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On Neutrosophic b-open sets in Neutrosophic topological space

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Abstract. Smarandache proposed the approach of neutrosophic sets. Neutrosophic sets deals with uncertain data. This paper defines the notion of neutrosophic b-open sets and neutrosophic b-closed sets and their properties are investigated. Further neutrosphic b-interior and neutrosphic b-closure operators are studied and their relationship with other operators are also discussed.

1. Introduction

Many real life problems in Business, Finance, Medical sciences, Engineering and Social sciences deals with uncertainties. There are difficulties in solving the uncertainties in these data by traditional mathematical models. There are approaches such as fuzzy sets [1], intuitionistic fuzzy sets [2], vague sets [3], and rough sets [4] which can be treated as mathematical tools to avert obstacles dealing with ambiguous data. But all these approaches have their implicit crisis in solving the problems involving indeterminant and inconsistent data due to inadequacy of parametrization tools. Moldstov [5] introduced soft set theory. Smarandache [6] studied the idea of neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy and persistent data.

Wang et al. [7] introduced single valued neutrosophic sets. Peng et al. [8] studied operations of neutrosophic numbers and introduced the idea of neutrosophic numbers. Neutrosophic topological space was introduced by Salama et.al. [9] in 2012. Further Neutrosophic topological spaces are studied in [10, 11, 12]. Maji [13] initiated the idea of neutrosophic soft set.

Applications of neutrosophic topology depend upon the properties of neutrosophic open sets, neutrosophic closed sets, neutrosophic interior operator and neutrosophic closure operator. Topologists studied the sets that are near to neutrosophic open sets and neutrosophic closed sets. In this order, Iswaraya et.al.[14] studied the concept of neutrosophic semi-open sets[NSO] and neutrosophic semi-closed sets[NSC]. In 2017, Imran et.al.[15] introduced neutrosophic semi-α open sets and studied their fundamental properties. Arokiarani et.al.[16] defined neutrosophic semi-open (resp. pre-open and α-open) functions and investigated their relations. Rao et.al.[17]

introduced neutrosophic pre-open sets. Here the notion of neutrosophic b-open sets are initiated and their properties are investigated. Further neutrosphic b interior and closure operators are studied and their relationship with other operators are also discussed.

2. Preliminaries

Definition 2.1 *[9] Let Z be a non empty set. A neutrosophic set(NS) U is defined as* $U = \{< z, \mathcal{T}_U(z), \mathcal{I}_U(z), \mathcal{F}_U(z) >: z \in Z\}$, *where* $\mathcal{T}, \mathcal{I}, \mathcal{F} : U \to]^-0, 1^+[and -0 \leq \mathcal{T}_U(z) + \mathcal{I}_U(z) + \mathcal{F}_U(z) \leq 3^+$

Remark 2.2 *[9] A neutrosophic set $U = \{< z, \mathcal{T}_U(z), \mathcal{I}_U(z), \mathcal{F}_U(z) : z \in Z\}$ can be identified to an ordered triple $< \mathcal{T}_U, \mathcal{I}_U, \mathcal{F}_U >$ in $]^-0, 1^+[$ on Z.*

Definition 2.3 *[9] The neutrosophic sets 0_N and 1_N in Z are represented as follows:*
(0_1) $0_N = \{< z, 0, 0, 1 >: z \in Z\}$
(0_2) $0_N = \{< z, 0, 1, 1 >: z \in Z\}$
(0_3) $0_N = \{< z, 0, 1, 0 >: z \in Z\}$
(0_4) $0_N = \{< z, 0, 0, 0 >: z \in Z\}$
(1_1) $1_N = \{< z, 1, 0, 0 >: z \in Z\}$
(1_2) $1_N = \{< z, 1, 0, 1 >: z \in Z\}$
(1_3) $1_N = \{< z, 1, 1, 0 >: z \in Z\}$
(1_4) $1_N = \{< z, 1, 1, 1 >: z \in Z\}$

Definition 2.4 *[9] Let $U =< \mathcal{T}_U, \mathcal{I}_U, \mathcal{F}_U >$ be a NS on Z, then the complement of the set U $[U^C]$ is defined as three type of complements :*
(C_1) $U^C = \{< z, 1 - \mathcal{T}_U(z), 1 - \mathcal{I}_U(z), 1 - \mathcal{F}_U(z) >: z \in Z\}$
(C_2) $U^C = \{< z, \mathcal{F}_U(z), \mathcal{I}_U(z), \mathcal{T}_U(z) >: z \in Z\}$
(C_3) $U^C = \{< z, \mathcal{F}_U(z), 1 - \mathcal{I}_U(z), \mathcal{T}_U(z) >: z \in Z\}$

Definition 2.5 *[9] If U and V are any two neutrosophic sets over Z then $U \subseteq V$ is defined as :*
$(1) U \subseteq V \Leftrightarrow \mathcal{T}_U(z) \leq \mathcal{T}_V(z), \mathcal{I}_U(z) \leq \mathcal{I}_V(z)$ *and* $\mathcal{F}_U(z) \geq \mathcal{F}_V(z) \forall z \in Z$
$(2) U \subseteq V \Leftrightarrow \mathcal{T}_U(z) \leq \mathcal{T}_V(z), \mathcal{I}_U(z) \geq \mathcal{I}_V(z)$ *and* $\mathcal{F}_U(z) \geq \mathcal{F}_V(z) \forall z \in Z$

Definition 2.6 *[9] If U and V are any two neutrosophic sets over Z then $U \cup V$ may be defined as :*
$(U_1) U \cup V =< z, \mathcal{T}_U(z) \vee \mathcal{T}_V(z), \mathcal{I}_U(z) \vee \mathcal{I}_V(z)$ *and* $\mathcal{F}_U(z) \wedge \mathcal{F}_V(z) >$
$(U_2) U \cup V =< z, \mathcal{T}_U(z) \vee \mathcal{T}_V(z), \mathcal{I}_U(z) \wedge \mathcal{I}_V(z)$ *and* $\mathcal{F}_U(z) \wedge \mathcal{F}_V(z) >$

Definition 2.7 *[9] If U and V are any two neutrosophic sets over Z then $U \cap V$ may be defined as :*
$(\mathcal{I}_1) U \cap V =< z, \mathcal{T}_U(z) \wedge \mathcal{T}_V(z), \mathcal{I}_U(z) \wedge \mathcal{I}_V(z)$ *and* $\mathcal{F}_U(z) \vee \mathcal{F}_V(z) >$
$(\mathcal{I}_2) U \cap V =< z, \mathcal{T}_U(z) \wedge \mathcal{T}_V(z), \mathcal{I}_U(z) \vee \mathcal{I}_V(z)$ *and* $\mathcal{F}_U(z) \vee \mathcal{F}_V(z) >$

Definition 2.8 *[9] Let τ be the collection of neutrosophic subsets of Z. Then τ is said to be neutrosophic topology [NT] on Z if*
(NT_1) $0_N, 1_N \in \tau$,
(NT_2) $U_1 \cap U_2 \in \tau$ *for any* $U_1, U_2 \in \tau$,
(NT_3) $\cup U_i \in \tau$ *for every* $\{U_i : i \in J\} \subseteq \tau$

Then (Z, τ) is the neutrosophic topological space[NTS] over Z. The members of τ are called neutrosophic open sets [NO]. A neutrosophic set H is closed if and only if H^C is neutrosophic open.

Definition 2.9 *[9] Let (Z, τ) be NTS and U be a NS in Z. Then the neutrosophic closure and neutrosophic interior of U are defined by*
$NCl(U) = \cap\{D : D \text{ is a NC set in } Z \text{ and } U \subseteq D\}$
$NInt(U) = \cup\{E : E \text{ is a NO set in } Z \text{ and } E \subseteq U\}$.

Definition 2.10 *Let (Z, τ) be a NTS and U is a NS. U is called*

(i) Neutrosophic α-open[Nα O] [16] set iff $U \subseteq NintNclNintU$

(ii) Neutrosophic semi-open[NSO] [14] set iff $U \subseteq NclNintU$

(iii) Neutrosophic pre-open[NPO] [17] set iff $U \subseteq NintNclU$

3. Neutrosophic b-open and b-closed sets

In this segment, the notion of neutrosophic b-open set and neutrosophic b-closed sets are introduced and also their properties are characterized.

Definition 3.1 *A NS U in a NTS Z is called*

(i) neutrosophic b-open (NBO) set iff $U \subseteq Nint(Ncl(U)) \cup Ncl(Nint(U))$

(ii) neutrosophic b-closed (NBC) set iff $U \supseteq Nint(Ncl(U)) \cap Ncl(Nint(U))$

It is obvious that $NPO(Z) \cup NSO(Z) \subseteq NBO(Z)$. The inclusion cannot be replaced with equalities.

Theorem 3.2 *For a NS U in a NTS Z*

(i) U is a NBO set iff U^C is a NBC set.

(ii) U is a NBC set iff U^C is a NBO set.

Proof: Obvious from the definition.

Definition 3.3 *Let (Z, τ) be a NTS and U be a NS over Z.*

(i) Neutrosophic b-interior of U briefly[Nbint(U)] is the union of all neutrosophic b-open sets of Z contained in U. That is, $Nbint(U) = \cup\{G : G \text{ is a NBO set in } Z \text{ and } G \subseteq U\}$.

(ii) Neutrosophic b-closure of U briefl[NScl(U)] is the intersection of all neutrosophic b-closed sets of Z contained in U. That is, $Nbcl(U) = \cap\{H : H \text{ is a NBC set in } Z \text{ and } K \supseteq U\}$.

Clearly $Nbcl(U)$ is the smallest neutrosophic b-closed set over Z which contains U and $Nbint(U)$ is the largest neutrosophic b-open set over Z which is contained in U.

Theorem 3.4 *Let U be a NS in a NTS Z. Then,*

(i) $(Nbint(U))^C = Nbcl(U^C)$

(ii) $(Nbcl(U))^C = Nbint(U^C)$

Proof: (i) Let U be a NS in NTS. Now $Nbint(U) = \cup\{D : D$ is a NBO set in Z and $D \subseteq U\}$. Then $(Nbint(U))^C = [\cup\{D : D$ is a NBO set in Z and $D \subseteq U\}]^C = \cap\{D^C : D^C$ is a NBC set in Z and $(U)^C \subseteq D^C\}$. Replacing D^C by M, we get $(Nbint(U))^C = \cap\{M : M$ is a NBC set in Z and $M \supseteq (U)^C\}$, $(Nbint(U))^C = Nbcl((U)^C)$. This proves (i).
Analogously (ii) can be proved.

Theorem 3.5 *In a neutrosophic topological space Z*

(i) Every neutrosophic pre-open set is a neutrosophic b-open set.

(ii) Every neutrosophic semi-open set is a neutrosophic b-open set.

Proof: (i) Let U be a NPO set in a NTS Z. Then $U \subseteq NintNcl(U)$ which implies $U \subseteq NintNcl(U) \cup NintU \subseteq NintNclU \cup NclNintU$. Thus U is a NBO set.
(ii) Let U be a NSO set in a NTS Z. Then $U \subseteq NclNint(U)$ which implies $U \subseteq NclNint(U) \cup NintU \subseteq NclNintU \cup NintNclU$. Thus U is a NBO set.

4. Properties of the neutrosophic b-interior and b-closure operator with other operators

Theorem 4.1 *Let U be a NS in NTS Z. Then*

(i) $NsclU = U \cup NintNclU$ and $NsintU = U \cap NclNintU$

(ii) $NpclU = U \cup NclNintU$ and $NpintU = U \cap NintNclU$

Proof: (i) $NsclU \supseteq NintNclNsclU \supseteq NintNclU$.
$U \cup NsclU = NsclU \supseteq U \cup NintNclU$.
So $U \cup NintNclU \subseteq NsclU$ ———(1)
Also $U \subseteq NsclU$, $NintNclU \subseteq NintNclNsclU \subseteq NsclU$.
$U \cup NintNclU \subseteq NsclU \cup U \subseteq NsclU$———(2)
From (1) and (2), $NsclU = U \cup NintNclU$.
$NsintU = U \cap NclNintU$ can be proved by taking the complement of $NsclU = U \cup NintNclU$.
This proves (i).
The proof for (ii) is analogous.

Theorem 4.2 *Let U be a NS in NTS. Then*

(i) $NbclU = NsclU \cap NpclU$

(ii) $NbintU = NsintU \cup NpintU$

Proof: (i) Since $NbclU$ is a NBO set.
We have $NbclU \supseteq NintNcl(NbclU) \cap NclNint(NbclU) \supseteq NintNclU \cap NclNintU$
and also $NbclU \supseteq U \cup NintNclU \cap NclNintU = NsclU \cap NpclU$.
The reverse inclusion is clear. Therefore $NbclU = NsclU \cap NpclU$.
Analogously (ii) can be proved.

Theorem 4.3 *Let U be a NS in NTS. Then*

(i) $NsclNsintU = NsintU \cup NintNclNintU$

(ii) $NsintNsclU = NsclU \cap NclNintNclU$

Proof: we have $NsclNsintU = NsintU \cup NintNcl(NsintU) = NsintU \cup Nint(Ncl[U \cap NclNintU]) \subseteq NsintU \cup Nint[NclU \cap Ncl(NintU)] = NsintU \cup Nint[Ncl(NintU)]$
To establish the opposite inclusion we observe that,
$Nscl(NsintU) = NsintU \cup NintNcl(NsintU) \supseteq NsintU \cup NintNcl(NintU)$.
Therefore we have $NsclNsintU = NsintU \cup NintNclNintU$.
This proves (i).
The proof for (ii) is analogous.

Theorem 4.4 *Let U be a NS in NTS. Then*

(i) $NpclNpintU = NpintU \cup NclNintU$

(ii) $NpintNpclU = NpclU \cap NintnclU$

Proof: we have $NpclNpintU = NpintU \cup NclNint(NpintU) = NpintU \cup NclNint[U \cap NintNclU]) = NpintU \cup Ncl[NintU \cap Nint(NintNclU)] = NpintU \cup Ncl[NintU]$
To establish the opposite inclusion we observe that,
$Nscl(NsintU) = NsintU \cup NintNcl(NsintU) \supseteq NsintU \cup NintNcl(NintU)$.
Therefore we have $NsclNsintU = NsintU \cup NintNclNintU$.
This proves (i).
Analogously (ii) can be proved..

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