# On Neutrosophic Crisp Topology via *N*-Topology

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**Abstract**. In this paper, we extend the neutrosophic crisp topological spaces into N–neutrosophic crisp topological spaces ($N_{nc}$-topological space). Moreover, we introduced new types of open and closed sets in N–neutrosophic crisp topological spaces. We also present $N_{nc}$-semi (open) closed sets, $N_{nc}$-preopen (closed) sets and $N_{nc}$-α-open (closed) sets and investigate their basic properties.

**Keywords:** $N_{nc}$-topology, N–neutrosophic crisp topological spaces, $N_{nc}$-semi (open) closed sets, $N_{nc}$-preopen (closed) sets, $N_{nc}$-α-open (closed) sets, $N_{nc}int(A)$, $N_{nc}cl(A)$.

## Introduction

The concept of non-rigid (fuzzy) sets introduced in 1965 by L. A. Zadeh [11] which revolutionized the field of logic and set theory. Since the need for supplementing the classical two-valued logic with respect to notions with rigid extension engendered the concept of fuzzy set. Soon after its advent, this notion has been utilized in different fields of research such as, decision-making problems, modelling of mental processes, that is, establishing a theory of fuzzy algorithms, control theory, fuzzy graphs, fuzzy automatic machine etc., and in general topology. Three years after the presence of the concept of fuzzy set, Chang [3] introduced and developed the theory of fuzzy topological spaces. Many researchers focused on this theory and they developed it further in different directions. Then another new notion called intuitionistic fuzzy set was established by Atanassov [2] in 1983. Coker [4] introduced the notion of intuitionistic fuzzy topological space. F. Smarandache introduced the concepts of neutrosophy and neutrosophic set ([7], [8]). A. A. Salama and S. A. Alblowi [5] introduced the notions of neutrosophic crisp set and neutrosophic crisp topological space. In 2014, A.A. Salama, F. Smarandache and V. Kroumov [6] presented the concept of neutrosophic crisp topological space (NCTS). W. Al-Omeri [1] also investigated neutrosophic crisp sets in the context of neutrosophic crisp topological Spaces. The geometric existence of N -topology was given by M. Lellis Thivagar et al. [10], which is a nonempty set equipped with N-arbitrary topologies. The notion of $N_n$-open (closed) sets and *N*-neutrosophic topological spaces are introduced by M. Lellis Thivagar, S. Jafari,V. Antonysamy and V. Sutha Devi. [9]

In this paper, we explore the possibility of expanding the concept of neutrosophic crisp topological spaces into N-neutrosophic crisp topological spaces ($N_{nc}$-topological space). Further, we develop the concept of open (closed) sets, semiopen (semiclosed) sets, preopen (preclosed) sets and α-open (α-closed) sets in the context of N-neutrosophic crisp topological spaces and investigate some of their basic properties.

## 1.Preliminaries

In this section, we discuss some basic definitions and properties of N -topological spaces and neutrosophic crisp topological spaces which are useful in sequel.

**Definition 1.1.** [6] Let X be a non-empty fixed set. A neutrosophic crisp set (NCS) A is an object having the form $A = \{A_1, A_2, A_3\}$, where $A_1, A_2$ and $A_3$ are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$.

**Definition 1.2.** [6] Types of *NCSs* $\phi_N$ and $X_N$ in $X$ are as follows:

1. $\phi_N$ may be defined in many ways as an $NCS$ as follows:

1. $\phi_N = (\phi, \phi, X)$ or
2. $\phi_N = (\phi, X, X)$ or
3. $\phi_N = (\phi, X, \phi)$ or
4. $\phi_N = (\phi, \phi, \phi)$.

2. $X_N$ may be defined in many ways as an $NCS$, as follows:

1. $X_N = (X, \phi, \phi)$ or
2. $X_N = (X, X, \phi)$ or
3. $X_N = (X, X, X)$.

**Definition 1.3.** [6] Let X be a nonempty set, and the NCSs A and B be in the form $A = \{A_1, A_2, A_3\}, B = \{B_1, B_2, B_3\}$. Then we may consider two possible definitions for subset $A \subseteq B$ which may be defined in two ways:

1. $A \subseteq B \Leftrightarrow A_1 \subseteq B_1,\ A_2 \subseteq B_2\ and\ B_3 \subseteq A_3$.
2. $A \subseteq B \Leftrightarrow A_1 \subseteq B_1,\ B_2 \subseteq A_2\ and\ B_3 \subseteq A_3$.

**Definition 1.4.** [6] Let X be a non-empty set and the NCSs A and B in the form $A = \{A_1, A_2, A_3\},\ B = \{B_1, B_2, B_3\}$. Then:

1. $A \cap B$ may be defined in two ways as an NCS as follows:

$i)\quad A \cap B = (A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3)$
$ii)\quad A \cap B = (A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3)$.

2. $A \cup B$ may be defined in two ways as an NCS, as follows:

$i)\quad A \cup B = (A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3)$
$ii)\quad A \cup B = (A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3)$.

**Definition 1.5.** [6] A neutrosophic crisp topology (NCT) on a non-empty set $X$ is a family $\Gamma$ of neutrosophic crisp subsets in $X$ satisfying the following axioms:

1. $\phi_N, X_N \in \Gamma$.
2. $A_1 \bigcap A_2 \in \Gamma$, for any $A_1$ and $A_2 \in \Gamma$.
3. $\bigcup A_j \in \Gamma, \forall \{A_j : j \in J\} \subseteq \Gamma$.

The pair $(X, \Gamma)$ is said to be a neutrosophic crisp topological space (NCTS) in X. Moreover, the elements in $\Gamma$ are said to be neutrosophic crisp open sets (NCOS). A neutrosophic crisp set F is closed (NCCS) if and only if its complement $F^c$ is an open neutrosophic crisp set.

**Definition 1.6.** [6] Let X be a non-empty set, and the NCSs A be in the form

$A = \{A_1, A_2, A_3\}$. *Then* $A^c$ may be defined in three ways as an *NCS*:

$i) A^c = < A_1^c, A_2^c, A_3^c >$ or
$ii) A^c = < A_3, A_2, A_1 >$ or
$iii) A^c = < A_3, A_2^c, A_1 >$.

## *2.N<sub>nc</sub>*-Topological Spaces

In this section, we introduce N–neutrosophic crisp topological spaces ($N_{nc}$-topological space) and discuss their basic properties. Moreover, we introduced new types of open and closed sets in the context of $N_{nc}$-topological spaces.

**Definition 2.1:** Let X be a non-empty set. Then ${}_{nc}\tau_1, {}_{nc}\tau_2, \ldots, {}_{nc}\tau_N$ are *N*-arbitrary crisp topologies defined on X and the collection

$$N_{nc}\tau = \{G \subseteq X : G = (\bigcup_{i=1}^{N} A_i) \cup (\bigcap_{i=1}^{N} B_i) \in N_{nc}\tau, \quad A_i, B_i \in_{nc} \tau_i\}$$

is called $N_{nc}$-topology on X if the following axioms are satisfied:

1. $\phi_N, X_N \in N_{nc}\tau$.

2. $\bigcup_{i=1}^{\infty} G_i \in N_{nc}\tau$ *for all* $\{G_i\}_{i=1}^{\infty} \in N_{nc}\tau$.

3. $\bigcap_{i=1}^{n} G_i \in N_{nc}\tau \quad for\ all\ \{G_i\}_{i=1}^{n} \in N_{nc}\tau.$

Then (X, $N_{nc}\tau$) is called $N_{nc}$-topological space on X. The elements of $N_{nc}\tau$ are known as

$N_{nc}$-open ($N_{nc}$- OS) sets on X and its complement is called $N_{nc}$-closed ($N_{nc}$- CS) sets on X. The elements of X are known as$N_{nc}$-sets ($N_{nc}$- S) on X.

**Remark 2.2:** Considering N = 2 in Definition 2.1, we get the required definition of bi-neutrosophic crisp topology on X. The pair (X, $2_{nc}\tau$) is called a bi-neutrosophic crisp topological space on X.

**Remark 2.3:** Considering N = 3 in Definition 2.1, we get the required definition of tri-neutrosophic crisp topology on X. The pair (X, $3_{nc}\tau$) is called a tri-neutrosophic crisp topological space on X.

**Example 2.4:**

$X = \{1,2,3,4\},\ {}_{nc}\tau_1 = \{\phi_N, X_N, \mathrm{A}\},\ {}_{nc}\tau_2 = \{\phi_N, X_N, \mathrm{B}\},{}_{nc}\tau_3 = \{\phi_N, X_N\}$
$\mathrm{A} =<\{3\},\{2,4\},\{1\}>, B =<\{1\},\{2\},\{2,3\}>,$
$\mathrm{A}\cup\mathrm{B} =<\{1,3\},\{2,4\},\varnothing>, A\cap B =<\varnothing,\{2\},\{1,2,3\}>,$ Then we get

$3_{nc}\tau = \{\varnothing_N, X_N, A, B, A\cup B, A\cap B\}$

which is a tri-neutrosophic crisp topology on *X*. The pair (*X*, $3_{nc}\tau$) is called a tri-neutrosophic crisp topological space on *X*.

**Example 2.5:**

$X = \{1,2,3,4\},\ {}_{nc}\tau_1 = \{\phi_N, X_N, \mathrm{A}\},\ {}_{nc}\tau_2 = \{\phi_N, X_N, \mathrm{B}\}$
$\mathrm{A} =<\{3\},\{2,4\},\{1\}>, B =<\{1\},\{2\},\{2,3\}>,$
$\mathrm{A}\cup\mathrm{B} =<\{1,3\},\{2,4\},\varnothing>, A\cap B =<\varnothing,\{2\},\{1,2,3\}>,$ Then

$2_{nc}\tau = \{\varnothing_N, X_N, A, B, A\cup B, A\cap B\}$

which is a bi-neutrosophic crisp topology on $X$. The pair $(X, 2_{nc}\tau)$ is called a bi-neutrosophic crisp topological space on $X$.

**Definition 2.6:** Let $(X, N_{nc}\tau)$ be a $N_{nc}$-topological space on $X$ and $A$ be an $N_{nc}$-set on $X$ then the $N_{nc}int(A)$ and $N_{nc}cl(A)$ are respectively defined as

(i) $N_{nc}int(A) = \cup \{G : G \subseteq A$ and $G$ is a $N_{nc}$-open set in $X\}$.

(ii) $N_{nc}cl(A) = \cap \{F: A \subseteq F$ and $F$ is a $N_{nc}$-closed set in $X\}$.

**Proposition 2.7:** Let $(X, N_{nc}\tau)$ be any $N_{nc}$-topological space. If A and B are any two $N_{nc}$-sets in $(X, N_{nc}\tau)$, so the $N_{nc}$-closure operator satisfies the following properties:

(i) $A \subseteq N_{nc}cl(A)$.

(ii) $A \subseteq B \Rightarrow N_{nc}cl(A) \subseteq N_{nc}cl(B)$.

(iii) $N_{nc}cl(A \cup B) = N_{nc}cl(A) \cup N_{nc}cl(B)$.

**Proof**

(i) $N_{nc}cl(A) = \cap \{G : G$ is a $N_{nc}$-closed set in $X$ and $A \subseteq G\}$. Thus, $A \subseteq N_{nc}cl(A)$.

(ii) $N_{nc}cl(B) = \cap \{G : G$ is a $N_{nc}$-closed set in $X$ and $B \subseteq G\} \supseteq \cap\{G :$ $G$ is a $N_{nc}$-closed set in $X$ and $A \subseteq G\} \supseteq N_{nc}cl(A)$. Thus, $N_{nc}cl(A) \subseteq N_{nc}cl(B)$.

(iii) $N_{nc}cl(A \cup B) = \cap\{G : G$ is a $N_k$-closed set in $X$ and $A \cup B \subseteq G\} =$ $(\cap\{G : G$ is a $N_{nc}$-closed set in $X$ and $A \subseteq G\}) \cup (\cap\{G : G$ is a $N_{nc}$-closed set in $X$ and $B \subseteq G\}) = N_{nc}cl(A) \cup N_{nc}cl(B)$. Thus, $N_{nc}cl(A \cup B) = N_{nc}cl(A) \cup N_{nc}cl(B)$.

**Proposition 2.8:** Let (X, $N_{nc}\tau$) be any $N_{nc}$-topological space. If A and B are any two $N_{nc}$-sets in (X, $N_{nc}\tau$ ), then the $N_{nc}int(A)$ operator satisfies the following properties:

(i) $N_{nc}int(A) \subseteq A$.

(ii) $A \subseteq B \Rightarrow N_{nc}int(A) \subseteq N_{nc}int(B)$.

(iii) $N_{nc}int(A \cap B) = N_{nc}int(A) \cap N_{nc}int(B)$.

(iv) $(N_{nc}cl(A))^c = N_{nc}int(A)^c$.

(v) $(N_{nc}int(A))^c = N_{nc}cl(A)^c$.

**Proof**

(i) $N_{nc}int(A) = \cup$ {G: $G$ is an $N_{nc}$-open set in $X$ and $G \subseteq A$ }. Thus, $N_{nc}int(A) \subseteq A$.

(ii) $N_{nc}int(B) = \cup$ {G: $G$ is a $N_{nc}$-open set in $X$ and $G \subseteq B$ }$\supseteq \cup${ $G$ : $G$ is an $N_{nc}$-open set in $X$ and $G \subseteq A$ } $\supseteq N_{nc}int(A)$. Thus, $N_{nc}int(A) \subseteq N_{nc}int(B)$.

(iii) $N_{nc}int(A \cap B) = \cup\{G : G$ is an $N_{nc}$-open set in $X$ and $A \cap B \supseteq G\}$ $= (\cup\{G : G$ is a $N_{nc}$-open set in $X$ and $A \supseteq G\}) \cap (\cup\{G : G$ is an $N_{nc}$-open set in $X$ and $B \supseteq G\}) = N_{nc}int(A) \cap N_{nc}int(B)$. Thus, $N_{nc}int(A \cap B) = N_{nc}int(A) \cap N_{nc}int(B)$.

(iv) $N_{nc}cl(A) = \cap$ {$G$: $G$ is an $N_{nc}$-closed set in $X$ and $A \subseteq G$}, $(N_{nc}cl(A))^c = \cup\{G^c : G^c$ is an $N_{nc}$-open set in $X$ and $A^c \supseteq G^c\} = N_{nc}int(A)^c$. Thus, $(N_{nc}cl(A))^c = N_{nc}int(A)^c$.

(v) $N_{nc}int(A) = \cup$ {$G$: $G$ is an $N_{nc}$-open set in $X$ and $A \supseteq G$}, $(N_{nc}int(A))^c = \cap\{G^c : G^c$ is $a$ $N_{nc}$-closed set in $X$ and $A^c \supseteq G^c\} = N_{nc}cl(A)^c$. Thus, $(N_{nc}int(A))^c = N_{nc}cl(A)^c$.

**Proposition 2.9:**

Let (X, $N_{nc}\tau$ ) be any $N_{nc}$-topological space. If A is a $N_{nc}$-sets in (X, $N_{nc}\tau$), the following properties are true:

(i) $N_{nc}cl(A)$ = A iff A is a $N_{nc}$-closed set.

(ii) $N_{nc}int(A)$ = A iff A is a $N_{nc}$-open set.

(iii) $N_{nc}cl(A)$ is the smallest $N_{nc}$-closed set containing A.

(iv) $N_{nc}int(A)$ is the largest $N_{nc}$-open set contained in A.

**Proof:** (i), (ii), (iii) and (iv) are obvious.

## 3.New open setes *in* $N_{nc}$-Topological Spaces

**Definition 3.1:** Let (X, $N_{nc}\tau$) be any $N_{nc}$-topological space. Let A be an $N_{nc}$-set in (X, $N_{nc}\tau$). Then A is said to be:

(i) A $N_{nc}$-preopen set ($N_{nc}$-P-OS) if $A\subseteq N_{nc}int(N_{nc}cl(A))$. The complement of an $N_{nc}$-preopen set is called an $N_{nc}$-preopen set in X. The family of all $N_{nc}$-P-OS (resp. $N_{nc}$-P-CS) of X is denoted by ($N_{nc}$POS(X)) (resp. $N_{nc}$PCS ).

(ii) An $N_{nc}$-semiopen set ($N_{nc}$-S-OS) if $A\subseteq N_{nc}cl(N_{nc}int(A))$. The complement of a $N_{nc}$-semiopen set is called a $N_{nc}$-semiopen set in X. The family of all $N_{nc}$-S-OS (resp. $N_{nc}$-S-CS) of X is denoted by ($N_{nc}$POS(X)) (resp. $N_{nc}$PCS ).

(iii) A $N_{nc}$-α-open set ($N_{nc}$-α-OS) if $A\subseteq N_{nc}int\,(N_{nc}cl(N_{nc}int(A)))$. The complement of a $N_{nc}$-α-open set is called a $N_{nc}$-α-open set in X. The family of all $N_{nc}$-α-OS (resp. $N_{nc}$-α-CS) of X is denoted by ($N_{nc}\alpha$OS(X)) (resp. $N_{nc}\alpha$CS ).

**Example 3.2:**

$X=\{a,b,c,d\}$, ${}_{nc}\tau_1=\{\phi_N, X_N, \text{A}\}$, ${}_{nc}\tau_2=\{\phi_N, X_N, \text{B}\}$

$A =< \{a\},\{b\},\{c\} >, B =< \{a\},\{b,d\},\{c\} >$, then we have $2_{nc}\tau = \{\varnothing_N, X_N, A, B\}$

which is a bi-neutrosophic crisp topology on *X*. Then the pair (*X*, $2_{nc}\tau$) is a bi-neutrosophic crisp topological space on *X*. If $H =< \{a,b\},\{c\},\{d\} >$,then H is a $N_{nc}$-P-OS but not $N_{nc}$-α-OS. It is clear that $H^c$ is a $N_{nc}$-P-CS. A is a $N_{nc}$-S-OS. It is clear that $A^c$ is a $N_{nc}$-S-CS. A is a $N_{nc}$-α-OS. It is clear that $A^c$ is a $N_{nc}$-α-CS.

**Definition 3.3:** Let (X, $N_{nc}\tau$ ) be a $N_{nc}$-topological space on X and A be a $N_{nc}$-set on X then

(i) $N_{nc}$-P-int(A) = ∪ {G: G ⊆ A and G is a $N_{nc}$-P-OS in X}.

(ii) $N_{nc}$-P-cl(A) = ∩ {F: A ⊆ F and F is a $N_{nc}$-P-CS in X}.

(iii) $N_{nc}$-S-int(A) = ∪ {G: G ⊆ A and G is a $N_{nc}$-S-OS in X}.

(iv) $N_{nc}$-S-cl(A) = ∩ {F: A ⊆ F and F is a $N_{nc}$-S-CS in X}.

(v) $N_{nc}$-α-int(A) = ∪ {G: G ⊆ A and G is a $N_{nc}$-α-OS in X}.

(vi) $N_{nc}$-α-cl(A) = ∩ {F: A ⊆ F and F is a $N_{nc}$-α-CS in X}.

In Proposition 3.4 and Proposition 3.5, by the notion $N_{nc}$-k-cl(A)( $N_{nc}$-k-int(A)), we mean $N_{nc}$-P-cl(A)( $N_{nc}$-P-int(A)) (if k = p ), $N_{nc}$-S-cl(A)( $N_{nc}$-S-int(A)) (if k = S) and $N_{nc}$-α-cl(A)( $N_{nc}$-α-int(A)) ( if k = α).

**Proposition 3.4:** Let (X, $N_{nc}\tau$) be any $N_{nc}$-topological space. If A and B are any two $N_{nc}$-sets in (X, $N_{nc}\tau$), then the $N_{nc}$-S-closure operator satisfies the following properties:

(i) A ⊆ $N_{nc}$-k-cl(A).

(ii) $N_{nc}$-k-int(A) ⊆ A.

(iii) A ⊆ B ⇒ $N_{nc}$-k-cl(A)⊆ $N_{nc}$-k-cl(B).

(iv) A ⊆ B ⇒ $N_{nc}$-k-int(A) ⊆ $N_{nc}$-k-int(B).

(v) $N_{nc}$-k-cl (A ∪ B) = $N_{nc}$-k-cl(A)∪$N_{nc}$-k-cl(B).

(vi) $N_{nc}$-k-int (A ∩ B) = $N_{nc}$-k-int(A) ∩ $N_{nc}$-k-int(B).

(vii) $(N_{nc}\text{-k-cl}(A))^c = N_{nc}\text{-k-cl}(A)^c$.

(viii) $(N_{nc}\text{-k-int}(A))^c = N_{nc}\text{-k-int}(A)^c$.

**Proposition 3.5:**

Let (X, $N_{nc}\tau$) be any $N_{nc}$-topological space. If A is an $N_{nc}$-sets in (X, $N_{nc}\tau$). Then the following properties are true:

(i) $N_{nc}$-k-cl(A)= A iff A is a $N_{nc}$-k-closed set.

(ii) $N_{nc}$-k-int(A)= A iff A is a $N_{nc}$-k-open set.

(iii) $N_{nc}$-k-cl(A) is the smallest $N_{nc}$-k-closed set containing A.

(iv) $N_{nc}$-k-int(A) is the largest $N_{nc}$-k-open set contained in A.

**Proof:** (i), (ii), (iii) and (iv) are obvious.

**Proposition 3.6:**

Let (X, $N_{nc}\tau$) be a $N_{nc}$-topological space on X. Then the following statements hold in whcih the equality of each statement are not true:

(i) Every $N_{nc}$- OS (resp. $N_{nc}$- CS) is a $N_{nc}$-α-OS (resp. $N_{nc}$-α-CS).

(ii) Every $N_{nc}$-α-OS (resp. $N_{nc}$-α-CS) is a $N_{nc}$-S-OS (resp. $N_{nc}$-S-CS).

(iii) Every $N_{nc}$-α-OS (resp. $N_{nc}$-α-CS) is a $N_{nc}$-P-OS (resp. $N_{nc}$-P-CS).

**Proposition 3.7:**

Let (X, $N_{nc}\tau$) be a $N_{nc}$-topological space on X, then the following statements hold, and the equality of each statement are not true:

(i) Every $N_{nc}$-OS (resp. $N_{nc}$-CS) is a $N_{nc}$-S-OS (resp. $N_{nc}$-S-CS).

(ii) Every $N_{nc}$-OS (resp. $N_{nc}$-CS) is a $N_{nc}$-P-OS (resp. $N_{nc}$-P-CS).

***Proof.***

(i) Suppose that A is a $N_{nc}$-OS. Then A=$N_{nc}int(A)$, and so $A \subseteq N_{nc}cl$(A)= $N_{nc}cl(N_{nc}int(A))$. so that A is a $N_{nc}$-S-OS.

(ii) Suppose that A is a $N_{nc}$-OS. Then A=$N_{nc}int(A)$, and since $A \subseteq N_{nc}cl$(A) so $A=N_{nc}int(A) \subseteq N_{nc}int(N_{nc}cl(A))$. so that A is a $N_{nc}$-P-OS.

**Proposition 3.8:**

Let (X, $N_{nc}\tau$) be a $N_{nc}$-topological space on X and A a $N_{nc}$-set on X. Then A is an $N_{nc}$-$\alpha$-OS (resp. $N_{nc}$-$\alpha$-CS) iff A is a $N_{nc}$-S-OS (resp. $N_{nc}$-S-CS) and $N_{nc}$-P-OS (resp. $N_{nc}$-P-CS).

***Proof.*** The necessity condition follows from the Definition 3.1. Suppose that A is both a $N_{nc}$-S-OS and a $N_{nc}$-P-OS. Then A$\subseteq N_{nc}cl(N_{nc}int(A))$ , and hence $N_{nc}cl(A) \subseteq N_{nc}cl(N_{nc}cl(N_{nc}int(A)))= N_{nc}cl(N_{nc}int(A))$.

It follows that A$\subseteq N_{nc}int(N_{nc}cl(A)) \subseteq N_{nc}int(N_{nc}cl(N_{nc}int(A)))$ , so that A is a $N_{nc}$-$\alpha$-OS.

**Proposition 3.9:**

Let (X, $N_{nc}\tau$) be an $N_{nc}$-topological space on X and A an $N_{nc}$-set on X. Then A is an $N_{nc}$-$\alpha$-CS iff A is an $N_{nc}$-S-CS and $N_{nc}$-P-CS.

***Proof.*** The proof is straightforward.

**Theorem 3.10:**

Let (X, $N_{nc}\tau$) be a $N_{nc}$-topological space on X and A a $N_{nc}$-set on X. If B is a $N_{nc}$-S-OS such that B $\subseteq$A$\subseteq$ $N_{nc}$int($N_{nc}$cl (A)), then A is a $N_{nc}$-$\alpha$-OS .

**Proof**. Since B is a $N_{nc}$-S-OS, we have B $\subseteq$ $N_{nc}$int($N_{nc}$cl (A)) . Thus, A $\subseteq$ $N_{nc}$int($N_{nc}$cl (B))$\subseteq$ $N_{nc}$int($N_{nc}$cl ($N_{nc}$cl ($N_{nc}$int( (B))))$\subseteq$ $N_{nc}$int($N_{nc}$cl ($N_{nc}$int( (B))))

$\subseteq N_{nc}int(N_{nc}cl\ (N_{nc}int(\ (A))))$ and therefore *A is a* $N_{nc}$-$\alpha$-OS.

**Theorem 3.11:**

Let $(X, N_{nc}\tau)$ be an $N_{nc}$-topological space on X and A be an $N_{nc}$-set on X. Then $A \in N_{nc}\alpha OS(X))$ iff there exists an $N_{nc}$- OS H such that $H \subseteq A \subseteq N_{nc}int\ (N_{nc}cl\ (A))$.

**Proposition 3.12:**

The union of any family of $N_{nc}\alpha OS(X)$ is a $N_{nc}\alpha OS(X)$.

***Proof.*** The proof is straightforward.

**Remark 3.13:**

The following diagram shows the relations among the different types of weakly neutrosophic crisp open sets that were studied in this paper:

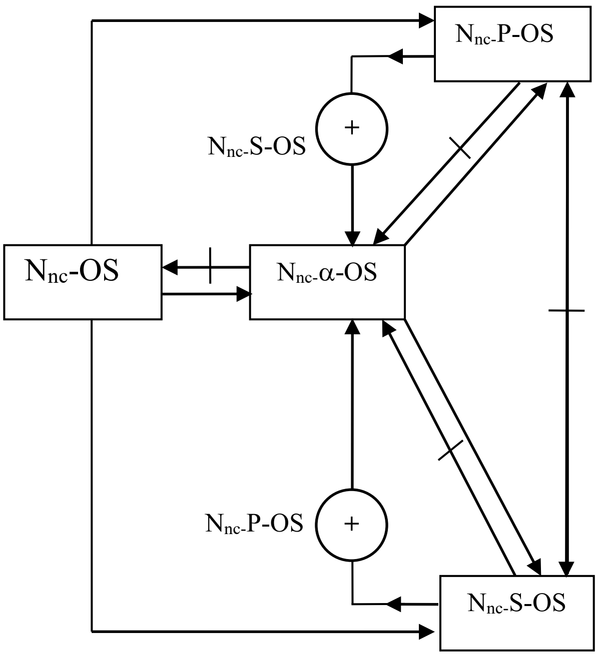


**Diagram (3.1)**

## Conclusion

In this work, we have introduced some new notions of N-neutrosophic crisp open (closed) sets called $N_{nc}$-semi (open) closed sets, $N_{nc}$-preopen (closed) sets, and $N_{nc}$-$\alpha$-open (closed) sets and studied some of their basic properties in the context of neutrosophic crisp topological spaces. The neutrosophic crisp semi-α-closed sets can be used to derive a new decomposition of neutrosophic crisp continuity.

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