

On Neutrosophic Crisp Supra Topological Spaces

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Abstract: The aim of this paper is to introduce the concepts of closure, interior, neighbourhood, base and local base of neutrosophic crisp supra topological spaces and some of its characterizations are investigated.

Keywords: Neutrosophic crisp supra topology, Neutrosophic crisp sets, Neutrosophic crisp supra open sets & neutrosophic crisp points.

I. INTRODUCTION

Zadeh introduced the concept of fuzzy set [14] in 1965. In 1983 K.Atanassov [13] introduced generalization of fuzzy set intuitionistic fuzzy set. Neutrosophy has the laid foundation for a whole family of new mathematical theories generalizing both their crisp and fuzzy counterparts, such as neutrosophic set theory [10, 9, 8].The idea of “neutrosophic set” was proposed by Smarandache [10, 8]. Neutrosophic operations have been developed by salama et al. [1, 3, 5, 7, 6, 4, 16, 11, 12, 1]. Salama and albłowi [3] define neutrosophic topological space and established some of its properties. Salama & Smarandache [2, 1, 10, 14] introduced the concepts of neutrosophic crisp sets, neutrosophic crisp operators have been investigated. In this paper we introduce the concepts of closure, interior, neighbourhood, base and local base of neutrosophic crisp supra topological spaces and some of its characterizations are investigated.

II. PRELIMINARIES

The definitions of neutrosophic crisp set (NCS), neutrosophic crisp types of $\varphi_N = \langle \varphi, \varphi, X \rangle$ & $X_N = \langle X, \varphi, \varphi \rangle$, neutrosophic crisp union, intersection, neutrosophic crisp subsets, neutrosophic crisp complement, family of union & intersection, neutrosophic crisp point and some properties of neutrosophic crisp sets was introduced by the authors [3,1].

III. NEUTROSOPHIC CRISP SUPRA TOPOLOGICAL SPACES

3.1. Definition:[16] A neutrosophic crisp supra topology (NCST) on a non empty set X is a family τ^μ of neutrosophic crisp subsets of X satisfying the following conditions:

$$(a) \varphi_N, X_N \in \tau^\mu$$

$$(b) \bigcup E_i \in \tau^\mu \forall \{E_i : i \in I\} \subseteq \tau^\mu$$

Then the pair (X, τ^μ) is called neutrosophic crisp supra topological space (NCSTS). Elements in τ^μ are called neutrosophic crisp supra open sets (NCSOS) and the complement of τ^μ are called neutrosophic crisp supra closed sets (NCSCS).

3.2. Example: Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^\mu = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle \{\delta_1, \eta_2\}, \varphi, \{\psi_3\} \rangle$, $M = \langle \{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\} \rangle$, $N = \langle \{\eta_2\}, \varphi, \{\delta_1, \psi_3\} \rangle$. Then (X, τ^μ) is a NCSTS.

3.3. Definition: Let (X, τ_1^μ) and (X, τ_2^μ) be two NCSTSs. We say that τ_1^μ is coarser than τ_2^μ or τ_2^μ is said to be finer than τ_1^μ ($\tau_1^\mu \subseteq \tau_2^\mu$) if $S \in \tau_2^\mu$ for every $S \in \tau_1^\mu$.



3.4. Definition: Let (X, τ^μ) be NCSTS and $E = \langle E_1, E_2, E_3 \rangle$ be any NCS of X then neutrosophic crisp supra closure of E (NCS-cl(E)) and neutrosophic crisp supra interior (NCS-int(E)) are defined by

$$(i) \text{ NCS-cl}(E) = \bigcap \{ \lambda : \lambda \text{ is a NCSCS in } X \text{ \& } E \subseteq \lambda \}.$$

$$(ii) \text{ NCS-int}(E) = \bigcup \{ \lambda^* : \lambda^* \text{ is a NCSOS in } X \text{ \& } \lambda^* \subseteq E \}.$$

3.5. Example: Let $X = \{ \delta_1, \eta_2, \psi_3, \sigma_4 \}$, $\tau^\mu = \{ \varphi_N, X_N, L, M, N \}$, where $L = \langle \{ \delta_1, \eta_2 \}, \varphi, \{ \psi_3 \} \rangle$, $M = \langle \{ \delta_1, \eta_2 \}, \varphi, \{ \sigma_4, \psi_3 \} \rangle$, $N = \langle \{ \eta_2 \}, \varphi, \{ \delta_1, \psi_3 \} \rangle$.

Then (X, τ^μ) is a NCSTS.

$$\text{Let } E = \langle \{ \eta_2 \}, \varphi, \{ \delta_1, \psi_3 \} \rangle.$$

$$\text{Then } \text{NCS-int}(E) = \langle \{ \eta_2 \}, \varphi, \{ \delta_1, \psi_3 \} \rangle \text{ and } \text{NCS-cl}(E) = \langle X, \varphi, \varphi \rangle.$$

3.6. Proposition: Let (X, τ^μ) be any NCSTS. If E and F are any NCSs of X Then the neutrosophic crisp supra closure operator satisfies the following.

$$(i) E \subseteq \text{NCS-cl}(E).$$

$$(ii) E \subseteq F \Rightarrow \text{NCS-cl}(E) \subseteq \text{NCS-cl}(F).$$

$$(iii) \text{NCS-cl}(E \cup F) = \text{NCS-cl}(E) \cup \text{NCS-cl}(F).$$

Proof:

$$(i) \text{NCS-cl}(E) = \bigcap \{ \lambda : \lambda \text{ is NCSCS in } X \text{ \& } E \subseteq \lambda \}.$$

$$\text{Thus } E \subseteq \text{NCS-cl}(E).$$

$$(ii) \text{NCS-cl}(F) = \bigcap \{ \lambda : \lambda \text{ is a NCSCS in } X \text{ \& } F \subseteq \lambda \} \supseteq \text{NCS-cl}(E)$$

$$= \bigcap \{ \lambda : \lambda \text{ is NCSCS in } X \text{ \& } E \subseteq \lambda \}.$$

$$\text{Thus } \text{NCS-cl}(E) \subseteq \text{NCS-cl}(F).$$

$$(iii) \text{NCS-cl}(E \cup F) = \bigcap \{ \lambda : \lambda \text{ is NCSCS in } X \text{ \& } E \cup F \subseteq \lambda \}$$

$$= (\bigcap \{ \lambda : \lambda \text{ is NCSCS in } X \text{ \& } E \subseteq \lambda \}) \cup (\bigcap \{ \lambda : \lambda \text{ is NCSCS in } X \text{ \& } F \subseteq \lambda \})$$

$$= \text{NCS-cl}(E) \cup \text{NCS-cl}(F). \text{ Thus } \text{NCS-cl}(E \cup F) = \text{NCS-cl}(E) \cup \text{NCS-cl}(F).$$

3.7. Proposition: Let (X, τ^μ) be any NCSTS. If E and F are any two NCSs in X . Then the neutrosophic crisp supra interior operator satisfies the following.

$$(i) \text{NCS-int}(E) \subseteq E$$

$$(ii) E \subseteq F \Rightarrow \text{NCS-int}(E) \subseteq \text{NCS-int}(F).$$

$$(iii) \text{NCS-int}(E \cap F) = \text{NCS-int}(E) \cap \text{NCS-int}(F).$$

$$(iv) (\text{NCS-cl}(E))^c = \text{NCS-int}(E)^c.$$

$$(v) (NCS-int(E))^c = NCS-cl(E)^c.$$

Proof:

$$(i) NCS-int(E) = \bigcup \{ \lambda^* : \lambda^* \text{ is a NCSOS in } X \text{ \& } \lambda^* \subseteq E \}.$$

Thus $NCS-int(E) \subseteq E$.

$$(ii) NCS-int(F) = \bigcup \{ \lambda^* : \lambda^* \text{ is a NCSOS in } X \text{ \& } \lambda^* \subseteq F \} \supseteq \bigcup \{ \lambda^* : \lambda^* \text{ is a NCSOS in } X \text{ \& } \lambda^* \subseteq E \} \supseteq NCS-int(E). \text{ Thus } NCS-int(E) \subseteq NCS-int(F).$$

$$(iii) NCS-int(E \cap F) = \bigcup \{ \lambda^* : \lambda^* \text{ is a NCSOS in } X \text{ \& } \lambda^* \subseteq E \cap F \}$$

$$= (\bigcup \{ \lambda^* : \lambda^* \text{ is a NCSOS in } X \text{ \& } \lambda^* \subseteq E \}) \cap (\bigcup \{ \lambda^* : \lambda^* \text{ is a NCSOS in } X \text{ \& } \lambda^* \subseteq F \})$$

$$= NCS-int(E) \cap NCS-int(F).$$

Thus $NCS-int(E \cap F) = NCS-int(E) \cap NCS-int(F)$.

$$(iv) NCS-cl(E) = \bigcap \{ \lambda : \lambda \text{ is a NCSCS in } X \text{ \& } E \subseteq \lambda \},$$

$$(NCS-cl(E))^c = \bigcup \{ \lambda^c : \lambda^c \text{ is a NCSOS in } X \text{ \& } E^c \supseteq \lambda^c \} = NCS-int(E)^c.$$

Thus $(NCS-cl(E))^c = NCS-int(E)^c$.

$$(v) NCS-int(E) = \bigcup \{ \lambda^* : \lambda^* \text{ is a NCSOS in } X \text{ \& } \lambda^* \subseteq E \},$$

$$(NCS-int(E))^c = \bigcap \{ \lambda^{*c} : \lambda^{*c} \text{ is a NCSCS in } X \text{ \& } \lambda^{*c} \supseteq E^c \}$$

$$= NCS-cl(E)^c.$$

Thus $(NCS-int(E))^c = NCS-cl(E)^c$.

3.8. Proposition: Let (X, τ^h) be any NCSTS and E is any NCS of X .

Then the following are holds.

$$(i) NCS-cl(E) = E \Leftrightarrow E \text{ is NCSCS.}$$

$$(ii) NCS-int(E) = E \Leftrightarrow E \text{ is NCSOS.}$$

(iii) $NCS-cl(E)$ is the smallest NCSCS containing E .

(iv) $NCS-int(E)$ is the largest NCSOS contained in E .

Proof: (i), (ii), (iii), and (iv) are obvious.

IV. NEUTROSOPHIC CRISP SUPRA NEIGHBOURHOOD

4.1. Definition: Let (X, τ^μ) be a neutrosophic crisp supra topological space and p_N be a neutrosophic crisp point (NCP) of X . A neutrosophic crisp set \mathcal{N}^* of X is called neutrosophic crisp supra neighbourhood (NCSNBD) of p_N if there exists an open set $G \in \tau^\mu$ such that $p_N \in G \subseteq \mathcal{N}^*$.

Thus a NCSNBD of p_N is either a superset or equal to the open set containing p_N .

The family consisting of all the NCSNBDs of a NCP $p_N \in X$ is called the system of NCSNBDs of X and denoted by $S \tau^\mu (p_N)$.

From the definition every NCSOS is a NCSNBD of each NCP. But a NCSNBD of a NCP need not be a NCSOS. Also every open set containing p_N is a neighbourhood of p_N .

The NCSNBD \mathcal{N}^* of a NCP is open if and only if \mathcal{N}^* is open.

4.2. Example: Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^\mu = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle \{\delta_1, \eta_2\}, \varphi, \{\psi_3\} \rangle$, $M = \langle \{\delta_1, \eta_2\}, \varphi, \{\psi_3, \sigma_4\} \rangle$, $N = \langle \{\eta_2\}, \varphi, \{\delta_1, \psi_3\} \rangle$.

Then (X, τ^μ) is a NCSTS.

Let $p_N = \langle \{\eta_2\}, \varphi, \{\delta_1, \psi_3, \sigma_4\} \rangle$ is NCP on X .

Clearly $p_N \in N$, so that the NCS \mathcal{N}^* of X is superset or equal to N are NCSNBDs of p_N .

For the NCP $p_N = \langle \{\delta_1\}, \varphi, \{\eta_2, \psi_3, \sigma_4\} \rangle$ of X , $p_N \in M$ and $M \subseteq L$.

Here L is also NCSNBD of p_N and also L is open.

4.3. Definition: Let (X, τ^μ) be a NCSTS and E be a neutrosophic crisp set of X . A subset \mathcal{N}^* of X is called neutrosophic crisp supra neighbourhood (NCSNBD) of E if there exists an open set $G \in \tau^\mu$ such that $E \subseteq G \subseteq \mathcal{N}^*$.

4.4. Example: Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^\mu = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle \{\psi_3\}, \{\eta_2\}, \{\delta_1, \sigma_4\} \rangle$, $M = \langle \{\delta_1, \psi_3\}, \{\eta_2\}, \{\sigma_4\} \rangle$, $N = \langle \{\delta_1, \psi_3\}, \varphi, \{\eta_2, \sigma_4\} \rangle$.

Then (X, τ^μ) is a NCSTS.

Let $E = \langle \{\psi_3\}, \varphi, \{\delta_1, \eta_2, \sigma_4\} \rangle$ be a NCS on X .

Then clearly $\mathcal{N}^* = \langle \{\psi_3\}, \{\eta_2\}, \{\delta_1\} \rangle$ is a NCSNBD of E since $E \subseteq L \subseteq \mathcal{N}^*$.

Also $E \subseteq L \subseteq M$ so M is a NCSNBD of E also M is NCSOS.

Hence NCSNBD of E is either a superset or equal to the NCSOS containing E .

The NCSNBD of a NCS need not be a NCSOS.

4.5. Theorem: Let (X, τ^μ) be a NCSTS and p_N be a NCP in X . Then

(i) Every p_N has at least one NCSNBD.

(ii) Every NCSNBD of p_N contains p_N .

(iii) Every set containing the NCSNBD of p_N is a NCSNBD of p_N .

(iv) The union of two NCSNBD of p_N is a NCSNBD of p_N .

(v) If \mathcal{N}^* is a NCSNBD of p_N then there exists a NCSNBD B of p_N which is a subset of \mathcal{N}^* such that B is a NCSNBD of each of its NCPs belongs to B .

Proof: Let (X, τ^μ) be a NCSTS and $S\tau^\mu (p_N)$ be the collection of all NCSNBDs of a NCP p_N of X .

(i) Clearly X_N is a NCSNBD of each NCP p_N of X .

(ii) Let $\mathcal{N}^* \in S\tau^\mu (p_N)$, by the definition of NCSNBD $p_N \in G$.

(iii) Let $\mathcal{N}^* \in S\tau^\mu (p_N)$, then there is a open set $G \in \tau^\mu$ such that $p_N \in G \subseteq \mathcal{N}^*$.

Suppose $N \subseteq \mathcal{N}^*$ then $p_N \in G \subseteq N$ so N is also a NCSNBD of p_N .

(iv) Let $T, U \in S\tau^\mu (p_N)$ then there exists a open sets $G_1, G_2 \in \tau^\mu$ such that $p_N \in G_1 \subseteq T$ and $p_N \in G_2 \subseteq U$.

So $p_N \in G_1 \cup G_2$ and $G_1 \cup G_2 \subseteq T \cup U$ and $G_1 \cup G_2 \in \tau^\mu$.

Hence $T \cup U$ is also a NCSNBD of p_N .

(v) Suppose $\mathcal{N}^* \in S\tau^\mu (p_N)$, then $\exists B \in \tau^\mu \ni p_N \in B \subseteq \mathcal{N}^*$.

Since B is NCSOS, it is a NCSNBD of each NCP belongs to B .

V. LOCAL BASE AND BASE OF NEUTROSOPHIC CRISP SUPRA TOPOLOGICAL SPACES

5.1. Definition: Let (X, τ^μ) be a NCSTS. A non-empty collection $\mathcal{B}(p_N)$ of NCSNBDs of NCP p_N of X is called a local base for the NCSNBD of p_N if for every NCSNBD \mathcal{N}^* of p_N there is a $\mathcal{B}^* \in \mathcal{B}(p_N)$ such that $\mathcal{B}^* \subseteq \mathcal{N}^*$. If $\mathcal{B}(p_N)$ is local base of p_N then the members of $\mathcal{B}(p_N)$ are called basic NCSNBD of p_N .

5.2. Example: Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^\mu = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle \{\delta_1, \eta_2\}, \varphi, \{\psi_3\} \rangle$, $M = \langle \{\delta_1, \eta_2\}, \varphi, \{\psi_3, \sigma_4\} \rangle$, $N = \langle \{\eta_2\}, \varphi, \{\delta_1, \psi_3\} \rangle$.

Then (X, τ^μ) is a NCSTS.

Let $p_N = \langle \{\delta_1\}, \varphi, \{\eta_2, \psi_3, \sigma_4\} \rangle$.

$\mathcal{B}(p_N) = \{S\}$, where $S = \langle \{\delta_1, \eta_2\}, \varphi, \{\psi_3\} \rangle$.

Let $\mathcal{N}^* = \langle \{\delta_1, \eta_2\}, \varphi, \varphi \rangle$ is NCSNBD of p_N .

Here $S \subseteq \mathcal{N}^*$. Hence $\mathcal{B}(p_N)$ is local base for p_N .

5.3. Theorem: Let (X, τ^μ) be a NCSTS and let p_N be a NCP of $x \in X$ and $p_N \in \tau^\mu$. Then the collection $\mathcal{B}(x)$ of all neutrosophic crisp supra open subsets of X containing x is local base at x .

Proof: Let \mathcal{N}^* be a NCSNBD of NCP p_N of $x \in X$. Then there exists a open set G such that $p_N \in G \subseteq \mathcal{N}^*$. Since G is open set in τ^μ containing x , $G \in \mathcal{B}(x)$. This shows that $\mathcal{B}(x)$ is a local base at x .

5.4. Definition: Let (X, τ^μ) be a NCSTS. A non empty collection \mathcal{B} of neutrosophic crisp sets of X is said to be base for τ^μ if $\mathcal{B} \subseteq \tau^\mu$ and every NCSNBD \mathcal{N}^* of p_N , there exists $\mathcal{B}^* \in \mathcal{B}$ such that $p_N \in \mathcal{B}^* \subseteq \mathcal{N}^*$.

5.5. Example: Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^\mu = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle \{\psi_3\}, \{\eta_2\}, \{\delta_1, \sigma_4\} \rangle$, $M = \langle \{\delta_1, \psi_3\}, \{\eta_2\}, \{\sigma_4\} \rangle$, $N = \langle \{\delta_1, \psi_3\}, \varphi, \{\eta_2, \sigma_4\} \rangle$.

Then (X, τ^μ) is a *NCSTS*.

Let $\mathcal{B} = \{L, N\} \subseteq \tau^\mu$ is a base for τ^μ . Since $p_N = \langle \{\psi_3\}, \varphi, \{\delta_1, \eta_2, \sigma_4\} \rangle \in L \subseteq \mathcal{N}^*$,

$p_N = \langle \{\delta_1\}, \varphi, \{\eta_2, \psi_3, \sigma_4\} \rangle \in M \subseteq \mathcal{N}^*$. Where $\mathcal{N}^* = \{\mathcal{G} \mid \mathcal{G} \supseteq L \& M\}$.

5.6. Theorem: Let (X, τ^μ) be a *NCSTS*. A non-empty collection \mathcal{B} of τ^μ is base for τ^μ iff every *NCSOS* in τ^μ can be expressed as the union of members of \mathcal{B} .

Proof: Let \mathcal{B} be a base for τ^μ and let $G \in \tau^\mu$.

Since G is open in τ^μ hence by the definition of base for each $p_N \in G$ there is $\mathcal{B}^* \in \mathcal{B}$ and $\mathcal{B}^* \subseteq G \Rightarrow G = \bigcup \{ \mathcal{B}^* \mid \mathcal{B}^* \in \mathcal{B} \text{ and } \mathcal{B}^* \subseteq G \}$.

Conversely, let $\mathcal{B} \subseteq \tau^\mu$ and let every *NCSOS* G be the union of members of \mathcal{B} .

We have to show that \mathcal{B} is base for τ^μ we have $\mathcal{B} \subseteq \tau^\mu$ and there exists a *NCSOS* $G \in \tau^\mu$ but G is the union of members of \mathcal{B} so \mathcal{B} is base for τ^μ .

VI. CONCLUSION

In this paper we introduced closure and interior of *NCSTS*. Also we introduced *NCSNBD* of *NCP* and local base, base of *NCST* also we investigated some of its basic properties. Finally we have huge scope for further research and we have planned to work some more concepts in *NCST*.

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