Hindawi
Journal of Mathematics
Volume 2021, Article ID 5594623, 13 pages
https://doi.org/10.1155/2021/5594623

Research Article

Optimal Solutions for Constrained Bimatrix Games with Payoffs Represented by Single-Valued Trapezoidal Neutrosophic Numbers

Mohamed Gaber[1], **Majed G. Alharbi**[2], **Abd Alwahed Dagestani**[3] and **El-Saeed Ammar**[4]

[1] *School of Mathematics and Statistics, Central South University, Changsha 410083, Hunan, China*
[2] *Department of Mathematics, College of Arts and Sciences, Methnab, Qassim University, Buridah, Saudi Arabia*
[3] *School of Business, Central South University, Changsha 410083, China*
[4] *Department of Mathematics Faculty of Science, Tanta University, Tanta, Egypt*

Correspondence should be addressed to Mohamed Gaber; mohamedgaber@csu.edu.cn

Received 25 January 2021; Revised 25 April 2021; Accepted 2 June 2021; Published 18 June 2021

Academic Editor: Parimala Mani

Single-valued neutrosophic set (SVNS) is considered as generalization and extension of fuzzy set, intuitionistic fuzzy set (IFS), and crisp set for expressing the imprecise, incomplete, and indeterminate information about real-life decision-oriented models. The theme of this research is to develop a solution approach to solve constrained bimatrix games with payoffs of single-valued trapezoidal neutrosophic numbers (SVTNNs). In this approach, the concepts and suitable ranking function of SVTNNs are defined. Hereby, the equilibrium optimal strategies and equilibrium values for both players can be determined by solving the parameterized mathematical programming problems, which are obtained from two novel auxiliary SVTNNs programming problems based on the proposed ranking approach of SVTNNs. Moreover, an application example is examined to verify the effectiveness and superiority of the developed algorithm. Finally, a comparison analysis between the proposed and the existing approaches is conducted to expose the advantages of our work.

1. Introduction

Constrained bimatrix games are nonzero-sum two-player noncooperative games which play a dominant role in many real-life applications such as in military, finance, economy, strategic welfares, cartel behaviour, management models, social problems or auctions, political voting systems, races, and development research [1, 2]. Usually, the constrained bimatrix game makes the assumption that the payoff values are described with crisp elements and exactly known by each player. However, players are not able to evaluate the games outcomes exactly due to the unavailability and ambiguity of information. To handle that, Zadeh [3] introduced the fuzzy set concept and since then various researchers have extended it to the different sets such as interval intuitionistic fuzzy set, IFS, linguistic interval IFS, and cubic IFS. Many scholars have studied various kinds of noncooperative games under uncertainty. For instance, Li et al. [4] proposed a bilinear programming algorithm for solving bimatrix games with intuitionistic fuzzy (IF) payoffs. Figueroa et al. [5] studied group matrix games with interval-valued fuzzy numbers payoffs. Jana et al. [6] introduced novel similarity measure to solve matrix games with dual hesitant fuzzy payoffs. Singh et al. [7] established 2-tuple linguistic matrix games. Zhou et al. [8] constructed novel matrix game with generalized Dempster-Shafer payoffs. Seikh et al. [9] solved matrix games with payoffs of hesitant fuzzy numbers. Han et al. [10] described new matrix game with Maxitive Belief information. Roy et al. [11] discussed Stackelberg game with payoffs of type-2 fuzzy numbers. Bhaumik et al. [12] solved

Prisoners' dilemma matrix game with hesitant interval-valued intuitionistic fuzzy-linguistic payoffs elements. Ammar et al. [13] studied bimatrix games with rough interval payoffs. Brikaa et al. [14] developed fuzzy multi-objective programming technique to solve fuzzy rough constrained matrix games. Bhaumik et al. [15] introduced multiobjective linguistic-neutrosophic matrix game with applications to tourism management. Brikaa et al. [16] applied resolving indeterminacy technique to find optimal solutions of multicriteria matrix games with IF goals. So far, as the authors are aware, there are only four articles that studied constraint bimatrix games. Jing-Jing et al. [17] proposed linear programming method for solving constrained bimatrix games with IF payoffs. Koorosh et al. [18] presented constrained bimatrix games and their application in wireless communications. Fanyong et al. [19] applied two approaches to solve the classical constrained bimatrix games. Bigdeli et al. [20] discussed constrained bimatrix games with fuzzy goals.

However, the IFS and fuzzy set theories are unable to deal with inconsistent and indeterminate data correctly. To consider that, Smarandache [21] introduced the theory of neutrosophic set (NS), defining the three components of indeterminacy, falsity, and truth; all lie in $]0^-, 1^+[$ and are independent. As NS is difficult to implement on realistic applications, Wang et al. [22] developed the single-valued neutrosophic set (SVNS) concept, which is an extension of the NS. Due to its importance, many scholars have applied the SVNS theory in various disciplines. For example, Garg [23] studied the analysis of decision-making based on sine trigonometric operational laws for SVNSs. Murugappan [24] presented neutrosophic inventory problem with immediate return for deficient items. Garg [25] proposed new neutrality aggregation operators with multiattribute decision-making (MADM) approach for single-valued neutrosophic numbers (SVNNs). Abdel-Basset et al. [26] investigated resource levelling model in construction projects with neutrosophic information. Garai et al. [27] discussed variance, standard deviation, and possibility mean of SVNNs with applications to MADM models. Broumi et al. [28] solved neutrosophic shortest path model by applying Bellman technique. Garg [29] proposed TOPSIS and clustering approaches to solve SVNNs decision-making model. Mullai et al. [30] presented inventory backorder model with neutrosophic environment. Garg et al. [31] studied MADM based on Frank Choquet Heronian mean operator for SVNSs. Leyva et al. [32] introduced a new problem of information technology project with neutrosophic information. Garg [33] presented nonlinear programming approach for solving MADM model with interval neutrosophic parameters. Sun et al. [34] developed new SVNN decision-making algorithms based on the theory of prospect. Garg [35] introduced biparametric distance measures on SVNSs and their applications in medical diagnosis and pattern recognition.

In the imprecise data game, players may encounter some assessment data that cannot be represented as real numbers when estimating the utility functions or uncertain subjects. Since SVNS has great superiority and flexibility in describing many uncertainties with complex environments, it is effective and convenient to represent the constrained bimatrix games with neutrosophic data. Due to decision-making growing requirements of expressing their judgments in a human friendly and neatly manner, it is important to extend the IF or fuzzy constrained bimatrix games into neutrosophic environment. The SVNS is an effective tool to satisfy the increasing requirement of higher uncertain and complicated constrained bimatrix game models. Probably, this is the first attempt of solving constrained bimatrix game with SVTNNs payoffs. The fundamental targets of this article are listed as follows:

(1) To propose a novel constrained bimatrix games model with SVTNNs payoffs
(2) To develop an effective algorithm for SVTNN constrained bimatrix games to obtain the optimal strategies for such games
(3) To formulate crisp linear optimization problems from the neutrosophic models based on the defined ambiguity and value indexes of SVTNN
(4) To present an application example to demonstrate the effectiveness and applicability of the proposed method
(5) To compare our results with other existing approaches

The remainder of the manuscript is summarized as follows. Section 2 introduces the concept, cut sets, and arithmetic operations of SVTNNs. Section 3 gives the concept of ambiguity and value indexes of SVTNNs and the ranking technique of SVTNNs. Section 4 formulates constrained bimatrix games with SVTNNs payoffs and the solution approach to solve such games. The illustrative example with comparative analysis is discussed in Section 5. Lastly, a short conclusion is given in Section 6.

2. Preliminaries

In the following, we introduce the basic concepts of fuzzy sets, IFSs, NSs, SVNSs, and SVNNs.

Definition 1 (see [36]). A fuzzy number $\widetilde{B} = (b_1, b_2, b_3, b_4)$ is said to be a trapezoidal fuzzy number (TFN), if its membership function $\delta_{\widetilde{B}}(y)$ is given by

$$\delta_{\widetilde{B}}(y) = \begin{cases} \dfrac{y - b_1}{b_2 - b_1}, & \text{if } b_1 \le y \le b_2, \\ 1, & \text{if } b_2 \le y \le b_3, \\ \dfrac{b_4 - y}{b_4 - b_3}, & \text{if } b_3 \le y \le b_4, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Definition 2 (see [37]). Suppose that Y is a universal set. An IFS $\widetilde{C}$ is defined as follows:

$$\widetilde{C} = \left\{\langle y, \delta_{\widetilde{C}}(y), \gamma_{\widetilde{C}}(y)\rangle: \quad y \in Y\right\}, \tag{2}$$

where $\gamma_{\widetilde{C}}: Y \longrightarrow [0,1]$ and $\delta_{\widetilde{C}}: Y \longrightarrow [0,1]$ are the non-membership degree and the membership degree of $y \in Y$ to the set $\widetilde{C} \subseteq Y$, such that $0 \le \delta_{\widetilde{C}}(y) + \gamma_{\widetilde{C}}(y) \le 1$, $\forall y \in Y$.

Definition 3 (see [22]). An SVNS $\widetilde{B}$ in a universe Y is defined by

$$\widetilde{B} = \left\{\langle y, T_{\widetilde{B}}(y), I_{\widetilde{B}}(y), F_{\widetilde{B}}(y)\rangle: \quad y \in Y\right\}, \tag{3}$$

where $T_{\widetilde{B}}(y): Y \longrightarrow [0,1]$, $I_{\widetilde{B}}(y): Y \longrightarrow [0,1]$, and $F_{\widetilde{B}}(y): Y \longrightarrow [0,1]$ such that $0 \le T_{\widetilde{B}}(y) + I_{\widetilde{B}}(y), + F_{\widetilde{B}}(y) \le 3$, $\forall y \in Y$. The values$F_{\widetilde{B}}(y), I_{\widetilde{B}}(y)$ and $T_{\widetilde{B}}(y)$, respectively, express the falsity membership, indeterminacy membership, and truth membership degree of y to $\widetilde{B}$.

Definition 4 (see [22]). An (α, β, γ)-cut set of SVNS $\widetilde{B}$, a crisp subset of $\mathbb{R}$, is given by

$$\widetilde{B}_{(\alpha,\beta,\gamma)} = \left\{y: T_{\widetilde{B}}(y) \ge \alpha, I_{\widetilde{B}}(y) \le \beta, F_{\widetilde{B}}(y) \le \gamma\right\}, \tag{4}$$

where $0 \le \alpha \le 1$, $0 \le \beta \le 1$, $0 \le \gamma \le 1$, and $0 \le \alpha + \beta + \gamma \le 3$.

Definition 5 (see [22]). An SVNS $\widetilde{B} = \left\{\langle y, T_{\widetilde{B}}(y), I_{\widetilde{B}}(y), F_{\widetilde{B}}(y)\rangle: y \in Y\right\}$ is called neutrosophic normal, if there exist at least three points $y_1, y_2, y_3 \in Y$ such that $T_{\widetilde{B}}(y_1) = I_{\widetilde{B}}(y_2) = F_{\widetilde{B}}(y_3) = 1$.

Definition 6 (see [22]). An SVNS $\widetilde{B} = \{\langle y, T_{\widetilde{B}}(y), I_{\widetilde{B}}(y), F_{\widetilde{B}}(y)\rangle: y \in Y\}$ is said to be neutrosophic convex, if, $\forall y_1, y_2 \in Y$ and $\xi \in [0,1]$, the following conditions are satisfied:

(i) $T_{\widetilde{B}}(\xi y_1 + (1-\xi)y_2) \ge \min(T_{\widetilde{B}}(y_1), T_{\widetilde{B}}(y_2))$
(ii) $I_{\widetilde{B}}(\xi y_1 + (1-\xi)y_2) \le \max(I_{\widetilde{B}}(y_1), I_{\widetilde{B}}(y_2))$
(iii) $F_{\widetilde{B}}(\xi y_1 + (1-\xi)y_2) \le \max(F_{\widetilde{B}}(y_1), F_{\widetilde{B}}(y_2))$

Definition 7 (see [22]). An SVNS $\widetilde{B} = \left\{\langle y, T_{\widetilde{B}}(y), I_{\widetilde{B}}(y), F_{\widetilde{B}}(y)\rangle: y \in Y\right\}$, is said to be single-valued neutrosophic number when

(1) $\widetilde{B}$ is neutrosophic normal
(2) $\widetilde{B}$ is neutrosophic convex
(3) $T_{\widetilde{B}}(y)$ is upper semicontinuous, $I_{\widetilde{B}}(y)$ is lower semicontinuous, and $F_{\widetilde{B}}(y)$ is lower semicontinuous
(4) The support of $\widetilde{B}$, that is, $S(\widetilde{B}) = \left\{\langle T_{\widetilde{B}}(y) > 0, I_{\widetilde{B}}(y) < 1, F_{\widetilde{B}}(y) < 1, \forall y \in Y\rangle\right\}$, is bounded

Definition 8 (see [38]). An SVTNN $\widetilde{b} = \langle (k,l,m,n); u_{\widetilde{b}}, v_{\widetilde{b}}, w_{\widetilde{b}}\rangle$ is a special neutrosophic set on the set of real numbers $\mathbb{R}$, whose truth membership, indeterminacy membership, and falsity membership are represented as

$$\mu_{\widetilde{b}}(y) = \begin{cases} \dfrac{(y-k)u_{\widetilde{b}}}{l-k}, & \text{if } k \le y < l, \\ u_{\widetilde{b}}, & \text{if } l \le y \le m, \\ \dfrac{(n-y)u_{\widetilde{b}}}{n-m}, & \text{if } m < y \le n, \\ 0, & \text{otherwise,} \end{cases}$$

$$\delta_{\widetilde{b}}(y) = \begin{cases} \dfrac{\left(l-y+(y-k)v_{\widetilde{b}}\right)}{l-k}, & \text{if } k \le y < l, \\ v_{\widetilde{b}}, & \text{if } l \le y \le m, \\ \dfrac{\left(y-m+(n-y)v_{\widetilde{b}}\right)}{n-m}, & \text{if } m < y \le n, \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

$$\eta_{\widetilde{b}}(y) = \begin{cases} \dfrac{\left(l-y+(y-k)w_{\widetilde{b}}\right)}{l-k}, & \text{if } k \le y < l, \\ w_{\widetilde{b}}, & \text{if } l \le y \le m, \\ \dfrac{\left(y-m+(n-y)w_{\widetilde{b}}\right)}{n-m}, & \text{if } m < y \le n, \\ 0, & \text{otherwise,} \end{cases}$$

respectively.

Definition 9 (see [38]). Let $\widetilde{c} = \langle (k_1, l_1, m_1, n_1); u_{\widetilde{c}}, v_{\widetilde{c}}, w_{\widetilde{c}}\rangle$ and $\widetilde{d} = \langle (k_2, l_2, m_2, n_2); u_{\widetilde{d}}, v_{\widetilde{d}}, w_{\widetilde{d}}\rangle$ be two SVTNNs and let $\lambda \ne 0$ be any real number. Then,

(1) $\widetilde{c} + \widetilde{d} = \langle (k_1 + k_2, l_1 + l_2, m_1 + m_2, n_1 + n_2); u_{\widetilde{c}} \wedge u_{\widetilde{d}}, v_{\widetilde{c}} \vee v_{\widetilde{d}}, w_{\widetilde{c}} \vee w_{\widetilde{d}}\rangle$
(2) $\widetilde{c}\widetilde{d} = \left\{ \langle (k_1k_2, l_1l_2, m_1m_2, n_1n_2); u_{\widetilde{c}} \wedge u_{\widetilde{d}}, v_{\widetilde{c}} \vee v_{\widetilde{d}}, w_{\widetilde{c}} \vee w_{\widetilde{d}}\rangle (n_1 > 0,\ n_2 > 0) \langle (k_1n_2, l_1m_2, m_1l_2, n_1k_2); u_{\widetilde{c}} \wedge u_{\widetilde{d}}, v_{\widetilde{c}} \vee v_{\widetilde{d}}, w_{\widetilde{c}} \vee w_{\widetilde{d}}\rangle (n_1 < 0, n_2 > 0) \langle (n_1n_2, m_1m_2, l_1l_2, k_1k_2); u_{\widetilde{c}} \wedge u_{\widetilde{d}}, v_{\widetilde{c}} \vee v_{\widetilde{d}}, w_{\widetilde{c}} \vee w_{\widetilde{d}}\rangle (n_1 < 0, n_2 < 0) \right.$
(3) $\lambda\widetilde{c} = \begin{cases} \langle (\lambda k_1, \lambda l_1, \lambda m_1, \lambda n_1); u_{\widetilde{c}}, v_{\widetilde{c}}, w_{\widetilde{c}}\rangle & (\lambda > 0) \\ \langle (\lambda n_1, \lambda m_1, \lambda l_1, \lambda k_1); u_{\widetilde{c}}, v_{\widetilde{c}}, w_{\widetilde{c}}\rangle & (\lambda < 0) \end{cases}$

Definition 10 (see [38]). Let $\widetilde{b} = \langle ((k_1, l_1, m_1, n_1), u_{\widetilde{b}}), ((k_2, l_2, m_2, n_2), v_{\widetilde{b}}), ((k_3, l_3, m_3, n_3), w_{\widetilde{b}})\rangle$ be an SVTNN. Then, $\langle \alpha, \beta, \gamma\rangle$-cut set of the SVTNN $\widetilde{b}$, represented by $\widetilde{b}_{\langle \alpha,\beta,\gamma\rangle}$, is given as

$$\widetilde{b}_{\langle\alpha,\beta,\gamma\rangle} = \left\{y\colon \mu_{\widetilde{b}}(y) \geq \alpha, \delta_{\widetilde{b}}(y) \leq \beta, \eta_{\widetilde{b}}(y) \leq \gamma, \quad y \in \mathbb{R}\right\}, \tag{6}$$

which satisfies the following conditions:

$$\begin{aligned} &0 \leq \alpha \leq u_{\widetilde{b}},\\ &v_{\widetilde{b}} \leq \beta \leq 1,\\ &w_{\widetilde{b}} \leq \gamma \leq 1,\\ &0 \leq \alpha + \beta + \gamma \leq 3. \end{aligned} \tag{7}$$

Obviously, any $\langle\alpha,\beta,\gamma\rangle$-cut set $\widetilde{b}_{\langle\alpha,\beta,\gamma\rangle}$ of an SVTNN $\widetilde{b}$ is a crisp subset over the set of real numbers $\mathbb{R}$.

Definition 11 (see [38]). Let $\widetilde{b} = \langle((k_1, l_1, m_1, n_1), u_{\widetilde{b}}), ((k_2, l_2, m_2, n_2), v_{\widetilde{b}}), ((k_3, l_3, m_3, n_3), w_{\widetilde{b}})\rangle$ be an SVTNN. Then, α-cut set of the SVTNN $\widetilde{b}$, represented by $\widetilde{b}_\alpha$, is given as

$$\widetilde{b}_\alpha = \left\{y\colon \mu_{\widetilde{b}}(y) \geq \alpha, \quad y \in \mathbb{R}\right\}, \tag{8}$$

where $\alpha \in [0, u_{\widetilde{b}}]$.

Obviously, any α-cut set $\widetilde{b}_\alpha$ of an SVTNN $\widetilde{b}$ is a crisp subset over the set of real numbers $\mathbb{R}$.

Here, any α-cut set of an SVTNN $\widetilde{b}$ for the truth membership function is a closed interval, represented by $\widetilde{b}_\alpha = [L^\alpha(\widetilde{b}), R^\alpha(\widetilde{b})]$.

Definition 12 (see [38]). Let $\widetilde{b} = \langle((k_1, l_1, m_1, n_1), u_{\widetilde{b}}), ((k_2, l_2, m_2, n_2), v_{\widetilde{b}}), ((k_3, l_3, m_3, n_3), w_{\widetilde{b}})\rangle$ be an SVTNN. Then, β-cut set of the SVTNN $\widetilde{b}$, represented by $\widetilde{b}_\beta$, is given as

$$\widetilde{b}_\beta = \left\{y\colon \delta_{\widetilde{b}}(y) \leq \beta, \quad y \in \mathbb{R}\right\}, \tag{9}$$

where $\beta \in [v_{\widetilde{b}}, 1]$.

Obviously, any β-cut set $\widetilde{b}_\beta$ of an SVTNN $\widetilde{b}$ is a crisp subset over the set of real numbers $\mathbb{R}$.

Here, any β-cut set of an SVTNN $\widetilde{b}$ for the indeterminacy membership function is a closed interval, represented by $\widetilde{b}_\beta = [L^\beta(\widetilde{b}), R^\beta(\widetilde{b})]$.

Definition 13 (see [38]). Let $\widetilde{b} = \langle((k_1, l_1, m_1, n_1), u_{\widetilde{b}}), ((k_2, l_2, m_2, n_2), v_{\widetilde{b}}), ((k_3, l_3, m_3, n_3), w_{\widetilde{b}})\rangle$ be an SVTNN. Then, γ-cut set of the SVTNN $\widetilde{b}$, represented by $\widetilde{b}_\gamma$, is given as

$$\widetilde{b}_\gamma = \left\{y\colon \eta_{\widetilde{b}}(y) \leq \gamma, \quad y \in \mathbb{R}\right\}, \tag{10}$$

where $\gamma \in [w_{\widetilde{b}}, 1]$.

Obviously, any γ-cut set $\widetilde{b}_\gamma$ of an SVTNN $\widetilde{b}$ is a crisp subset over the set of real numbers $\mathbb{R}$.

Here, any γ-cut set of an SVTNN $\widetilde{b}$ for the falsity membership function is a closed interval, represented by $\widetilde{b}_\gamma = [L^\gamma(\widetilde{b}), R^\gamma(\widetilde{b})]$.

3. Characteristics and the Ranking Approach for SVTNNs

3.1. Value and Ambiguity of SVTNNs. Here, we introduce the basic definitions of value and ambiguity indices of SVTNN.

Definition 14 (see [38]). Let $\widetilde{b} = \langle((k_1, l_1, m_1, n_1), u_{\widetilde{b}}), ((k_2, l_2, m_2, n_2), v_{\widetilde{b}}), ((k_3, l_3, m_3, n_3), w_{\widetilde{b}})\rangle$ be an SVTNN and let $\widetilde{b}_\alpha = [L^\alpha(\widetilde{b}), R^\alpha(\widetilde{b})]$, $\widetilde{b}_\beta = [L^\beta(\widetilde{b}), R^\beta(\widetilde{b})]$, and $\widetilde{b}_\gamma = [L^\gamma(\widetilde{b}), R^\gamma(\widetilde{b})]$ be any α-cut set, β-cut set, and γ-cut set of the SVTNN $\widetilde{b}$, respectively. Then, we have the following.

(1) The value of the SVTNN $\widetilde{b}$ for α-cut set, represented by $V_\mu(\widetilde{b})$, is given as

$$V_\mu(\widetilde{b}) = \int_0^{u_{\widetilde{b}}} \left(L^\alpha(\widetilde{b}) + R^\alpha(\widetilde{b})\right) h(\alpha)\mathrm{d}\alpha, \tag{11}$$

where $h(\alpha) \in [0,1]$ $(\alpha \in [0, u_{\widetilde{b}}])$, $h(0) = 0$, and $h(\alpha)$is nondecreasing and monotonic of $\alpha \in [0, u_{\widetilde{b}}]$.

(2) The value of the SVTNN $\widetilde{b}$ for β-cut set, represented by $V_\delta(\widetilde{b})$, is given as

$$V_\delta(\widetilde{b}) = \int_{v_{\widetilde{b}}}^{1} \left(L^\beta(\widetilde{b}) + R^\beta(\widetilde{b})\right) f(\beta)\mathrm{d}\beta, \tag{12}$$

where $f(\beta) \in [0,1]$ $(\beta \in [v_{\widetilde{b}}, 1])$, $f(1) = 0$, and $f(\beta)$ is nondecreasing and monotonic of $\beta \in [v_{\widetilde{b}}, 1]$.

(3) The value of the SVTNN $\widetilde{b}$ for γ-cut set, represented by $V_\eta(\widetilde{b})$, is given as

$$V_\eta(\widetilde{b}) = \int_{w_{\widetilde{b}}}^{1} \left(L^\gamma(\widetilde{b}) + R^\gamma(\widetilde{b})\right) g(\gamma)\mathrm{d}\gamma, \tag{13}$$

where $g(\gamma) \in [0,1]$ $(\gamma \in [w_{\widetilde{b}}, 1])$, $g(1) = 0$, and $g(\gamma)$ is nondecreasing and monotonic of $\gamma \in [w_{\widetilde{b}}, 1]$.

Definition 15 (see [38]). Let $\widetilde{b} = \langle((k_1, l_1, m_1, n_1), u_{\widetilde{b}}), ((k_2, l_2, m_2, n_2), v_{\widetilde{b}}), ((k_3, l_3, m_3, n_3), w_{\widetilde{b}})\rangle$ be an SVTNN and let $\widetilde{b}_\alpha = [L^\alpha(\widetilde{b}), R^\alpha(\widetilde{b})]$, $\widetilde{b}_\beta = [L^\beta(\widetilde{b}), R^\beta(\widetilde{b})]$, and $\widetilde{b}_\gamma = [L^\gamma(\widetilde{b}), R^\gamma(\widetilde{b})]$ be any α-cut set, β-cut set, and γ-cut set of the SVNN $\widetilde{b}$, respectively. Then, we have the following.

(1) The ambiguities of the SVTNN $\widetilde{b}$ for α-cut set, represented by $A_\mu(\widetilde{b})$, are given as

$$A_\mu(\widetilde{b}) = \int_0^{u_{\widetilde{b}}} \left(R^\alpha(\widetilde{b}) - L^\alpha(\widetilde{b})\right) h(\alpha)\mathrm{d}\alpha, \tag{14}$$

where $h(\alpha) \in [0,1]$ $(\alpha \in [0, u_{\widetilde{b}}])$, $h(0) = 0$, and $h(\alpha)$is nondecreasing and monotonic of $\alpha \in [0, u_{\widetilde{b}}]$.

(2) The ambiguities of the SVTNN $\widetilde{b}$ for β-cut set, represented by $A_\delta(\widetilde{b})$, are given as

$$A_\delta(\widetilde{b}) = \int_{v_{\widetilde{b}}}^{1} \left(R^\beta(\widetilde{b}) - L^\beta(\widetilde{b})\right) f(\beta)\mathrm{d}\beta, \tag{15}$$

where $f(\beta) \in [0,1]$ $(\beta \in [v_{\widetilde{b}}, 1])$, $f(1) = 0$, and $f(\beta)$ is nondecreasing and monotonic of $\beta \in [v_{\widetilde{b}}, 1]$.

(3) The ambiguities of the SVTNN $\widetilde{b}$ for γ-cut set, represented by $A_\eta(\widetilde{b})$, are given as

$$A_\eta(\widetilde{b}) = \int_{w_{\widetilde{b}}}^{1} \left(R^\gamma(\widetilde{b}) - L^\gamma(\widetilde{b})\right) h(\gamma) \mathrm{d}\gamma, \tag{16}$$

where $g(\gamma) \in [0,1]$ $(\gamma \in [w_{\widetilde{b}}, 1])$, $g(1) = 0$a, and $g(\gamma)$ is nondecreasing and monotonic of $\gamma \in [w_{\widetilde{b}}, 1]$.

Here, the weighting functions $h(\alpha)$, $f(\beta)$, and $g(\gamma)$ can be supposed according to the decision-making model nature. Suppose that $h(\alpha) = \alpha$, $f(\beta) = 1 - \beta$, and $g(\gamma) = 1 - \gamma$.

Let $\widetilde{b} = \langle (k, l, m, n); u_{\widetilde{b}}, v_{\widetilde{b}}, w_{\widetilde{b}} \rangle$ be an SVTNN. Then the value and ambiguity indices, using the above descriptions, are constructed as

$$\begin{aligned}
V_\mu(\widetilde{b}) &= \frac{(k + 2l + 2m + n) u_{\widetilde{b}}^2}{6}, \quad A_\mu(\widetilde{b}) = \frac{(n - k + 2m - 2l) u_{\widetilde{b}}^2}{6},\\
V_\delta(\widetilde{b}) &= \frac{(k + 2l + 2m + n)\left(1 - v_{\widetilde{b}}\right)^2}{6}, \quad A_\delta(\widetilde{b}) = \frac{(n - k + 2m - 2l)\left(1 - v_{\widetilde{b}}\right)^2}{6},\\
V_\eta(\widetilde{b}) &= \frac{(k + 2l + 2m + n)\left(1 - w_{\widetilde{b}}\right)^2}{6}, \quad A_\eta(\widetilde{b}) = \frac{(n - k + 2m - 2l)\left(1 - w_{\widetilde{b}}\right)^2}{6}.
\end{aligned} \tag{17}$$

3.2. A Ranking Approach of an SVTNN Based on Value and Ambiguity Indices. This section provides a ranking approach of SVTNNs based on the ambiguity and value indices of SVTNNs in a similar way to those of SVNNs introduced by A. Bhaumik et al. [39].

Definition 16. Let $\widetilde{b} = \langle (k, l, m, n); u_{\widetilde{b}}, v_{\widetilde{b}}, w_{\widetilde{b}} \rangle$ be an SVTNN. The weighted value ambiguity index for an SVTNN $\widetilde{b}$ is given as

$$R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{b}) = \left[\lambda_1 V_\mu(\widetilde{b}) + (1 - \lambda_1) A_\mu(\widetilde{b})\right] + \left[\lambda_2 V_\delta(\widetilde{b}) + (1 - \lambda_2) A_\delta(\widetilde{b})\right] + \left[\lambda_3 V_\eta(\widetilde{b}) + (1 - \lambda_3) A_\eta(\widetilde{b})\right], \tag{18}$$

with $\lambda_1, \lambda_2, \lambda_3 \in [0,1]$.

Definition 17. Let $\widetilde{c}$ and $\widetilde{d}$ be two SVTNNs and let $\lambda_1, \lambda_2, \lambda_3 \in [0,1]$. For the weighted value ambiguity index of the SVTNNs $\widetilde{c}$ and $\widetilde{d}$, the ranking order of $\widetilde{c}$ and $\widetilde{d}$ is given as follows:

(1) if $R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{c}) >_N R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{d})$, then $\widetilde{c} >_N \widetilde{d}$

(2) if $R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{c}) <_N R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{d})$, then $\widetilde{c} <_N \widetilde{d}$

(3) if $R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{c}) =_N R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{d})$, then $\widetilde{c} =_N \widetilde{d}$

where "$>_N$" and "$<_N$" are neutrosophic versions of the order relations "$>$" and "$<$" in the real line, respectively.

4. Constrained Bimatrix Games with SVTNNs Payoffs and Solution Method

Let us consider the constrained bimatrix game with SVTNNs payoffs. Suppose that $T_1 = \{\xi_1, \xi_2, \ldots, \xi_\kappa\}$ and $T_2 = \{\eta_1, \eta_2, \ldots, \eta_\ell\}$ are pure strategies sets for two players I and II, respectively. When player II selects pure strategy $\eta_j \in T_2$ and player I selects pure strategy $\xi_i \in T_1$, at the situation (ξ_i, η_j), player II gains payoff and player I gains payoff, which are expressed with SVTNNs as $\widetilde{\mathbf{C}} = (\widetilde{c}_{ij})_{\kappa\times\ell}$ and $\widetilde{\mathbf{D}} = (\widetilde{d}_{ij})_{\kappa\times\ell}$, where each $\widetilde{c}_{ij} = \langle (a_{ij}, b_{ij}, f_{ij}, h_{ij}); u_{\widetilde{c}_{ij}}, v_{\widetilde{c}_{ij}}, w_{\widetilde{c}_{ij}} \rangle$ and $\widetilde{d}_{ij} = \langle (k_{ij}, l_{ij}, m_{ij}, n_{ij}); u_{\widetilde{d}_{ij}}, v_{\widetilde{d}_{ij}}, w_{\widetilde{d}_{ij}} \rangle$ $(i = 1, 2, \ldots, \kappa; j = 1, 2, \ldots, \ell)$ are SVTNNs defined as above. The mixed strategies vectors are represented as $\mathbf{r} = (r_1, r_2, \ldots, r_\kappa)^T$ and $\mathbf{s} = (s_1, s_2, \ldots, s_\ell)^T$, where r_i $(i = 1, 2, \ldots, \kappa)$ and s_j $(j = 1, 2, \ldots, \ell)$ are probabilities for both players selecting their pure strategies $\xi_i \in T_1$ and $\eta_j \in T_2$, respectively. The mixed strategies r_i and s_j are affiliated with the strategies sets (convex polyhedron) which are described by some inequalities and equations. Let $R = \{\mathbf{r}\colon \mathbf{G}^T\mathbf{r} \geq \mathbf{p},\ \mathbf{r} \geq 0\}$ represent the strategy constraint set of player I, where $\mathbf{p} = (p_1, p_2, \ldots, p_e)^T$, $\mathbf{G} = (g_{in})_{\kappa\times e}$, and e is a positive integer. Let $S = \{\mathbf{s}\colon \mathbf{Hs} \geq \mathbf{q},\ \mathbf{s} \geq 0\}$ express the strategy constraint set of player II, where $\mathbf{q} = (q_1, q_2, \ldots, q_b)$, $\mathbf{H} = (h_{mj})_{b\times\ell}$, and b is a positive integer. Note that $\mathbf{G}^T\mathbf{r} \geq \mathbf{p}$ contains $\sum_{i=1}^{\kappa} r_i = 1$, since $\sum_{i=1}^{\kappa} r_i = 1$ is equivalent to $\sum_{i=1}^{\kappa} r_i \geq 1$ and $-\sum_{i=1}^{\kappa} r_i \geq -1$. Similarly, $\mathbf{Hs} \geq \mathbf{q}$ contains $\sum_{j=1}^{\ell} s_j = 1$. In the sequel, the above SVTNN constrained bimatrix game is simply denoted by $(\widetilde{\mathbf{C}}, \widetilde{\mathbf{D}})$ for short.

Without loss of generality, suppose that both players I and II, respectively, select mixed strategies $r \in R$ and $s \in S$ in order to maximize their own payoffs; then their expected payoffs can be obtained as follows:

$$E_1(\mathbf{r},\mathbf{s},\widetilde{\mathbf{C}}) = \mathbf{r}^T\widetilde{\mathbf{C}}\mathbf{s} = \sum_{i=1}^{\kappa}\sum_{j=1}^{\ell} r_i\widetilde{c}_{ij}s_j,$$
$$E_2(\mathbf{r},\mathbf{s},\widetilde{\mathbf{D}}) = \mathbf{r}^T\widetilde{\mathbf{D}}\mathbf{s} = \sum_{i=1}^{\kappa}\sum_{j=1}^{\ell} r_i\widetilde{d}_{ij}s_j. \quad (19)$$

Definition 18 (see [40]). If $(\mathbf{r}^*,\mathbf{s}^*) \in R\times S$ satisfies the following conditions:

$$\mathbf{r}^{*T}\widetilde{\mathbf{C}}\mathbf{s}^* = \min_{s\in S}\mathbf{r}^{*T}\widetilde{\mathbf{C}}\mathbf{s} = \max_{r\in R}\min_{s\in S}\mathbf{r}^T\widetilde{\mathbf{C}}\mathbf{s},$$
$$\mathbf{r}^{*T}\widetilde{\mathbf{D}}\mathbf{s}^* = \min_{r\in R}\mathbf{r}^T\widetilde{\mathbf{D}}\mathbf{s}^* = \max_{s\in S}\min_{r\in R}\mathbf{r}^T\widetilde{\mathbf{D}}\mathbf{s}, \quad (20)$$

for any mixed strategies $\mathbf{r}\in R$ and $\mathbf{s}\in S$, then $\mathbf{r}^*$ and $\mathbf{s}^*$ are called equilibrium strategies, and $U^* = \mathbf{r}^{*T}\widetilde{\mathbf{C}}\mathbf{s}^*$ and $W^* = \mathbf{r}^{*T}\widetilde{\mathbf{D}}\mathbf{s}^*$ are called equilibrium values of players I and II, respectively.

Theorem 1. *If $(\mathbf{r}^*,\mathbf{y}^*)$ and $(\mathbf{s}^*,\mathbf{z}^*)$ are the optimal solutions of the following linear programming problems:*

$$\max\{\mathbf{q}^{\mathbf{T}}\mathbf{y}\}$$
$$\text{s.t.}\begin{cases}\mathbf{H}^{\mathbf{T}}\mathbf{y} \le_N \widetilde{\mathbf{C}}^{\mathbf{T}}\mathbf{r},\\ \mathbf{G}^{\mathbf{T}}\mathbf{r}\ge\mathbf{p},\\ \mathbf{r}\ge\mathbf{0},\\ \mathbf{y}\ge\mathbf{0},\end{cases} \quad (21)$$

$$\max\{\mathbf{p}^{\mathbf{T}}\mathbf{z}\},$$
$$\text{s.t.}\begin{cases}\mathbf{G}\mathbf{z} \le_N \widetilde{\mathbf{D}}\mathbf{s},\\ \mathbf{H}\mathbf{s}\ge\mathbf{q},\\ \mathbf{s}\ge\mathbf{0},\\ \mathbf{z}\ge\mathbf{0},\end{cases} \quad (22)$$

respectively, then $\mathbf{r}^*$ and $\mathbf{s}^*$ are equilibrium strategies of the SVTNN constrained bimatrix game $(\widetilde{\mathbf{C}},\widetilde{\mathbf{D}})$, and $U^* = \mathbf{q}^{\mathbf{T}}\mathbf{y}^* = \mathbf{r}^{*T}\widetilde{\mathbf{C}}\mathbf{s}^*$ and $W^* = \mathbf{p}^{\mathbf{T}}\mathbf{z}^* = \mathbf{r}^{*T}\widetilde{\mathbf{D}}\mathbf{s}^*$ are equilibrium values of players I and II, respectively.

Proof:. The proof of this theorem is similar to the proof given by Jing-Jing et al. [17].

It is obvious that the two players often cannot calculate the payoffs accurately in each situation, and the game values of the SVTNN constrained bimatrix games are not equal to $\mathbf{q}^{\mathbf{T}}\mathbf{y}$ in (21) and $\mathbf{p}^{\mathbf{T}}\mathbf{z}$ in (22). The two players may allow some violations on the set of constraints $\mathbf{H}^{\mathbf{T}}\mathbf{y}\le_N\widetilde{\mathbf{C}}^{\mathbf{T}}\mathbf{r}$ and $\mathbf{G}\mathbf{z}\le_N\widetilde{\mathbf{D}}\mathbf{s}$.

Therefore, the equilibrium strategies $\mathbf{r}^*$ and $\mathbf{s}^*$ and equilibrium values U^* and W^* of the SVTNN constrained bimatrix games are equal to the optimal values and optimal solutions of (23 and 24) as follows:

$$\max\{\mathbf{q}^{\mathbf{T}}\mathbf{y}\},$$
$$\text{s.t.}\begin{cases}\mathbf{H}^{\mathbf{T}}\mathbf{y} - \widetilde{\mathbf{C}}^{\mathbf{T}}\mathbf{r} \le_N (1-\rho)\widetilde{\mathbf{m}},\\ \mathbf{G}^{\mathbf{T}}\mathbf{r}\ge\mathbf{p},\\ \mathbf{r}\ge\mathbf{0},\\ \mathbf{y}\ge\mathbf{0},\end{cases} \quad (23)$$

$$\max\{\mathbf{p}^{\mathbf{T}}\mathbf{z}\},$$
$$\text{s.t.}\begin{cases}\mathbf{G}\mathbf{z} - \widetilde{\mathbf{D}}\mathbf{s} \le_N (1-\rho)\widetilde{\mathbf{n}},\\ \mathbf{H}\mathbf{s}\ge\mathbf{q},\\ \mathbf{s}\ge\mathbf{0},\\ \mathbf{z}\ge\mathbf{0},\end{cases} \quad (24)$$

respectively, where $\widetilde{\mathbf{m}} = (\widetilde{m}_1,\widetilde{m}_2,\ldots,\widetilde{m}_\ell)^T$, $\widetilde{\mathbf{n}} = (\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_k)^T$, and all the vectors elements of $\widetilde{\mathbf{m}}$ and $\widetilde{\mathbf{n}}$ are SVTNNs that are approximately equal to zero, which represent the maximum violations that the two players may permit on the set of constraints. The parameter $\rho\ (0\le\rho\le 1)$ is a real number.

Applying the ranking approach of SVTNNs, as proposed in Subsection 3.2, the SVTNN mathematical programming problems (equations (23) and (24)) can be transformed into the following parameterized programming problems:

$$\max\{\mathbf{q}^{\mathbf{T}}\mathbf{y}\},$$
$$\text{s.t.}\begin{cases}\mathbf{H}^{\mathbf{T}}\mathbf{y} - R_{\lambda_1,\lambda_2,\lambda_3}\left(\widetilde{\mathbf{C}}^{\mathbf{T}}\right)\mathbf{r} \le_N (1-\rho)R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{\mathbf{m}}),\\ \mathbf{G}^{\mathbf{T}}\mathbf{r}\ge\mathbf{p},\\ \mathbf{r}\ge\mathbf{0},\\ \mathbf{y}\ge\mathbf{0},\end{cases} \quad (25)$$

$$\max\{\mathbf{p}^{\mathbf{T}}\mathbf{z}\},$$
$$\text{s.t.}\begin{cases}\mathbf{G}\mathbf{z} - R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{\mathbf{D}})\mathbf{s} \le_N (1-\rho)R_{\lambda_1,\lambda_2,\lambda_3}(\widetilde{\mathbf{n}}),\\ \mathbf{H}\mathbf{s}\ge\mathbf{q},\\ \mathbf{s}\ge\mathbf{0},\\ \mathbf{z}\ge\mathbf{0},\end{cases} \quad (26)$$

respectively.

For given $\lambda_1,\lambda_2,\lambda_3\in[0,1]$, solving equations (25) and (26), we can obtain the optimal game values $\mathbf{q}^{\mathbf{T}}\mathbf{y}^*(\rho)$ and $\mathbf{p}^{\mathbf{T}}\mathbf{z}^*(\rho)$ and the optimal solutions $(\mathbf{r}^*(\rho),\mathbf{y}^*(\rho))$ and $(\mathbf{s}^*(\rho),\mathbf{z}^*(\rho))$, respectively. □

Theorem 2. *If $(\mathbf{r}^*(\rho),\mathbf{y}^*(\rho))$ and $(\mathbf{s}^*(\rho),\mathbf{z}^*(\rho))(\lambda_1,\lambda_2,\lambda_3\in[0,1])$ are optimal solutions of equations (25) and (26), respectively, then $\mathbf{r}^*(\rho)$ and $\mathbf{s}^*(\rho)$ are equilibrium strategies, and $U^* = \mathbf{q}^{\mathbf{T}}\mathbf{y}^*(\rho)$ and $W^* = \mathbf{p}^{\mathbf{T}}\mathbf{z}^*(\rho)$ are equilibrium values of both players for SVTNN constrained bimatrix games, respectively.*

Table 1: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.2, 0.3, 0.5)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	4.868	(0.4, 0.6)	4.855
0.1	(0.75, 0.25)	4.854	(0.4, 0.6)	4.849
0.2	(0.75, 0.25)	4.839	(0.4, 0.6)	4.844
0.3	(0.75, 0.25)	4.825	(0.4, 0.6)	4.838
0.4	(0.75, 0.25)	4.811	(0.4, 0.6)	4.832
0.5	(0.75, 0.25)	4.797	(0.4, 0.6)	4.827
0.6	(0.75, 0.25)	4.782	(0.4, 0.6)	4.821
0.7	(0.75, 0.25)	4.768	(0.4, 0.6)	4.815
0.8	(0.75, 0.25)	4.754	(0.4, 0.6)	4.809
0.9	(0.75, 0.25)	4.739	(0.4, 0.6)	4.804
1.0	(0.75, 0.25)	4.725	(0.4, 0.6)	4.798

Table 2: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.4, 0.5, 0.6)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	6.223	(0.4, 0.6)	6.666
0.1	(0.75, 0.25)	6.204	(0.4, 0.6)	6.658
0.2	(0.75, 0.25)	6.186	(0.4, 0.6)	6.649
0.3	(0.75, 0.25)	6.168	(0.4, 0.6)	6.642
0.4	(0.75, 0.25)	6.149	(0.4, 0.6)	6.634
0.5	(0.75, 0.25)	6.132	(0.4, 0.6)	6.626
0.6	(0.75, 0.25)	6.113	(0.4, 0.6)	6.617
0.7	(0.75, 0.25)	6.095	(0.4, 0.6)	6.609
0.8	(0.75, 0.25)	6.076	(0.4, 0.6)	6.601
0.9	(0.75, 0.25)	6.059	(0.4, 0.6)	6.593
1.0	(0.75, 0.25)	6.04	(0.4, 0.6)	6.585

Table 3: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.5, 0.5, 0.5)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	6.232	(0.4, 0.6)	6.801
0.1	(0.75, 0.25)	6.214	(0.4, 0.6)	6.792
0.2	(0.75, 0.25)	6.196	(0.4, 0.6)	6.784
0.3	(0.75, 0.25)	6.178	(0.4, 0.6)	6.776
0.4	(0.75, 0.25)	6.16	(0.4, 0.6)	6.767
0.5	(0.75, 0.25)	6.143	(0.4, 0.6)	6.759
0.6	(0.75, 0.25)	6.125	(0.4, 0.6)	6.751
0.7	(0.75, 0.25)	6.107	(0.4, 0.6)	6.743
0.8	(0.75, 0.25)	6089	(0.4, 0.6)	6.735
0.9	(0.75, 0.25)	6.071	(0.4, 0.6)	6.726
1.0	(0.75, 0.25)	6.054	(0.4, 0.6)	6.718

5. Application Example

In this section, an example of the company development strategy choice model adapted from Jing-Jing et al. [17] is used to illustrate the solution procedure of a constrained bimatrix game with payoffs of SVTNNs.

Table 4: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.6, 0.4, 0.7)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	6.785	(0.4, 0.6)	7.309
0.1	(0.75, 0.25)	6.765	(0.4, 0.6)	7.301
0.2	(0.75, 0.25)	6.746	(0.4, 0.6)	7.292
0.3	(0.75, 0.25)	6.726	(0.4, 0.6)	7.283
0.4	(0.75, 0.25)	6.706	(0.4, 0.6)	7.274
0.5	(0.75, 0.25)	6.687	(0.4, 0.6)	7.265
0.6	(0.75, 0.25)	6.667	(0.4, 0.6)	7.256
0.7	(0.75, 0.25)	6.647	(0.4, 0.6)	7.247
0.8	(0.75, 0.25)	6.628	(0.4, 0.6)	7.238
0.9	(0.75, 0.25)	6.608	(0.4, 0.6)	7.229
1.0	(0.75, 0.25)	6.588	(0.4, 0.6)	7.219

Table 5: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.7, 0.6, 0.8)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	7.861	(0.4, 0.6)	8.732
0.1	(0.75, 0.25)	7.838	(0.4, 0.6)	8.721
0.2	(0.75, 0.25)	7.815	(0.4, 0.6)	8.709
0.3	(0.75, 0.25)	7.792	(0.4, 0.6)	8.699
0.4	(0.75, 0.25)	7.769	(0.4, 0.6)	8.688
0.5	(0.75, 0.25)	7.746	(0.4, 0.6)	8.677
0.6	(0.75, 0.25)	7.723	(0.4, 0.6)	8.666
0.7	(0.75, 0.25)	7.7	(0.4, 0.6)	8.655
0.8	(0.75, 0.25)	7.678	(0.4, 0.6)	8.644
0.9	(0.75, 0.25)	7.655	(0.4, 0.6)	8.633
1.0	(0.75, 0.25)	7.632	(0.4, 0.6)	8.622

5.1. The Company Development Strategy Choice Model. "We consider two companies E_1 and E_2 (i.e., players I and II). In order to improve the two companies competitiveness, both players have two strategies: introducing the advanced equipment ξ_1 or η_1 and introducing the senior talent ξ_2 or η_2. When player I chooses pure strategies ξ_1 and ξ_2, he wants to invest 7 million and 5 million dollars, respectively. Due to a lack of fund, player I can invest up to 6.5 million dollars, which means that player I has a constraint, $7r_1 + 5r_2 \leq 6.5$, when selecting strategy. Likewise, player II wants to invest 4 million and 6.5 million dollars when he chooses pure strategies η_1 and η_2, respectively. However, due to a lack of fund, player II can invest up to 5.5 million dollars. Namely, player II has a constraint, $4s_1 + 6.5s_2 \leq 5.5$, when choosing strategies." This is a typical SVTN constrained bimatrix game. According to the previous description of the matrix game model, the two players' constrained strategy sets are given as follows:

$$\begin{aligned} R &= \{r \mid 7r_1 + 5r_2 \leq 6.5, r_1 + r_2 = 1, \quad r_1 \geq 0, r_2 \geq 0\}, \\ S &= \{s \mid 4s_1 + 6.5s_2 \leq 5.5, s_1 + s_2 = 1, \quad s_1 \geq 0, s_2 \geq 0\}, \end{aligned} \tag{27}$$

respectively. The SVTNNs payoff matrices of the two players are given by

$$\tilde{\mathbf{C}} = \begin{pmatrix} \langle(6,7,9,1);0.9,0.2,0.4\rangle & \langle(3.5,5,7,9);0.5,0.4,0.2\rangle \\ \langle(3,5,6,8);0.6,0.5,0.1\rangle & \langle(5,6.5,8,10);0.7,0.3,0.5\rangle \end{pmatrix},$$
$$\tilde{\mathbf{D}} = \begin{pmatrix} \langle(5,6.5,8,9);0.8,0.2,0.3\rangle & \langle(4,5,7,8.5);0.8,0.3,0.1\rangle \\ \langle(3.5,4.5,6,7.5);0.6,0.4,0.2\rangle & \langle(6,7,8,9);0.9,0.1,0.4\rangle \end{pmatrix}. \tag{28}$$

The vectors of the constraints and the coefficient matrices are given by

$$\begin{aligned} \mathbf{G} &= \begin{pmatrix} -7 & 1 & -1 \\ -5 & 1 & -1 \end{pmatrix}, \\ \mathbf{H}^T &= \begin{pmatrix} -4 & 1 & -1 \\ -6.5 & 1 & -1 \end{pmatrix}, \\ \mathbf{p} &= \begin{pmatrix} -6.5 & 1 & -1 \end{pmatrix}^T, \\ \mathbf{q} &= \begin{pmatrix} -5.5 & 1 & -1 \end{pmatrix}^T. \end{aligned} \tag{29}$$

Let the two players select $\tilde{m}_1 = \tilde{m}_2 = \langle(0.18, 0.1, 0.21, 0.13); 0.7, 0.2, 0.1\rangle$ and $\tilde{n}_1 = \tilde{n}_2 = \langle(0.04, 0.1, 0.13, 0.02); 0.8, 0.2, 0.3\rangle$, respectively.

5.2. The Solution Procedure. Applying the ranking approach presented in Section 3 to the SVTN constrained bimatrix game, we have

$$\begin{aligned} R_{\lambda_1,\lambda_2,\lambda_3}(\tilde{C}) &= \begin{pmatrix} 5.4\lambda_1 + 4.267\lambda_2 + 2.4\lambda_3 + 2.715 & 1.125\lambda_1 + 1.62\lambda_2 + 2.88\lambda_3 + 1.979 \\ 1.56\lambda_1 + 1.083\lambda_2 + 3.51\lambda_3 + 1.657 & 2.94\lambda_1 + 2.94\lambda_2 + 1.5\lambda_3 + 1.64 \end{pmatrix}, \\ R_{\lambda_1,\lambda_2,\lambda_3}(\tilde{D}) &= \begin{pmatrix} 3.84\lambda_1 + 3.84\lambda_2 + 2.94\lambda_3 + 2.065 & 2.987\lambda_1 + 2.287\lambda_2 + 3.78\lambda_3 + 2.748 \\ 1.5\lambda_1 + 1.5\lambda_2 + 2.667\lambda_3 + 1.587 & 5.4\lambda_1 + 5.4\lambda_2 + 2.4\lambda_3 + 1.65 \end{pmatrix}. \end{aligned} \tag{30}$$

According to equations (25) and (26), we can formulate the optimization problems with four parameters $\lambda_1, \lambda_2, \lambda_3 \in [0,1]$, and $\rho \in [0,1]$ as follows:

$$\begin{aligned} &\text{maximize}\{-5.5y_1 + y_2 - y_3\}, \\ &\text{subject to} \begin{cases} -4y_1 + y_2 - y_3 - (5.4\lambda_1 + 4.267\lambda_2 + 2.4\lambda_3 + 2.715)r_1 - (1.56\lambda_1 + 1.083\lambda_2 + 3.51\lambda_3 + 1.657)r_2 \\ \le (0.062\lambda_1 + 0.081\lambda_2 + 0.103\lambda_3 + 0.055)(1-\rho), \\ -6.5y_1 + y_2 - y_3 - (1.125\lambda_1 + 1.62\lambda_2 + 2.88\lambda_3 + 1.979)r_1 - (2.94\lambda_1 + 2.94\lambda_2 + 1.5\lambda_3 + 1.64)r_2 \\ \le (0.062\lambda_1 + 0.081\lambda_2 + 0.103\lambda_3 + 0.055)(1-\rho), \\ 7r_1 + 5r_2 \le 6.5, \\ r_1 + r_2 = 1, \\ y_1, y_2, y_3, r_1, r_2 \ge 0, \end{cases} \end{aligned} \tag{31}$$

$$\begin{aligned} &\text{maximize}\{-6.5z_1 + z_2 - z_3\}, \\ &\text{subject to} \begin{cases} -7z_1 + z_2 - z_3 - (3.84\lambda_1 + 3.84\lambda_2 + 2.94\lambda_3 + 2.065)s_1 - (2.987\lambda_1 + 2.287\lambda_2 + 3.78\lambda_3 + 2.748)s_2 \\ \le (0.051\lambda_1 + 0.051\lambda_2 + 0.039\lambda_3 + 0.012)(1-\rho), \\ -5z_1 + z_2 - z_3 - (1.5\lambda_1 + 1.5\lambda_2 + 2.667\lambda_3 + 1.587)s_1 - (5.4\lambda_1 + 5.4\lambda_2 + 2.4\lambda_3 + 1.65)s_2 \\ \le (0.051\lambda_1 + 0.051\lambda_2 + 0.039\lambda_3 + 0.012)(1-\rho), \\ 4s_1 + 6.5s_2 \le 5.5, \\ s_1 + s_2 = 1, \\ z_1, z_2, z_3 s_1, s_2 \ge 0. \end{cases} \end{aligned} \tag{32}$$

Table 6: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.7, 0.7, 0.7)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	7.856	(0.4, 0.6)	8.866
0.1	(0.75, 0.25)	7.833	(0.4, 0.6)	8.855
0.2	(0.75, 0.25)	7.81	(0.4, 0.6)	8.844
0.3	(0.75, 0.25)	7.787	(0.4, 0.6)	8.833
0.4	(0.75, 0.25)	7.765	(0.4, 0.6)	8.822
0.5	(0.75, 0.25)	7.742	(0.4, 0.6)	8.811
0.6	(0.75, 0.25)	7.719	(0.4, 0.6)	8.799
0.7	(0.75, 0.25)	7.696	(0.4, 0.6)	8.789
0.8	(0.75, 0.25)	7.674	(0.4, 0.6)	8.778
0.9	(0.75, 0.25)	7.651	(0.4, 0.6)	8.767
1.0	(0.75, 0.25)	7.628	(0.4, 0.6)	8.756

Table 7: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.8, 0.7, 0.9)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	8.673	(0.4, 0.6)	9.765
0.1	(0.75, 0.25)	8.648	(0.4, 0.6)	9.752
0.2	(0.75, 0.25)	8.622	(0.4, 0.6)	9.739
0.3	(0.75, 0.25)	8.597	(0.4, 0.6)	9.727
0.4	(0.75, 0.25)	8.571	(0.4, 0.6)	9.715
0.5	(0.75, 0.25)	8.546	(0.4, 0.6)	9.703
0.6	(0.75, 0.25)	8.521	(0.4, 0.6)	9.690
0.7	(0.75, 0.25)	8.495	(0.4, 0.6)	9.678
0.8	(0.75, 0.25)	8.469	(0.4, 0.6)	9.665
0.9	(0.75, 0.25)	8.444	(0.4, 0.6)	9.653
1.0	(0.75, 0.25)	8.419	(0.4, 0.6)	9.641

Table 8: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.8, 0.8, 0.8)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	8.667	(0.4, 0.6)	9.899
0.1	(0.75, 0.25)	8.642	(0.4, 0.6)	9.887
0.2	(0.75, 0.25)	8.617	(0.4, 0.6)	9.874
0.3	(0.75, 0.25)	8.591	(0.4, 0.6)	9.862
0.4	(0.75, 0.25)	8.567	(0.4, 0.6)	9.849
0.5	(0.75, 0.25)	8.542	(0.4, 0.6)	9.837
0.6	(0.75, 0.25)	8.516	(0.4, 0.6)	9.824
0.7	(0.75, 0.25)	8.491	(0.4, 0.6)	9.812
0.8	(0.75, 0.25)	8.466	(0.4, 0.6)	9.799
0.9	(0.75, 0.25)	8.441	(0.4, 0.6)	9.787
1.0	(0.75, 0.25)	8.416	(0.4, 0.6)	9.774

Table 9: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.9, 0.8, 0.8)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	8.946	(0.4, 0.6)	10.288
0.1	(0.75, 0.25)	8.92	(0.4, 0.6)	10.275
0.2	(0.75, 0.25)	8.894	(0.4, 0.6)	10.262
0.3	(0.75, 0.25)	8.869	(0.4, 0.6)	10.249
0.4	(0.75, 0.25)	8.843	(0.4, 0.6)	10.236
0.5	(0.75, 0.25)	8.817	(0.4, 0.6)	10.223
0.6	(0.75, 0.25)	8.791	(0.4, 0.6)	10.21
0.7	(0.75, 0.25)	8.765	(0.4, 0.6)	10.197
0.8	(0.75, 0.25)	8.739	(0.4, 0.6)	10.184
0.9	(0.75, 0.25)	8.714	(0.4, 0.6)	10.171
1.0	(0.75, 0.25)	8.688	(0.4, 0.6)	10.158

Table 10: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (0.9, 0.9, 0.9)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	9.479	(0.4, 0.6)	10.932
0.1	(0.75, 0.25)	9.452	(0.4, 0.6)	10.918
0.2	(0.75, 0.25)	9.424	(0.4, 0.6)	10.904
0.3	(0.75, 0.25)	9.397	(0.4, 0.6)	10.89
0.4	(0.75, 0.25)	9.369	(0.4, 0.6)	10.876
0.5	(0.75, 0.25)	9.341	(0.4, 0.6)	10.862
0.6	(0.75, 0.25)	9.314	(0.4, 0.6)	10.848
0.7	(0.75, 0.25)	9.286	(0.4, 0.6)	10.835
0.8	(0.75, 0.25)	9.258	(0.4, 0.6)	10.821
0.9	(0.75, 0.25)	9.231	(0.4, 0.6)	10.807
1.0	(0.75, 0.25)	9.203	(0.4, 0.6)	10.793

Table 11: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (1.0, 0.9, 0.8)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	9.488	(0.4, 0.6)	11.066
0.1	(0.75, 0.25)	9.461	(0.4, 0.6)	11.052
0.2	(0.75, 0.25)	9.434	(0.4, 0.6)	11.038
0.3	(0.75, 0.25)	9.407	(0.4, 0.6)	11.024
0.4	(0.75, 0.25)	9.379	(0.4, 0.6)	11.01
0.5	(0.75, 0.25)	9.352	(0.4, 0.6)	10.996
0.6	(0.75, 0.25)	9.325	(0.4, 0.6)	10.982
0.7	(0.75, 0.25)	9.298	(0.4, 0.6)	10.968
0.8	(0.75, 0.25)	9.27	(0.4, 0.6)	10.954
0.9	(0.75, 0.25)	9.243	(0.4, 0.6)	10.94
1.0	(0.75, 0.25)	9.216	(0.4, 0.6)	10.926

For different values $\lambda_1, \lambda_2, \lambda_3$, and ρ, the equilibrium strategies and the equilibrium values of both players can be obtained by solving equations (31) and (32), as depicted in Tables 1–12.

It can be easily seen from Table 1 that when $\lambda_1 = 0.2, \lambda_2 = 0.3, \lambda_3 = 0.5$, and $\rho = 0$, the equilibrium value and the equilibrium strategy for player I are $U^* = \mathbf{q}^{\mathbf{T}}\mathbf{y}^* = 4.868$ and $\mathbf{r}^* = (0.75, 0.25)^T$, respectively; and the equilibrium value and the equilibrium strategy for player II are $W^* = \mathbf{p}^{\mathbf{T}}\mathbf{z}^* = 4.855$ and $\mathbf{s}^* = (0.4, 0.6)^T$, respectively. The results indicate that different optimal solutions can be obtained for different values of $\lambda_1, \lambda_2, \lambda_3$, and ρ. Thus, it is essential to take all the parameters into consideration.

Table 12: The equilibrium strategies and the equilibrium values of player I and player II when $(\lambda_1, \lambda_2, \lambda_3) = (1.0, 1.0, 1.0)$.

ρ	Player I		Player II	
	$\mathbf{r}^*$	U^*	$\mathbf{s}^*$	W^*
0	(0.75, 0.25)	10.291	(0.4, 0.6)	11.965
0.1	(0.75, 0.25)	10.261	(0.4, 0.6)	11.949
0.2	(0.75, 0.25)	10.231	(0.4, 0.6)	11.934
0.3	(0.75, 0.25)	10.201	(0.4, 0.6)	11.919
0.4	(0.75, 0.25)	10.171	(0.4, 0.6)	11.903
0.5	(0.75, 0.25)	10.141	(0.4, 0.6)	11.888
0.6	(0.75, 0.25)	10.111	(0.4, 0.6)	11.873
0.7	(0.75, 0.25)	10.081	(0.4, 0.6)	11.858
0.8	(0.75, 0.25)	10.051	(0.4, 0.6)	11.842
0.9	(0.75, 0.25)	10.021	(0.4, 0.6)	11.827
1.0	(0.75, 0.25)	9.99	(0.4, 0.6)	11.812

Table 13: The equilibrium strategies and the equilibrium values of player I and player II when $\theta = 0$.

ρ	Player I		Player II	
	r^*	U^*	s^*	W^*
0	(0.75, 0.25)	6.809	(0.4, 0.6)	7.496
0.1	(0.75, 0.25)	6.787	(0.4, 0.6)	7.486
0.2	(0.75, 0.25)	6.764	(0.4, 0.6)	7.476
0.3	(0.75, 0.25)	6.742	(0.4, 0.6)	7.467
0.4	(0.75, 0.25)	6.719	(0.4, 0.6)	7.457
0.5	(0.75, 0.25)	6.697	(0.4, 0.6)	7.447
0.6	(0.75, 0.25)	6.674	(0.4, 0.6)	7.438
0.7	(0.75, 0.25)	6.652	(0.4, 0.6)	7.427
0.8	(0.75, 0.25)	6.629	(0.4, 0.6)	7.418
0.9	(0.75, 0.25)	6.607	(0.4, 0.6)	7.408
1.0	(0.75, 0.25)	6.585	(0.4, 0.6)	7.398

Table 14: The equilibrium strategies and the equilibrium values of player I and player II when $\theta = 0.2$.

ρ	Player I		Player II	
	r^*	U^*	s^*	W^*
0	(0.75, 0.25)	6.144	(0.4, 0.6)	6.891
0.1	(0.75, 0.25)	6.124	(0.4, 0.6)	6.882
0.2	(0.75, 0.25)	6.105	(0.4, 0.6)	6.873
0.3	(0.75, 0.25)	6.085	(0.4, 0.6)	6.864
0.4	(0.75, 0.25)	6.066	(0.4, 0.6)	6.854
0.5	(0.75, 0.25)	6.046	(0.4, 0.6)	6.846
0.6	(0.75, 0.25)	6.027	(0.4, 0.6)	6.837
0.7	(0.75, 0.25)	6.007	(0.4, 0.6)	6.828
0.8	(0.75, 0.25)	5.988	(0.4, 0.6)	6.819
0.9	(0.75, 0.25)	5.968	(0.4, 0.6)	6.810
1.0	(0.75, 0.25)	5.949	(0.4, 0.6)	6.801

5.3. Comparison Analysis. In this subsection, the proposed ranking approach is compared with three other approaches that were introduced by Khalifa [41], Ye [42], and Garai et al. [43].

We compare our results with those of Khalifa [41], where a score function is described by

Table 15: The equilibrium strategies and the equilibrium values of player I and player II when $\theta = 0.4$.

ρ	Player I		Player II	
	r^*	U^*	s^*	W^*
0	(0.75, 0.25)	5.478	(0.4, 0.6)	6.285
0.1	(0.75, 0.25)	5.462	(0.4, 0.6)	6.277
0.2	(0.75, 0.25)	5.445	(0.4, 0.6)	6.269
0.3	(0.75, 0.25)	5.429	(0.4, 0.6)	6.261
0.4	(0.75, 0.25)	5.412	(0.4, 0.6)	6.253
0.5	(0.75, 0.25)	5.396	(0.4, 0.6)	6.245
0.6	(0.75, 0.25)	5.379	(0.4, 0.6)	6.237
0.7	(0.75, 0.25)	5.363	(0.4, 0.6)	6.228
0.8	(0.75, 0.25)	5.346	(0.4, 0.6)	6.220
0.9	(0.75, 0.25)	5.329	(0.4, 0.6)	6.212
1.0	(0.75, 0.25)	5.313	(0.4, 0.6)	6.204

$$S(\tilde{b}) = \frac{1}{16}(k + l + m + n)\left(u_{\tilde{b}} + \left(1 - v_{\tilde{b}}\right) + \left(1 - w_{\tilde{b}}\right)\right). \tag{33}$$

Here, $\tilde{b} = \langle (k, l, m, n); u_{\tilde{b}}, v_{\tilde{b}}, w_{\tilde{b}} \rangle$ expresses an SVTNN. Based on this score function, we obtain a set of linear optimization models as follows:

$$\begin{aligned} &\max\{-5.5y_1 + y_2 - y_3\},\\ &\text{s.t.} \begin{cases} -4y_1 + y_2 - y_3 - 4.744r_1 - 2.75r_2 \le 0,\\ -6.5y_1 + y_2 - y_3 - 2.909r_1 - 3.503r_2 \le 0,\\ 7r_1 + 5r_2 \le 6.5,\\ r_1 + r_2 = 1,\\ y_1, y_2, y_3, r_1, r_2 \ge 0, \end{cases}\\ &\max\{-6.5z_1 + z_2 - z_3\},\\ &\text{s.t.} \begin{cases} -7z_1 + z_2 - z_3 - 4.097s_1 - 3.675s_2 \le 0,\\ -5z_1 + z_2 - z_3 - 2.688s_1 - 4.5s_2 \le 0,\\ 4s_1 + 6.5s_2 \le 5.5,\\ s_1 + s_2 = 1,\\ z_1, z_2, z_3 s_1, s_2 \ge 0. \end{cases} \end{aligned} \tag{34}$$

Using the Simplex technique, we can obtain that the equilibrium value and the equilibrium strategy for player I are $U^* = \mathbf{q}^{\mathrm{T}}\mathbf{y}^* = 3.533$ and $\mathbf{r}^* = (0.75, 0.25)^T$, respectively; and the equilibrium value and the equilibrium strategy for player II are $W^* = \mathbf{p}^{\mathrm{T}}\mathbf{z}^* = 3.775$ and $\mathbf{s}^* = (0.4, 0.6)$, respectively, although this approach provides the same optimal solutions as our results.

We compare our results with those of Jun Ye [42], where the score function is given by

$$S(\tilde{b}) = \frac{1}{12}(k + l + m + n)\left(2 + u_{\tilde{b}} - v_{\tilde{b}} - w_{\tilde{b}}\right). \tag{35}$$

Here, $\tilde{b} = \langle (k, l, m, n); u_{\tilde{b}}, v_{\tilde{b}}, w_{\tilde{b}} \rangle$ expresses an SVTNN. Based on this score function, we obtain the following mathematical programming models:

Table 16: The equilibrium strategies and the equilibrium values of player I and player II when $\theta = 0.6$.

ρ	Player I		Player II	
	r^*	U^*	s^*	W^*
0	(0.75, 0.25)	4.813	(0.4, 0.6)	5.679
0.1	(0.75, 0.25)	4.799	(0.4, 0.6)	5.672
0.2	(0.75, 0.25)	4.786	(0.4, 0.6)	5.665
0.3	(0.75, 0.25)	4.772	(0.4, 0.6)	5.658
0.4	(0.75, 0.25)	4.759	(0.4, 0.6)	5.651
0.5	(0.75, 0.25)	4.745	(0.4, 0.6)	5.643
0.6	(0.75, 0.25)	4.732	(0.4, 0.6)	5.636
0.7	(0.75, 0.25)	4.718	(0.4, 0.6)	5.629
0.8	(0.75, 0.25)	4.704	(0.4, 0.6)	5.622
0.9	(0.75, 0.25)	4.691	(0.4, 0.6)	5.614
1.0	(0.75, 0.25)	4.677	(0.4, 0.6)	5.607

$$\begin{aligned}
&\max \quad \{-5.5y_1 + y_2 - y_3\}\\
&\text{s.t.} \quad \begin{cases} -4y_1 + y_2 - y_3 - 6.325r_1 - 3.667r_2 \le 0,\\ -6.5y_1 + y_2 - y_3 - 3.879r_1 - 4.671r_2 \le 0,\\ 7r_1 + 5r_2 \le 6.5,\\ r_1 + r_2 = 1,\\ y_1, y_2, y_3, r_1, r_2 \ge 0, \end{cases}\\
&\max \quad \{-6.5z_1 + z_2 - z_3\},\\
&\text{s.t.} \quad \begin{cases} -7z_1 + z_2 - z_3 - 5.463s_1 - 4.9s_2 \le 0,\\ -5z_1 + z_2 - z_3 - 3.583s_1 - 6s_2 \le 0,\\ 4s_1 + 6.5s_2 \le 5.5,\\ s_1 + s_2 = 1,\\ z_1, z_2, z_3 s_1, s_2 \ge 0. \end{cases}
\end{aligned} \tag{36}$$

Using the Simplex technique, we can obtain the equilibrium value and the equilibrium strategy for player I as $U^* = \mathbf{q}^{\mathrm{T}}\mathbf{y}^* = 4.71$ and $\mathbf{r}^* = (0.75, 0.25)^T$, respectively; and the equilibrium value and the equilibrium strategy for player II are $W^* = \mathbf{p}^{\mathrm{T}}\mathbf{z}^* = 5.033$ and $\mathbf{s}^* = (0.4, 0.6)$, respectively, although this approach provides the same optimal solutions as our results.

Finally, we compare our results with those of Garai et al. [43], where the ranking function is described by

$$M(\widetilde{b}) = \frac{1}{6}(k + 2l + 2m + n)\left(\theta u_{\widetilde{b}}^2 + (1-\theta)\left(1 - v_{\widetilde{b}}\right)^2 + (1-\theta)\left(1 - w_{\widetilde{b}}\right)^2\right). \tag{37}$$

Here, $\widetilde{b} = \langle (k, l, m, n); u_{\widetilde{b}}, v_{\widetilde{b}}, w_{\widetilde{b}} \rangle$ represents an SVTNN. Based on this ranking function, we can get a set of optimization models as follows:

$$\begin{aligned}
&\text{maximize} \quad \{-5.5y_1 + y_2 - y_3\},\\
&\text{subject to} \quad \begin{aligned}[t] &-4y_1 + y_2 - y_3 - (8.16667 - 1.55167\theta)r_1 - (5.83 - 3.85\theta)r_2 \le (0.22475 - 0.1488\theta)(1-\rho),\\ &-6.5y_1 + y_2 - y_3 - (6.08333 - 4.5625\theta)r_1 - (5.42667 - 1.83333\theta)r_2 \le (0.22475 - 0.1488\theta)(1-\rho),\\ &7r_1 + 5r_2 \le 6.5,\\ &r_1 + r_2 = 1,\\ &y_1, y_2, y_3, r_1, r_2 \ge 0. \end{aligned}\\
&\text{maximize} \quad \{-6.5z_1 + z_2 - z_3\},\\
&\text{subject to} \quad \begin{aligned}[t] &-7z_1 + z_2 - z_3 - (8.09833 - 3.51167\theta)s_1 - (7.90833 - 4.015\theta)s_2 \le (0.0979333 - 0.0424667\theta)(1-\rho),\\ &-5z_1 + z_2 - z_3 - (5.33333 - 3.41333\theta)s_1 - (8.775 - 2.7\theta)s_2 \le (0.0979333 - 0.0424667\theta)(1-\rho),\\ &4s_1 + 6.5s_2 \le 5.5,\\ &s_1 + s_2 = 1,\\ &z_1, z_2, z_3 s_1, s_2 \ge 0. \end{aligned}
\end{aligned} \tag{38}$$

By solving the above mathematical programming models, we obtain the following tabulated optimal solutions, given in Tables 13–18. From the results shown in Tables 1–18, the optimal strategies obtained by different

Table 17: The equilibrium strategies and the equilibrium values of player I and player II when $\theta = 0.8$.

ρ	Player I		Player II	
	r^*	U^*	s^*	W^*
0	(0.75, 0.25)	4.147	(0.4, 0.6)	5.017
0.1	(0.75, 0.25)	4.137	(0.4, 0.6)	5.010
0.2	(0.75, 0.25)	4.126	(0.4, 0.6)	5.003
0.3	(0.75, 0.25)	4.116	(0.4, 0.6)	4.997
0.4	(0.75, 0.25)	4.105	(0.4, 0.6)	4.991
0.5	(0.75, 0.25)	4.094	(0.4, 0.6)	4.985
0.6	(0.75, 0.25)	4.084	(0.4, 0.6)	4.978
0.7	(0.75, 0.25)	4.073	(0.4, 0.6)	4.972
0.8	(0.75, 0.25)	4.063	(0.4, 0.6)	4.965
0.9	(0.75, 0.25)	4.052	(0.4, 0.6)	4.959
1.0	(0.75, 0.25)	4.042	(0.4, 0.6)	4.953

Table 18: The equilibrium strategies and the equilibrium values of player I and player II when $\theta = 1.0$.

ρ	Player I		Player II	
	r^*	U^*	s^*	W^*
0	(0.75, 0.25)	3.482	(0.4, 0.6)	4.287
0.1	(0.75, 0.25)	3.474	(0.4, 0.6)	4.281
0.2	(0.75, 0.25)	3.467	(0.4, 0.6)	4.276
0.3	(0.75, 0.25)	3.459	(0.4, 0.6)	4.270
0.4	(0.75, 0.25)	3.451	(0.4, 0.6)	4.265
0.5	(0.75, 0.25)	3.443	(0.4, 0.6)	4.259
0.6	(0.75, 0.25)	3.436	(0.4, 0.6)	4.253
0.7	(0.75, 0.25)	3.429	(0.4, 0.6)	4.248
0.8	(0.75, 0.25)	3.421	(0.4, 0.6)	4.242
0.9	(0.75, 0.25)	3.413	(0.4, 0.6)	4.237
1.0	(0.75, 0.25)	3.406	(0.4, 0.6)	4.231

ranking approaches are the same as those of the proposed approach. So, the proposed approach is feasible and effective.

6. Conclusion

The constrained bimatrix games with payoffs of SVTNNs are studied and constructed in this article. The ranking order relation, important theorems, and arithmetic operations of SVTNNs are outlined. Novel neutrosophic optimization problems for both players are established from the arithmetic operations of SVTNNs and solution method for SVTNNs constrained bimatrix games. Based on the ranking approach of SVTNNs presented in this article, the neutrosophic optimization problems for both players are converted into crisp parameterized problems, which are solved to obtain the equilibrium optimal strategies and equilibrium values for two players. Moreover, the ranking approach proposed in this article is demonstrated with a numerical simulation. Finally, our article is the first to study the constrained bimatrix games under neutrosophic environment and provide algorithm and practicable application for SVTNNs constrained bimatrix games.

In the future, we will study game theory under other types of uncertain environment such as linguistic neutrosophic, interval neutrosophic, linguistic interval neutrosophic, and linguistic interval intuitionistic neutrosophic. Furthermore, we will apply the proposed ranking approach to other areas such as pattern recognition, supply chain, risk evaluation, teacher selection, and optimization models.

Data Availability

The data used to support the findings of this research are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

References

[1] J. Von Neumann and D. Morgenstern, *The Theory of Games in Economic Bahavior*, John Wiley & Sons, Inc., New York, NY, USA, 1944.

[2] I. C. Hung, K. H. Hsia, and L. W. Chen, “Fuzzy differential game of guarding a movable territory,” *Information Sciences*, vol. 91, pp. 113–131, 1996.

[3] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[4] D.-F. Li, “Bi-matrix games with payoffs of intuitionistic fuzzy sets and bilinear programming method,” in *Decision and Game Theory in Management with Intuitionistic Fuzzy Sets*, pp. 421–441, Springer, Berlin, Germany, 2014.

[5] J. C. Figueroa-García and A. Mehra, S. Chandra, “Optimal solutions for group matrix games involving interval-valued fuzzy numbers,” *Fuzzy Sets and Systems*, vol. 362, pp. 55–70, 2019.

[6] J. Jana and S. K. Roy, “Dual hesitant fuzzy matrix games: based on new similarity measure,” *Soft Computing*, vol. 23, no. 18, pp. 8873–8886, 2018.

[7] A. Singh, A. Gupta, and A. Mehra, “Matrix games with 2-tuple linguistic information,” *Annals of Operations Research*, vol. 287, no. 2, pp. 895–910, 2020.

[8] L. Zhou and F. Xiao, “A new matrix game with payoffs of generalized Dempster-Shafer structures,” *International Journal of Intelligent Systems*, vol. 34, no. 9, pp. 2253–2268, 2019.

[9] M. R. Seikh, S. Karmakar, and M. Xia, “Solving matrix games with hesitant fuzzy pay-offs,” *Iranian Journal of Fuzzy Systems*, vol. 17, no. 4, pp. 25–40, 2020.

[10] Y. Han and Y. Deng, “A novel matrix game with payoffs of Maxitive Belief Structure,” *International Journal of Intelligent Systems*, vol. 34, no. 4, pp. 690–706, 2019.

[11] S. K. Roy and S. K. Maiti, “Reduction methods of type-2 fuzzy variables and their applications to Stackelberg game,” *Applied Intelligence*, vol. 50, pp. 1398–1415, 2020.

[12] A. Bhaumik, S. K. Roy, and G. W. Weber, “Hesitant interval-valued intuitionistic fuzzy-linguistic term set approach in Prisoners’ dilemma game theory using TOPSIS: a case study on Human-trafficking,” *Central European Journal of Operations Research*, vol. 28, no. 2, pp. 797–816, 2020.

[13] E. Ammar and M. G. Brikaa, “Solving bi-matrix games in tourism planning management under rough interval approach,” *International Journal of Mathematical Sciences and Computing(IJMSC)*, vol. 4, no. 44–62, 2019.

[14] M. G. Brikaa, Z. Zheng, and E.-S. Ammar, “Fuzzy multi-objective programming approach for constrained matrix games with payoffs of fuzzy rough numbers,” *Symmetry*, vol. 11, no. 5, p. 702, 2019.

[15] A. Bhaumik, S. K. Roy, and G. W. Weber, “Multi-objective linguistic-neutrosophic matrix game and its applications to tourism management,” *Journal of Dynamics & Games*, vol. 8, no. 2, pp. 101–118, 2021.

[16] M. G. Brikaa, Z. Zheng, and E.-S. Ammar, “Resolving indeterminacy approach to solve multi-criteria zero-sum matrix games with intuitionistic fuzzy goals,” *Mathematics*, vol. 8, no. 3, p. 305, 2020.

[17] J.-J. An and D.-F. Li, “A linear programming approach to solve constrained bi-matrix games with intuitionistic fuzzy payoffs,” *International Journal of Fuzzy Systems*, vol. 21, no. 3, pp. 908–915, 2019.

[18] K. Firouzbakht, G. Noubir, and M. Salehi, “Linearly constrained bimatrix games in wireless communications,” *IEEE Transactions on Communications*, vol. 64, no. 1, pp. 429–440, 2015.

[19] F. Meng and J. Zhan, “Two methods for solving constrained bi-matrix games,” *The Open Cybernetics and Systemics Journal*, vol. 8, no. 1, 2014.

[20] H. Bigdeli, H. Hassanpour, and J. Tayyebi, “Constrained bimatrix games with fuzzy goals and its application in nuclear negotiations,” *Iranian Journal of Numerical Analysis and Optimization*, vol. 8, no. 1, pp. 81–110, 2018.

[21] F. Smarandache, “Neutrosophic set a generalization of the intuitionistic fuzzy set,” *International Journal of Pure and Applied Mathematics*, vol. 24, pp. 287–297, 2005.

[22] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, “Single valued neutrosophic sets,” *Multispace & Multistructure*, vol. 4, pp. 410–413, 2010.

[23] H. Garg, “Decision making analysis based on sine trigonometric operational laws for single-valued neutrosophic sets and their applications,” *Applied and Computational Mathematics*, vol. 19, no. 2, pp. 255–276, 2020.

[24] M. Murugappan, “Neutrosophic inventory model under immediate return for deficient items,” *Annals of Optimization Theory and Practice*, vol. 3, no. 4, pp. 1–9, 2020.

[25] G. Harish, “Novel neutrality aggregation operators-based multiattribute group decision making method for single-valued neutrosophic numbers,” *Soft Computing*, vol. 24, pp. 10327–10349, 2020.

[26] M. Abdel-Basset, M. Ali, and A. Atef, “Resource levelling problem in construction projects under neutrosophic environment,” *The Journal of Supercomputing*, vol. 76, no. 2, pp. 964–988, 2020.

[27] T. Garai, S. Dalapati, H. Garg, and T. K. Roy, “Possibility mean, variance and standard deviation of single-valued neutrosophic numbers and its applications to multi-attribute decision-making problems,” *Soft Computing*, vol. 24, no. 24, pp. 18795–18809, 2020.

[28] S. Broumi, A. Dey, M. Talea et al., “Shortest path problem using Bellman algorithm under neutrosophic environment,” *Complex & Intelligent Systems*, vol. 5, no. 4, pp. 409–416, 2019.

[29] H. Garg, “Algorithms for single-valued neutrosophic decision making based on TOPSIS and clustering methods with new distance measure,” *AIMS Mathematics*, vol. 5, no. 3, pp. 2671–2693, 2020.

[30] M. Mullai and R. Surya, “Neutrosophic inventory backorder problem using triangular neutrosophic numbers,” *Neutrosophic Sets and Systems*, vol. 31, pp. 148–155, 2020.

[31] H. Garg and Nancy, “Multiple criteria decision making based on frank choquet heronian mean operator for single-valued neutrosophic sets,” *Applied and Computational Mathematics*, vol. 18, no. 2, pp. 163–188, 2019.

[32] M. Leyva-Vázquez, M. A. Quiroz-Martínez, E. Portilla-Castell, Y. Hechavarría-Hernández, and J. R. González-Caballero, “A new model for the selection of information technology project in a neutrosophic environment,” *Neutrosophic Sets and Systems*, vol. 32, no. 1, pp. 344–360, 2020.

[33] H. Garg and Nancy, “Non-linear programming method for multi-criteria decision making problems under interval neutrosophic set environment,” *Applied Intelligence*, vol. 48, no. 8, pp. 2199–2213, 2018.

[34] R. Sun, J. Hu, and X. Chen, “Novel single-valued neutrosophic decision-making approaches based on prospect theory and their applications in physician selection,” *Soft Computing*, vol. 23, no. 1, pp. 211–225, 2019.

[35] H. Garg and Nancy, “Some new biparametric distance measures on single-valued neutrosophic sets with applications to pattern recognition and medical diagnosis,” *Information*, vol. 8, no. 4, p. 162, 2017.

[36] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, NY, USA, 1980.

[37] K. Atanassov, “Intuitionistic fuzzy sets,” *Theory and Applications, Physica*, Academic Press, New York, NY, USA, 1999.

[38] I. Deli and Y. Şubaş, “A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems,” *International Journal of Machine Learning and Cybernetics*, vol. 8, no. 4, pp. 1309–1322, 2017.

[39] A. Bhaumik, S. Kumar, and D. Li, “(α, β, γ)—cut set based ranking approach to solving bi-matrix games in neutrosophic environment,” *Soft Computing*, vol. 25, no. 8, pp. 1–11, 2020.

[40] W. Fei and D.-F. Li, “Bilinear programming approach to solve interval bimatrix games in tourism planning management,” *International Journal of Fuzzy Systems*, vol. 18, no. 3, pp. 504–510, 2016.

[41] H. A. Khalifa, “An approach for solving two-person zero-sum matrix games in neutrosophic environment,” *Journal of Industrial and Systems Engineering*, vol. 12, no. 2, pp. 186–198, 2019.

[42] J. Ye, “Some weighted aggregation operators of trapezoidal neutrosophic numbers and their multiple attribute decision making method,” *Informatica*, vol. 28, no. 2, pp. 387–402, 2017.

[43] T. Garai, H. Garg, and T. K. Roy, “A ranking method based on possibility mean for multi-attribute decision making with single valued neutrosophic numbers,” *Journal of Ambient Intelligence and Humanized Computing*, vol. 11, pp. 5245–5258, 2020.