India, Kashmir), 0.6,0.1,0.1, − 0.2, − 0.2, − 0.7), ((Palestine, Israel),0.1,0.0,0.5, − 0.4, 0.0, − 0.1)},
B_5 = {((Pakistan, Afghanistan),0.6, 0.2, 0.3, − 0.5, − 0.2, − 0.2), ((Kashmir, Afghanistan),0.5, 0.3, 0.4, − 0.6, − 0.2, − 0.3), ((Pakistan, Iraq), 0.5,0.1,0.2, − 0.6, − 0.2, − 0.3), ((Iraq, Afghanistan), 0.5,0.2,0.3, − 0.6, − 0.2, − 0.2)},
B_6 = {((America, Pakistan),0.4,0.2,0.2,− 0.2,− 0.1, − 0.6), ((America, Afghanistan),0.4,0.2,0.1,− 0.3,− 0.2, − 0.5)}.

Obviously, (B, B_1, B_2, B_3, B_4, B_5, B_6) is a BSVNGS as shown in Fig. 10.

In BSVNGS, shown in Fig. 10, every edge demonstrates most dominating and attention-getting undercover reason for global terrorism due to those contiguous countries. For instance, attention-getting undercover reason for global terrorism due to Pakistan and America relationship is the drone attacks of America on Pakistan, participation of America in this reason's strength, weakness and indeterminacy is 40%, 20% and 20%, respectively, and participation of Pakistan in its strength, weakness and indeterminacy is 20%, 10% and 60%, respectively. This Pakistan–America edge does not represent direct terrorism from contiguous countries but it points out dominating reason for global terrorism due to Pakistan–America relationships. This BSVNGS also highlights that sovereignty issues of some countries or estates are also increasing global terrorism and tells its intensity, weakness and uncertainty due to both occupying and occupied countries. For instance, sovereignty issues of Kashmir and Palestine are also causing global terrorism. Furthermore, it also helps any country's government that what is main reason for terrorism due to its relationships with other countries and what is percentage of its strength, weakness and ambiguity from its side and other country's side.

Moreover, most frequent relation in this BSVNGS is the benefits of powerful countries. It shows that powerful countries are destroying global peace to achieve their political benefits. A BSVNGS of all countries can be very helpful for anti-terrorism organizations that which perspective is to be considered to control global terrorism. According to this BSVNGS, they should pressurize powerful countries and solve sovereignty issues to maintain global peace.

General procedure of this application is explained by the following algorithm.

Algorithm 3

1. Begin
2. Input membership values $B(u_i)$ of n number of countries $u_1, u_2, \ldots, u_n$ which are either victims of terrorist or said to be responsible for global terrorism.
3. Input the adjacency matrix of countries with respect to $R_1, R_2, \ldots, R_m$ terrorism reasons due to relationships among all pairs of vertices(i.e., pairs of countries).
4. Follow steps 3 and 4 of Algorithm 2.
5. $T^P_{R_i}, F^P_{R_i}, I^P_{R_i}$ values of an edge between two different vertices (countries) u_k and u_l show participation of vertex (country) u_k in strength, weakness and indeterminacy of most dominating and attention-getting terrorist reason R_i due to relationships between them, whereas $T^N_{R_i}, F^N_{R_i}, I^N_{R_i}$ are corresponding values of vertex (country) u_l.
6. End.

4 Conclusions

Graph theory is widely used for dealing with structural information in different domains, including computer science, electrical engineering, operations research, network routing, and transportation, pattern recognition and image interpretation. However, in many cases, some aspects of a graph-theoretic problem may be uncertain. In such cases, it is natural to deal with the uncertainty using the methods of fuzzy sets and bipolar single-valued neutrosophic sets. Bipolar single-valued neutrosophic models are more flexible and practical than fuzzy models. A bipolar neutrosophic graph constitutes a generalization of the notion bipolar fuzzy graph. In this research paper, we have introduced the idea of bipolar single-valued neutrosophic graph structure and discussed many relevant notions. We also have investigated worthwhile applications of bipolar single-valued neutrosophic graph structure in decision making. In the future, we aim to generalize our notions to (1) rough neutrosophic structures, (2) soft rough neutrosophic graph structures, (3) bipolar neutrosophic soft graph structures and (4) rough neutrosophic hypergraph structures.

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Compliance with ethical standards

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