On Neutro-LA-semihypergroups and Neutro-Hv-LA-semigroups

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On Neutro-LA-semihypergroups and Neutro-H_v-LA-semigroups

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Abstract. In this paper, we extend the notion of LA-semihypergroups (resp. H_v-LA-semigroups) to neutro-LA-semihypergroups (respectively, neutro-H_v-LA-semigroups). Anti-LA-semihypergroups (respectively, anti-H_v-LA-semigroups) are studied and investigated some of their properties. We show that these new concepts are different from classical concepts by several examples. These are particular cases of the classical algebraic structures generalized to neutroalgebraic structures and antialgebraic structures (Smarandache, 2019).

Keywords: Hyperoperation, Neutrohyperoperation, Antihyperoperation, LA-semihypergroup, Neutro-LA-semihypergroup, Anti-LA-semihypergroup, H_v-LA-semigroup, Neutro-H_v-LA-semigroup, Anti-H_v-LA-semigroup.

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1. Introduction

Kazim and Naseeruddin [13] provided the concept of left almost semigroup (abbreviated as LA-semigroup). They generalized some useful results of semigroup theory. Later, Mushtaq [17] and others further investigated the structure and added many useful results to the theory of LA-semigroups; see also [1, 3, 11, 14, 18, 19, 26].

A hypergroup as a generalization of the notion of a group, was introduced by F. Marty [16] in 1934. Some valuable books in hyperstructures have published [4, 5, 6, 7, 27]. In 1990, Vougiouklis introduced the concept of H_v-structures in Fourth AHA Congress as a generalization of the well-known algebraic hyperstructures. Two books on algebraic H_v-structure or weak hyperstructure have been published [7, 27].

Hila and Dine [10] introduced the notion of LA-semihypergroups as a generalization of semigroups, semihypergroups, and LA-semigroups. Yaqoob, Corsini and Yousafzai [28] extended the work of Hila and Dine. Gulistan, Yaqoob and Shahzad, [9] introduced the notion of H_v-LA-semigroups as LA-semihypergroups. They showed that every LA-semihypergroup is an H_v-LA-semigroup and each LA-semigroup endowed with an equivalence relation can induced an H_v-LA-semigroup and they investigated isomorphism theorem with the help of regular relations.

In 2019 and 2020, within the field of neutrosophy, Smarandache [21, 22, 23] generalized the classical algebraic structures to neutroalgebraic structures (or neutroalgebras) whose operations and axioms are partially true, partially indeterminate, and partially false as extensions of partial algebra, and to antialgebraic structures (or antialgebras) {whose operations and axioms are totally false}. And in general, he extended any classical structure, in no matter what field of knowledge, to a neutrostructure and an antistructure. These are new fields of research within neutrosophy.

Smarandache in [23] revisited the notions of neutroalgebras and antialgebras, where he studied partial algebras, Universal algebras, Effect algebras and Boole′s partial algebras, and showed that neutroalgebras are generalization of partial algebras. Further, he extended the classical hyperalgebra to n-ary hyperalgebra and its alternatives n-ary neutrohyperalgebra and n-ary antihyperalgebra [25].

The notion of neutrogroup was defined and studied by A.A.A. Agboola in [2]. A. Rezaei et al. introduced the notions of neutrosemihypergroup and antisemihypergroup [20]. Recently, S. Mirvakili et al. extend the notion of H_v-semigroups to neutro-H_v-semigroups and anti-H_v-semigroups and investigated many of their properties [15].

In this paper, the concept of neutro-LA-semihypergroups(resp. neutro-H_v-LA-semigroup) and anti-LA-semihypergroups (resp. anti-H_v-LA-semigroup) is formally presented. Moreover, We characterize LA-semihypergroups (resp. H_v-LA-semigroup), neutro-LA-semihypergroups (resp. neutro-H_v-LA-semigroup) and anti-LA-semihypergroups (resp. anti-H_v-LA-semigroup) of order 2.

2. Preliminaries

In this section we recall some basic notions and results regarding to LA-semigroups, LA-semihypergroups and H_v-LA-semigroups.

A groupoid $(H, \circ)$ is a non-empty set H together with a map $\circ : H \times H \to H$ called (binary) operation. The structure $(H, \circ)$ is called a groupoid.

Definition 2.1. [13] A groupoid $(H, \circ)$ is called an LA-semigroup, if $(a \circ b) \circ c = (c \circ b) \circ a$, for all $a, b, c \in H$.

Example 2.2. [17] Let $(\mathbb{Z}, +)$ denote the commutative group of integers under addition. Define a binary operation $\circ$ in $\mathbb{Z}$ as follows:

$$a \circ b = b - a, \ \forall a, b \in \mathbb{Z},$$

where $-$ denotes the ordinary subtraction of integers. Then $(\mathbb{Z}, \circ)$ is an LA-semigroup.

Definition 2.3. ([4, 6]) A hypergroupoid $(H, \circ)$ is a non-empty set H together with a map $\circ : H \times H \to P^*(H)$ called (binary) hyperoperation, where $P^*(H)$ denotes the set of all non-empty subsets of H. The hyperstructure $(H, \circ)$ is called a hypergroupoid and image of the pair (x, y) is denoted by $x \circ y$.

If A and B are non-empty subsets of H and $x \in H$, then by $A \circ B$, $A \circ x$, and $x \circ B$ we mean $A \circ B = \bigcup\limits_{a \in A, b \in B} a \circ b$, $A \circ x = A \circ \{x\}$ and $x \circ B = \{x\} \circ B$.

Definition 2.4. ([4, 6]) (1) A hypergroupoid $(H, \circ)$ is called a semihypergroup if it satisfies the following:

(A) $(\forall a, b, c \in H)(a \circ (b \circ c) = (a \circ b) \circ c)$.

(2) A hypergroupoid $(H, \circ)$ is called an H_v-semigroup if it satisfies the following:

(WA) $(\forall a, b, c \in H)(a \circ (b \circ c) \cap (a \circ b) \circ c) \neq \emptyset$.

Definition 2.5. ([9, 10]) (1) A hypergroupoid $(H, \circ)$ is called a Left Almost semihypergroup or an LA-semihypergroup if it satisfies the following:

(LA) $(\forall a, b, c \in H)(a \circ b) \circ c = (c \circ b) \circ a$.

(2) A hypergroupoid $(H, \circ)$ is called a Left Almost H_v-semigroup or H_v-LA-semigroup if it satisfies the following:

(WLA) $(\forall a, b, c \in H)(a \circ b) \circ c \cap (c \circ b) \circ a \neq \emptyset$.

Example 2.6. ([4, 6]) Let H be a nonempty set and for all $x, y \in H$, we define $x \circ y = H$. Then $(H, \circ)$ is a semihypergroup and an LA-semihypergroup.

We define the commutative law on $(H, \circ)$ as follows:

(C) $(\forall a, b \in H)(a \circ b = b \circ a)$.

Also, we define the weak commutative law on $(H, \circ)$ as follows:

(WC) $(\forall a, b \in H)(a \circ b \cap b \circ a \neq \emptyset)$.

Theorem 2.7. *Let $(H, \circ)$ be a commutative hypergroupoid. Then $(H, \circ)$ is an LA-semihypergroup if and only if $(H, \circ)$ is a semihypergroup.*

Proof. Let $x, y, z \in H$. Then by commutativity we have

$$(x \circ y) \circ z = x \circ (y \circ z) \Leftrightarrow (x \circ y) \circ z = (z \circ y) \circ x.$$

□

EXAMPLE 2.8. Let $H = \{a, b\}$. Define the hyperoperation $\circ$ on H with the following Cayley table.

$\circ$	a	b
a	H	a
b	H	b

Then $(H, \circ)$ is a semihypergroup, but is not an LA-semihypergroup.

EXAMPLE 2.9. Let $H = \{a, b\}$. Define the hyperoperation $\circ$ on H with the following Cayley table.

$\circ$	a	b
a	H	H
b	a	a

Then $(H, \circ)$ is an LA-semihypergroup, but is not a semihypergroup.

Theorem 2.10. *Let $(H, \circ)$ be a commutative hypergroupoid. Then $(H, \circ)$ is an H_v-LA-semigroup if and only if $(H, \circ)$ is an H_v-semigroup*

Proof. Let $x, y, z \in H$. Then by commutativity we have

$$(x \circ y) \circ z \cap x \circ (y \circ z) \neq \emptyset \Leftrightarrow (x \circ y) \circ z \cap (z \circ y) \circ x \neq \emptyset.$$

□

EXAMPLE 2.11. Let $H = \{a, b\}$. Define the hyperoperation $\circ$ on H with the following Cayley table.

$\circ$	a	b
a	a	a
b	b	H

Then $(H, \circ)$ is an H_v-semigroup, but is not an H_v-LA-semigroup.

EXAMPLE 2.12. Let $H = \{a, b\}$. Define the hyperoperation $\circ$ on H with the following Cayley table.

$\circ$	a	b
a	a	b
b	H	a

Then $(H, \circ)$ is an H_v-LA-semigroup, but is not an H_v-semigroup.

3. On Neutro-LA-semihypergroups, Neutro-H_v-LA-semigroups, Anti-LA-semihypergroups and Anti-H_v-LA-semigroups

F. Smarandache generalized the classical algebraic structures to the neutroalgebraic structures and antialgebraic structures. **neutro-sophication of an item** C (that may be a concept, a space, an idea, an operation, an axiom, a theorem, a theory, etc.) means to split C into three parts (two parts opposite to each other, and another part which is the neutral / indeterminacy between the opposites), as pertinent to neutrosophy $\{(< A >, < neutA >, < antiA >)$, or with other notation $(T, I, F)\}$, meaning cases where C is partially true (T), partially indeterminate (I), and partially false (F). While **anti-sophication of** C means to totally deny C (meaning that C is made false on its whole domain) (for detail see Smarandache [21, 22, 24, 25]).

Neutro-sophication of an axiom on a given set X, means to split the set X into three regions such that: on one region the axiom is true (we say degree of truth T of the axiom), on another region the axiom is indeterminate (we say degree of indeterminacy I of the axiom), and on the third region the axiom is false (we say degree of falsehood F of the axiom), such that the union of the regions covers the whole set, while the regions may or may not be disjoint, where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$. Anti-sophication of an axiom on a given set X, means to have the axiom false on the whole set X (we say total degree of falsehood F of the axiom), or $(0, 0, 1)$.

Similarly for the **neutro-sophication of an operation** defined on a given set X, means to split the set X into three regions such that on one region the operation is well-defined (or inner-defined) (we say degree of truth T of the operation), on another region the operation is indeterminate (we say degree of indeterminacy I of the operation), and on the third region the operation is outer-defined (we say degree of falsehood F of the operation), such that the union of the regions covers the whole set, while the regions may or may not be disjoint, where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$.

Anti-sophication of an operation on a given set X, means to have the operation outer-defined on the whole set X (we say total degree of falsehood F of the axiom), or $(0, 0, 1)$.

In this section we will define the **neutro-LA-semihypergroups** and **anti-LA-semihypergroups**.

Definition 3.1. Neutrohyperoperation (Neutrohyperlaw) A neutrohyperoperation is a map $\circ : H \times H \to P(U)$ where U is a universe of discourse that contains H that satisfies the below neutro-sophication process.

The neutro-sophication (degree of well-defined, degree of indeterminacy, degree of outer-defined) of the hyperoperation is the following neutrohyperoperation:

(NHA) $(\exists x, y \in H)(x \circ y \in P^*(H))$ and $(\exists x, y \in H)(x \circ y$ is an indeterminate subset, or $x \circ y \notin P^*(H))$.

The neutro-sophication (degree of truth, degree of indeterminacy, degree of falsehood) of the LA-semihypergroup axiom is the following neutrohyperLA-semihypergroup:

(NLA) $(\exists a, b, c \in H$ such that $(a, b, c) \neq (x, x, x)$ or $(a, b, c) \neq (x, y, x))$ $((a \circ b) \circ c = (c \circ b) \circ a)$ and $(\exists d, e, f \in H$ such that $(d, e, f) \neq (x, x, x)$ or $(d, e, f) \neq (x, y, x))((d \circ e) \circ f \neq (f \circ e) \circ d$ or $(d \circ e) \circ f =$ indeterminate, or $(f \circ e) \circ d =$ indeterminate).

Also, The neutro-sophication (degree of truth, degree of indeterminacy, degree of falsehood) of the H_v-LA-semigroup axiom is the following neutrohyperH_v-LA-semigroup:

(NWLA) $(\exists a, b, c \in H$ such that $(a, b, c) \neq (x, x, x)$ or $(a, b, c) \neq (x, y, x))$ $((a \circ b) \circ c \cap (c \circ b) \circ a \neq \emptyset)$ and $(\exists d, e, f \in H$ such that $(d, e, f) \neq (x, x, x)$ or $(d, e, f) \neq (x, y, x))((d \circ e) \circ f \cap (f \circ e) \circ d = \emptyset$ or $(d \circ e) \circ f =$ indeterminate, or $(f \circ e) \circ d =$ indeterminate).

We define the neutrohypercommutativity (NC) on $(H, \circ)$ as follows:

(NC) $(\exists a, b \in H)(a \circ b = b \circ a)$ and $(\exists c, d \in H)(c \circ d \neq d \circ c$, or $c \circ d =$ indeterminate, or $d \circ c =$ indeterminate).

Also, we define the neutrohyperweakcommutativity (NWC) on $(H, \circ)$ as follows:

(NWC) $(\exists a, b \in H)(a \circ b \cap b \circ a \neq \emptyset)$ and $(\exists c, d \in H)(c \circ d \cap d \circ c = \emptyset$, or $c \circ d =$ indeterminate, or $d \circ c =$ indeterminate).

Now, we define a neutrohyperalgebraic system $S =< H, F, A >$, where H is a set or neutrosophic set, F is a set of the hyperoperations (hyperlaws), and A is the set of hyperaxioms, such that there exists at least one neutrohyperoperation (neutrohyperlaw) or at least one neutrohyperaxiom, and no antihyperoperation (antihyperlaw) and no antihyperaxiom.

Definition 3.2. Antihyperoperation {Antihyperlaw (AHL)}

The antihyper-sophication (totally outer-defined) of the hyperoperation (hyperlaw) gives the definition of antihyperoperation antihyperlaw (AHL):

(AHL) $(\forall x, y \in H)(x \circ y \notin P^*(H))$.

The antihyper-sophication (totally false) of the LA-semihypergroup:

(ALA) $(\forall a, b, c \in H$ *such that* $(a, b, c) \neq (x, x, x)$ *or* $(a, b, c) \neq (x, y, x))((a \circ b) \circ c) \neq (c \circ b) \circ a)$.

Also, the antihyper-sophication (totally false) of the H_v-LA-semigroup:

(AWLA) $(\forall a, b, c \in H$ *such that* $(a, b, c) \neq (x, x, x)$ *or* $(a, b, c) \neq (x, y, x))((a \circ b) \circ c) \cap (c \circ b) \circ a = \emptyset)$.

We define the anticommutativity (AC) on $(H, \circ)$ as follows:

(AC) $(\forall a, b \in H$ with $a \neq b)(a \circ b \neq b \circ a)$.

Also, we define the antiweakcommutativity (AWC) on $(H, \circ)$ as follows:

(AWC) $(\forall a, b \in H \text{ with } a \neq b)(a \circ b \cap b \circ a = \emptyset)$.

Definition 3.3. (1) A neutro-LA-semihypergroup is an alternative of LA-semihypergroup that has at least one (NLA), with no antihyperoperation.

(2) A neutro-H_v-LA-semigroup is an alternative of H_v-LA-semigroup that has at least one (WNLA), with no antihyperoperation.

(3) An anti-LA-semihypergroup is an alternative of LA-semihypergroup that has at least one (ALA) or an (AHL) axiom.

(4) An anti-H_v-LA-semigroup is an alternative of H_v-LA-semigroup that has at least one (WALA) or an (AHL) axiom.

Remark 3.4. If hyperoperation $\circ$ in Definition 3.3 is operation, then we have neutro-LA-semigroup and anti-LA-semigroup.

Example 3.5. (i) Let $H = \{a, b, c\}$ and $U = \{a, b, c, d\}$ a universe of discourse that contains H. Define the neutrohyperoperation $\circ$ on H with the following Cayley table.

$\circ$	a	b	c
a	a	a	a
b	a	a	$\{a, b, d\}$
c	c	?	H

Then $(H, \circ)$ is a neutrosemihypergroup.

Example 3.6. (i) Let $\mathbb{N}$ be the set of natural numbers except 0. Define hyper-Low $\circ$ on $\mathbb{N}$ by $x \circ y = \{\frac{x^2}{x^2+1}, y\}$. Then $(\mathbb{N}, \circ)$ is an anti-LA-semihypergroup. (AHL) is valid, since for all $x, y \in \mathbb{N}$, $x \circ y \notin P^*(\mathbb{N})$. Thus, (AHL) holds.

(ii) Let $H = \{a, b\}$. Define the hyperoperation $\circ$ on H with the following Cayley table.

$\circ$	a	b
a	b	a
b	a	a

Then $(H, \circ)$ is a commutative anti-LA-semihypergroup.

(iii) Let $H = \{a, b\}$. Define the hyperoperation $\circ$ on H with the following Cayley table.

$\circ$	a	b
a	a	a
b	b	b

Then $(H, \circ)$ is an anticommutative anti-LA-semihypergroup.

Theorem 3.7. (1) *Every LA-semihypergroup is an H_v-LA-semigroup.*

(2) *Every anti-H_v-LA-semigroup is an anti-LA-semihypergroup.*

(3) *Every neutro-H_v-LA-semigroup is a neutro-LA-semihypergroup or an anti-LA-semihypergroup.*

(4) *Every H_v-LA-semigroup is an LA-semihypergroup or a neutro-LA-semihypergroup or an anti-LA-semihypergroup.*

EXAMPLE 3.8. [9] The Converse of part (1) of Theorem 3.7 is not true. Consider $H = \{x, y, z\}$ and define a hyperoperation $\circ$ on H by the following table:

$\circ$	x	y	z
x	x	$\{x, z\}$	H
y	$\{x, z\}$	x	x
z	$\{x, y\}$	x	$\{x, z\}$

Then $(H, \circ)$ is an H_v-LA-semigroup which is not an LA-semihypergroup and not an H_v-semigroup. Indeed, we have

$$\{x, y\} = z \circ (y \circ y) \;\&\; (z \circ y) \circ y = \{z\}.$$

Thus, $z \circ (y \circ y) \cap (z \circ y) \circ y \neq \emptyset$. Therefore $(H, \circ)$ is not an H_v-semigroup. Also,

$$\{x, y, z\} = (x \circ y) \circ z \neq (z \circ y) \circ x = \{x, y\}$$

Thus, $(H, \circ)$ is not an LA-semihypergroup.

EXAMPLE 3.9. The Converse of part (2) of Theorem 3.7 is not true. Consider $H = \{x, y, z\}$ and define a hyperoperation $\circ$ on H by the following table:

$\circ$	a	b
a	a	H
b	a	a

Then $(H, \circ)$ is a commutative anti-LA-semihypergroup and is not anti-H_v-LA-semigroup.

Let $(H, \circ)$ is a hypergroupoid. Then the hyperoperation $*$ defined as follows:

$$x * y = y \circ x, \ \forall x, y \in H.$$

$(H, *)$ in is called dual hypergroupoid of $(H, \circ)$. It is easy to see that:

Theorem 3.10. *$(H, \circ)$ is a semihypergroup if and only if $(H, \circ)$ is a semihypergroup.*

Theorem 3.10 for LA-semihypergroups is not true.

EXAMPLE 3.11. Let $(H = \{a, b\}, \circ)$ be an LA-semihypergroup of order 2 when the hyperoperation $\circ$ defined on H with the following Cayley table.

$\circ$	a	b
a	H	H
b	a	a

But $(H, *)$ is not an LA-semihypergroup.

Proposition 3.12. *Let $(H, \circ_H)$ and $(G, \circ_G)$ be two neutro-LA-semihypergroups (resp. anti-LA-semihypergroups). Then $(H \times G, *)$ is a neutro-LA-semihypergroup (resp. anti-LA-semihypergroups), where $*$ is defined on $H \times G$ by: for any*

$(x_1, y_1), (x_2, y_2) \in H \times G$

$$(x_1, y_1) * (x_2, y_2) = (x_1 \circ_H x_2, y_1 \circ_G y_2).$$

Note that if $(H, \circ)$ is a neutro-LA-semihypergroup, then if there is a non-empty set $H_1 \subseteq H$, such that $(H_1, \circ)$ is an LA-semihypergroup, we call it **Smarandache LA-semihypergroup**.

Suppose $(H, \circ_H)$ and $(G, \circ_G)$ be two hypergroupoids. A function $f : H \to G$ is called a homomorphism if, for all $a, b \in H$, $f(a \circ_H b) = f(a) \circ_G f(b)$.

Proposition 3.13. *Let $(H, \circ_H)$ be an LA-semihypergroup(H_v-LA-semigroup), $(G, \circ_G)$ be a neutro-LA-semihypergroup (neutro-H_v-LA-semigroup) and $f : H \to G$ be a homomorphism. Then $(f(H), \circ_G)$ is an LA-semihypergroup (H_v-LA-semigroup), where $f(H) = \{f(h) : h \in H\}$.*

Proof. Assume that $(H, \circ_H)$ is an LA-semihypergroup (H_v-LA-semigroup) and $x, y, z \in f(H)$. Then there exist $h_1, h_2, h_3 \in f(H)$ such that $f(h_1) = x$, $f(h_2) = y$ and $f(h_3) = z$, and so we have

$$\begin{aligned}
(x \circ_G y) \circ_G z) &= (f(h_1) \circ_G f(h_2)) \circ_G f(h_3) \\
&= f(h_1 \circ_G h_2) \circ_H f(h_3) \\
&= f((h_1 \circ_H h_2) \circ_H h_3) \\
&= f((h_3 \circ_H h_2) \circ_H h_1) \\
&= f(h_3 \circ_H h_2) \circ_G f(h_1) \\
&= (f(h_3) \circ_G f(h_2)) \circ_G f(h_1) \\
&= (x \circ_G y) \circ_G z.
\end{aligned}$$

□

Definition 3.14. Let $(H, \circ_H)$ and $(G, \circ_G)$ be two hypergroupoids. A bijection $f : H \to G$ is an isomorphism if it conserves the multiplication (i.e. $f(a \circ_H b) = f(a) \circ_G f(b)$) and write $H \cong G$. A bijection $f : H \to G$ is an antiIsomorphism if for all $a, b \in H$, $f(a \circ_H b) \neq f(b) \circ_G f(a)$. A bijection $f : H \to G$ is a neutroIsomorphism if there exist $a, b \in H$, $f(a \circ_H b) = f(b) \circ_G f(a)$, i.e. degree of truth (T), there exist $c, d \in H$, $f(c \circ_H d)$ or $f(c) \circ_G f(d)$ are indeterminate, i.e. degree of Indeterminacy (I), and there exist $e, h \in H$, $f(e \circ_H h) \neq f(e) \circ_G f(h)$, i.e. degree of falsehood (F), where (T, I, F) are different from $(1, 0, 0)$ and $(0, 0, 1)$, and $T, I, F \in [0, 1]$.

Proposition 3.15. *Let $(H_i, \circ)$, where $i \in \Lambda$, be a family of neutro-LA-semihypergroups (neutro-H_v-LA-semigroups). Then $(\bigcap\limits_{i \in \Lambda} H_i, \circ)$ is a neutro-LA-semihypergroup (neutro-H_v-LA-semigroup) or an anti-LA-semihyperGgroup (anti-H_v-LA-semigroup) or an LA-semihypergroups (H_v-LA-semigroup).*

Proof. It is trivial. □

Proposition 3.16. *Let $(H_i, \circ)$ be a family of anti-LA-semihypergroupsn (anti-H_v-LA-semigroups), where $i \in \Lambda$. Then $(\bigcap_{i\in\Lambda} H_i, \circ)$ is an anti-LA-semihypergroup (anti-H_v-LA-semigroup).*

Proof. It is trivial. □

Note that if $(H, \circ)$ is a neutro-LA-semihypergroup (neutro-H_v-LA-semigroup) and $(G, \circ)$ is an anti-LA-semihypergroup (anti-H_v-LA-semigroup), then $(H \cap G, \circ)$ is an anti-LA-semihypergroupb (anti-H_v-LA-semigroup). Also, let $(H, \circ_H)$ be a neutro-LA-semihypergroup (neutro-H_v-LA-semigroup) and $(G, \circ_G)$ be an anti-LA-semihypergroup (anti-H_v-LA-semigroup) and $H \cap G = \emptyset$. Define hyperoperation $\circ$ on $H \biguplus G$ by:

$$x \circ y = \begin{cases} x \circ_H y & \text{if } x, y \in H; \\ x \circ_G y & \text{if } x, y \in G; \\ \{x, y\} & \textit{otherwise}. \end{cases}$$

Then $(H \biguplus G, \circ)$ is a neutro-LA-semihypergroup (neutro-H_v-LA-semigroup), but it is not an anti-LA-semihypergroup (anti-H_v-LA-semigroup).

Proposition 3.17. *Let $(H, \circ)$ be an anti-LA-semihypergroup (anti-H_v-LA-semigroup) and $e \in H$. Then $(H \cup \{e\}, *)$ is a neutro-LA-semihypergroup (neutro-H_v-LA-semigroup), where $*$ is defined on $H \cup \{e\}$ by:*

$$x * y = \begin{cases} x \circ_H y & \text{if } x, y \in H; \\ \{e, x, y\} & \textit{otherwise}. \end{cases}$$

Proof. It is straightforward. □

4. Characterization of groupoids of order 2

In the next results we use the operation $\circ : H \times H \to H$.

In this section, let $\circ$ be an operation on $H = \{a, b\}$ and $(A_{11}, A_{12}, A_{21}, A_{22})$ inside of the below Cayley table:

$\circ$	a	b
a	A_{11}	A_{12}
b	A_{21}	A_{22}

Lemma 4.1. *Let $(H = \{a, b\}, \circ_H)$ and $(G = \{a', b'\}, \circ_G)$ be two groupoids with the Cayley tables $(h_{11}, h_{12}, h_{21}, h_{22})$ and $(g_{11}, g_{12}, g_{21}, g_{22})$ respectively. Then $H \cong G$ if and only if for all $i, j \in \{1, 2\}$,*

$$g_{ij} = h'_{ij}$$

or

$$g_{ij} = G \setminus k'_{ij},$$

where $k_{12} = h_{21}, k_{21} = h_{12}, k_{11} = h_{22}$ and $k_{22} = h_{11}$.

Lemma 4.2. *Any commutative semigroup of order 2 is an LA-semiGrouop.*

Theorem 4.3. *Every LA-semigroup of order 2 is commutative.*

Proof. Let $(H = \{a, b\}, \circ)$ be an LA-semigroup. We have

(1) $(a \circ b) \circ b = (b \circ b) \circ a$,
(2) $(a \circ a) \circ b = (b \circ a) \circ a$.

Let $a \circ b = a$. Then by (1), $a = (b \circ b) \circ a$. If $b \circ b = b$, then we obtain $b \circ a = a$, and so $(H, \circ)$ is commutative. So we have $b \circ b = a$. Hence $a \circ a = a$ and by (2), $a = (b \circ a) \circ a$. If $b \circ a = b$, then we obtain $a = b$. and this is a contradiction. Thus, $b \circ a = a$, and so $(H, \circ)$ is commutative.
Now, let $a \circ b = a$. Then by the similar way we obtain $(H, \circ)$ is commutative. □

Corollary 4.4. *Any LA-semigroup of order 2 is commutative.*

Corollary 4.5. *Any LA-semigroup of order 2 is a semigroup.*

Corollary 4.6. *There is no Non-commutative LA-semigroup of order 2.*

Theorem 4.7. [8] *There exist 5 semigroups $(H = \{a, b\}, \circ_i)$, $i = 1, \cdots, 5$, of order 2 by the following Cayley table (up to isomorphism).*

$\circ_1$	a	b
a	a	a
b	a	a

$\circ_2$	a	b
a	a	a
b	a	b

$\circ_3$	a	b
a	b	a
b	a	b

$\circ_4$	a	b
a	a	a
b	b	b

$\circ_5$	a	b
a	a	b
b	a	b

Theorem 4.8. *There exist 3 LA-semigroups of order 2 (up to isomorphism).*

Proof. By Corollary 5.2, the only LA-semigroups of order 2 are commutative semigroups of order 2, and so $(H, \circ_i)$, for $i = 1, \cdots, 3$, from Theorem 4.7 are LA-semigroups of order 2 (up to isomorphism). □

Theorem 4.9. *There exist 5 anti-LA-semigroups of order 2 (up to isomorphism).*

Proof. Let $(H = \{a, b\}, \circ)$ be an anti-LA-semigroup. Then We have

(1) $(a \circ b) \circ b \neq (b \circ b) \circ a$,
(2) $(a \circ a) \circ b \neq (b \circ a) \circ a$.

Also, Then we have one of the following Cases:

(3) $(a \circ b = a \ \& \ b \circ a = a)$,
(4) $(a \circ b = a \ \& \ b \circ a = b)$,
(5) $(a \circ b = b \ \& \ b \circ a = a)$,
(6) $(a \circ b = b \ \& \ b \circ a = b)$.

Case 3: By (1) and (2) we have

(7) $a \neq (b \circ b) \circ a$,
(8) $(a \circ a) \circ b \neq a \circ a$.

If $a \circ a = a$, then (8) implies that $a \circ b \neq a$, which is a contradiction. If $a \circ a = b$, then (8) implies that $b \circ b \neq b$, and so $b \circ b = a$. We set $\circ_1 := \circ$ and therefore we we have an anti-LA-semigroup as follows:

$\circ_1$	a	b
a	b	a
b	a	a

Case 4: By (1) and (2) we have

(9) $a \neq (b \circ b) \circ a$,
(10) $(a \circ a) \circ b \neq b$.

If $a \circ a = b$, then (8) implies that $b \circ b \neq b$, and so $b \circ b = a$. We set $\circ_2 := \circ$ and therefore we we have an anti-LA-semigroup as follows:

$\circ_2$	a	b
a	b	a
b	b	a

If $a \circ a = a$, then by (9) $b \circ b \neq a$ so $b \circ b = b$. We set $\circ_3 := \circ$ and therefore we we have an anti-LA-semigroup as follows:

$\circ_3$	a	b
a	a	a
b	b	b

Case 5: By (1) and (2) we have

(11) $b \circ b \neq (b \circ b) \circ a$,
(12) $(a \circ a) \circ b \neq a \circ a$.

If $a \circ a = a$, then $b \circ b = a$ or $b \circ b = b$. Let $b \circ b = a$. Then by (11) $a \circ a = b$ and this is a contradiction. If $b \circ b = b$, then we set $\circ_4 := \circ$ and therefore we have an anti-LA-semigroup as follows:

$\circ_4$	a	b
a	a	b
b	a	b

If $a \circ a = b$, then (12) implies that $b \circ b = a$. We set $\circ_5 := \circ$ and therefore we have an anti-LA-semigroup as follows:

$\circ_5$	a	b
a	b	b
b	a	a

Case 6: By (1) and (2) we have

(13) $b \circ b \neq (b \circ b) \circ a$,
(14) $(a \circ a) \circ b \neq b$.

If $a \circ a = b$, then (14) implies that $b \circ b = a$. We set $\circ_6 := \circ$ and therefore we have an anti-LA-semigroup as follows:

$\circ_6$	a	b
a	b	b
b	b	a

If $a \circ a = a$, then (14) implies that $a \circ b = a$, which is a contradiction. $(H, \circ_1) \cong (H, \circ_6)$, and so we have anti-$LA$-semigroups $(H, \circ_i)$, for $i = 1, \cdots, 5$, of order 2. □

Corollary 4.10. *There exists 1 commutative anti-LA-semigroup of order 2 (up to isomorphism).*

Corollary 4.11. *There exist 4 Non-commutative anti-LA-semigroups of order 2 (up to isomorphism).*

Theorem 4.12. *Every neutro-LA-semigroup of order 2 is Non-commutative.*

Proof. Let $(H = \{a, b\}, \circ)$ be a commutative neutro-LA-semigroup. Then We have

(1) $(a \circ b) \circ b \neq (b \circ b) \circ a$,
(2) $(a \circ a) \circ b = (b \circ a) \circ a$,
(3) $a \circ b = b \circ a$,

or

(4) $(a \circ b) \circ b = (b \circ b) \circ a$,
(5) $(a \circ a) \circ b \neq (b \circ a) \circ a$,
(6) $a \circ b = b \circ a$.

Case 2: If $a \circ b = a = b \circ a$, then we have

(7) $a \neq (b \circ b) \circ a$,
(8) $(a \circ a) \circ b = a \circ a$,

or

(9) $a = (b \circ b) \circ a$,
(10) $(a \circ a) \circ b \neq a \circ a$.

Now, (7) implies that $b \circ b = a$ and $a \circ a = b$, which is a contradiction with (8). Also, (10) implies that $a \circ a = b$ and $b \circ b = a$ and this is a contradiction with (9).

Case 2: If $a \circ b = b = b \circ a$, then by the similar way of Case 2 we prove that there is no commutative neutro-LA-semigroup of order 2. □

Theorem 4.13. *There exist 2 neutro-LA-semigroups of order 2 (up to isomorphism).*

Proof. Let $(H = \{a, b\}, \circ)$ be a neutro-LA-semigroup. Then by Theorem 4.12 $(H = \{a, b\}, \circ)$ is a Non-commutative neutro-LA-semigroup. Now, we have

(1) $(a \circ b) \circ b \neq (b \circ b) \circ a$,
(2) $(a \circ a) \circ b = (b \circ a) \circ a$,

or

(3) $(a \circ b) \circ b = (b \circ b) \circ a$,
(4) $(a \circ a) \circ b \neq (b \circ a) \circ a$.

Case 1: If $a \circ b = a$ and $b \circ a = b$, then by (1), (2), (3) and (4) we obtain

(5) $a \neq (b \circ b) \circ a$,
(6) $(a \circ a) \circ b = b$,

or

(7) $a = (b \circ b) \circ a$,
(8) $(a \circ a) \circ b \neq b$.

Let (5) and (6) be true. If $a \circ a = a$, then $a \circ b = a$ and this is a contradiction. If $a \circ a = b$, then using (6), $b \circ b = b$. We set $\circ_1 := \circ$ and therefore we have a neutro-LA-semigroup as follows:

$\circ_1$	a	b
a	b	a
b	b	b

Let (7) and (8) be true. If $b \circ b = b$, then $b \circ a = a$ and this is a contradiction. If $b \circ b = a$ by (7), $a \circ a = a$. We set $\circ_2 := \circ$ and therefore we have a neutro-LA-semigroup as follows:

$\circ_2$	a	b
a	a	a
b	b	a

Case 2: If $a \circ b = b$ and $b \circ a = a$, then by (1), (2), (3) and (4) we obtain

(9) $b \circ b \neq (b \circ b) \circ a$,
(10) $(a \circ a) \circ b = a \circ a$,

or

(11) $b \circ b = (b \circ b) \circ a$,
(12) $(a \circ a) \circ b \neq a \circ a$.

Let (9) and (10) be true. If $a \circ a = a$, then $a \circ b = a$ and this is a contradiction. If $a \circ a = b$ by (10), $b \circ b = b$. We set $\circ_3 := \circ$ and therefore we have a neutro-LA-semigroup as follows:

$\circ_3$	a	b
a	b	b
b	a	b

Let (11) and (12) be true. If $b \circ b = b$, then $b \circ a = b$ and this is a contradiction. If $b \circ b = a$ by (11), $a \circ a = a$. We set $\circ_4 := \circ$ and therefore we have a neutroLA-semigroup as follows:

$\circ_4$	a	b
a	a	b
b	a	a

It is not to difficult to see that $(H, \circ_1) \cong (H, \circ_2)$ and $(H, \circ_3) \cong (H, \circ_4)$. Therefore there exist 2 neutro-LA-semigroups of order 2 up to isomorphism. □

Now, by the above results in this section, we obtain the number of anti-LA-semigroups, neutro-LA-semigroups and LA-semigroups of order 2 (classes up to isomorphism).

Table 1. Classification of the groupoids of order 2

	C	AC
LA-semigroups	3	0
neutro-LA-semigroups	0	2
Anti-LA-semigroup	1	4

5. Characterization of hypergroupoids of order 2

In the next results we use the hyperoperation instead of neutrohyperoperation.

In this section, let $\circ$ be a hyperoperation on $H = \{a, b\}$ and $(A_{11}, A_{12}, A_{21}, A_{22})$ inside of the below Cayley table:

$\circ$	a	b
a	A_{11}	A_{12}
b	A_{21}	A_{22}

Lemma 5.1. *Let $(H = \{a, b\}, \circ_H)$ and $(G = \{a', b'\}, \circ_G)$ be two hypergroupoids with the Cayley tables $(H_{11}, H_{12}, H_{21}, H_{22})$ and $(G_{11}, G_{12}, G_{21}, G_{22})$ respectively. Then $H \cong G$ if and only if for all $i, j \in \{1, 2\}$, $G_{ij} = H'_{ij}$ or*

$$G_{ij} = \begin{cases} K'_{ij} & \text{if } K_{ij} = H; \\ G \setminus K'_{ij} & \text{if } K_{ij} \neq H. \end{cases}$$

where $K_{11} = H_{22}$, $K_{12} = H_{12}$, $K_{21} = H_{21}$ and $K_{22} = H_{11}$.

Proof. It is straightforward. □

Lemma 5.2. *Let $(H, \circ)$ be a groupoid of order 2. $(H, \circ)$ is an LA-semihypergroup if and only if it is a commutative semigroup.*

Theorem 5.3. *If $(H, \circ)$ is an anti-H_v-LA-semigroup of order 2, then $\circ$ is an operation and $(H, \circ)$ is anti-LA-semigroup.*

Theorem 5.4. *There exists one antiweakcommutative LA-semihypergroup of order 2, up to isomorphism.*

Proof. Let $(H = \{a, b\}, \circ)$ be an antiweakcommutative LA-semihypergroup. Then we have $(a \circ b = a \;\&\; b \circ a = b)$ or $(a \circ b = b \;\&\; b \circ a = a)$. Suppose $a \circ b = a$ and $b \circ a = b$. Since $(H = \{a, b\}, \circ)$ is an LA-semihypergroup, we get

(1) $(a \circ b) \circ b = (b \circ b) \circ a$,
(2) $(a \circ a) \circ b = (b \circ a) \circ a$.

Thus, by (1) and (2) we obtain

(3) $a = (b \circ b) \circ a$,
(4) $(a \circ a) \circ b = b$.

Now, if $b \circ b = a$, then (3) implies that $a \circ a = a$. So (4) implies that $a \circ b = b$ and this is a contradiction. If $b \circ b = b$, then (3) implies that $b \circ a = a$ and this is a contradiction. Finally, if $b \circ b = H$, then (3) implies that $a = H$ and this is a contradiction.

Let $a \circ b = b$ and $b \circ a = a$. Then using (1) and (2) we have

(5) $b \circ b = (b \circ b) \circ a$,
(6) $(a \circ a) \circ b = a \circ a$.

If $a \circ b = b$ and $b \circ a = a$, then using (5) and (6), we get $b \circ b \neq a$, $b \circ b \neq b$, $a \circ a \neq a$ or $a \circ a \neq a$. Therefore $b \circ b = H = a \circ a$. □

Theorem 5.5. *There exist 3 anticommutative LA-semihypergroups of order 2, up to isomorphism.*

Proof. Let $(H = \{a, b\}, \circ)$ be an antiweakcommutative LA-semihypergroup. Then we have $(a \circ b \neq b \circ a$. Then we have

(1) $(a \circ b = a \;\&\; b \circ a = b)$,
(2) $(a \circ b = b \;\&\; b \circ a = a)$,
(3) $(a \circ b = a \;\&\; b \circ a = H)$,
(4) $(a \circ b = b \;\&\; b \circ a = H)$,
(5) $(a \circ b = H \;\&\; b \circ a = a)$,
(6) $(a \circ b = H \;\&\; b \circ a = b)$.

By proof of Theorem 5.4, case (1) can not admitting an LA-semihypergroup. By proof of Theorem5.4, case (2) admitting a LA-semihypergroup $(H, \circ_1)$ with the following Cayley table:

$\circ_1$	a	b
a	H	b
b	a	H

Case 3: Let $a \circ b = a$ and $b \circ a = H$. Since $(H = \{a, b\}, \circ)$ is an LA-semihypergroup, so

(7) $(a \circ b) \circ b = (b \circ b) \circ a$,
(8) $(a \circ a) \circ b = (b \circ a) \circ a$.

Thus, by (3), (7) and (8) we obtain

(9) $a = (b \circ b) \circ a$,

(10) $(a \circ a) \circ b = H$.

Now, if $b \circ b = a$, then (9) implies that $a \circ a = a$. So (10) implies that $a \circ b = H$, which is a contradiction. If $b \circ b = b$, then (9) implies that $b \circ a = a$ and this is a contradiction. Finally, if $b \circ b = H$, then (9) implies that $a = H$, which is a contradiction.

Case 4: Let $a \circ b = b$ and $b \circ a = H$. Thus by (4), (7) and (8) we obtain

(11) $b \circ b = (b \circ b) \circ a$,

(12) $(a \circ a) \circ b = H$.

Now, if $b \circ b = a$, then (11) implies that $a \circ a = a$. So (12) implies that $a \circ b = H$, which is a contradiction. If $b \circ b = b$, then (11) implies that $b \circ a = b$ and this is a contradiction. So we have $b \circ b = H$. By (4) and (12) we obtain $a \circ a = b$ or $a \circ a = H$. Therefore we obtain two hyperoperations and we call these two hyperoperation $\circ_2$ and $\circ_3$, respectively as follows:

$\circ_2$	a	b
a	H	b
b	H	H

$\circ_3$	a	b
a	b	b
b	H	H

Case 5: Let $a \circ b = H$ and $b \circ a = a$. Thus, by (4), (7) and (8) we obtain

(13) $H = (b \circ b) \circ a$,

(14) $(a \circ a) \circ b = a \circ a$.

Now, if $a \circ a = b$, then (14) implies that $b \circ b = b$. So (13) implies that $b \circ a = H$, which is a contradiction. If $a \circ a = a$, then (14) implies that $a \circ b = a$ and this is a contradiction. So we have $a \circ a = H$. By (5) and (13) we obtain ($b \circ b = a$ or $b \circ b = H$. Therefore we obtain two hyperoperations and we call these two hyperoperation $\circ_4$ and $\circ_5$, respectively as follows:

$\circ_4$	a	b
a	H	H
b	a	H

$\circ_5$	a	b
a	H	H
b	a	a

Case 6: Let $a \circ b = H$ & $b \circ a = b$. Thus by (4), (7) and (8) we obtain

(15) $H = (b \circ b) \circ a$,

(16) $(a \circ a) \circ b = b$.

Now, if $a \circ a = b$, then (16) implies that $b \circ b = b$. So (15) implies that $b \circ a = H$ and this is a contradiction. If $a \circ a = a$, then (16) implies that $a \circ b = a$ and this is a contradiction. So we have $a \circ a = H$. By (5) and (16) we obtain $H = b$, which is a contradiction.

It is easy to see that $(H, \circ_2) \cong (H, \circ_4)$ and $(H, \circ_3) \cong (H, \circ_5)$. Therefore $(H, \circ_1)$, $(H, \circ_2)$ and $(H, \circ_4)$ are anticommutative LA-semihypergroups of order 2. □

Corollary 5.6. *There is no anticommutative LA-semigroup of order 2.*

Theorem 5.7. *There is no proper anti-H_v-LA-semigroup of order 2.*

Proof. Let $(H = \{a, b\}, \circ)$ be a proper anti-H_v-LA-semigroup. Then We have

(1) $(a \circ b) \circ b \cap (b \circ b) \circ a = \emptyset$,
(2) $(a \circ a) \circ b \cap (b \circ a) \circ a = \emptyset$.

Since anti-H_v-LA-semigroup $(H = \{a, b\}, \circ)$ is proper, we have one of the following cases:

(3) $a \circ a = H$,
(4) $a \circ b = H$,
(5) $b \circ a = H$,
(6) $b \circ b = H$.

Case 3: If $b \circ a = a$ or $b \circ a = H$, then $(b \circ a) \circ a = H$ and this is a contradiction with (2). If $b \circ a = b$, then $(b \circ a) \circ a = b$ and $(a \circ a) \circ b = H \circ b$. Thus, by (2) we have $a \circ b = a$ and $b \circ b = a$. Then $(a \circ b) \circ b = a$ and $(b \circ b) \circ a = H$, which is a contradiction with (1).

Case 4: Then $(a \circ b) \circ b = H$ and this is a contradiction with (1).

Case 5: Then $(b \circ a) \circ a = H$, which is a contradiction with (2).

Case 6: If $a \circ b = b$ or $a \circ b = H$, then $(a \circ b) \circ b = H$ and this is a contradiction with (2). If $a \circ b = a$, then $(a \circ b) \circ b = a$ and $(b \circ b) \circ a = H \circ a$. So by (1) we have $a \circ a = b$ and $b \circ a = b$. Then $(b \circ a) \circ a = b$ and $(a \circ a) \circ b = H$, which is a contradiction with (2). □

Theorem 5.8. *There exist 4 commutative anti-LA-semihypergroups of order 2.*

Proof. Let $(H = \{a, b\}, \circ)$ be a commutative anti-LA-semihypergroup. Then We have

(1) $(a \circ b) \circ b \neq (b \circ b) \circ a$,
(2) $(a \circ a) \circ b \neq (b \circ a) \circ a$,
(3) $a \circ b = b \circ a$.

Now, we have 3 Cases:

Case 1: Let $a \circ b = a = b \circ a$. then

(4) $(a \neq (b \circ b) \circ a$,
(5) $(a \circ a) \circ b \neq a \circ a$.

If $a \circ a = a$, then (3) and (5) implies that $a \neq a$ and this is contradiction. If $a \circ a = b$, then (5) implies that $b \notin b \circ b$ and so $a \in b \circ b$. Then $b \circ b = a$ or $b \circ b = H$. Therefore we obtain two commutative anti-LA-semihypergroups with two hyperoperations $\circ_1$ and $\circ_2$ as follows:

$\circ_1$	a	b
a	b	a
b	a	H

$\circ_2$	a	b
a	b	a
b	a	a

If $a \circ a = H$, then by (5) we have

(6) $H \circ b \neq H$,

since $a \circ b = a$, using (6), we have $H \circ b = a$. Thus, $b \circ b = b$. Then we have a commutative anti-LA-semihypergroup with the following Cayley table.

$\circ_3$	a	b
a	H	a
b	a	b

Case 2: Let $a \circ b = b = b \circ a$. In the similar way we obtain 3 commutative anti-LA-semihypergroups isomorphism with anti-LA-semihypergroups in Case 1.

Case 3: Let $a \circ b = H = b \circ a$, by (1) and (2) we have

(7) $H \neq (b \circ b) \circ a$,

(8) $(a \circ a) \circ b \neq H$.

Thus, (7) and (8) imply that $a \circ a \neq H \neq b \circ b$, $a \circ a \neq a$ and $b \circ b \neq b$. So $a \circ a = b$ and $b \circ b = a$. Therefore we have a commutative anti-LA-semihypergroup with the following Cayley table.

$\circ_4$	a	b
a	b	H
b	H	a

Then we have 4 commutative anti-LA-semihypergroups of order 2 up to isomorphism. □

Now, we have a generalization of Theorem 4.12.

Theorem 5.9. *There is no commutative neutro-H_v-LA-semigroups of order 2.*

Proof. Let $(H = \{a, b\}, \circ)$ be a weakcommutative neutro-H_v-LA-semigroup. Then we have

(1) $(a \circ b) \circ b \cap (b \circ b) \circ a = \emptyset$,

(2) $(a \circ a) \circ b \cap (b \circ a) \circ a \neq \emptyset$,

(3) $a \circ b = b \circ a$.

Or

(4) $(a \circ b) \circ b \cap (b \circ b) \circ a \neq \emptyset$,

(5) $(a \circ a) \circ b \cap (b \circ a) \circ a = \emptyset$.

(6) $a \circ b = b \circ a$.

Case 1: If $a \circ b = H = b \circ a$, then it is a contradiction with (1) and (5).

Case 2: If $a \circ b = a = b \circ a$, then we have

(7) $a \cap (b \circ b) \circ a = \emptyset$,

(8) $(a \circ a) \circ b \cap a \circ a \neq \emptyset$,

or

(9) $a \cap (b \circ b) \circ a \neq \emptyset$,
(10) $(a \circ a) \circ b \cap a \circ a = \emptyset$.

Now, (7) implies that $b \circ b = a$ and $a \circ a = b$ and this is a contradiction with (8). Also, (10) implies that $a \circ a = b$ and $b \circ b = a$, which is a contradiction with (9).

Case 3: If $a \circ b = b = b \circ a$, then by the similar way of Case 2 we can prove that there is no a weakcommutative neutro-H_v-LA-semigroup of order 2. □

Theorem 5.10. *There exist 2 weakcommutative neutro-H_v-LA-semigroups of order 2.*

Proof. Let $(H = \{a, b\}, \circ)$ be a weakcommutative neutro-H_v-LA-semigroup. Then we have

(1) $(a \circ b) \circ b \cap (b \circ b) \circ a = \emptyset$,
(2) $(a \circ a) \circ b \cap (b \circ a) \circ a \neq \emptyset$,

or

(3) $(a \circ b) \circ b \cap (b \circ b) \circ a \neq \emptyset$,
(4) $(a \circ a) \circ b \cap (b \circ a) \circ a = \emptyset$.

Theorem 5.9 and weakcommutativity imply that

(5) $b \circ a \neq a \circ b = H$,

or

(6) $a \circ b \neq b \circ a = H$.

Now, we have the following Cases:

Case 1: Let (1), (2) and (5) be true. Then $H \cap (b \circ b) \circ a = \emptyset$ and this is a contradiction.

Case 2: Let (1), (2) and (6) be true. By (6) and (1) we have $b \circ b = a$. Using (6) we have $a \circ b = a$ or $a \circ b = b$. If $a \circ b = a$, then by (1) we obtain $a \circ a = b$. Thus, we have a weakcommutative neutro-H_v-LA-semigroup with the following Cayley table.

$\circ$	a	b
a	b	a
b	H	a

If $a \circ b = b$, then by (1) we obtain $a \circ a = b$. So we have a weakcommutative neutro-H_v-LA-semigroup with the following Cayley table.

$\circ$	a	b
a	b	b
b	H	a

Case 3: Let (3), (4) and (5) are true. By (5) and (4) we have $a \circ a = b$. by (5) we have $b \circ a = a$ or $b \circ a = b$. If $b \circ a = a$, then by (4) we obtain $b \circ b = a$. So we

have a weakcommutative neutro-H_v-LA-semigroup with the following Cayley table.

$\circ$	a	b
a	b	H
b	a	a

If $b \circ a = b$, then by (4) we obtain $b \circ b = a$. Then we have a weakcommutative neutro-H_v-LA-semigroup with the following Cayley table.

$\circ$	a	b
a	b	H
b	a	a

Case 1: Let (3), (4) and (6) are true. Then $(a \circ a) \circ H \circ a = \emptyset$. This is a contradiction. $\square$

Theorem 5.11. *There exists one antiweakcommutative LA-semihypergroup of order 2, up to isomorphism.*

Proof. Let $(H = \{a, b\}, \circ)$ be an antiweakcommutative LA-semihypergroup. Then We have

(1) $(a \circ b) \circ b = (b \circ b) \circ a$, or (2) $(a \circ a) \circ b = (b \circ a) \circ a$, or (3) $a \circ b \cap b \circ a = \emptyset$,

Using (3) we get $a \circ b \neq H$ and $b \circ a \neq H$. Then

(4) $a \circ b = a, \ b \circ a = b,$

or

(5) $a \circ b = b, \ b \circ a = a.$

If (4) is true, then

(6) $a = (b \circ b) \circ a,$

(7) $(a \circ a) \circ b = b.$

If $b \circ b = a$ or $b \circ b = b$ or $b \circ b = H$, then we have a contradiction.

So, let (5) be true. Then

(8) $b \circ b = (b \circ b) \circ a,$

(9) $(a \circ a) \circ b = a \circ a.$

So, if $b \circ b = a$, then $a \circ a = b$. By (9) we have $b = a$, which is a contradiction. If $b \circ b = b$, then $b \circ a = b$ this is a contradiction with (5). If $a \circ a = b$ or $a \circ a = a$, then by the similar way we have a contradiction. Thus, $b \circ b = H$, and so $a \circ a = H$. Therefore we have an antiweakcommutative LA-semihypergroup with the following Cayley table.

$\circ$	a	b
a	H	b
b	a	H

$\square$

Using the above results in the sections 4 and 5, we can characterize 45 non-isomorphic classes hypergroupoids of the order 2. We obtain anti-LA-semihypergroups, neutro-LA-semihypergroups, LA-semihypergroups, anti-H_v-LA-semigroups, neutro-H_v-LA-semigroups, H_v-LA-semigroups of order 2 (classes up to isomorphism).

TABLE 2. Classification of the hypergroupoids of order 2

	C	WC	AC	AWC
LA-semihypergroups	9	11	3	1
H_v-LA-semigroups	13	30	20	3
Neutro-LA-semihypergroup	2	9	10	3
Neutro-H_v-LA-semigroup	0	2	7	5
Anti-LA-semihypergroup	3	13	18	8
Anti-H_v-LA-semigroup	1	1	4	4

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Human participants and/or animals This article does not contain any studies with human participants or animals performed by the authors.

Authors' contributions All authors read and approved the final manuscript.

References

1. M. Akram, N. Yaqoob and M. Khan, *On (m, n)-ideals in LA-semigroups*, Applied Mathematical Sciences, 7(44) (2013) 2187–2191.
2. A.A.A. Agboola, *Introduction to neutrogroups*, International Journal of Neutrosophic Science, 6(1) (2020) 41–47.
3. J.R. Cho, J. Jezek and T. Kepka, *Paramedial groupoids*, Czechoslovak Mathematical Journal, 49(2) (1999) 277–290.
4. P. Corsini, *Prolegomena of hypergroup theory*, Aviani Editor, (1993).
5. P. Corsini, V. Leoreanu-Fotea, *Applications of hyperstruture theory*, Kluwer Academic Publishers, (2003).
6. B. Davvaz, *Semihypergroup theory*, Elsevier, (2016).
7. B. Davvaz and T. Vougiouklis, *A walk through weak hyperstructures; H_v-structure*, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, (2019).
8. A. Distler, *Classification and eumeration of finite semigroups*, Ph.D. Thesis, University of St Andrews, (2010).
9. M. Gulistan, N. Yaqoob and M. Shahzad, *A note on H_v-LA-semigroups*, U.P.B. Sci. Bull., Series A, 77(33) (2015) 93–106.
10. K. Hila and J. Dine, *On hyperideals in left almost semihypergroups*, ISRN Algebra, Article ID 953124 (2011) 8 pages.
11. P. Holgate, *Groupoids satisfying a simple invertive law*, The Mathematics Student, 61(1-4) (1992) 101–106.

12. J. Jantosciak, *Transposition hypergroups: Noncommutative join spaces*, J. Algebra, 187 (1997) 97–119.
13. M.A. Kazim and M. Naseeruddin, *On almost semigroups*, The Aligarh Bulletin of Mathematics, 2 (1972) 1–7.
14. M. Khan and N. Ahmad, *Characterizations of left almost semigroups by their ideals*, Journal of Advanced Research in Pure Mathematics, 2(3) (2010) 61–73.
15. S. Mirvakili, F. Smarandache and A. Rezaei, *On neutro-H_v-semigroups*, Journal of Intelligent & Fuzzy Systems, 42 (2022) 1289–299.
16. F. Marty, *Sur une generalization de la notion de groupe*, Huitieme congress de Mathematiciens, Scandinaves, Stockholm, (1934) 45–49.
17. Q. Mushtaq and S.M. Yusuf, *On LA-semigroups*, The Aligarh Bulletin of Mathematics, 8 (1978) 65–70.
18. Q. Mushtaq and S.M. Yusuf, *On locally associative LA-semigroups*, The Journal of Natural Sciences and Mathematics, 19(1) (1979) 57–62.
19. P.V. Protic and N. Stevanovic, *AG-test and some general properties of Abel-Grassmann's groupoids*, Pure Mathematics and Applications, 6(4) (1995) 371–383.
20. A. Rezaei, F. Smarandache, S. Mirvakili, *Applications of (neutro/anti) sophications to semihypergroups*, Journal of Mathematics Volume 2021, Article ID 6649349, 7 pages.
21. F. Smarandache, *Neutroalgebra is a generalization of partial algebra*, International Journal of Neutrosophic Science (IJNS), 2(1) (2020) 8–17.
22. F. Smarandache, *Introduction to neutroalgebraic structures and antialgebraic structures, in Advances of Standard and Nonstandard Neutrosophic Theories*, Pons Publishing House Brussels, Belgium, Ch. 6, (2019) 240–265.
23. F. Smarandache, *Introduction to neutroalgebraic structures and antialgebraic structures (revisited)*, Neutrosophic Sets and Systems, 31 (2020) 1–16.
24. F. Smarandache, *Neutrosophy / Neutrosophic probability, set, and logic*, American Research Press, (1998).
25. F. Smarandache, *Extension of hypergraph to n-superhypergraph and to plithogenic n-superhypergraph, and extension of hyperalgebra to n-ary (classical- / neutro- / anti-) hyperalgebra*, neutrosophic sets and systems, 33 (2020) 290–296.
26. N. Stevanovic and P.V. Protic, *Composition of Abel-Grassmann's 3-bands*, Novi Sad Journal of Mathematics, 34(2) (2004) 175–182.
27. T. Vougiouklis, *Hyperstructures and their representations*, Hadronic Press, Inc. (1994) (see page 31).
28. N. Yaqoob, P. Corsini and F. Yousafzai, *On intra-regular left almost semihypergroups with pure left identity*, Journal of Mathematics, Article ID 510790 (2013) 10 pages.