On Neutrosophic Extended Triplet Group Action

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ON NEUTROSOPHIC EXTENDED TRIPLET GROUP ACTION

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BY

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Supervisor

Assoc. Prof. Dr. Necati OLGUN

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Moges Mekonnen SHALLA

ABSTRACT

ON NEUTROSOPHIC EXTENDED TRIPLET GROUP ACTION

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M.Sc. in Mathematics
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This thesis discusses neutrosophic extended triplet (NET) direct product, semi-direct product and NET group actions. The aim is to give a clear introduction that provides a solid foundation for further studies into the subject. We introduce NET internal and external direct and semi-direct products for NET group by utilizing the notion of NET set theory of Smarandache. We also give examples and discuss their difference with the classical one. The action of NETG N on a NET set X is given and the difference between right and left NETG actions are briefly discussed. Then, we define fixed points, orbits and stabilizers on NET set with examples. We show that all units modulo n isn't NETG. Furthermore, we give and proof the fundamental theorem about NETG actions. Finally, we give conclusions.

Key Words: NET set, Direct Products of NETG, NET Internal Direct Product, NET External Direct Product, NET Semi-Direct Product, NETG Action, Fundamental Theorem About NETG Actions, Burnside's Lemma.

ÖZET

NÖTROSOFİK GENİŞLETİLMİŞ ÜÇLÜ GRUP ETKİSİ ÜZERİNE

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Danışman: Doç. Dr. Necati OLGUN
Temmuz 2019
61 sayfa

Bu tezde, nötrosofik genişletilmiş üçlü (NET) direkt çarpımı, yarı-direkt çarpımı ve NET grup etkilerini tartışılmaktadır. Amaç, konuyla iligili daha ileri çalışmalar için sağlam bir temel sağlayan net bir giriş yapmaktır. Smarandache'in NET küme teorisini kullanarak NETG için NET iç ve dış direkt ve yarı-direkt çarpımlarını tanıtırız. Ayrıca örnekler vererek klasik ile aralarındaki farklarını tartışırız. N NET grubun X kümesi üzerindeki etkisi verilmiş ve sağ ve sol NETG etkileri arasındaki fark kısaca tartışılmıştır. Daha sonra, NET küme üzerinde sabit noktalar, yörüngeler ve dengeleyeciler tanımlanmıştır. Modulo n'nin bütün birimlerinin NETG olduğunu gösterilmiştir. Ayrıca, NETG etkileri ile ilgili temel teoremi verilmiş ve ispat edilmiştir. Son olarak, sonuçlar verilmiştir.

Anahtar Kelimeler: NET küme, NETG'nun Direkt Çarpımları, NET Dış Direkt Çarpım, NET İç Direkt Çarpım, NET Yarı-Direkt Çarpım, NETG Etkisi İle İlgili Temel Teorem, Burniside Lemması.

''Dedicated to my family''

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LIST OF SYMBOLS

$\in$	Element
$\cap$	Intersection
$\subseteq$	Subset
$\leq$	Neutrosophic extended triplet subgroup
0_N	Neutral element of additive neutrosophic extended triplet group
1_N	Neutral element of multiplicative NET group
X_ρ	Neutrosophic extended triplet semi-direct product
$Orb_{(x,neut(x),anti(x))}$	Neutrosophic extended triplet orbit
$Stab_{(x,neut(x),anti(x))}$	Neutrosophic extended triplet stabilizer
$Fix_{(x,neut(x),anti(x))}$	Neutrosophic extended triplet fixed point
Z_N	Neutrosophic extended triplet center
$\trianglelefteq$	Neutrosophic extended triplet normal subgroup
$\rtimes$	Neutrosophic extended triplet internal semi-direct product

LIST OF ABBREVIATIONS

NT	Neutrosophic Triplet
NET	Nueutrosophic Extended Triplet
NTG	Neutrosophic Triplet Group
NETG	Neutrosophic Extended Triplet Group

CHAPTER I

INTRODUCTION

1.1 Motivation of Study

Galois is well known as the first researcher associating group theory and field theory, along the theory particularily called Galois theory. The concept of groupoid gives a more flexible and powerful approach to the concept of symmetry (see [1]). Symmetry groups come out in the review of combinatorics outline and algebraic number theory, along with physics and chemistry. For instance, Burnside's lemma can be utilized to compute combinatorial objects related along symmetry groups. A group action is a precise method of solving the technique wither the elements of a group meet transformations of any space in a method such protects the structure of a certain space. Just as there is a natural similarity among the set of a group elements and the set of space transformations, a group can be explained as acting on the space in a canonical way. A familiar method of defining no-canonical groups is to express a homomorphism f from a group G to the group of symmetries (an object is invariant to some of different transformations; including reflection, rotation) of a set X. The action of an element $g \in G$ on a point $x \in X$ is supposed to be similiar to the action of its image $f(g) \in Sym(X)$ on the point x. The stabilizers of the action are the vertex groups, and the orbits of the action are the elements, of the action groupoid. Some other facts about group theory can be revealed in [2-5].

Neutrosophy is a new branch of philosophy, presented by Florentic Smarandache [6] in 1980, which studies the interactions with different ideational spectra in our everyday life. A NET is an object of the structure $(x, e^{neut(x)}, e^{anti(x)})$, for $x \in N$, was firstly presented by Florentin Smarandache [7-9] in 2016. In this theory, the extended neutral and the extended opposites can similar or non-identical from the classical unitary element and inverse element respectively. The NETs are depend on real triads : (friend, neutral, enemy), (pro, neutral, against), (accept, pending, reject), and in general $(x, neut(x), anti(x))$ as in neutrosophy is a conclusion of Hegel's dialectics that is depend on x and $anti(x)$. This theory acknowledges every concept or idea x together along its opposite $anti(x)$ and along their spectrum of neutralities $neut(x)$ among them. Nutrosophy is the foundation of neutrosophic logic, neutrosophic set, neutrosophic probablity, and neutrosophic statistics that are utilized or applied in engineering (like software and information fusion), medicine, military, airspace, cybernetics, physics. Kandasamy and Smarandache [10] introduced many new neutrosophic notions in graphs and applied it to the case of neutrosophic cognitive and relational maps. The same researchers [11] were introduced the concept of neutrosophic algebraic structures for groups, loops, semigroups and groupoids and also their $N-$algebraic structures in 2006. Smarandache and Mumtaz Ali [12] proposed neutrosophic triplets and by utilizing these they defined NTG and the application areas of NTGs. They also define NT field [13] and NT in physics [14]. Smarandache investigated physical structures of hybrid NT ring [15]. Zhang et al [16] examined the Notion of cancellable NTG and group coincide in 2017. Şahın and Kargın [17], [18] firstly introduced new structures called NT normed space and NT inner product respectively. Smarandache et al [19] studied new algebraic structure called NT G-module which is constructed on NTGs and NT vector spaces.

This work deals with direct and semi-direct products of NETGs and NETG action. We provide basic definitions, notations, facts, and examples about NETs which play an important role to define and build new algebraic structures. Then, the concept of NET internal and external direct and semi-direct products are given and their difference between the classical structures are briefly discussed. Furthermore, we define the action of NETG N on a NET set X and deal about the difference between left and right NETG actions. Then, NET fixed point, orbit, stabilizer, centralizer and conjugation are defined. Finally, the fundamental theorem about NETG actions and Burnside's lemma are given and proved.

CHAPTER 2

PRELIMINARIES

Since some properties of NETs are used in this work, it is important to have a keen knowlege of NETs. We will point out some few NETs and concepts of NET group, NT normal subgroup, and NT cosets according to what needed in this work.

2.1 Neutrosophic Triplet

Definition 2.1.1 [12,14] A NT has a form $\left(x,\ neut\left(x\right),\ anti\left(x\right)\right)$, for $\left(x, neut\left(x\right), anti\left(x\right)\right) \in N$, accordingly $neut\left(x\right)$ and $anti\left(x\right) \in N$ are neutral and opposite of x, that is different from the unitary element, thus : $x * neut(x) = neut(x) * x = x$ and $x * anti(x) = anti(x) * x = neut(x)$ respectively. In general, x may have one or more than one neut's and one or more than one anti's.

Example 2.1.1 Let's construct the table for (x) modulo 14.

Table 2.1 The table of of (x) modulo 14.

*	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
2	0	2	4	6	8	10	12	0	2	4	6	8	10	12
3	0	3	6	9	12	1	4	7	10	13	2	5	8	11
4	0	4	8	12	2	6	10	0	4	8	12	2	6	8
5	0	5	10	1	6	11	2	7	12	3	8	13	4	9
6	0	6	12	4	10	2	8	0	6	12	4	10	2	8
7	0	7	0	7	0	7	0	7	0	7	0	7	0	7
8	0	8	2	10	4	12	6	0	8	2	10	4	12	6
9	0	9	4	13	8	3	12	7	2	11	6	1	10	5
10	0	10	6	2	12	8	4	0	10	6	2	12	8	4
11	0	11	8	5	2	13	10	7	4	1	12	9	6	3

12	0	12	10	8	6	4	2	0	12	10	8	6	4	2
13	0	13	12	11	10	9	8	7	6	5	4	3	2	1

The neutrosophic triplets of (x) modulo 14 are

$(0,0,0),(0,0,1),(0,0,2),(0,0,3),(0,0,4),(0,0,5),(0,0,6),(0,0,7),(0,0,8),(0,0,9),(0,0,10),$
$(0,0,11),(0,0,12),(0,0,13),(2,8,4),(2,8,11),(4,8,9),(4,8,12),(4,8,13),(6,8,6),(6,8,13),$
$(7,7,7),(8,8,8),(9,1,11),(10,8,5),(10,8,12),(11,1,9),(12,8,3),(12,8,10),(13,1,13).$

Note: Here U is a universe of discourse and $(N,*)$ is a set included in it, endowed with well defined binary law.

2.2 Neutrosophic Extended Triplet

Definition 2.2.1 [8, 14] A NET is a NT, defined as definition 1, but where the neutral of x (symbolized by $e^{neut(x)}$ and called "extended neutral") is equal to the classical unitary element. As a consequence, the "extended opposite" of x, symbolized by $e^{anti(x)}$ is also same to the classical inverse element. Thus, a NET has a form $(x, e^{neut(x)}, e^{anti(x)})$, for $x \in N$, where $e^{neut(x)}$ and $e^{anti(x)}$ in N are the extended neutral and negation of x respectively, thus :

$$x * e^{neut(x)} = e^{neut(x)} * x = x,$$

which can be the same or non-identical from the classical unitary element if any and

$$x * e^{anti(x)} = e^{anti(x)} * x = e^{neut(x)}.$$

Generally, for each x ∈ N there are one or more $e^{neut(x)}$'s and $e^{anti(x)}$'s.

Example 2.2.1 The neutrosophic extended triplets of (x) modulo 14 from example 2.2.1 are :

$(0,0,0),(0,0,1),(0,0,2),(0,0,3),(0,0,4),(0,0,5),(0,0,6),(0,0,7),(0,0,8),(0,0,9),(0,0,10),$
$(0,0,11),(0,0,12),(0,0,13),(1,1,1),(2,8,4),(2,8,11),(4,8,9),(4,8,12),(4,8,13),(6,8,6),$
$(6,8,13),(7,7,7),(8,8,8),(9,1,11),(10,8,5),(10,8,12),(11,1,9),(12,8,3),(12,8,10),(13,1,13).$

2.3 Neutrosophic Triplet Group

Definition 2.3.1 [12, 14] Suppose $(N,*)$ is a NT set. Subsequently $(N,*)$ is called a NTG, if the axioms given below are holds.

(1) $(N,*)$ is well-defined, i.e. for and

$(x, neut(x), anti(x)), (y, neut(y), anti(y) \in N$,

one has $(x, neut(x), anti(x)) * (y, neut(y), anti(y) \in N$.

(2) $(N,*)$ is associative, i.e. for any

one has

$$(x, neut(x), anti(x)) * (y, neut(y), anti(y) * (z, neut(z), anti(z)) \in N.$$

Example 2.3.1 Let's construct the table for (x) modulo 8 under addition.

Table 2.3 The table of of (x) modulo 8

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

The neutrosophic triplet sets of (x) modulo 8 are :

$\{(0,0,0),(1,0,7),(2,0,6),(3,0,5),(4,0,4),(5,0,3),(6,0,2),(7,0,1)\}$. Thus, each elements of (x) modulo 8 has neutral and anti-neutral element. We can easily see that the given operation is well defined and it also holds associativity.

Generally, NTG is not a group, for the reason that, it doesn't include the unitary element, nor inverse elements of group theory. We assume, that the neutrosophic neutrals are used instead of unitary element, and the neutrosophic negation used instead of inverse elements. In other hand, all NETGs are a group in classical way.

Theorem 2.3.1 [22] Let $(N,*)$ be a commutative NET relating to $*$ and $(a, neut(a), anti(a)), (b, neut(b), anti(b)) \in N$;

(i) $neut(a) * neut(b) = neut(a*b)$;

(ii) $anti(a) * anti(b) = anti(a*b)$;

2.4 Neutrosophic Extended Triplet Group

Definition 2.4.1 [8, 14] Assume $(N,*)$ is a NET strong set. Subsequently $(N,*)$ is called a NETG, if the axioms given below are holds.

(1) $(N,*)$ is well-defined, i.e. for any

$(x, neut(x), anti(x)), (y, neut(y), anti(y) \in N$,

one has $(x, neut(x), anti(x)) * (y, neut(y), anti(y) \in N$.

(2) $(N,*)$ is associative,

i.e. for any $(x, neut(x), anti(x)), (y, neut(y), anti(y)), (z, neut(z), anti(z)) \in N$,

one has

$$(x, neut(x), anti(x)) * \left((y, neut(y), anti(y)) * (z, neut(z), anti(z))\right)$$
$$= \left((x, neut(x), anti(x)) * (y, neut(y), anti(y))\right) * (z, neut(z), anti(z)).$$

Example 2.4.1 Let's construct the table for (x) modulo 8 under multiplication.

Table 2.4 The table of of (x) modulo 8.

$\times$	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

Here, each elements of (x) modulo 8 does not satisfy the rules given above.

Therefore let's see the elements of units modulo 8.

Consider the set of integers t meeting the following conditions of units modulo n.

i. $1 \leq t < 8.$

ii. t and 8 are relatively prime.

So the $u(8) = \{1,3,5,7\}$ is a NETG under multiplication modulo 8. Let's construct a NETG table for $u(8).$

Table 2.5 The table of $u(8).$

$\times$	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

The NETs of $u(8)$ are $(1,1,1),(3,1,3),(5,1,5),(7,1,7).$ Thus, each elements of $u(8)$ has extended neutral and extended anti-neutral element. We can easily see that the given operation is well defined and it also holds associativity.

In NETG, the $e^{neut(x)}$'s replace the unitary element, and the $e^{anti(x)}$'s replace the inverse elements of group. In so when NETG includes a group, subsequently NETG enriches the structure of a group, since there may be elements along two or more $e^{neut(x)}$'s and two or more $e^{anti(x)}$'s.

2.5 Neutro-homomorphism

Definition 2.5.1 [23] Assume that $(N_1,*)$ and $(N_2,\circ)$ are two NETG's. A mapping $f: N_1 \to N_2$ is called a neutro-homomorphism if:

(1) For any $(x, neut(x), anti(x)),(y, neut(y), anti(y) \in N_1,$ we have

$$f\left((x, neut(x), anti(x)) * (y, neut(y), anti(y))\right)$$
$$= f\left((x, neut(x), anti(x))\right) * f\left((y, neut(y), anti(y))\right)$$

(2) If $(x, neut(x), anti(x))$ is a NET from N_1, Then

$$f\left(neut(x)\right) = neut\left(f(x)\right) \text{ and } f\left(anti(x)\right) = anti\left(f(x)\right).$$

2.6 Neutrosophic Extended Triplet Subgroup

Definition 2.6.1 [21] Assume that $(N_1, *)$ is a NETG and H is a subset of N_1. H is called a NET subgroup of N if itself forms a NETG under $*$. On other hand it means :

(1) $e^{neut(x)}$ lies in H.

(2) For any $(x, neut(x), anti(x)), (y, neut(y), anti(y) \in H$,

$$(x, neut(x), anti(x)) * (y, neut(y), anti(y) \in H.$$

(3) If $(x, neut(x), anti(x)) \in H$, then $e^{anti(x)} \in H$.

We write $H \leq N$ when ever H is a NET subgroup of N. $\emptyset \neq H \subseteq N$ holding the axioms (2) and (3) above will be a NET subgroup as we may take $(x, neut(x), anti(x)) \in H$ and then (2) gives $e^{anti(x)} \in H$ after which (3) gives $x * e^{anti(x)} = e^{neut(x)} \in H$.

Example 2.6.1 Let's construct the table for (x) modulo 4 under addition.

Table 2.6 Table of (x) modulo 4

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

The NETs are : (0,0,0), (1,0,3), (2,0,2), (3,0,1). $< \left\{ \overline{(0,0,0)}, \overline{(2,0,2)} \right\}, + >$ is a NET subgroup of $< Z_4, + >$.

2.7 Neutrosophic Extended Triplet Normal Subgroup

Definition 2.7.1 [21] A NET subgroup H of a NETG N is called a NT normal subgroup of N if

$(x, neut(x), anti(x))H = H(x, neut(x), anti(x)), \forall (x, neut(x), anti(x)) \in N$ and we represent it as $H \trianglelefteq N$.

Example 2.7.1 The Cayley table for a NETG *N* is shown below.

Table 2.7 The Cayley table of *N*.

*	e	a	b	c	d	f	g	h	i	j	k	l
e	e	a	b	c	d	f	g	h	i	j	k	l
a	a	b	e	k	f	l	h	j	c	g	i	d
b	b	e	a	i	l	d	j	g	k	h	c	f
c	c	l	g	d	e	i	k	a	j	f	b	h
d	d	h	k	e	c	j	b	l	f	i	g	a
f	f	j	i	a	k	g	e	d	l	c	h	b
g	g	c	l	j	h	e	f	k	b	a	d	i
h	h	k	d	g	j	a	l	i	e	b	f	c
i	i	f	j	l	b	k	c	e	h	d	a	g
j	j	i	f	h	g	b	d	c	a	e	l	k
k	k	d	h	f	a	c	i	b	g	l	e	j
l	l	g	c	b	i	h	a	f	d	k	j	e

The NETs of N are ;

(e,e,e), (a,e,b), (b,e,a), (c,e,d), (d,e,c), (f,e,g), (g,e,f), (h,e,i), (i,e,h), (j,e,j), (k,e,k), (l,e,l).

Now let's take the subset $H = \{(e,e,e), (j,e,j), (k,e,k), (l,e,l)\}$ is a NET subgroup of N. Then, let's show that *H* is NT normal in *N*.

Note : A NET subgroup H of N is NT normal if its left and right NT cosets coexist, that is

$$(n, neut(n), anti(n))H = H(n, neut(n), anti(n))$$

for all $(n, neut(n), anti(n))$ in N. We have obtained

left NT cosets

$$(e,e,e)H = \{(e,e,e),(j,e,j),(k,e,k),(l,e,l)\}$$

$$(a,e,b)H = \{(a,e,b),(g,e,f),(i,e,h),(d,e,c)\}$$

$$(b,e,a)H=\{(b,e,a),(h,e,i),(c,e,d),(f,e,g)\}$$

right NT cosets

$$H(e,e,e)=\{(e,e,e),(j,e,j),(k,e,k),(l,e,l)\}$$

$$H(a,e,b)=\{(a,e,b),(i,e,h),(d,e,c),(g,e,f)\}$$

$$H(b,e,a)=\{(b,e,a),(f,e,g),(h,e,i),(c,e,d)\}$$

Hence,

$$(n,neut(n),anti(n))H=H(n,neut(n),anti(n))$$

for all $(n,neut(n),anti(n))\in N$. Therefore, H is NT normal in N.

2.8 Neutrosophic Triplet Cosets

Definition 2.8.1 [21] Suppose N is a NETG and $H\subseteq N$. $\forall(x,neut(x),anti(x))\in N$, the NET set $(x,neut(x),anti(x))(h,neut(h),anti(h))\,/\,(h,neut(h),anti(h))\in H$ is represented by $(x,neut(x),anti(x))H$. Similarly,

$$H(x,neut(x),anti(x))=(x,neut(x),anti(x))(h,neut(h),anti(h))\,/\,(h,neut(h),anti(h))\in H$$

and

$$\big((x,neut(x),anti(x))H\big)(x,neut(x),anti(x))^{-1}=\big((x,neut(x),anti(x))(h,neut(h),anti(h))\big)$$
$$(x,neut(x),anti(x))^{-1}\,/\,(h,neut(h),anti(h))\in H.$$

Where as $(h,neut(h),anti(h))\le N$, $(x,neut(x),anti(x))H$ is called the left NT coset of $H\in N$ involving $(x,neut(x),anti(x))$, and $H(x,neut(x),anti(x))$ is called the right NT coset of $H\in N$ involving $(x,neut(x),anti(x))$. If so the element $(x,neut(x),anti(x))$ is called the NT coset representative of $(x,neut(x),anti(x))H$ or $H(x,neut(x),anti(x))$. $|(x,neut(x),anti(x))H|$ and $|H(x,neut(x),anti(x))|$ are utilized to represent the number of elements in $(x,neut(x),anti(x))H$ or $H(x,neut(x),anti(x))$, respectively.

Example 2.8.1 Consider the NET subgroup $H=\{(0,0,0),(2,0,6),(4,0,4),(6,0,2)\}$ of $N=\mathbf{Z}_8$ (which is an additive NETG in example 2.3.1) with NETs $N=\{(0,0,0),(1,0,7),(2,0,6),(3,0,5),(4,0,4),(5,0,3),(6,0,2),(7,0,1)\}$. The left NT cosets are

$$\{(0,0,0)+h/h\in H\}=\{(2,0,6)+h/h\in H\}=\{(4,0,4)+h/h\in H\}$$
$$=\{(6,0,2)+h/h\in H\}=\{(0,0,0),(2,0,6),(4,0,4),(6,0,2)\},$$

$$\{(1,0,7)+h/h\in H\}=\{(3,0,5)+h/h\in H\}=\{(5,0,3)+h/h\in H\}$$
$$=\{(7,0,1)+h/h\in H\}=\{(1,0,7),(3,0,5),(5,0,3),(7,0,1)\},$$

so $G/H=\{\{(1,0,7),(3,0,5),(5,0,3),(7,0,1)\},\{(0,0,0),(2,0,6),(4,0,4),(6,0,2)\}\}$.

The right cosets are

$$\{h+(0,0,0)/h\in H\}=\{h+(2,0,6)/h\in H\}=\{h+(4,0,4)/h\in H\}$$
$$=\{h+(6,0,2)/h\in H\}=\{(0,0,0),(2,0,6),(4,0,4),(6,0,2)\},$$

$$\{h+(1,0,7)/h\in H\}=\{h+(3,0,5)/h\in H\}=\{h+(5,0,3)/h\in H\}$$
$$=\{h+(7,0,1)/h\in H\}=\{(1,0,7),(3,0,5),(5,0,3),(7,0,1)\},$$

so $H/G=\{\{(0,0,0),(2,0,6),(4,0,4),(6,0,2)\},\{(1,0,7),(3,0,5),(5,0,3),(7,0,1)\}\}$.

CHAPTER 3

DIRECT PRODUCTS OF NEUTROSOPHIC EXTENDED TRIPLET GROUP

The notion of a direct product of groups are one of the vital notions of group theory. It plays a vital position in the study of the structure of a groups. It is an operation that proceeds two groups G and H and builds another group, generally symbolized $G \times H$. Just as a direct product of groups play a vital position in the classical group theory, direct products of NETG play the same role in the theory of NETs. In this section, we define NET internal and external direct products. Then, we give propositions and proof them.

3.1 Direct Products Of NETG

Definition 3.1.1 Assume that we have two neutrosophic extended triplet groups H and K, and $N = H \times K$ is the NET cartesian product of H and K, in other words

$$N = \left((h_1, neut(h_1), anti(h_1)), (k_1, neut(k_1), anti(k_1))\right), \begin{pmatrix} (h_2, neut(h_2), anti(h_2)), \\ (k_2, neut(k_2), anti(k_2)) \end{pmatrix}$$
$$= \left(h_1 * h_2, neut(h_1 * h_2), anti(h_1 * h_2)\right), \left(k_1 * k_2, neut(k_1 * k_2), anti(k_1 * k_2)\right) \in H \times K.$$

Clearly N is closed under multiplication, it is obvious to see associativity and it has a neutral element denoted by

$$1_N = (1_H, 1_K)$$

and the anti neutrals of $\left((h, neut(h), anti(h)), (k, neut(k), anti(k))\right)$ is $\left(anti(h)), anti(k))\right)$, respectively.

Definition 3.1.2 Suppose that H, K are two NETGs. The NETG $N = H \times K$ with binary operation described componentwise as denoted in definition (3.1.1) is called the "neutrosophic extended triplet direct product" of H and K.

Example 3.1.1 Find the NET direct product of two NETG Z_2 and Z_3. Since $Z_2=\{0,1\}$ and $Z_3=\{0,1,3\}$, the NETs Z_2 is $(0,0,0),(1,0,1)$ and the NETs of Z_3 is $(0,0,0),(1,0,2),(2,0,1)$. The NET direct products are

$$Z_2 \times Z_3 = \left\{ \begin{matrix} ((0,0,0),(0,0,0)),((0,0,0),(1,0,2)),((0,0,0),(2,0,1)),((1,0,1),(0,0,0)), \\ ((1,0,1),(1,0,2)),((1,0,1),(2,0,1)) \end{matrix} \right\}.$$

3.2 Neutrosophic Extended triplet internal direct product

Definition 3.2.1 If a NETG N contains neutrosophic triplet normal subgroups H and K as shown $N=HK$ and $H\cap K=\{1_N\}$, we call N is the "neutrosophic triplet internal direct product" of H and K.

Example 3.2.1 Examine the NETG $(Z_6,+)$ and the following NET subgroups:

$$H=\{(0,0,0),(2,0,4),(4,0,2)\}$$
$$K=\{(0,0,0),(3,0,3)\}.$$

Note that $\left\{ \begin{matrix} (h,neut(h),anti(h))*(k,neut(k),anti(k)):(h,neut(h),anti(h))\in H, \\ (k,neut(k),anti(k))\in K \end{matrix} \right\}$
$=N.$

That means $\{(0,0,0),(2,0,4),(4,0,2)+(0,0,0),(3,0,3)\}$

$$=\{(0,0,0),(1,0,5),(2,0,4),(3,0,3),(4,0,2),(5,0,1)\}.$$

So the first condition is met. Also the neutral for z_6 is 0_N and $H\cap K=0_N=\{(0,0,0)\}$ so the second condition is met. Lastly Z_6 is an abelian so the third condition is met.

Table 3.2 The table of Z_6

+	0	1	2	3	4	5
0	0	1	2	3	4	5

1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

The formed NETs of Z_6 is $\{(0,0,0),(1,0,5),(2,0,4),(3,0,3),(4,0,2),(5,0,1)\}$.

Proposition 3.2.1 If N is the neutrosophic triplet internal direct product of H and K, subsequently N is neutro-isomorphic to the neutrosophic triplet external direct product $H \times K$.

Proof To put on that N is neutro-isomorphic to $H \times K$, we describe the succeeciding map

$$f : H \times K \to N,$$

$$f\left((h, neut(h), anti(a)),(k, neut(k), anti(k))\right)$$

$$=\left(h*k, neut(h*k), anti(h*k)\right)...(1)$$

First remark that if $\left((h, neut(h), anti(h)\right) \in H, \left(k, neut(k), anti(k)\right) \in K$, then

$$\begin{aligned}&\left((h*k, neut(h*k), anti(h*k)\right)\\&=\left((k*h, neut(k*h), anti(k*h)\right).\end{aligned}$$

Actually, we have utilizing that both NETGs K and H are neutrosophic triplet normal that

$$\left((h, neut(h), anti(h))(k, neut(k), anti(k)\left(h, neut(h), anti(h)\right)^{-1}\right)\left((k, neut(k), anti(k)\right)^{-1} \in K,$$

$$\left((h,neut(h),anti(h))(k,neut(k),anti(k)\left(h,neut(h),anti(h)\right)^{-1}\right)\left((k,neut(k),anti(k)\right)^{-1}\in H$$

Implying that

$$\left((h,neut(h),anti(h))(k,neut(k),anti(k)\left(h,neut(h),anti(h)\right)^{-1}\right)\left((k,neut(k),anti(k)\right)^{-1}$$
$$\in K\cap H=\{1_N\}.$$

At the same time let us show that f is a NETG neutro-isomorphism.

1. This a NETG neutro-homomorphism onwards

$$f\begin{pmatrix}(h,neut(h),anti(h)),(k,neut(k),anti(k)),(h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),\\ anti(k^{'}))\end{pmatrix}$$

$$=f\left((h*h^{'},neut(h*h^{'}),anti(h*h^{'})),(k*k^{'},neut(k*k^{'}),anti(k*k^{'}))\right) \text{ by . .. (1)}$$

$$=(h,neut(h),anti(h))\left((h^{'}*k),neut(h^{'}*k),anti(h^{'}*k))\right)(k^{'},neut(k^{'}),anti(k^{'}))$$

$$=(h,neut(h),anti(h))\left((k*h^{'}),neut(k*h^{'}),anti(k*h^{'}))\right)(k^{'},neut(k^{'}),anti(k^{'}))$$

$$=f\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)f\begin{pmatrix}(h^{'},neut(h^{'}),anti(h^{'})),\\ (k^{'},neut(k^{'}),anti(k^{'}))\end{pmatrix}.$$

2. Let us show that the map f is injective. First we have to check that its neutro-kernel is trivial. Actually, if

$$f\left((h,neut(h),anti(h)),(k,neut(k),anti(k)\right)=1_N$$

Then

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k)\right)=1_N$$

$$\Rightarrow\left(h,neut(h),anti(h)\right)=\left(k,neut(k),anti(k)\right)^{-1}$$

$$\Rightarrow\left(h,neut(h),anti(h)\right)\in K$$
$$\Rightarrow\left(h,neut(h),anti(h)\right)\in H\cap K=\{1_N\}$$

We have then that

$$\left(h,neut(h),anti(h)\right)=\left(k,neut(k),anti(k)\right)=\{1_N\}$$

which proves that the neutro-kernel is $\{(1_N,1_N)\}$.

3. Lastly it's obvious to see that f is surjective since $N = HK$. Briefly record that the definitions of NET external and internal are assuredly unlimited to two NETGs. We can totally describe them for n NETGs as $H_1,...,H_n$.

3.3 Neutrosophic Extended Triplet External Direct Product

Definition 3.3.1 If $H_1,...,H_n$ are randomNETGs the NET external direct product of $H_1,...,H_n$ is

$$N = H_1 \times H_2 \times ... \times H_n$$

which is the NET cartesian product with componentwise multiplication.

Example 3.3.1 Let NETG $u(8) = \{1,3,5,7\}$ from example 2.4.1 and $u(12) = \{1,5,7,11\}$ under multiplication modulo 8 and mudulo 12 respectively. Let's construct a NETG table for $u(12)$.

Table 3.3 The table of $u(12)$

$\times$	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

The NETs of $u(8)$ are $(1,1,1),(3,1,3),(5,1,5),(7,1,7)$ and the NETs of $u(12)$ are $(1,1,1),(5,1,5),(7,1,7),(11,1,11)$.

Now let's see the NET external direct products of
$u(8) \times u(12) = \big((1,1,1),(1,1,1)\big), \big((1,1,1),(5,1,5)\big), \big((1,1,1),(7,1,7)\big), \big((1,1,1),(11,1,11)\big),$
$\big((3,1,3),(1,1,1)\big), \big((3,1,3),(5,1,5)\big), \big((3,1,3),(7,1,7)\big), \big((3,1,3),(11,1,11)\big), \big((5,1,5),(1,1,1)\big),$
$\big((5,1,5),(5,1,5)\big), \big((5,1,5),(7,1,7)\big), \big((5,1,5),(11,1,11)\big), \big((7,1,7),(1,1,1)\big), \big((7,1,7),(3,1,3)\big),$
$\big((7,1,7),(5,1,5)\big), \big((7,1,7),(7,1,7)\big), \big((7,1,7),(11,1,11)\big).$

Definition 3.3.2 If N contains NET normal subgroups $H_1,...,H_n$ as shown

$$N = H_1 \cdots H_n$$

and every n can be symbolized as

$$\left(h, neut(h), anti(h)\right) \ldots \left(h_n, neut(h_n), anti(h_n)\right)$$

particularly, we call N is the neutrosophic extended triplet internal direct product of $H_1, \ldots, H_n.$

There is a small distiniction between neutrosophic extended triplet internal product as we see in the definition, since in this instance of two NET subgroups, the condition dedicated briefly record that each n can be symbolized particularly as

$$\left(h_1, neut(h_1), anti(h_1)\right)\left(h_2, neut(h_2), anti(h_2)\right),$$

but alternately that the intersection of of the two NET subgroups is $\{(1_N)\}$. The following proposition indicates the relation among those two points of view.

Proposition 3.3.1 Assume that $N = H_1 \cdots H_n$ thus every H_i is a NET normal subgroup of N. The succeecing axioms are equivalent.

I. N is the neutrosophic extended triplet direct product of the H_i.

II. $H_1 H_2 \cdots H_{i-1} \cap H_i = \{1_N\}, \ \forall i = 1, \ldots, n.$

Proof Let's show I. $\Leftrightarrow$ II.

I. $\Rightarrow$ II. Let's suppose that N is the neutrosophic extended triplet internal direct product of the H_i, in other words all element in N can be inscribed particularly as a product of elements in H_i. Let's assume

$$\left(n, neut(n), anti(n)\right) \in H_1 H_2 \cdots H_{i-1} \cap H_i = \left\{\left(1_N\right)\right\}.$$

We obtain that

$$\left(n, neut(n), anti(n)\right) \in H_1 H_2 \cdots H_{i-1},$$

this is particularly expressed as

$$\left(n,neut(n),anti(n)\right)=\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)...$$
$$\left(h_{i-1},neut(h_{i-1}),anti(h_{i-1})\right)1_N H_i..1_N H_n,\left(h_j,neut(h_j),anti(h_j)\right)\in H_j.$$

On the other hand,

$$\left(n,neut(n),anti(n)\right)\in H_i$$

thus

$$\left(n,neut(n),anti(n)\right)=\left(1_N\right)H_1 n...\left(1_N\right)H_{i-1} n$$

and by unicity of the representation, we have

$$\left(h_j,neut(h_j),anti(h_j)\right)=\left(1_N\right) \text{ for all } j \text{ and } \left(n,neut(n),anti(n)\right)=\left(1_N\right).$$

$\text{II.}\Rightarrow\text{I.}$ conversely, let us assume that

$$\left(n,neut(n),anti(n)\right)\in N$$

can be written either

$$\left(n,neut(n),anti(n)\right)=\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)...$$
$$\left(h_n,neut(h_n),anti(h_n)\right),\left(h_j,neut(h_j),anti(h_j)\right)\in H_j,$$

or

$$\left(n,neut(n),anti(n)\right)=\left(k_1,neut(k_1),anti(k_1)\right)\left(k_2,neut(k_2),anti(k_2)\right)...$$
$$\left(k_n,neut(k_n),anti(k_n)\right),\left(k_j,neut(k_j),anti(k_j)\right)\in H_j.$$

Remember that whereby every H_j are NET normal subgroups, subsequantly

$$\left(h_i,neut(h_i),anti(h_i)\right)\left(h_j,neut(h_j),anti(h_j)\right)$$

$$=\left(h_j,neut(h_j),anti(h_j)\right)\left(h_i,neut(h_i),anti(h_i)\right),\left(h_i,neut(h_i),anti(h_i)\right)\in H_i,$$
$$\left(h_j,neut(h_j),anti(h_j)\right)\in H_j.$$

In other words, we can do the succeeding manipulations.

$$\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)...\left(h_n,neut(h_n),anti(h_n)\right)$$

$$=\left(k_1,neut(k_1),anti(k_1)\right)\left(k_2,neut(k_2),anti(k_2)\right)...\left(k_n,neut(k_n),anti(k_n)\right)$$

$$\Leftrightarrow\left(h_2,neut(h_2),anti(h_2)\right)...\left(h_n,neut(h_n),anti(h_n)\right)$$

$$=\left(\left(h_1,neut(h_1),anti(h_1)\right)^{-1}...\left(k_1,neut(h_1),anti(h_1)\right)\right)\left(k_2,neut(k_2),anti(k_2)\right)...\left(k_n,neut(k_n),anti(k_n)\right)$$

$$\Leftrightarrow\left(h_3,neut(h_3),anti(h_3)\right)...\left(h_n,neut(h_n),anti(h_n)\right)$$

$$=\left(\left(h_1,neut(h_1),anti(h_1)\right)^{-1}...\left(k_1,neut(h_1),anti(h_1)\right)\right)\begin{pmatrix}\left(h_2,neut(h_2),anti(h_2)\right)^{-1}\\ \left(k_2,neut(k_2),anti(k_2)\right)\end{pmatrix}\left(k_3,neut(k_3),anti(k_3)\right)...\left(k_n,neut(k_n),anti(k_n)\right)$$

and likewise and then so long as we achieve

$$\left(h_n,neut(h_n),anti(h_n)\right)\left(k_n,neut(k_n),anti(k_n)\right)^{-1}\ ...\ (1)$$

$$=\left(h_1,neut(h_1),anti(h_1)\right)^{-1}\left(k_1,neut(k_1),anti(k_1)\right)...\left(h_{n-1},neut(h_{n-1}),anti(h_{n-1})\right)^{-1}\left(k_{n-1},neut(k_{n-1}),anti(k_{n-1})\right).$$

Until now the left handside (1) refers to H_n although the right handside refers to $H_1 \cdots H_{n-1}$, we obtain such

$$\left(h_n,neut(h_n),anti(h_n)\right)\left(k_n,neut(k_n),anti(k_n)\right)^{-1}\in H_n \cap H_1 \cdots H_{n-1}=\{1_N\}$$

signfying that

$$\left(h_n,neut(h_n),anti(h_n)\right)=\left(k_n,neut(k_n),anti(k_n)\right).$$

We end this by repeating the procedure. Let’s prove this for the conditions of two NETGs. We’ve noticed overhead that the NET cartesian product of two NETGs H and K endowed in relation to a NETG structure by taking in mind componentwise binary operartion

$$\left(h_1,neut(h_1),anti(h_1)\right),\left(k_1,neut(k_1),anti(k_1)\right)$$

$$=\left(h_1 * h_1, neut(h_1 * h_1), anti(h_1 * h_1)\right), \left(k_1 * k_1, neut(k_1 * k_1), anti(k_1 * k_1)\right) \in H \times K.$$

The preference of this binary operation of course decides the structures of $N = H \times K,$ and exceptionaly, we've noticed such the neutro-isomorphic duplicates of NETGs H and K in N are NET normal subgroups. Contrarily that one may describe a NET internal direct product, we have to suppose that we've two NET normal subgroups.

Now let's examine a further overall setting, thus the NET subgroup K does't need to be NET normal, for whatever we have to describe another binary operation on the NET cartesian product $H \times K.$ This'll take us to the definition of NET internal and external semi-direct product.

Remember that a neutro-automorphism of a NETG H is a bijective NETG neutro-homomorphism from $H \to H.$ It's obvious to realize such the set of neutro-automorphism of H shapes a NETG according to the composition of maps and identify element the neutrality map $1_H.$ We symbolize it by $Aut(1_H).$

Proposition 3.3.2 Suppose that H and K are NETGs, and

$$\rho : K \to Aut(H), \left(k, neut(k), anti(k)\right) \mapsto \rho\left(k, neut(k), anti(k)\right)$$

are a NETG neutro-homomorphism. Subsequently the binary operation

$$\left(H \times K\right) \times \left(H \times K\right) \to \left(H \times K\right),$$

$$\left((h, neut(h), anti(h)), (k, neut(k), anti(k))\right), \begin{pmatrix} (h^{'}, neut(h^{'}), anti(h^{'})), (k^{'}, neut(k^{'}), \\ anti(k^{'})) \end{pmatrix}$$

$$\to \begin{pmatrix} (h, neut(h), anti(h)) \rho((k, neut(k), anti(k)) \left((h^{'}, neut(h^{'}), anti(h^{'}))\right) \\ (k, neut(k), anti(k))(k^{'}, neut(k^{'}), anti(k^{'})) \end{pmatrix}$$

endows $H \times K$ with a NETG structure, with neutral element $\left(1_H, 1_K\right).$

Proof let's realize such the closure property is holds.

1) Neutrality : Let's prove that $\left(1_H, 1_K\right)$ is the neutral element. We have

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\left(1_H,1_K\right)$$
$$=\left((h,neut(h),anti(h))\rho(k,neut(k),anti(k))\left(1_H\right),(k,neut(k),anti(k))\right)$$
$$=\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right) \text{ for all } (h,neut(h),anti(h)) \in H,$$
$$(k,neut(k),anti(k)) \in K, \text{ whereby } \rho(k,neut(k),anti(k))$$

is a NETG neutro-homomorphism. We also have

$$\left(1_H,1_K\right)\left((h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

$$=\left(\rho 1_H(h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

$$=\left((h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

for all $(h^{'},neut(h^{'}),anti(h^{'})) \in H, (k^{'},neut(k^{'}),anti(k^{'})) \in K,$ wherby ρ being a NETG neutro-homomorphism, it maps 1_K to $1_{Aut(K)}=N_{1H}$.

2) Anti-neutrality : Let $\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right) \in H \times K$

and let us show that

$$\left(\rho^{-1}{}_{(k,neut(k),anti(k))}\left((h,neut(h),anti(h))\right)^{-1},\left(k,neut(k),anti(k)\right)\right)^{-1}$$

is the anti-neutral of

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right).$$

We have

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\begin{pmatrix} \rho^{-1}(k,neut(k),anti(k))\begin{pmatrix} h,neut(h) \\ ,anti(h) \end{pmatrix}^{-1} \\ \left(k,neut(k),anti(k)\right)^{-1} \end{pmatrix}$$

$$=\left(h,neut(h),anti(h)\right)\rho_{(k,neut(k),anti(k))}\begin{pmatrix} \rho^{-1}(k,neut(k),anti(k)) \\ \left(h,neut(h),anti(h)\right)^{-1},1_K \end{pmatrix}$$
$$=\left(\left(h,neut(h),anti(h)\right)\left(h,neut(h),anti(h)\right)^{-1},1_K\right)=\left(1_H,1_K\right).$$

We also have

$$\left(\rho^{-1}{}_{(k,neut(k),anti(k)}\left(h,neut(h),anti(h)\right)^{-1},\left(k,neut(k),anti(k)\right)^{-1}\right)$$
$$\left((h,neut(h),anti(h))(k,neut(k),anti(k)\right)$$
$$=\left(\begin{array}{l}\rho^{-1}{}_{(k,neut(k),anti(k)}\left(h,neut(h),anti(h)\right)^{-1}\rho\left(k,neut(k),anti(k)\right)^{-1}\\ \left(h,neut(h),anti(h)\right),1_K\end{array}\right)$$
$$=\left(\begin{array}{l}\rho\left(k,neut(k),anti(k)\right)^{-1}\left(h,neut(h),anti(h)\right)^{-1}\rho\left(k,neut(k),anti(k)\right)^{-1}\\ \left(h,neut(h),anti(h)\right)^{-1}\rho\left(k,neut(k),anti(k)\right)^{-1}\left(h,neut(h),anti(h)\right),1_K\end{array}\right).$$

Using that

$$\rho^{-1}{}_{\left(k,neut(k),anti(k)\right)}=\rho\left(k,neut(k),anti(k)\right)^{-1}$$

whereby ρ is a NETG neutro-homomorphism. Instantly

$$\left(\begin{array}{l}\rho\left(k,neut(k),anti(k)\right)^{-1}\left(h,neut(h),anti(h)\right)^{-1}\rho\left(k,neut(k),anti(k)\right)^{-1}\\ \left(h,neut(h),anti(h)\right),1_K\end{array}\right)$$
$$=\left(\rho\left(k,neut(k),anti(k)\right)^{-1}\left(h,neut(h),anti(h)\right)^{-1}\left(h,neut(h),anti(h)\right),1_K\right)$$
$$=\left(\rho\left(k,neut(k),anti(k)\right)^{-1}\left(1_H\right),1_K\right)$$
$$=\left(1_H,1_K\right)$$

using that $\rho\left(k,neut(k),anti(k)\right)^{-1}$ is a NETG neutro-homomorphism for all $\left(k,neut(k),anti(k)\right)\in K.$

3) Associativity : Lastly let's check that the following condition holds, we've

$$\left(\begin{array}{l}(h,neut(h),anti(h)),(k,neut(k),anti(k)),(h',neut(h'),anti(h')),\\ (k',neut(k'),anti(k'))\end{array}\right)$$
$$\left((h'',neut(h''),anti(h'')),(k'',neut(k''),anti(k''))\right)$$
$$=\left(\begin{array}{l}(h,neut(h),anti(h)),\rho(k,neut(k),anti(k)),(h',neut(h'),anti(h')),\\ (k,neut(k),anti(k)),(k',neut(k'),anti(k'))\end{array}\right)$$
$$\left((h'',neut(h''),anti(h'')),(k'',neut(k''),anti(k''))\right)$$

$$= \begin{pmatrix} (h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}(h',neut(h'),anti(h')), \\ \rho_{(k,neut(k),anti(k))}(k',neut(k'),anti(k')) \end{pmatrix}$$

$$\begin{pmatrix} (h'',neut(h''),anti(h'')),(k,neut(k),anti(k)),(k',neut(k'),anti(k')), \\ (k'',neut(k''),anti(k'')) \end{pmatrix},$$

while conversely

$$((h,neut(h),anti(h)),(k,neut(k),anti(k))) \begin{pmatrix} (h',neut(h'),anti(h')),(k',neut(k') \\ ,anti(k'))(h'',neut(h''),anti(h'')), \\ (k'',neut(k''),anti(k'')) \end{pmatrix}$$

$$= ((h,neut(h),anti(h)),(k,neut(k),anti(k)))$$
$$\begin{pmatrix} (h',neut(h'),anti(h')),\rho_{(k',neut(k'),anti(k'))}(h'',neut(h''), \\ anti(h'')),(k',neut(k'),anti(k'))(k'',neut(k''),anti(k'')) \end{pmatrix}$$

$$= \begin{pmatrix} (h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))} \begin{pmatrix} (h',neut(h'),anti(h'))\rho_{(k',neut(k'),anti(k'))} \\ \big((h'',neut(h''),anti(h''))\big) \end{pmatrix}, \\ (k,neut(k),anti(k))\big((k',neut(k'),anti(k'))(k'',neut(k''),anti(k''))\big) \end{pmatrix}$$

whereby K is a NETG, we have

$$\big((k,neut(k),anti(k))(k',neut(k'),anti(k'))\big)(k'',neut(k''),anti(k''))$$
$$= (k,neut(k),anti(k))\big((k',neut(k'),anti(k'))(k'',neut(k''),anti(k''))\big).$$

Mark that by seeing at the first component

$$\rho_{(k,neut(k),anti(k))(k',neut(k'),anti(k'))}$$
$$= \rho_{(k,neut(k),anti(k))} \circ \rho_{(k',neut(k'),anti(k'))}$$

utilizing that ρ is a NETG neutro-homomorphism, therefore

$$(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\big((h',neut(h'),anti(h'))\big)$$
$$\rho_{(k,neut(k),anti(k))}(k',neut(k'),anti(k')\big((h'',neut(h''),anti(h''))\big)$$

$$= (h, neut(h), anti(h))\rho_{(k,neut(k),anti(k))}\big((h',neut(h'),anti(h'))\big)$$

$$\rho_{(k,neut(k),anti(k))}\rho_{(k,neut(k),anti(k))}\begin{pmatrix}\rho_{(k',neut(k'),anti(k'))}\\ \big((h'',neut(h''),anti(h''))\big)\end{pmatrix}.$$

Furthermore, $\rho_{(k,neut(k),anti(k))}$ is a NETG neutro-homomorphism, yielding

$$(h, neut(h), anti(h))\rho_{(k,neut(k),anti(k))}\big((h',neut(h'),anti(h'))\big)$$

$$\rho_{(k,neut(k),anti(k))}\Big(\rho_{(k',neut(k'),anti(k'))}\Big)\big((h'', neut(h''), anti(h''))\big)$$

$$= (h, neut(h), anti(h))\rho_{(k,neut(k),anti(k))}\begin{pmatrix}(h',neut(h'),anti(h'))\\ \rho_{(k',neut(k'),anti(k'))}\\ \big((h'',neut(h''),anti(h''))\big)\end{pmatrix}$$

which concludes the proof. Now let’s define the first NET semi-direct product.

CHAPTER 4

SEMI-DIRECT PRODUCTS OF NEUTROSOPHIC EXTENDED TRIPLET GROUP

In general, the NET direct product is not enough because the operation between elements of the two NET subgroups is always commutative. On other hand, if N is a NETG, H is a NT normal subgroup, K is a NET subgroup (K need not be NT normal like in a NET direct product), $K \cap N = 1_N$, then N must be a NET semi-direct product. (The operation between elements of H and K need not be commutative.) So, we can argue that the NET semi-direct product classifies all NETGs constructed in this way.

4.1 NETG external semi-direct Product

Definition 4.1.1 Suppose that H and K are two NETGs, and

$$\rho : K \to Aut(H)$$

is a NETG neutro-homomorphism. The set $H \times K$ endowed in a relation to the binary operation

$$\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big)\big((h', neut(h'), anti(h')), (k', neut(k'), anti(k'))\big)$$
$$\to \begin{pmatrix} (h, neut(h), anti(h))\rho_{(k, neut(k), anti(k))}\big((h', neut(h'), anti(h'))\big), \\ (k, neut(k), anti(k))(k', neut(k'), anti(k')) \end{pmatrix}$$

is a NETG N called a “NET external semi-direct product of NETGs H and K” b

ρ, symbolized by $N = H \chi_\rho K$.

Example 4.1.1 The NET set $L = H \times N$, where H, N are NETGs and $N \le AutH$ is the NET external semi-direct product of H and N when equipped with the following operation, defined by the action

$$\theta : N \to AutH :$$

$$\big((h_1, neut(h_1), anti(h_1)), (n_1, neut(n_1), anti(n_1))\big)$$

$$=\begin{pmatrix}(h_1,neut(h_1),anti(h_1))\theta_{(n_1,neut(n_1),anti(n_1))}\left((h_2,neut(h_2),anti(h_2))\right),\\ (n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\end{pmatrix}$$

$$=\begin{pmatrix}(h_1,neut(h_1),anti(h_1))(n_1,neut(n_1),anti(n_1))\left((h_2,neut(h_2),anti(h_2))\right),\\ (n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\end{pmatrix},$$

for all $(h_1,neut(h_1),anti(h_1)),(h_2,neut(h_2),anti(h_2))\in H$ and all $(n_1,neut(n_1),anti(n_1))$, $(n_2,neut(n_2),anti(n_2))\in N.$

Definition 4.1.2 Let N be a NETG in a relation to NET subgroups H and $K.$We say that N is the "NET internal semi-direct product of H and K" if H is a NET normal subgroup of $N,$ thus $HK = N$ and $H\cap K=\{1_N\}.$It is symbolized by

$$N = H \rtimes K.$$

Example 4.1.2 Let's show that the dihedral NETG D_{2n} is the NET internal semi-direct product of two of its NET subgroups : the NET subgroup of rotations of a regular $n-$gon, and the NET subgroup generated by a single reflection of the same regular $n-$gon. If $D_{2n}=<(a,neut(a),anti(a)),(x,neut(x),anti(x))>,$ where $(a,neut(a),anti(a))$ generates the NET subgroup $<(a,neut(a),anti(a))>$ of rotations and $(x,neut(x),anti(x))$ generates the NET subgroup $<(x,neut(x),anti(x))>,$ then we know that $(a,neut(a),anti(a))^n=1_N$ and $(x,neut(x),anti(x))^2=1_N,$ where 1_N is the neutral symmetry. We know that $\{1_N\}=<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>;$ we also know that, if x is a reflection and a a rotation, then

$$(x,neut(x),anti(x))(a,neut(a),anti(a)) = (a,neut(a),anti(a))^{n-1}(x,neut(x),anti(x)).$$

Being D_{2n} the NETG of all symmetries of a regular $n-$gon, it contains all and only the rotations and reflections of the $n-$gon itself; this fact, combined with the fact that $\{1_N\}=<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>,$ allows us to deduce

$$|<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>|=|D_{2n}|.$$

Since $<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>\leq D_{2n}$, it follows $<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>=D_{2n}$. Finally, we obtain

$$\begin{aligned}&(x,neut(x),anti(x))(a,neut(a),anti(a))(x,neut(x),anti(x))^{-1}\\&=(a,neut(a),anti(a))^{n-1}\in<(a,neut(a),anti(a))>;\end{aligned}$$

thus, $<(a,neut(a),anti(a))>$ is NT normal. Therefore

$$D_{2n}=<(a,neut(a),anti(a))>\rtimes<(x,neut(x),anti(x))>.$$

Lemma 4.1.1 Assume that N is a NETG with NET subgroups H and K. Assume that $N=HK$ and $H\cap K=\{1_N\}$. Subsequently all element $(n,neut(n),anti(n))$ of N can be inscribed particularly in the form $(h,neut(h),anti(h))(k,neut(k),anti(k))$, for $(h,neut(h),anti(h))\in H$ and $(k,neut(k),anti(k))\in K$.

Proof Since $N=HK$, we know that $(n,neut(n),anti(n))$ can be written as $(h,neut(h),anti(h))(k,neut(k),anti(k))$. Assume it can also be inscribed $(h',neut(h'),anti(h'))(k',neut(k'),anti(k'))$. Then

$$(h,neut(h),anti(h))(k,neut(k),anti(k))=(h',neut(h'),anti(h'))(k',neut(k'),anti(k'))$$

so

$$\begin{aligned}&(h',neut(h'),anti(h'))^{-1}(h,neut(h),anti(h))=(k',neut(k'),anti(k'))(k,neut(k),anti(k))^{-1}\\&\in H\cap K=\{1_N\}.\end{aligned}$$

In case

$$(h,neut(h),anti(h))=(h',neut(h'),anti(h'))$$

and

$$(k,neut(k),anti(k))=(k',neut(k'),anti(k')).$$

The NET internal and external direct products were two sides of the similar objects, consequently are the NET internal and external semi-direct products. If $N=H\chi_{\rho}K$ is the NET external semi-direct product of NETGS H and K, subsequently $\overline{H}=H\times\{1\}$ is a NET normal subgroup of N and it's obvious that N is the NET internal semi-direct product of $H\times\{1\}$ and $\{1\}\times K$. Because of this we can go from NET external to internal semi-direct products. The following conclusion goes in the another way, from NET internal to external semi-direct products.

Proposition 4.1.1 Suppose that N is a NETG with NET subgroups H and K, and N is the NET internal semi-direct product of H and K. Then $N\simeq H\chi_{\rho}K$ where

$$\rho: K\to Aut(H)$$

is stated by

$$\rho_{(k,neut(k),anti(k))}\big((h,neut(h),anti(h))\big)=(k,neut(k),anti(k))(h,neut(h),anti(h))\big((k,neut(k),anti(k))\big)^{-1},$$
$$(h,neut(h),anti(h))\in H,(k,neut(k),anti(k))\in K.$$

Proof Note that $\rho_{(k,neut(k),anti(k))}$ refers to $Aut(H)$ where H is NET normal. By the lemma 4.1.1, all element $(n,neut(n),anti(n))$ of N can be inscribed particularly in terms of

$$(h,neut(h),anti(h))(k,neut(k),anti(k)),$$

with $(h,neut(h),anti(h))\in H$ and $(k,neut(k),anti(k))\in K$. So that, the map

$$\varphi: H\chi_{\rho}K\to N,$$

$$\varphi\big((h,neut(h),anti(h))(k,neut(k),anti(k))\big)=(h,neut(h),anti(h))(k,neut(k),anti(k))$$

is a bijection. It only remains to prove such this bijection is a neutro-homomorphism. Stated

$$\big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big)$$

and

$$\big((h',neut(h'),anti(h')),(k',neut(k'),anti(k'))\big)\text{in } H\chi_{\rho}K.$$

We have

$$\varphi\left(\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\begin{pmatrix}(h',neut(h'),anti(h')),(k',neut(k'),\\ anti(k'))\end{pmatrix}\right)$$

$$=\varphi\left(\begin{pmatrix}(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\left((h',neut(h'),anti(h'))\right),\\ (k,neut(k),anti(k))(k',neut(k'),anti(k'))\end{pmatrix}\right)$$

$$=\varphi\begin{pmatrix}(h,neut(h),anti(h))(k,neut(k),anti(k))(h',neut(h'),anti(h'))\\ (k,neut(k),anti(k))^{-1},(k,neut(k),anti(k))(k',neut(k'),anti(k'))\end{pmatrix}$$

$$=(h,neut(h),anti(h))(k,neut(k),anti(k))(h',neut(h'),anti(h'))(k',neut(k'),anti(k'))$$

$$=\varphi\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\varphi\begin{pmatrix}(h',neut(h'),anti(h')),\\ (k',neut(k'),anti(k'))\end{pmatrix}.$$

Therefore φ is a NETG neutro-homomorphism, which ends the proof. Shortly, we obtain such all NET internal semi-direct product is neutro-isomorphic to any NET external semi-direct product, when φ is conjugation.

CHAPTER 5

NEUTROSOPHIC EXTENDED TRIPLET GROUP ACTION

A NETG action is a representation of the elements of a NETG as a symmetries of a NET set. It is a precise method of solving the technique in which the elements of a NETG meet transformations of any space in a method that maintains the structure of that space. Just as a group action plays an important role in the classical group theory, NETG action enacts identical role in the theory of NETG theory.

5.1 Left NETG Action

Definition 5.1.1 An action of N on X (left NETG action) is a map $N \times X \to X$ denoted

$$\big((n, neut(n), anti(n)), (x, neut(x), anti(x))\big) \to (n, neut(n), anti(n))(x, neut(x), anti(x))$$

as shown

$$1(x, neut(x), anti(x)) = (x, neut(x), anti(x))$$

and

$$\begin{aligned}&(n, neut(n), anti(n))\big((h, neut(h), anti(h))(x, neut(x), anti(x))\big)\\&= \big((n, neut(n), anti(n))(h, neut(h), anti(h))\big)(x, neut(x), anti(x))\end{aligned}$$

for all $(x, neut(x), anti(x))$ in X and $(n, neut(n), anti(n)), (h, neut(h), anti(h))$ in $N.$ Given a NET action of N on X, we call X a $N-$set. A $N-$map between $N-$sets X and Y is a map $f : X \to Y$ of NET sets that respects the $N-$action, meaning that,

$$\begin{aligned}&f\big((n, neut(n), anti(n))(x, neut(x), anti(x))\big)\\&= (n, neut(n), anti(n))\, f\big((x, neut(x), anti(x))\big)\end{aligned}$$

for all $(x, neut(x), anti(x))$ in X and $(n, neut(n), anti(n))$ in $N.$ To give a NET action of N on X is equivalent to giving a NETG neutro-homomorphism from N to the

NETG of bijections of X.

Note that a NETG action is not the same thing as a binary structure, we combine two elements of X to get a third element of X (we combine two apples and get an apple). In a NETG action, we combine an element of N with an element of X to get an element of X (we combine an apple and an orange and get another orange).

It is critical to note that

$(n,neut(n),anti(n))\cdot\big((h,neut(h),anti(h))\cdot(x,neut(x),anti(x))\big)$ has two actions of N on elements of X. Under other conditions

$\big((n,neut(n),anti(n))(h,neut(h),anti(h))\big)\cdot(x,neut(x),anti(x))$

has one multiplication in the NETG $\big((n,neut(n),anti(n))(h,neut(h),anti(h))\big)$ and then one action of an element of N on X.

Example 5.1.1 For a NET subgroup $H\subset N$, consider the left NT coset space $N/H=\{(a,neut(a),anti(a))H:(a,neut(a),anti(a))\in N\}$. (We do not care wether or not $H\lhd N$, as we are just thinking about N/H as a set.) Let N act on N/H by left multiplication. That is for $(n,neut(n),anti(n))\in N$ and a left NT coset $(a,neut(a),anti(a))H$ ($(a,neut(a),anti(a))\in N$), set

$$(n,neut(n),anti(n))\cdot(a,neut(a),anti(a))H=(n,neut(n),anti(n))(a,neut(a),anti(a))H$$
$$=\left\{\begin{matrix}(n,neut(n),anti(n))(y,neut(y),anti(y)):\\(y,neut(y),anti(y))\in(a,neut(a),anti(a))H\end{matrix}\right\}.$$

This is an action of N on N/H, since

$$1_N(a,neut(a),anti(a))H=(a,neut(a),anti(a))H$$

and

$$(n_1,neut(n_1),anti(n_1))\cdot\big((n_2,neut(n_2),anti(n_2))\cdot(a,neut(a),anti(a))H\big)$$
$$=(n_1,neut(n_1),anti(n_1))\cdot\big((n_2,neut(n_2),anti(n_2))(a,neut(a),anti(a))H\big)$$

$$= (n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))(a, neut(a), anti(a))H$$
$$= \big((n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))\big)(a, neut(a), anti(a))H.$$

Note (Groups acting independently by multiplication).

All NETG acts independently like so, NET set $N = N$ and $X = N.$ Then for

$$(n, neut(n), anti(n)) \in N$$

and

$$(n, neut(n), anti(n)) \in X = N,$$

we define $(n, neut(n), anti(n)) \cdot \big((n, neut(n), anti(n))\big)$

$$= (n, neut(n), anti(n))\big((n, neut(n), anti(n))\big) \in X = N.$$

Example 5.1.2 Each NETG N acts independently $(X = N)$ by left multiplication functions. In other words, we set

$$\pi_{(n,neut(n),anti(n))} : N \to N$$

by

$$\pi_{(n,neut(n),anti(n))}\big((h,neut(h),anti(h))\big) = (n,neut(n),anti(n))(h,neut(h),anti(h))$$

for all $(n, neut(n), anti(n)) \in N$ and $(h, neut(h), anti(h)) \in H$. Subsequently, the axioms for being a NETG action are

$$1_N (h,neut(h),anti(h)) = (h,neut(h),anti(h))$$

for all $(h, neut(h), anti(h)) \in N$ and

$$(n_1,neut(n_1),anti(n_1))\big((n_2,neut(n_2),anti(n_2))(h,neut(h),anti(h)\big)$$

$$= \big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)(h,neut(h),anti(h))$$

for all $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2)),(h,neut(h),anti(h)) \in N,$ which are both true whereby 1_N is a neutrality and multiplication in N is associative.

The notation for the NET effect of N is

$$\pi(n, neut(n), anti(n))$$

or

$$\pi(n, neut(n), anti(n))\big((x, neut(x), anti(x))\big)$$

simply as

$$(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$$

or

$$(n, neut(n), anti(n))(x, neut(x), anti(x)).$$

In this explanation, the conditions for the left NETG action take the succeeding shape :

i. for all $(x, neut(x), anti(x)) \in X, 1_N(x, neut(x), anti(x)) = (x, neut(x), anti(x))$.

ii. for every $(n_1, neut(n_1), anti(n_1)), (n_2, neut(n_2), anti(n_2)) \in N$ and $(x, neut(x), anti(x)) \in X$,

$$(n_1, neut(n_1), anti(n_1)) \cdot \big((n_2, neut(n_2), anti(n_2)) \cdot (x, neut(x), anti(x))\big)$$

$$= \big((n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2))\big) \cdot (x, neut(x), anti(x)).$$

Theorem 5.1.1 Let a NETG action N act on the NET set X. If $(x, neut(x), anti(x)) \in X$, $(n, neut(n), anti(n)) \in N$, and

$$(y, neut(y), anti(y)) = (n, neut(n), anti(n))(x, neut(x), anti(x)),$$

then

$$(x, neut(x), anti(x)) = (n, neut(n), anti(n))^{-1} \cdot (y, neut(y), anti(y)).$$

If

$$(x, neut(x), anti(x)) \neq (x', neut(x'), anti(x'))$$

then

$$\begin{aligned}&(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x)) \\ &\neq (n, neut(n), anti(n)) \cdot (x', neut(x'), anti(x')).\end{aligned}$$

Proof From $(y, neut(y), anti(y)) = (n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$ we get

$$\begin{aligned}&(n, neut(n), anti(n))^{-1} \cdot (y, neut(y), anti(y)) \\ &= (n, neut(n), anti(n))^{-1} \left((n, neut(n), anti(n))(x, neut(x), anti(x)) \right)\end{aligned}$$

$$= \left((n, neut(n), anti(n))^{-1} (n, neut(n), anti(n)) \right) (x, neut(x), anti(x))$$

$$= 1_N (x, neut(x), anti(x))$$

$$= (x, neut(x), anti(x)).$$

To show $(x, neut(x), anti(x)) \neq (x', neut(x'), anti(x')) \Rightarrow$

$$(n, neut(n), anti(n))(x, neut(x), anti(x)) \neq (n, neut(n), anti(n))(x', neut(x'), anti(x')),$$

we show the contrapositive : if

$$(n, neut(n), anti(n))(x, neut(x), anti(x)) = (n, neut(n), anti(n))(x', neut(x'), anti(x'))$$

then applying $(n, neut(n), anti(n))^{-1}$ to both sides gives

$$\begin{aligned}&(n, neut(n), anti(n))^{-1} \cdot \left((n, neut(n), anti(n)) \cdot (x, neut(x), anti(x)) \right) \\ &= (n, neut(n), anti(n))^{-1} \cdot \left((n, neut(n), anti(n)) \cdot (x', neut(x'), anti(x')) \right)\end{aligned}$$

so

$$\begin{aligned}&\left((n, neut(n), anti(n))^{-1} (n, neut(n), anti(n)) \right) \cdot (x, neut(x), anti(x)) \\ &= \left((n, neut(n), anti(n))^{-1} (n, neut(n), anti(n)) \right) \cdot (x', neut(x'), anti(x'))\end{aligned}$$

so

$$(x, neut(x), anti(x)) = (x', neut(x'), anti(x')).$$

On the other hand to imagine action of a NETG on a NET set is such it’s a definite neutro-homomorphism. On hand are the facts.

Theorem 5.1.2 Actions of the NETG N on the NET set X are indentical NETG neutro-homomorphisms from $N \to Sym(X)$, the NETG of permutaions of X.

Proof Assume we've an action of N on the NET set X. We observe $(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x))$ as a function of $(x, neut(x), anti(x))$ (with $(n, neut(n), anti(n))$ fixed). That is, for each $(n, neut(n), anti(n)) \in N$ we have a function

$$\pi_{(n,neut(n),anti(n))} : X \to X$$

by

$$\pi_{(n,neut(n),anti(n))}\big((x, neut(x), anti(x))\big) = (n, neut(n), anti(n)) \cdot (x, neut(x), anti(x)).$$

The axiom $1_N \cdot (x,neut(x),anti(x)) = (x,neut(x),anti(x))$ says π_1 is the neutrality function on X. The axiom

$$(n_1,neut(n_1),anti(n_1))\big((n_2,neut(n_2),anti(n_2)) \cdot (x,neut(x),anti(x)\big)$$

$$=\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big) \cdot (x,neut(x),anti(x))$$

says

$$\pi_{(n_1,neut(n_1),anti(n_1))} \circ \pi_{(n_2,neut(n_2),anti(n_2))}$$
$$= \pi_{(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))},$$

so structure of functions on X match multiplication in N. Additionally, $\pi_{(n,neut(n),anti(n))}$ is an invertible function whereby $\pi_{(n_1,neut(n_1),anti(n_1))^{-1}}$ is an anti-neutral: the composite of $\pi_{(n_1,neut(n_1),anti(n_1))}$ and $\pi_{(n_1,neut(n_1),anti(n_1))^{-1}}$ is π_1, which is the neutral function on X. Therefore, $\pi_{(n_1,neut(n_1),anti(n_1))} \in Sym(X)$ and $(n, neut(n), anti(n)) \to \pi_{(n_1,neut(n_1),anti(n_1))}$ is a neutro-homomorphism $N \to Sym(X)$.

Contrariwise, assume we've a homomorphism $f : N \to Sym(X)$. For every $(n, neut(n), anti(n))$, we have a permutation $f\big((n, neut(n), anti(n))\big)$ on X, and

$$f\left((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\right)$$

$$= f\left((n_1,neut(n_1),anti(n_1))\right)\circ f\left((n_2,neut(n_2),anti(n_2))\right).$$

Setting

$$(n, neut(n), anti(n))\cdot(x, neut(x), anti(x))$$

$$= f\left((n, neut(n), anti(n))\right)\left((x, neut(x), anti(x))\right)$$

introduces a NETG action of N on X, whereby the neutro-homomorphism properties of f submits the defining properties of a NETG action. From this view point, the NET set of $(n, neut(n), anti(n)) \in N$ that act trivially

$$\left((n, neut(n), anti(n))\cdot(x, neut(x), anti(x))\right) = (x, neut(x), anti(x))$$

for all $(x, neut(x), anti(x)) \in X$ is straightforwardly the neutrosophic kernel of the neutro-homomorphism $N \to Sym(X)$ related to the action. Consequently the above mentioned $(n, neut(n), anti(n))$ such act trivially on X are assumed to lie in the neutrosophic kernel of the action.

Example 5.1.3 To build N act independently by conjugation, take $X = N$ and let

$$(n, neut(n), anti(n))\cdot(x, neut(x), anti(x))$$
$$=(n,neut(n),anti(n))(x,neut(x),anti(x))(n,neut(n),anti(n))^{-1}.$$

Here, $(n, neut(n), anti(n)) \in N$ and $(x, neut(x), anti(x)) \in N$. Since

$$1_N\cdot(x, neut(x), anti(x))$$
$$=1_N(x,neut(x),anti(x))1_N{}^{-1}$$
$$= (x, neut(x), anti(x))$$

and

$$(n_1, neut(n_1), anti(n_1)) \cdot \big((n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x))\big)$$
$$= (n_1, neut(n_1), anti(n_1)) \cdot$$
$$\Big((n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x))(n_2,neut(n_2),anti(n_2))^{-1}\Big)$$
$$=(n_1,neut(n_1),anti(n_1))$$
$$\Big((n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x))(n_2,neut(n_2),anti(n_2))^{-1}\Big)$$
$$(n_1,neut(n_1),anti(n_1))^{-1}$$
$$=\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)(x,neut(x),anti(x))$$
$$\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)^{-1}$$
$$= \big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big) \cdot (x, neut(x), anti(x)),$$

neutrosophic conjugation is a NET action.

Definition 5.1.2 Assume such N is a NETG and X is a NET set. A right NETG action of N on X is a rule for merging elements $(n, neut(n), anti(n)) \in N$ and elements $(x, neut(x), anti(x)) \in X$, symbolized by

$$(n, neut(n), anti(n)) \cdot (x, neut(x), anti(x)),$$
$$(n, neut(n), anti(n)) \cdot \big((x, neut(x), anti(x))\big) \in X \text{ for all } (x, neut(x), anti(x)) \in X$$

and

$(n, neut(n), anti(n)) \in N.$ We also need the succeecing conditionss.

i. $(x,neut(x),anti(x))1_N=(x,neut(x),anti(x))$ for all $(x, neut(x), anti(x)) \in X$.

ii. $\big((x,neut(x),anti(x))\cdot(n_2,neut(n_2),anti(n_2))\big)\cdot(n_1,neut(n_1),anti(n_1))$
$=(x,neut(x),anti(x))\big((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\big)$

for all $(x, neut(x), anti(x)) \in X$

and

$$(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2))\in N.$$

Remark 5.1.1 Left NETG actions are not very distinct from right NETG actions. The only distinction exists in condition (ii).

- For left NETG actions, implementing $(n_2, neut(n_2), anti(n_2))$ to an element and then applying $(n_1, neut(n_1), anti(n_1))$ to the result is the same as applying $(n_1, neut(n_1), anti(n_1))(n_2, neut(n_2), anti(n_2)) \in N$.
- For right NETG actions applying $(n_2, neut(n_2), anti(n_2))$ and then $(n_1, neut(n_1), anti(n_1))$ is the same as applying $(n_2, neut(n_2), anti(n_2))(n_1, neut(n_1), anti(n_1)) \in N$.

Let us see the example of a right NETG action (beyond the Rubik's cube example, which as we wrote things is a right NETG action). Also it is easy to do matrices multplying vectors from the right.

Example 5.1.4 (A NETG acting on a NET set of NT cosets). Assume such N is a NETG and H is a NET subgroup. Examine the NET set $X = \{Ha / (a, neut(a), anti(a)) \in N\}$ of right NT cosets of H. Subsequently N acts on X by right multiplication, That is, we describe

$$\begin{aligned} &\left(H(a, neut(a), anti(a))\right) \cdot (n, neut(n), anti(n)) \\ &= H\left((a, neut(a), anti(a))(n, neut(n), anti(n))\right) \end{aligned}$$

for $(n, neut(n), anti(n)) \in N$ and $H(a, neut(a), anti(a)) \in X$. First let's chect that this is well defined, hence assume such $H(a, neut(a), anti(a)) = H(a', neut(a'), anti(a'))$, then $(a', neut(a'), anti(a'))(a, neut(a), anti(a))^{-1} \in H$. Now, we have to prove that

$$\begin{aligned} &H\left((a, neut(a), anti(a))(n, neut(n), anti(n))\right) \\ &= H\left((a', neut(a'), anti(a'))(n, neut(n), anti(n))\right) \end{aligned}$$

for any $(n, neut(n), anti(n)) \in N$. But $(a', neut(a'), anti(a'))(a, neut(a), anti(a))^{-1} \in H$ so that

$$\begin{aligned} &(a', neut(a'), anti(a'))(n, neut(n), anti(n)) \\ &= \left((a', neut(a'), anti(a'))(a, neut(a), anti(a))^{-1}\right)\begin{pmatrix} (a, neut(a), \\ anti(a))(n, neut(n), anti(n)) \end{pmatrix} \\ &\in H\left((a, neut(a), anti(a))(n, neut(n), anti(n))\right) \end{aligned}$$

So that

$$(a', neut(a'), anti(a'))(n, neut(n), anti(n)) \in H\begin{pmatrix}(a, neut(a), anti(a))(n, neut(n), \\ anti(n))\end{pmatrix}.$$

But certainly $H\left((a', neut(a'), anti(a'))(n, neut(n), anti(n))\right)$ also contains

$$\begin{aligned}&1_N\left((a', neut(a'), anti(a'))(n, neut(n), anti(n))\right)\\ &= (a', neut(a'), anti(a'))(n, neut(n), anti(n)).\end{aligned}$$

Thus the two cosets

$$H\left((a, neut(a), anti(a))(n, neut(n), anti(n))\right)$$

and

$$H\left((a', neut(a'), anti(a'))(n, neut(n), anti(n))\right)$$

have the elements $(a', neut(a'), anti(a'))(n, neut(n), anti(n))$ in common. This proves that

$$\begin{aligned}&H\left((a, neut(a), anti(a))(n, neut(n), anti(n))\right)\\ &= H\left((a', neut(a'), anti(a'))(n, neut(n), anti(n))\right)\end{aligned}$$

since NT cosets are either same or separate.

Now we've proved that this is well defined, we have to show it is also an action. Definetly axiom (i) is holds since

$$\left(H(a, neut(a), anti(a))\right)\cdot 1_N = H\left((a, neut(a), anti(a))1_N\right) = H(a, neut(a), anti(a)).$$

Lastly, we have to show axiom (ii). Assume such

$$(n_1, neut(n_1), anti(n_1)), (n_2, neut(n_2), anti(n_2)) \in N.$$

Then

$$
\begin{aligned}
&\left(\left(H(a,neut(a),anti(a))\right)\cdot(n_2,neut(n_2),anti(n_2))\right)\cdot(n_1,neut(n_1),anti(n_1))\\
&=\left(H\left((a,neut(a),anti(a))(n_2,neut(n_2),anti(n_2))\right)\right)\cdot(n_1,neut(n_1),anti(n_1))\\
&=H\left(\left((a,neut(a),anti(a))(n_2,neut(n_2),anti(n_2))\right)\right)(n_1,neut(n_1),anti(n_1))\\
&=H\left((a,neut(a),anti(a))\left((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\right)\right)\\
&=\left(H(a,neut(a),anti(a))\right)\cdot\left((n_2,neut(n_2),anti(n_2))(n_1,neut(n_1),anti(n_1))\right)
\end{aligned}
$$

which proves (ii) and ends the proof.

Of course, N also acts on the set of left NT cosets of H by multiplication on the left.

Definition 5.1.3 A NETG action of N on X is called NET faithful if distinct elements of N act on X in dis-similar methods: when

$$(n_1,neut(n_1),anti(n_1))\neq(n_2,neut(n_2),anti(n_2))$$

in N, there is an $(x,neut(x),anti(x))\in X$ such that

$$(n_1,neut(n_1),anti(n_1))\cdot(x,neut(x),anti(x))\neq(n_2,neut(n_2),anti(n_2))\cdot(x,neut(x),anti(x)).$$

Note that when we say $(n_1,neut(n_1),anti(n_1))$ and $(n_2,neut(n_2),anti(n_2))$ act distinctly, we signfy they act distinctly somewhere, not all place. This is consistent with what it signfys to say two functions are disjoint. They take distinct values somewhere, not all place.

Example 5.1.5 The action of N independently by left multiplication is faithful : distinct elements send 1_N to distinct places.

Example 5.1.6 When H is a NET subgroup of N and N acts on N/H left multiplication $(n_1,neut(n_1),anti(n_1))$ and $(n_2,neut(n_2),anti(n_2))$ in N act in the similar method on N/H exactly when

$$
\begin{aligned}
&(n_1,neut(n_1),anti(n_1))(n,neut(n),anti(n))H\\
&=(n_2,neut(n_2),anti(n_2))(n,neut(n),anti(n))H
\end{aligned}
$$

for all $(n,neut(n),anti(n))\in N$, which means

$$(n_2, neut(n_2), anti(n_2))^{-1}(n_1, neut(n_1), anti(n_1)) \in \bigcap_{(n,neut(n),anti(n))}$$
$$\in N(n, neut(n), anti(n))H(n, neut(n), anti(n))^{-1}.$$

So the left multiplication action of N on N/H is NET faithful in the case that the NET subgroups $(n, neut(n), anti(n))H(n, neut(n), anti(n))^{-1}$ (as $(n, neut(n), anti(n))$ varies) have trivial intersection.

Viewing NETG actions as neutro-homomorphisms, a NET faithful action of N on X is an injective neutro-homomorphism $N \to Sym(X)$. Non faithful actions are not injective as NETG neutro-homomorphisms, and many important homomorphisms are not injective.

Remark 5.1.2 What we've been calling a NETG action could be a left and right NETG action. The difference among left and right actions is how a product $(n, neut(n), anti(n))(n', neut(n'), anti(n'))$ acts : in a left action $(n', neut(n'), anti(n'))$ acts first and $(n, neut(n), anti(n))$ acts second, while in a right action $(n, neut(n), anti(n))$ acts first and $(n', neut(n'), anti(n'))$ acts second.

We can introduce the NET conjugate of $(h, neut(h), anti(h))$ by $(n, neut(n), anti(n))$ as

$$(n, neut(n), anti(n))(h, neut(h), anti(h))(n, neut(n), anti(n))$$

instead

$$(n, neut(n), anti(n))(h, neut(h), anti(h))(n, neut(n), anti(n))^{-1},$$

and this convention fits well with the right NET conjugation action but not left action : setting

$$(h, neut(h), anti(h))^{(n,neut(n),anti(n))}$$
$$= (n, neut(n), anti(n))^{-1}(h, neut(h), anti(h))(n, neut(n), anti(n))$$

we have

$$(h, neut(h), anti(h))^{1_N} = (h, neut(h), anti(h))$$

and

$$\left((h,neut(h),anti(h))^{(n_1,neut(n_1),anti(n_1))}\right)^{(n_2,neut(n_2),anti(n_2))}$$
$$=(h,neut(h),anti(h))^{(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))}.$$

The distinction among left and right actions of a NETG is mostly unreal, whereby subsetituting $(n,neut(n),anti(n))$ with $(n,neut(n),anti(n))^{-1}$ in the NETG changes left actions into right actions and contrarily since inversion backwards the order of multiplication in N. So for us "NETG action" means "left NETG action".

5.2 NET Orbit and Stabilizers

Definition 5.2.1 Let a NETG N act on NET set X. For each $(x,neut(x),anti(x))\in X$, its orbit is

$$Orb(x,neut(x),anti(x))=\{(n,neut(n),anti(n))(x,neut(x),anti(x)):(n,neut(n),anti(n))\in N\}$$
$$\subset X$$

and its stabilizer is

$$Stab(x,neut(x),anti(x))=\{(n,neut(n),anti(n))\in N:(n,neut(n),anti(n))(x,neut(x),anti(x))\}$$
$$\subset N.$$

(The stabilizer of NET $(x,neut(x),anti(x))$ is symbolized by $N(x,neut(x),anti(x))$, where N is NETG.)

We call $(x,neut(x),anti(x))$ a NET fixed point for the action when

$$(n,neut(n),anti(n))\cdot(x,neut(x),anti(x))=(x,neut(x),anti(x))$$

for every $(n,neut(n),anti(n))\in N$, that is, when

$$Orb(x,neut(x),anti(x))=\{(x,neut(x),anti(x))\}$$

(or equivalently, when $Stab(x,neut(x),anti(x))=N$). The orbit of NETs of a point is a geometric notion: it is the NET set of places where the points can be moved by the NETG action. Under other conditions, the stabilizer of a NET of a point is an algebraic notion: it is the NET set of NETG elements that fix the point. Mostly we'll denote the elements of X as points and we'll denote the size of a NET orbit as its length.

Definition 5.2.2 Let N be a NETG, $(n, neut(n), anti(n)) \in N,$ and let H be a NET subgroup of $N.$

$$(a, neut(a), anti(a))H(a, neut(a), anti(a))^{-1}$$
$$=\left\{\begin{matrix}(a, neut(a), anti(a))(h, neut(h), anti(h))(a, neut(a), anti(a))^{-1}: \\ (h, neut(h), anti(h)) \in H\end{matrix}\right\}$$

is called a NET conjugate of H and the NET center of N is

$$Z\,N=\left\{\begin{matrix}(a,neut(a),anti(a))\in N:(a,neut(a),anti(a))(n,neut(n),anti(n)) \\ =(n,neut(n),anti(n))(a,neut(a),anti(a)):\forall(n,neut(n),anti(n))\in N\end{matrix}\right\}.$$

Remark 5.2.1 When we imagine about a NET set as a geometric object, it is useful to describe to its elements as points. For instance, when we imagine about N / H as a NET set on which N acts, it is helpful to imagine about the NT cosets of H, which are the elements N / H, as the points in $N / H.$ Simultaneously, though, a NT coset is a NET subset of $N.$

All of our applications of NETG actions to group theory will flow from the similarities among NET orbits, stabilizers, and fixed points, which we now build explicit in our the following fundamental examples of NETG actions.

Example 5.2.1 When a NETG N acts independently by conjugation,

a) the NET orbit of $(a, neut(a), anti(a))$ is

$$Orb_{(a,neut(a),anti(a))}=\left\{\begin{matrix}(n,neut(n),anti(n))(a,neut(a),anti(a)) \\ (n,neut(n),anti(n))^{-1}:(n,neut(n),anti(n))\in N\end{matrix}\right\},$$

which is the conjugacy class of $(a, neut(a), anti(a)),$

b) $$Stab_{(a,neut(a),anti(a))}=\left\{\begin{matrix}(n,neut(n),anti(n)):(n,neut(n),anti(n)) \\ (a,neut(a),anti(a))(n,neut(n),anti(n))^{-1} \\ =(a,neut(a),anti(a))\end{matrix}\right\}$$

c) $Z(a,neut(a),anti(a)) = \begin{cases} (n,neut(n),anti(n)) \\ :(n,neut(n),anti(n))(a,neut(a),anti(a)) \end{cases}$
$=(a,neut(a),anti(a))(n,neut(n),anti(n)) \}$

is the NET centralizer of $(a,neut(a),anti(a))$.

d) $(a,neut(a),anti(a))$ is a NET fixed point when it commutes with all elements of N, and thus the NET fixed points of conjugation form the NET center of N, and thus the NET fixed points of NET conjugation form the center of N.

Example 5.2.2 When H acts on N by conjugation,

i. the orbit of $(a,neut(a),anti(a))$ is

$$Orb(a,neut(a),anti(a)) = \left\{ \begin{array}{l} (h,neut(h),anti(h))(a,neut(a),anti(a)) \\ (h,neut(h),anti(h))^{-1} : (h,neut(h),anti(h)) \in H \end{array} \right\},$$

which has no special name (elements of N that are $H-$conjugate to $(a,neut(a),anti(a))$),

ii.
$$\begin{aligned} Stab(a,neut(a),anti(a)) &= \{(h,neut(h),anti(h)) : \\ &(h,neut(h),anti(h))(a,neut(a),anti(a))(h,neut(h),anti(h))^{-1} \\ &= (h,neut(h),anti(h)) \ \} \\ &= \{(h,neut(h),anti(h)) : (h,neut(h),anti(h))(a,neut(a),anti(a)) \\ &= (a,neut(a),anti(a))(h,neut(h),anti(h)) \ \} \end{aligned}$$

is the elements of H commuting with $(a,neut(a),anti(a))$ (this is $H \cap Z((a,neut(a),anti(a)))$ is the NET centralizer of $(a,neut(a),anti(a))$ in N).

iii. $(a,neut(a),anti(a))$ is a NET fixed point when it commutes with all elements of H, so the NET fixed points of $H-$conjugation on N shape the NET centralizer of H in N.

5.3 The Fundamental Theorem About NETG Actions

Theorem 5.3.1 Let a NETG N act on a NET set X.

a. Different NET orbits of the action are disjoint and form a partion of X.

b. For each $(x,neut(x),anti(x))\in X$, $Stab(x,neut(x),anti(x))$ is a NET subgroup of N and

$$Stab(n,neut(n),anti(n))(x,neut(x),anti(x)) = (n,neut(n),anti(n))$$
$$Stab(x,neut(x),anti(x))Stab(n,neut(n),anti(n))(n,neut(n),anti(n))^{-1}$$

for all $(n,neut(n),anti(n))\in N$.

c. For each $(x,neut(x),anti(x))\in X$, there is a bijection

$$Orb(x,neut(x),anti(x)) \to N/Stab(x,neut(x),anti(x))$$

by

$$\begin{aligned}&(n,neut(n),anti(n))(x,neut(x),anti(x))\\ &\to (n,neut(n),anti(n))Stab(x,neut(x),anti(x)).\end{aligned}$$

More concretely,

$$\begin{aligned}&(n,neut(n),anti(n))(x,neut(x),anti(x))\\ &= (n',neut(n'),anti(n'))(x,neut(x),anti(x))\end{aligned}$$

in the case that $(n,neut(n),anti(n))$ and $(n',neut(n'),anti(n'))$ lie in the similar NET coset of $Stab(x,neut(x),anti(x))$, and different NT left cosets of $Stab(x,neut(x),anti(x))$ correspond to different points in $Orb(x,neut(x),anti(x))$. In particular, if $(x,neut(x),anti(x))$ and $(y,neut(y),anti(y))$ are in the same NET orbit then

$$\left\{\begin{aligned}&(n,neut(n),anti(n))\in N : (n,neut(n),anti(n))(x,neut(x),anti(x))\\ &= (y,neut(y),anti(y))\end{aligned}\right\}$$

is a NT left coset of $Stab(x,neut(x),anti(x))$,

and

$$\left|Orb(x,neut(x),anti(x))\right| = \left[N : Stab(x,neut(x),anti(x))\right].$$

Parts b and c Show the role of conjugate NET subgroups and neutrosophic triplet cosets of a NET subgroup when working with NETG actions. The formula in part c that relates the length of a NET orbit to the index in N of a NET stabilizer for a point in the NET orbit, is named the NET orbit-stabilzer formula.

Proof :

a. We show distinct NET orbits in a NETG action are not equal by showing that two NET orbits that overlap must coexist.

Assume $Orb(x,neut(x),anti(x))$ and $Orb(y,neut(y),anti(y))$ have a common element $(z,neut(z),anti(z))$.

$$(z,neut(z),anti(z)) = (\eta_1, neut(\eta_1), anti(\eta_1))(x,neut(x),anti(x))$$
$$(z,neut(z),anti(z)) = (\eta_2, neut(\eta_2), anti(\eta_2))(y,neut(y),anti(y)).$$

We want to show $Orb(x,neut(x),anti(x))$ and $Orb(y,neut(y),anti(y))$. It suffices to show $Orb(x,neut(x),anti(x)) \subset Orb(y,neut(y),anti(y))$, since then we can switch the roles of $(x,neut(x),anti(x))$ and $(y,neut(y),anti(y))$ to obtain the converse insertion. For each point $(u,neut(u),anti(u)) \in Orb(x,neut(x),anti(x))$, write

$$(u,neut(u),anti(u)) = (n,neut(n),anti(n))(x,neut(x),anti(x))$$

for some $(n,neut(n),anti(n)) \in N$. Since

$$\begin{aligned}
&(x,neut(x),anti(x)) \\
&= (n_1,neut(n_1),anti(n_1))^{-1}(z,neut(z),anti(z)),(u,neut(u),anti(u)) \\
&= (u,neut(u),anti(u))\Big((n_1,neut(n_1),anti(n_1))^{-1}(z,neut(z),anti(z))\Big) \\
&= \Big((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}\Big)(z,neut(z),anti(z)) \\
&= \Big((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}\Big)\begin{pmatrix}(n_2,neut(n_2),anti(n_2)) \\ (y,neut(y),anti(y))\end{pmatrix} \\
&= \Big((n,neut(n),anti(n))(n_1,neut(n_1),anti(n_1))^{-1}(n_2,neut(n_2),anti(n_2))\Big) \\
&(y,neut(y),anti(y)),
\end{aligned}$$

which shows us that $(u,neut(u),anti(u)) \in Orb(y,neut(y),anti(y))$. Therefore $Orb(x,neut(x),anti(x)) \subset Orb(y,neut(y),anti(y))$. Every element of X is in some

NET orbit (its own NET orbits), so the NET orbits partition X into disjoint NET subsets.

b. To see that $Stab_{(x,neut(x),anti(x))}$ is a NET subgroup of N, we've $1_N \in Stab_{(x,neut(x),anti(x))}$ since $1_N(x,neut(x),anti(x))=(x,neut(x),anti(x))$, and if $(n_1,neut(n_1),anti(n_1)),(n_2,neut(n_2),anti(n_2)) \in Stab_{(x,neut(x),anti(x))}$,

then

$$\begin{aligned}
&\big((n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\big)(x,neut(x),anti(x)) \\
&=(n_1,neut(n_1),anti(n_1))\big((n_2,neut(n_2),anti(n_2))(x,neut(x),anti(x))\big) \\
&=(n_1,neut(n_1),anti(n_1))(x,neut(x),anti(x)) \\
&=(x,neut(x),anti(x)),
\end{aligned}$$

so $(n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2)) \in Stab_{(x,neut(x),anti(x))}$. Thus $Stab_{(x,neut(x),anti(x))}$ is closed under multiplication.

Lastly,

$$\begin{aligned}
&(n_1,neut(n_1),anti(n_1))(x,neut(x),anti(x))=(x,neut(x),anti(x)) \\
&\Rightarrow (n,neut(n),anti(n))^{-1}\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big) \\
&=(n,neut(n),anti(n))^{-1}(x,neut(x),anti(x)) \\
&\Rightarrow (x,neut(x),anti(x))=(n,neut(n),anti(n))^{-1}(x,neut(x),anti(x)),
\end{aligned}$$

so $Stab_{(x,neut(x),anti(x))}$ is closed under inversion.

To prove

$$\begin{aligned}
&Stab_{(n,neut(n),anti(n))(x,neut(x),anti(x))} \\
&=(n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}(n,neut(n),anti(n))^{-1},
\end{aligned}$$

for all $(x,neut(x),anti(x)) \in X$ and $(n,neut(n),anti(n)) \in N$,

observe that

$$(h,neut(h),anti(h))\in Stab_{(n,neut(n),anti(n))(x,neut(x),anti(x))}$$
$$\Leftrightarrow (h,neut(h),anti(h))\cdot\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big)$$
$$=(n,neut(n),anti(n))(x,neut(x),anti(x))$$
$$\Leftrightarrow \big((h,neut(h),anti(h))(n,neut(n),anti(n))\big)(x,neut(x),anti(x))$$
$$=(n,neut(n),anti(n))(x,neut(x),anti(x))$$
$$\Leftrightarrow (n,neut(n),anti(n))^{-1}\begin{pmatrix}\big((h,neut(h),anti(h))(n,neut(n),anti(n))\big)\\ (x,neut(x),anti(x))\end{pmatrix}$$
$$=(n,neut(n),anti(n))^{-1}\big((n,neut(n),anti(n))(x,neut(x),anti(x))\big)$$
$$\Leftrightarrow \big((n,neut(n),anti(n))^{-1}(h,neut(h),anti(h))(n,neut(n),anti(n))\big)$$
$$(x,neut(x),anti(x))=(x,neut(x),anti(x))$$
$$\Leftrightarrow (n,neut(n),anti(n))^{-1}(h,neut(h),anti(h))(n,neut(n),anti(n))$$
$$\in Stab_{(x,neut(x),anti(x))}$$
$$\Leftrightarrow (h,neut(h),anti(h))\in (n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}$$
$$(n,neut(n),anti(n))^{-1},$$

So

$$Stab_{(x,neut(x),anti(x))}(x,neut(x),anti(x))$$
$$=(n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}(n,neut(n),anti(n))^{-1}.$$

c. The condition

$$(n,neut(n),anti(n))(x,neut(x),anti(x))$$
$$=(n',neut(n'),anti(n'))(x,neut(x),anti(x))$$

is equivalent to

$$(x,neut(x),anti(x))$$
$$=\big((n,neut(n),anti(n))^{-1}(n',neut(n'),anti(n'))\big)(x,neut(x),anti(x)),$$

which means $(n,neut(n),anti(n))^{-1}(n',neut(n'),anti(n'))\in Stab_{(x,neut(x),anti(x))}$, or

$$(n',neut(n'),anti(n'))\in (n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}.$$

Therefore $(n,neut(n),anti(n))$ and $(n',neut(n'),anti(n'))$ have the same effect on $(x,neut(x),anti(x))$ in the case that $(n,neut(n),anti(n))$ and $(n',neut(n'),anti(n'))$ lie in the similar NT coset of $Stab_{(x,neut(x),anti(x))}$. (Recall that for all NET subgroups H and

$$N, (n', neut(n'), anti(n')) \in (n, neut(n), anti(n))H \Leftrightarrow$$
$$(n', neut(n'), anti(n'))H = (n, neut(n), anti(n))H.)$$

Whereby $Orb_{(x,neut(x),anti(x))}$ consists of the points $(n, neut(n), anti(n))(x, neut(x), anti(x))$ for varying $(n, neut(n), anti(n)),$ and we showed elements of N have the similar effect on $(x, neut(x), anti(x))$ if and only if they lie in the similar NT left coset of $Stab_{(x,neut(x),anti(x))}$, we get a bijection between the points in the NET orbit of $(x, neut(x), anti(x))$ and the NT left cosets of $Stab_{(x,neut(x),anti(x))}$ by

$$(n, neut(n), anti(n))(x, neut(x), anti(x))$$
$$\rightarrow (n, neut(n), anti(n))Stab_{(x,neut(x),anti(x))}.$$

Therefore the cardinality of the NET orbit of $(x, neut(x), anti(x)),$ which is $\left|Orb_{(x,neut(x),anti(x))}\right|$ equals the cardinality of the NT left cosets of $Stab_{(x,neut(x),anti(x))}$ in $N.$

Remark 5.3.1 That the NET orbits of a NETG action are a partition results in a NETG theory : conjugacy classes are a partitioning of a NETG and the NT left cosets of a NET subgroup partition the NETG. The first result utilizes the action of a NETG independently by NET conjugation, having NET conjugacy classes as its NET orbits. The second result utilizes the right inverse multiplication action of the NET subgroup on the NETG.

Corollary 5.3.1 Let a finite NETG act on a NET set.

a) The length of every NET orbit divides the size of $N.$

b) Points in a common NET orbit have conjugate stabilizers, and in particular the size of the NET stabilizer is the similar for all points in a NET orbit.

Proof a) The length of NET orbit is an index of a NET subgroup, so it divides $|N|.$

b) If $(x, neut(x), anti(x))$ and $(y, neut(y), anti(y))$ are in the same NET orbit, write

$$(y, neut(y), anti(y)) = (n, neut(n), anti(n))(x, neut(x), anti(x)).$$

Then

$$\begin{aligned}Stab_{(y,neut(y),anti(y))} &= Stab_{(n,neut(n),anti(n))(x,neut(x),anti(x))} \\ &= (n,neut(n),anti(n))Stab_{(x,neut(x),anti(x))}(n,neut(n),anti(n))^{-1},\end{aligned}$$

so the NET stabilizers of $(x, neut(x), anti(x))$ and $(y, neut(y), anti(y))$ are conjugate NET subgroups.

A converse of part b is not generally true : points with NET conjugate stabilizers need not be in the same NET orbit. Even points with the same NET stabilizer need nor be in the same NET orbit. For example, if N acts on itself trivially then all poiints have NET stabilizer N and all orbits have size 1.

Corollary 5.3.2 Let a NETG N acts on a NET set X, where X is finite. Let the distinct NET orbits of X be symbolized by $(x_1,neut(x_1),anti(x_1)),\ldots,(x_t,neut(x_t),anti(x_t))$. Then

$$|X| = \sum_{i=1}^{t} \left|Orb(x_i, neut(x_i), anti(x_i))\right| = \sum_{i=1}^{t} \left[N : Stab(x_i, neut(x_i), anti(x_i))\right].$$

Proof The NET set X can be written as the union of its NET orbits, which are mutually disjoint. The NET orbit-stabilizer formula tells us how large each NET orbit is.

Example 5.3.1 As an application of the NET orbit-stabilizer formula we describe why $|HK| = {|H||K|}/{|H \cap K|}$ for NET subgroups H and K of a finite NETG N. At this point

$$HK = \begin{Bmatrix} (h, neut(h), anti(h)), (k, neut(k), anti(k)) : (h, neut(h), anti(h)) \in H, \\ (K, neut(K), anti(K)) \in K \end{Bmatrix}$$

is the NET set of products, which ususally is just a subset of N. To count the size of HK, let the direct product of NETG $H \times K$ act on the NET set HK like this :

$$\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big) \cdot (x, neut(x), anti(x))$$
$$= (h, neut(h), anti(h))(x, neut(x), anti(x))(h, neut(h), anti(h))^{-1},$$

which gives us a NETG action (the NETG is $H \times K$ and the NET set is HK). There is only 1 NET orbit wherby

$$1_N = 1_N 1_N \in HK$$

and

$$(h, neut(h), anti(h)), (k, neut(k), anti(k))$$
$$= \Big((h, neut(h), anti(h)), (k, neut(k), anti(k))^{-1}\Big) \cdot 1_N.$$

So that the NET orbit-stabilizer formula shows us

$$|HK| = \frac{|H \times K|}{|Stab 1_N|}$$

$$= \frac{|H||K|}{\left|\left\{\begin{array}{l}\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big) : (h, neut(h), anti(h)), (k, neut(k), anti(k)) \cdot 1_N \\ = 1_N\end{array}\right\}\right|}.$$

The condition $\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big) \cdot 1_N = 1_N$ means

$$(h, neut(h), anti(h))(k, neut(k), anti(k))^{-1} = 1_N,$$

so

$$Stab 1_N = \big\{\big((h, neut(h), anti(h))(h, neut(h), anti(h))\big) : (h, neut(h), anti(h)) \in H \cap K\big\}.$$

So that

$$|Stab 1_N| = |H \cap K|$$

and

$$|HK| = {|H||K|}/{|H \cap K|}.$$

Theorem 5.3.2 Burnside's Lemma

Let a finite NETG N act on a finite NET set X in relation to r NET orbits. Subsequently r is the average number of NET fixed points of the elements of the NETG.

$$r=\frac{1}{|N|}\sum_{(n,neut(n),anti(n))\in N}\left|Fix_{(n,neut(n),anti(n))}(X)\right|,$$

where

$$Fix_{(n,neut(n),anti(n))}(X)=\left\{\begin{matrix}(x,neut(x),anti(x))\in X:(n,neut(n),anti(n)) \\ (x,neut(x),anti(x))=(x,neut(x),anti(x))\end{matrix}\right\}$$

is the NET set of elements of X fixed by $(n, neut(n), anti(n))$.

Don't confuse the NET set $Fix_{(n,neut(n),anti(n))}(X)$ in relation to the NET fixed points of the action : $Fix_{(n,neut(n),anti(n))}(X)$ is only the points fixed by the elements $(n, neut(n), anti(n))$. The NET set of NET fixed points for the action of N is the intersection of the NET sets $Fix_{(n,neut(n),anti(n))}(X)$ as $(n, neut(n), anti(n))$ runs over the NETG.

Proof We will count

$$\left\{\begin{matrix}\left((n,neut(n),anti(n)),(x,neut(x),anti(x))\right)\in N\times X: \\ (n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x))\end{matrix}\right\}$$

in two ways.

By counting over $(n, neut(n), anti(n))$'s first we have to add up the number of $(x, neut(x), anti(x))'s$ with

$$(n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x)),$$

so

$$\left|\left\{\begin{matrix}\big((n,neut(n),anti(n)),(x,neut(x),anti(x))\big)\in N\times X: \\ (n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x))\end{matrix}\right\}\right|$$

$$=\sum_{(n,neut(n),anti(n))\in N}\left|Fix_{(n,neut(n),anti(n))}(X)\right|.$$

Next we count over the $(x,neut(x),anti(x))$'s and have to add up the number of $(n,neut(n),anti(n))$'s with

$$(n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x)),$$

i.e., with

$$(n,neut(n),anti(n))\in Stab_{(x,neut(x),anti(x))}:$$

$$\left|\left\{\begin{matrix}\big((n,neut(n),anti(n)),(x,neut(x),anti(x))\big)\in N\times Y: \\ (n,neut(n),anti(n))(x,neut(x),anti(x))=(x,neut(x),anti(x))\end{matrix}\right\}\right|$$

$$=\sum_{(X,neut(X),anti(X))\in X}\left|Stab_{(x,neut(x),anti(x))}\right|.$$

Equating these two counts gives

$$=\sum_{(n,neut(n),anti(n))\in N}\left|Fix_{(n,neut(n),anti(n))}(X)\right|$$
$$=\sum_{(X,neut(X),anti(X))\in X}\left|Stab_{(x,neut(x),anti(x))}\right|.$$

By the NET orbit-stabilizer formula,

$$|N| \Big/ \left|Stab_{(x,neut(x),anti(x))}\right| = \left|Orb_{(x,neut(x),anti(x))}\right|,$$

so

$$\sum_{(n,neut(n),anti(n))\in N} \left|Fix_{(n,neut(n),anti(n))}(X)\right|$$
$$= \sum_{(X,neut(X),anti(X))\in X} \frac{|N|}{\left|Orb_{(x,neut(x),anti(x))}\right|}.$$

Divide by $|N|$:

$$\frac{1}{|N|}\sum_{(n,neut(n),anti(n))\in N} \left|Fix_{(n,neut(n),anti(n))}(X)\right|$$
$$= \sum_{(x,neut(x),anti(x))\in X} \frac{1}{\left|Orb_{(x,neut(x),anti(x))}\right|}.$$

Let's examine the benefaction to the right side from points in a single NET orbit. If a NET orbit has n points in it, subsequently the sum over the points in that NET orbit is a sum of $\frac{1}{n}$ for n terms, and in other words equal to 1. Consequently the part of the sum over points in a NET orbit is 1, which makes the sum on the right side equal to the number of NET orbits, which is r.

Definition 5.3.1 Two actions of NETG N on a NET sets X and Y are called NET equivalent if there is a bijection $f: X \to Y$ as shown

$$f\left((n,neut(n),anti(n))(x,neut(x),anti(x))\right)$$
$$=(n,neut(n),anti(n))\,f\left((x,neut(x),anti(x))\right)$$

for all $(n,neut(n),anti(n)) \in N$ and $(x,neut(x),anti(x)) \in X$.

Actions of N on two NET sets are equivalent when N permutes elements in the similar method on the two NET sets following matching up the NET sets properly. When $f: X \to Y$ is a NET equivalence of NETG actions on X and Y,

$$(n,neut(n),anti(n))(x,neut(x),anti(x)) = (x,neut(x),anti(x))$$

if and only if

$$(n,neut(n),anti(n))\left(f\left((x,neut(x),anti(x))\right)\right) = f\left((x,neut(x),anti(x))\right),$$

so the NET stabilizer subgroups of $(x, neut(x), anti(x)) \in X$ and $f(x, neut(x), anti(x)) \in Y$ are the same.

Example 5.3.2 Let H and K be NET subgroup of N. The NETG N acts by left multiplication on N/H and N/K. If H and K are NET conjugate subgroups then these actions are equivalent: fix a representation $K=(n_0,neut(n_0),anti(n_0))H(n_0,neut(n_0),anti(n_0))^{-1}$ for some $(n_0,neut(n_0),anti(n_0))\in N$ and let

$$f : N/H \to N/K$$

by

$$f\left((n, neut(n), anti(n))H\right) = (n, neut(n), anti(n))(n_0, neut(n_0), anti(n_0))^{-1}K.$$

This is well-defined (independent of the NT coset representatives for $(n, neut(n), anti(n))H$) since, for $(h, neut(h), anti(h)) \in H$,

$f\left((n,neut(n),anti(n))h,neut(h),anti(h))H\right)$
$=(n,neut(n),anti(n))(h,neut(h),anti(h))(n_0,neut(n_0),anti(n_0))^{-1}K$
$=(n,neut(n),anti(n))(h,neut(h),anti(h))(n_0,neut(n_0),anti(n_0))^{-1}$
$H(n_0,neut(n_0),anti(n_0))^{-1}$
$=(n,neut(n),anti(n))H(n_0,neut(n_0),anti(n_0))^{-1}$
$= (n, neut(n), anti(n))(n_0, neut(n_0), anti(n_0))^{-1}K.$

There can be multiple equivalences between two equivalent NETG actions, just as there can be multiple neutro-isomorphisms between two isomorphic NETGs. If H and K are not NET conjugate then the actions have the same NET stabilizer subgroup, but the NET stabilizer subgroups of left NT cosets in N/H are NET conjugate to K, and none of the former and the latter are equal.

Theorem 5.3.3 An action of N that has one NET orbit is equivalent to the left multiplication action of N on some left NT coset space of N.

Proof Assume that N acts on the NET set X in relation to one NET orbit. $Fix_{(x_0,neut(x_0),anti(x_0))} \in X$ and let $H = Stab_{(x_0,neut(x_0),anti(x_0))}$. We will Show the action of N on X is equivalent to the left multiplication action of N on N/H.

Every $(x, neut(x), anti(x)) \in X$ has the form $(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0))$ for some $(n, neut(n), anti(n)) \in N$, and all elements in a left NT coset $(n, neut(n), anti(n))H$ have the same effect on $(x_0,neut(x_0),anti(x_0))$: for all $(h, neut(h), anti(h)) \in H$,

$$\begin{aligned}&\big((n,neut(n),anti(n))(h,neut(h),anti(h))\big)\big((x_0,neut(x_0),anti(x_0))\big)\\&=(n,neut(n),anti(n))\big((h,neut(h),anti(h))(x_0,neut(x_0),anti(x_0))\big).\end{aligned}$$

Let $f : N/H \to X$ by

$$f\big((n,neut(n),anti(n))H\big)=(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0)).$$

This is well defined, as we just saw. Moreover,

$$\begin{aligned}&\big((n, neut(n), anti(n)) \cdot (n', neut(n'), anti(n'))H\big)\\&= (n, neut(n), anti(n))\, f\,\big((n', neut(n'), anti(n'))H\big)\end{aligned}$$

since both sides equal

$$(n,neut(n),anti(n))(n',neut(n'),anti(n'))\big((n,neut(n),anti(n))\cdot(x_0,neut(x_0),anti(x_0))\big).$$

We will show f is a bijection. Since X has one NET orbit,

$$\begin{aligned}X &= \{(n,neut(n),anti(n))(x_0,neut(x_0),anti(x_0)):(n,neut(n),anti(n))\in N\}\\&= \{ f\,\big((n, neut(n), anti(n))H\big) : (n, neut(n), anti(n)) \in N\},\end{aligned}$$

so f is onto.

If $f\big((n_1,neut(n_1),anti(n_1))H\big)=f\big((n_2,neut(n_2),anti(n_2))H\big)$ then

$$\begin{aligned}&(n_1, neut(n_1), anti(n_1))(x_0, neut(x_0), anti(x_0))\\&= (n_2, neut(n_2), anti(n_2))(x_0, neut(x_0), anti(x_0)),\end{aligned}$$

so

$$(n_2, neut(n_2), anti(n_2))^{-1}(n_1, neut(n_1), anti(n_1))(x_0, neut(x_0), anti(x_0))$$
$$= (x_0, neut(x_0), anti(x_0)).$$

Since $(x_0,neut(x_0),anti(x_0))$ has NET stabilizer H,

$$(n_2,neut(n_2),anti(n_2))^{-1}(n_1,neut(n_1),anti(n_1))\in H,$$

so

$$(n_1,neut(n_1),anti(n_1))H=(n_2,neut(n_2),anti(n_2))H.$$

Consequently f is one – to –one.

A special condition of this theorem tells that an action of N is equivalent to the left multiplication action of N independently in the case that the action has one NET orbit and the NET stabilizer subgroup are trivial.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

The most important point of this thesis is first to define the NETs and subsequently use these NETs to describe the NET internal and external direct and semi-direct products of NETG. As in classical group theory, in neutrosophic extended triplet group building blocks for finite NET groups is simple NET groups. One way to make this simple NETG to larger group is NET direct product. We also explained some special properties of this newly born algebraic structures and their application to NETG. Then, NETG action as a reprsentation of the elementsof a NET group as a symmetries of a NET set is introduced and the fundamental theorem about NETG actions is given and proved. Furthermore, we defined orbits, stabilizers, fixed points, conjugation and centralizer for NETG. As an addition, we allow rise to a new field called NT Structures (such as neutrosophic extended triplet direct product, semi-direct product, and neutrosophic extended triplet group action. Another researchers can work on the application of NETG theory to NT vector spaces (representation of the NETG), number theory, analysis, geometry, and topological spaces.

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