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On Studying Certain Fuzzy Neutrosophic Matrices and Operators

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ABSTRACT

In this paper, some types of neutrosophic fuzzy matrices have been detailed and the following operators $\oplus$, $\circledast$, $\ominus$ are defined for this neutrosophic fuzzy matrices. Certain Properties based on these operators $\oplus$, $\circledast$, $\ominus$ have been presented. In addition, some results on the existing operators like $\vee$, $\wedge$ along with these operators are also put forth. Numerical examples are also provided.

Keywords: Neutrosophic set; Neutrosophic fuzzy matrix

Mathematical Subject Classification (2010):15B15

1. Introduction

Neutrosophic set is a new mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In 1988, Smarandache introduce the concept of a neutrosophic set from a philosophical point of view. The neutrosophic set is a powerful general framework that generalizes the concept of fuzzy set and intutionistic fuzzy set. Each element had three associated defining functions, namely the membership function (T), indeterminacy function (I), the non-membership function (F) defined on the universe of discourse X, the three functions are completely independent. The theory has been found extensive application in various fields [] for dealing with indeterminate and inconsistent information in real world.

Fuzzy matrices play an important role in science and technology. However, the classical matrix theory sometimes fails to solve the problems involving uncertainities, occurring in an imprecise environment. In [7] Thomasan introduced fuzzy matrices to represent fuzzy relation in a system based on fuzzy set theory and discussed about the convergence of powers of fuzzy matrix. In 2004, Kandasamy and Smarandache [5] introduced fuzzy relational maps and neutrosophic relational maps. In 2014, Smarandache [6] introduced a type of neutrosophic matrices, called Square neutrosophic matrices, whose entries are of the form a+ I b (neutrosophic number), where a,b are the elements of [0,1] and I is an indeterminate such that $I^n = I$, n being a positive integer.

The paper is organized as follows: Section 2 deals with the preliminaries of Neutrosophic fuzzy set. In Section 3, after reviewing the concepts of Neutrosophic fuzzy set, some special types of Neutrosophic fuzzy matrices are introduced. Section 4 demonstrate these operators $\oplus$, $\circledast$, $\ominus$ on Neutrosophic fuzzy matrix and investigate some properties based on these operators. Numerical examples presented in section 5. Conclusion is given in last section.

2. Preliminaries

Definition 2.1. (Neutrosophic set) Let X be a non -empty set. A neutrosophic set A is an object having the form A = $\{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ where the functions μ_A: X $\to]^-0, 1^+[$, σ_A: X $\to]^-0, 1^+[$, ν_A: X $\to]^-0, 1^+[$ denote the degree of membership function , degree of indeterminacy function, degree of non-membership function respectively of each element $x \in X$ to the set A and $^-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+$, for each $x \in X$.

Definition 2.2. (Neutrosophic fuzzy set) Let X be a non - empty set. A neutrosophic fuzzy set A is an object having the form A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions μ_A: X $\to$[0,1] , σ_A: X$\to$[0,1] ,ν_A: X$\to$[0,1] denote the degree of membership function , degree of indeterminacy function , degree of non-membership function respectively of each element $x \in X$ to the set A and $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$, for each $x \in X$.

Definition 2.3. (Complement of a neutrosophic fuzzy set) Let X be a non-empty set and let A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and B = $\{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$ two neutrosophic fuzzy sets on X then

(i) A $\subseteq$ B iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.

(ii) A = B iff A $\subseteq$ B and B $\subseteq$ A.

(iii) $\bar{A} = \{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$.

(iv) $A \ominus$B = $\{\langle x, \mu_A(x)\rangle \wedge \nu_B(x)$, $\sigma_A(x) \wedge$ (1- $\sigma_B(x)$), $\nu_A(x) \vee \mu_B(x)\}$.

Definition 2.4. (Emptyneutrosophic fuzzy set) A Neutrosophic fuzzy set A over the universe X is said to be null or empty Neutrosophic fuzzy set if $\mu_A(x) = 0, \sigma_A(x) = 0, \nu_A(x) = 1$ for all $x \in X$.It is denoted by 0_N.

Definition 2.5. (Absolute neutrosophic fuzzy set)

A Neutrosophic fuzzy set A over the universe X is said to be absolute neutrosophic fuzzy set if $\mu_A(x) = 1, \sigma_A(x) = 1, \nu_A(x) = 0$ for all $x \in X$. It is denoted by I_N.

Definition 2.6. (Union and Intersection of neutrosophic fuzzy sets) Let X be a non empty set and let A_i $i \in J$ be an arbitrary family of Neutrosophic fuzzy sets in X where $A_i = \{\langle x, \mu_{A_i}(x), \sigma_{A_i}(x), \nu_{A_i}(x)\rangle : x \in X\}$, then

(i) $\cup A_i = \{\langle x, \vee_{i \in J} \mu_A(x), \vee_{i \in J} \sigma_A(x), \wedge_{i \in J} \nu_A(x)\rangle$

(ii) $\cap A_i = \{\langle x, \wedge_{i\in J}\mu_A(x), \wedge_{i\in J}\sigma_A(x), \vee_{i\in J}\nu_A(x)\rangle$

3. Fundamental neutrosophic fuzzy matrices

Definition 3.1. (Neutrosophic fuzzy matrix) Neutrosophic fuzzy matrix of order m × n is defined as A = $(a_{ij})_{m \times n}$ where a_{ij} = ($\mu_a(x)$, $\sigma_a(x)$, $\gamma_a(x)$) is the ij th element of A where $\mu_a(x)$ denote the degree of membership function, $\sigma_a(x)$ denote the degree of indeterminacy function and $\gamma_a(x)$ denote the degree of non membership function respectively.

Definition 3. 2. (Null neutrosophic fuzzy matrix) Neutrosophic fuzzy matrix is said to be Null Neutrosophic fuzzy matrix if all its entries are zero, i.e., all elements are (0,0,1).

Definition 3.3. (Unit neutrosophic fuzzy matrix) A square Neutrosophic fuzzy matrix is said to be unit Neutrosophic fuzzy matrix ifa_{ii} = (1,1,0) and a_{ij}= (0,0,1),i $\neq j$ for all i,j. It is denoted by I.

Definition 3.4 (Symmetric neutrosophic fuzzy matrix) A square Neutrosophic fuzzy matrix is said to be symmetric Neutrosophic fuzzy matrix if $a_{ij} = a_{ji}$.

Definition 3.5. (Triangular neutrosophic fuzzy matrix) A square Neutrosophic fuzzy matrix is said to be triangular Neutrosophic fuzzy matrix if either a_{ij} = (0,0,1) for all i$>$ $j$or a_{ij} = (0,0,1) for all i$<$ j: i,j =1,2,…,nz.

A square Neutrosophic fuzzy matrix is said to be upper triangular Neutrosophic fuzzy matrix if either a_{ij} = (0,0,1) for all i$> j$ and is said to be lower triangular Neutrosophic fuzzy matrix if a_{ij} = (0,0,1) for all i$< j$: i, j =1,2,…,n.

3.1. Special neutrosophic fuzzy matrices

Definition 3.1.1. Vandermonde neutrosophic fuzzy matrix

A Vandermonde Neutrosophic fuzzy matrix of order n is of the form

$$V = \begin{bmatrix} (0,0,1) & a_1 & a_1^2 \ldots & a_1^{n-1} \\ (0,0,1) & a_2 & a_2^2 \ldots & a_1^{n-1} \\ \vdots & \vdots & \vdots & \ddots \vdots \\ (0,0,1) & a_n & a_n^2 \ldots & a_n^{n-1} \end{bmatrix}$$

A Vandermonde Neutrosophic fuzzy matrix is also called an alternate neutrosophic fuzzy matrix. Sometimes, the transpose of an alternate fuzzy matrix is known as the Vandermon defuzzy matrix.

Definition 3.1.2. Moore neutrosophic fuzzy matrix

A moore fuzzy neutrosophic matrix is a Neutrosophic fuzzy matrix of the form

$$M = \begin{bmatrix} x_1 & x_1^q & \dots & x_1^{q^{n-1}} \\ x_2 & x_2^q & \dots & x_2^{q^{n-1}} \\ \vdots & \vdots & \vdots & \ddots \vdots \\ x_m & x_m^q & \dots & x_m^{q^{n-1}} \end{bmatrix}$$

Definition 3.1.3. Anti –diagonal neutrosophic fuzzy matrix

Anti – diagonal is the diagonal of a fuzzy matrix starting from the lower left corner to the upper right corner of the matrix. An anti-diagonal neutrosophic fuzzy matrix is a Neutrosophic fuzzy matrix where all the entries are zero except the anti – diagonal.

$$A = \begin{bmatrix} a & (0,0,1) & (0,0,1) \\ (0,0,1) & b & (0,0,1) \\ (0,0,1) & (0,0,1) & c \end{bmatrix}$$

Neutrosophic fuzzy matrix is a square Neutrosophic fuzzy matrix which has zeros in all entries except the first row, first column, and the main diagonal. The general form of an Arrow head matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & (0,0,1) & (0,0,1) \\ a_{31} & (0,0,1) & a_{33} & (0,0,1) \\ a_{41} & (0,0,1) & (0,0,1) & a_{44} \end{bmatrix}$$

Definition 3.1.5. Band neutrosophic fuzzy matrix

A band Neutrosophic fuzzy matrix is a sparse Neutrosophic fuzzy matrix whose non zero entries are confined to a diagonal band, comprising the main diagonal and zero or more diagonals on either side. The general form of a Band matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & (0,0,1) & (0,0,1) & (0,0,1) \\ a_{21} & a_{22} & a_{23} & (0,0,1) & (0,0,1) \\ (0,0,1) & a_{32} & a_{33} & a_{34} & (0,0,1) \\ (0,0,1) & (0,0,1) & a_{43} & a_{44} & a_{45} \\ (0,0,1) & (0,0,1) & (0,0,1) & a_{54} & a_{55} \end{bmatrix}$$

Let us consider a $n \times n$ fuzzy matrix A = (a_{ij}). If all the elements of the matrix are zero outside a diagonally bordered band whose range is determined by the constants p and q as follows:

$a_{ij} = 0$ if j < $i - p$ or $j > i + q, p, q \geq 0$, then the quantities p and q are called the lower and upper bandwidth, respectively. The bandwidth of the matrix is the maximum of p and q , that is it is the number r such that $a_{ij} = 0$ if $|i - j| > r$.

A Neutrosophic fuzzy matrix is called a banded neutrosophic fuzzy matrix if its bandwidth is reasonably small. A band Neutrosophic fuzzy matrix with p = q=0 is a diagonal Neutrosophic fuzzy matrix. A band Neutrosophic fuzzy matrix with p = q = 1 is a tridiagonal Neutrosophic fuzzy matrix. When p = q = 2, it is a pentadiagonal Neutrosophic fuzzy matrix. If p = 0, q = n-1, we get an upper triangular Neutrosophic fuzzy matrix. Similiarly, for p = n-1,q = 0, we obtain a lower triangular Neutrosophic fuzzy matrix.

Definition 3.1.6. Binary neutrosophic fuzzy matrix
A Binary Neutrosophic fuzzy matrix or a Boolean fuzzy neutrsophic matrix is a being Neutrosophic fuzzy matrix with entries either zero or one.

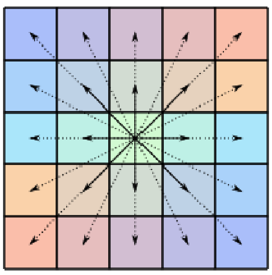
Definition 3.1.7. Permutation neutrosophic fuzzy matrix
A Permutation Neutrosophic fuzzy matrix is a square fuzzy neutrsophic matrix obtained from the same size identity matrix by a permutation of rows. Every row and column has a single (1,1,0) with (0,0,1) 's everywhere else.

Definition 3.1.8. Exchange neutrosophic fuzzy matrix
The exchange fuzzy neutrsophic matrix is an anti-diagonal fuzzy neutrsophic matrix in which all the entries in the anti- diagonal are (1,1,0) and all the other elements are (0,0,1). It is also a special case of permutation Neutrosophic fuzzy matrix.

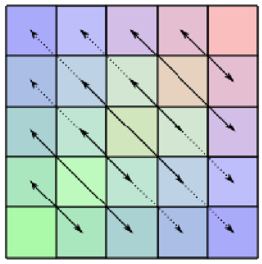
Definition 3.1.9. Bisymmetric neutrosophic fuzzy matrix
A bisymmetric Neutrosophic fuzzy fuzzy matrix is a square fuzzy neutrsophic matrix which is symmetric about both of its main diagonals. More precisely an n×n matrix B is bisymmetric Neutrosophic fuzzy fuzzy matrix if it satisfies both B = B^T and BE = EB where E is the n×n exchange fuzzy neutrsophic matrix.

$$B = \begin{bmatrix} a\,b\,c\,d\,e \\ f\,b\,g\,h\,d \\ c\,g\,i\,g\,c \\ d\,h\,g\,f\,b \\ e\,d\,c\,b\,a \end{bmatrix}$$

Definition 3.1.10. Centro symmetric neutrosophic fuzzy matrix
A matrix which is symmetric about its center is called centro symmetric fuzzy neutrsophic matrix. More precisely A = $[A_{ij}]$ is centro symmeteric when its entries satisfy $A_{i,j} = A_{m1-i+1,m-j+1}$ for $1 \leq i, j \leq m$. (i.e) If E denotes the m×m matrix with all the counterdiagonal elements as (1,1,0) and (0,0,1) elsewhere ,then a matrix A is centrosymmetric if and only if AE = EA. The matrix E sometimes named as the exchange matrix.



Definition 3.1.11. Persymmetric neutrosophic fuzzy matrix
A persymmetric fuzzy neutrsophic matrix is a square fuzzy neutrsophic matrix which is symmetric in the northeast-to-southeast diagonal or a square fuzzy neutrsophic matrix such that the values on each line perpendicular to the main diagonal are the same for a given line.



Definition 3.1.12. Block neutrosophic fuzzy matrix
A Block Neutrosophic fuzzy matrix is a fuzzy neutrsophic matrix which is obtained using smaller matrices called blocks.It is also called partitioned fuzzy neutrsophic matrices.
For example, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ where A,B,C,D are fuzzy neutrsophic matrices,is a Block Neutrosophic fuzzy matrix.

Definition 3.1.13. Sparseneutrosophic fuzzy matrix
A Neutrosophic fuzzy matrix in which most of its elements are zero is called a sparse fuzzy neutrsophic matrix. Also, if most of the elements are nonzero, then the neutrosophic fuzzy matrix is said to be dense.

Definition 3.1.14. Toeplitzneutrosophic fuzzy matrix
A $n \times n$ Neutrosophic fuzzy matrix in which the negative sloping diagonal elements are constants is called a Toeplitzneutrosophic fuzzy matrix . Its of the form

$$T = \begin{bmatrix} x_0 & x_{-1} x_{-2} \cdots x_{-(n-1)} & & & \\ x_1 & x_0 & x_{-1} & \ddots & \ddots \\ x_2 & x_1 & x_0 & \ddots & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots \\ x_{n-1} & \ddots & \ddots & \ddots & x_0 \end{bmatrix}$$

4. Operators on neutrosophic fuzzy matrices
Let $\widetilde{M} = (M_{ij})$ and $\widetilde{N} = (N_{ij})$ be two Neutrosophic fuzzy matrices of order $n \times n$ then

(i) M⊕ N = $[(m_{ij} + n_{ij}) - (m_{ij} \,.\, n_{ij})]$
(ii) M ⊛ N = $[m_{ij} \,.\, n_{ij}]$
(iii) M⊖N = $[m_{ij} - n_{ij}]$
(iv) M' = $[m_{ji}]$
(v) M $\wedge N$= $[m_{ij} \wedge n_{ij}]$
(vi) M$\vee N$ = $[m_{ij} \vee n_{ij}]$

Property 1. If M and N are symmetric Neutrosophic fuzzy matrices then M⊕N, M ⊛ N, M⊖N and N⊖M are symmetric Neutrosophic fuzzy matrices.
Proof: Let M = $[m_{ij}]$ and N = $[n_{ij}]$ be two symmetric Neutrosophic fuzzy matrices
Therefore $m_{ij} = m_{ji}$ and $n_{ij} = n_{ji}$
Let c_{ij} be the ijth element of M⊕N
$c_{ij} = (m_{ij} + n_{ij}) - (m_{ij} \,.\, n_{ij}) = (m_{ji} + n_{ji}) - (m_{ji} \,.\, n_{ji}) = c_{ji}$

Hence M⊕N is symmetric Neutrosophic fuzzy matrices
Let d_{ij} be the ijth element of M⊛ N
$d_{ij}= m_{ij} .n_{ij} = m_{ji} . n_{ji} = d_{ji}$
Hence M⊛N is symmetric Neutrosophic fuzzy matrix.
Let e_{ij} be the ijth element of M ⊖ N
$e_{ij} = (m_{ij} - n_{ij}) = (m_{ji} - n_{ji}) = e_{ji}$
Hence N⊖M is symmetric Neutrosophic fuzzy matrix.
Let f_{ij} be the ijth element of M ⊖ N
$f_{ij} = (n_{ij} - m_{ij}) = (n_{ji} - m_{ji}) = f_{ji}$
Hence N ⊖M is symmetric Neutrosophic fuzzy matrix. ■

Property 2. Let M, N be any two Neutrosophic fuzzy matrices, then
(i) (M ⊛ N)′ ′= M ′ ⊛ N′
(ii) (M ⊕ N)′ ′= M ′ ⊕ N′
Proof: (i) Let c_{ij} and d_{ij} be the ijth element of M ⊛N and M ′ ⊛ N′ respectively.
$e_{ij} = c_{ji}$ is the ijth element of (M ⊛ N)′. Then $c_{ij}= m_{ij} . n_{ij}$. Thus $c_{ij} = m_{ji} . n_{ji}$. and $d_{ij}= m_{ji} . n_{ji}$. Therefore $d_{ij} = e_{ij}$ for all i,j.
Hence (M ⊛ N)′′= M ′ ⊛ N′.
(ii) Let c_{ij} and d_{ij} be the ijth element of M ⊕N and M ′ ⊕ N′ respectively.
Therefore $e_{ij} = c_{ji}$ is the ijth element of (M ⊕ N)′.
Then. $c_{ij}= (m_{ij} + n_{ij}) - (m_{ij} . n_{ij})$ and $d_{ij} = (m_{ji} + n_{ji}) - (m_{ji} . n_{ji})$
$e_{ij} = (m_{ji} + n_{ji}) - (m_{ji} . n_{ji}) = d_{ij}$.
Hence (M ⊕ N)′ ′= M′ ⊕N′. ■

Property3. Let M, N, P be three Neutrosophic fuzzy matrices, then
(i) M⊕N = N⊕M
(ii) M ⊛ N = N ⊛ M
(iii) (M⊕ N) ⊕P = M ⊕(N ⊕P)
(iv) (M ⊛ N) ⊛ P = M ⊛(N ⊛ P)
Proof:
(i) M⊕N =[$(m_{ij} + n_{ij}) - (m_{ij} . n_{ij})$] = [$(n_{ij} + m_{ij}) - (n_{ij} . m_{ij})$] = N⊕ M
(ii) M ⊛ N = $m_{ij} . n_{ij} = n_{ij} . m_{ij}$ = N ⊛ M
(iii) M⊕N = [$(m_{ij} + n_{ij}) - (m_{ij} . n_{ij})$] = r_{ij} and P = p_{ij}
(M⊕ N) ⊕P = [$(r_{ij} + p_{ij})- (r_{ij} .p_{ij})$] = M ⊕(N⊕P).
(iv) (M ⊛ N) ⊛ P = $(m_{ij} . n_{ij})\, p_{ij} = m_{ij} . n_{ij} .p_{ij} =m_{ij} .(n_{ij} .p_{ij})$ = M ⊛ (N ⊛ P)
(M⊕N) ⊕P = M ⊕(N⊕P). ■

Property 4. If M is a symmetric Neutrosophic fuzzy matrix then I_n⊕(M⊕M‘) is symmetric Neutrosophic fuzzy matrix .

Proof: M ⊕M‘ = [$(m_{ij} + m_{ji}) - (m_{ij} . m_{ji})$] and I_n⊕(M ⊕M‘) = t_{ij}, where $t_{ii} = 1$ and
$t_{ij} = [(m_{ij} + m_{ji}) - (m_{ij} . m_{ji})]$ for i$\neq j$.

Now, $t_{ji} = [(m_{ji} + m_{ij}) - (m_{ji} \, . \, m_{ij})] = t_{ij}$.That is each diagonal elements of $I_n \oplus$(M $\oplus$M') is 1 and all non diagonal elements are $(m_{ij} + m_{ji}) - (m_{ij} \, . \, m_{ji})$.

Hence $I_n \oplus$ (M$\oplus$M') is symmetric Neutrosophic fuzzy matrix. ■

Property 5. For any two Neutrosophic fuzzy matrices M and N , (M ∨ N) ∨ (M⊖N) = (M ∨ N).

Proof: Let c_{ij} , d_{ij} and e_{ij} be the ijth elements of (M∨N), (M⊖N) and (M ∨ N) ∨ (M⊖N) respectively.

Let $c_{ij} = \{m_{ij} \vee n_{ij}\} = \{ \mu_{m_{ij}} \vee \mu_{n_{ij}} , \sigma_{m_{ij}} \vee \sigma_{n_{ij}}, \gamma_{m_{ij}} \wedge \gamma_{n_{ij}}\}$

$d_{ij} = \{ \mu_{m_{ij}} \wedge \gamma_{n_{ij}} , \sigma_{m_{ij}} \wedge (1-\sigma_{n_{ij}}), \gamma_{m_{ij}} \vee \mu_{n_{ij}}\}$

$e_{ij} = \{ \mu_{m_{ij}} \vee \mu_{n_{ij}} , \sigma_{m_{ij}} \vee \sigma_{n_{ij}}, \gamma_{m_{ij}} \wedge \gamma_{n_{ij}}\} \vee \{\mu_{m_{ij}} \wedge \gamma_{n_{ij}} , \sigma_{m_{ij}} \wedge (1-\sigma_{n_{ij}}), \gamma_{m_{ij}} \vee \mu_{n_{ij}}\} = c_{ij}$.

Therefore (M ∨ N) ∨ (M⊖N) = (M ∨ N). ■

Property 6. Let M, N and P be three Neutrosophic fuzzy matrices, then M ⊕ **(**N∨P) = (M ⊕N) ∨ (M⊕P).

Proof: Let c_{ij} , d_{ij},e_{ij},f_{ij} *and* g_{ij} be the ijth elements of (N∨P), (M ⊕N), (M ⊕P),M ⊕ **(**N ∨ P) and (M ⊕N) ∨ (M⊕P) respectively.

Let $c_{ij} = \max \{ n_{ij} \vee p_{ij} \}$, $d_{ij} = [(m_{ij} + n_{ij}) - (m_{ij} \, . \, n_{ij})]$, $e_{ij} = [(n_{ij} + p_{ij}) - (n_{ij} \, . \, p_{ij})]$

$$f_{ij} = m_{ij} \oplus \max \{n_{ij} \vee p_{ij}\} = \begin{cases} [(m_{ij} + n_{ij}) - (m_{ij} \, . \, n_{ij})] \, , n_{ij} > p_{ij} \\ [(m_{ij} + p_{ij}) - (m_{ij} \, . \, p_{ij})] \, , n_{ij} \leq p_{ij} \end{cases}$$

$$g_{ij} = \max \{d_{ij} \vee e_{ij}\} = \begin{cases} [(m_{ij} + n_{ij}) - (m_{ij} \, . \, n_{ij})] \, , d_{ij} > e_{ij} \\ [(m_{ij} + p_{ij}) - (m_{ij} \, . \, p_{ij})] \, , d_{ij} \leq e_{ij} \end{cases}$$

Therefore, M ⊕ **(**N ∨ P) = (M ⊕N) ∨ (M⊕P).■

5. Numerical examples

Example 5.1.

M = $\begin{bmatrix} (0.2,0.3,0.5) & (0.1,0.3,0.5) \\ (0.1,0.3,0.5) & (0.4,0.3,0.3) \end{bmatrix}$, N = $\begin{bmatrix} (0.1,0.4,0.5) & (0.3,0.6,0.7) \\ (0.3,0.6,0.7) & (0.2,0.4,0.5) \end{bmatrix}$ are symmetric Neutrosophic fuzzy matrices.

(i) M ⊕ N = $\begin{bmatrix} (0.27,0.58, 0.75) & (0.27,0.72, 0.85) \\ (0.27,0.72, 0.85) & (0.27,0.58, 0.75) \end{bmatrix}$ is symmetric neutrosophic fuzzy matrix.

(ii) M ⊛N = $\begin{bmatrix} (0.02,0.12,0.25) & (0.03,0.18,0.35) \\ (0.03,0.18,0.35) & (0.02,0.12,0.25) \end{bmatrix}$ is symmetric neutrosophic fuzzy matrix.

(iii) $M \ominus N = \begin{bmatrix} (0.2,0.3,0.5) & (0.1,0.3,0.5) \\ (0.1,0.3,0.5) & (0.4,0.3,0.3) \end{bmatrix}$ is symmetric neutrosophic fuzzy matrix.

(iv) $N \ominus M = \begin{bmatrix} (0.1,0.4,0.5) & (0.3,0.6,0.7) \\ (0.3,0.6,0.7) & (0.2,0.4,0.5) \end{bmatrix}$ is symmetric neutrosophic fuzzy matrix.

Example 5.2.

$M = \begin{bmatrix} (0.3,0.2,0.5) & (0.4,0.3,0.3) \\ (0.3,0.1,0.6) & (0.2,0.3,0.5) \end{bmatrix}$, $N = \begin{bmatrix} (0.2,0.1,0.7) & (0.3,0.4,0.3) \\ (0.4,0.5,0.1) & (0.2,0.4,0.4) \end{bmatrix}$

$M' = \begin{bmatrix} (0.3,0.2,0.5) & (0.3,0.1,0.6) \\ (0.4,0.3,0.3) & (0.2,0.3,0.5) \end{bmatrix}$, $N' = \begin{bmatrix} (0.2,0.1,0.7) & (0.4,0.5,0.1) \\ (0.3,0.4,0.3) & (0.2,0.4,0.4) \end{bmatrix}$

(i) $(M \circledast N)' = \begin{bmatrix} (0.06,0.02,0.35) & (0.12,0.12,0.09) \\ (0.12,0.05,0.06) & (0.04,0.12,0.20) \end{bmatrix} = M' \circledast N'$

(ii) $(M \oplus N)' = \begin{bmatrix} (0.44,0.28,0.85) & (0.58,0.58,0.51) \\ (0.58,0.45,0.54) & (0.36,0.58,0.7) \end{bmatrix} = M' \oplus N'$.

Example 5.3. Consider M and N in Example 5.2

$M \vee N = \begin{bmatrix} (0.2,0.1,0.7) & (0.3,0.3,0.3) \\ (0.3,0.1,0.6) & (0.2,0.3,0.5) \end{bmatrix}$, $M \ominus N = \begin{bmatrix} (0.3,0.2,0.2) & (0.3,0.3,0.3) \\ (0.1,0.1,0.6) & (0.2,0.3,0.5) \end{bmatrix}$

$(M \vee N) \vee (M \ominus N) = \begin{bmatrix} (0.2,0.1,0.7) & (0.3,0.3,0.3) \\ (0.3,0.1,0.6) & (0.2,0.3,0.5) \end{bmatrix} = (M \vee N)$.

6. Conclusion

Up to this point we introduce Neutrosophic fuzzy matrix and some special types Neutrosophic fuzzy matrix and also we investigated the properties of Neutrosophic fuzzy matrix based on these operators $\oplus$, $\circledast$, $\ominus$. Further some future works are necessary to deal with determinants and properties of determinants of such kinds of matrices.

Conflict of Interests

The authors have declared that no Conflict of Interest exists.

REFERENCES

1. S.A.Albowi, A.A.Salama and Mohmed Esia, New concepts of neutrosophic sets, IJMCAR, 3 (2013) 95-102.
2. R.Antony and H.Alemayehu, A note on special matrices, Italian Journal of Applied Mathematics, 35 (2015) 587-604.
3. K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986) 87-96.
4. S.Broumi and F.Smarandache, Correlation coefficient of interval neutrosophic set, Applied Mechanics and Materials, 436 (2013) 511-517.
5. M.Dhar, S.Broumi and F.Smarandache, A note on square neutrosophic fuzzy matrices, Neutrosophic Sets and Systems, 3 (2014) 37-41.
6. P.Majumdar and S.K.Samanta, On similarity and entropy of neutrosophic sets, Journal of Intelligent & Fuzzy Systems, 26(3) (2014) 1245–1252.

7. M Pal, Fuzzy matrices with fuzzy rows and columns, Journal of Intelligent & Fuzzy Systems, 30 (1) (2016) 561-573.
8. J.J.Peng, J.Q.Wang, H.Y.Zhang and X.H.Chen, An outranking approach for multicriteria decision-making problems with simplified neutrosophic sets, Applied Soft Computing, 25 (2014) 336–346.
9. R.Sahin and A.Kucuk, Subsethood measure for single valued neutrosophic sets, Journal of Intelligent & Fuzzy Systems, 2014, DOI:10.3233/IFS-141304
10. A.A.Salama and S.A.Alblowi, Neutrosophic set and neutrosophic topological spaces, IOSR Journal of Mathematics, 3(4) (2012) 31-35.
11. A.K.Shyamal and M.Pal, Two new operators on fuzzy matrices, J. Appl. Math. and Computing, 15 (2004) 91-107.
12. F.Smarandache, Neutrosophic logic-A generalization of the intuitionistic fuzzy logic, Multispace and Multistructure Neutrosophic Transdisciplinarity, 4 (2010) 396-402.
13. F.Smarandache, Neutrosophic set-a generalization of the intuitionistic fuzzy logic, MMNT, 4 (2010) 403-409.
14. M.G.Thomasan, Convergence of powers of a fuzzy matrix, J. Math Anal. Appl, 57 (1997) 476-480.
15. H.Wang, F.Smarandache, Y.Q.Zhang, et al., Interval neutrosophic sets and logic: Theory and applications in computing, Hexix: Arizona, (2005) 6-14.
16. H.Wang, F.Smarandache, Y.Q.Zhang and R.Sunderraman, Single valued neutrosophic sets, Multispace Multistruct, 4 (2010) 410–413.
17. J.Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, International Journal of General Systems, 42(4) (2013) 386–394.
18. J.Ye, Improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making, Journal of Intelligent & Fuzzy Systems, 27 (2014) 2453–2462.
19. J.Ye, Amulticriteria decision-making method using aggregation operators for simplified neutrosophic sets, Journal of Intelligent & Fuzzy Systems, 26 (5) (2014) 2459-2466.
20. L.A.Zadeh, Fuzzy sets, Inform and Control, 8 (1965) 338-353.