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Optimization of roadway support schemes with likelihood-based MABAC method

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HIGHLIGHTS

- LNNs are counseled to state qualitative decision making information.
- A new likelihood-based comparison method is proposed to overcome extant limitations.
- The MABAC method is extended with LNNs to select ideal roadway support scheme.
- A combined weight model is constructed to compute the criteria weight values.

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ABSTRACT

Whether a roadway support scheme is reasonable or not significantly affects the safety of workers and the mining efficiency of mines. This paper concentrates on the optimization of roadway support schemes with group decision making methods. At first, in order to convey the decision makers' preference more fully and aptly, assessment information for schemes is transformed into LNNs (linguistic neutrosophic numbers). For the purpose of overcoming the weakness of the extant comparison method, the likelihood of LNNs is advised. After that, an extended MABAC (multi-attributive border approximation area comparison) method based on likelihood is presented. Three stages are contained in this modified approach. Stage 1 aims to obtain the normalized decision making matrix; Stage 2 builds an inclusive criteria weight model to compute the weighted decision making matrix; Stage 3 gets the ranking order of alternatives by defining three approximation areas. Subsequently, a case study is given to confirm the practicability of the proposed method. At last, sensitive analysis and comparative analysis are conducted, followed by some discussions and conclusions.

1. Introduction

Mineral resources are the fundamental guarantees for industrial production and economic growth [1]. In order to exploit underground mineral resources, it is necessary to establish some channels between the surface and ore body by excavating plenty of roadways [2,3]. However, the surrounding rock mass can generate plenty of displacement after the roadway is excavated. If the movement is not able to be effectively controlled, some geological disasters such as collapse, large deformation and rockburst, would happen [4]. Hence, the quality of support directly affects the safety of workers and economic benefit of mines [5–7].

There are many types of roadway support schemes, including bolt support, shotcrete support, mesh support, u-shaped steel support and combined support [8,9]. As the engineering geological conditions in different regions are various and influencing factors are complicated, there is a great challenge to select suitable roadway support schemes. Besides, selecting the optimal support scheme is a complex system in the decision making process, because most of the influencing factors are of strong fuzziness [10]. Generally, the selection of roadway support schemes can be deemed as a fuzzy multi-criteria decision making problem.

Pamučar and Ćirović [11] initiated a novel decision making approach, the MABAC (multi-attributive border approximation area comparison) approach, to rank transport and handling resources in logistics centers. The idea of this method is to make the results as precise as possible by computing the potential gains and losses values. The calculation process of MABAC method is forthright, logical and methodical. Consequently, Peng and Yang [12] chose suitable projects based on Pythagorean fuzzy Choquet operators and MABAC method. Debnath [13] adopted MABAC to deal with the issue of selecting strategic project portfolios. Gigović et al. [14] constructed a MABAC-based model to determine the location of wind farms.

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Furthermore, plenty of researchers integrated MABAC with various fuzzy sets or other decision making methods to solve ranking problems in reality [15]. For example, Xue et al. [16] applied this method with interval intuitionistic fuzzy numbers to handle the optimal selection of materials for a certain produce. Shi et al. [17] combined MABAC with the cloud model to evaluate healthcare waste treatment technologies. Peng and Dai [18,19] incorporated MABAC with hesitant fuzzy soft set, interval neutrosophic set and interval fuzzy soft set, respectively. Yu et al. [20] modified MABAC method under interval type-2 fuzzy situation to obtain the rank of hotels on a tourism website. Pamučar et al. [21] extended MABAC with interval-valued fuzzy-rough numbers to pick out the best fire-fighting helicopter.

The purpose of this paper is to establish a new multi-criteria decision making model for optimizing the roadway support schemes. There are several motivations of writing this paper. Firstly, using linguistic phrases is a common and direct way to express preference in the process of evaluation. Faced with the complex environment of mine roadway, finding a suitable kind of linguistic extension to relevantly describe uncertainty of assessment information is chief. Secondly, it is a pity that the MABAC method, as an uncomplicated and operative technique, is not joined in some extended linguistic sets for tackling qualitative decision making problems. Finally, the traditional MABAC method in the existent literature usually employs distance measures to decide the upper, border and lower approximation areas. Other eligible measures should be advocated to improve the flexibility of MABAC method.

The core innovations and contributions of this paper are as follows.

(1) LNNs (linguistic neutrosophic numbers) are counseled to state qualitative decision making information. Thereby all of the linguistic true, hesitant and false membership degrees are included to make the evaluation more pertinent.

(2) The extant comparison rule of LNNs is on the basis of score and accuracy functions. However, there are some drawbacks, which are elaborated in Section 3. Therefore, a new comparison method based on likelihood is tested as a proposal to overcome these disadvantages.

(3) Inspired by LNNs and their likelihood, an extended MABAC method is integrated with LNNs to cope with the problem of ranking roadway support schemes under uncertainty. The likelihood of LNNs serves as an indicator or reference to determine which approximation area a particular object is in. In addition, the ranking results with different values of parameter λ (the identification coefficient) under several changed semantics are discussed and analyzed.

(4) Selecting the optimal roadway support scheme is regarded as a multi-criteria decision making problem. As a result, some evaluation criteria are accepted. But how to obtain the criteria weights scientifically is another important issue. Since neither the objective data nor the subjective preference should be ignored, a combined weight model, which takes notice of both objective and subjective information, is developed.

The remainder of this paper is designed as follows. Section 2 introduces the concepts of linguistic variables, linguistic scale functions and LNNs in brief. Section 3 proposes the likelihood of LNNs, and proves some important properties. Section 4 presents a likelihood-based MABAC method with three stages. Section 5 provides an illustration to optimize roadway support schemes using the modified approach from Section 4. Section 6 makes sensitivity and comparison analyses, and some discussions are also completed. Section 7 comes to final conclusions.

2. Preliminaries

Some related reviews on linguistic variables, linguistic scale functions and LNNs are conducted in this section.

Definition 1 (*[22]*). Given some independent linguistic evaluation phrases (linguistic variables s_i), the set of them can be regarded as a linguistic term set, denoted as $S^* = \left\{s_i^* | i = 0, 1, \ldots, 2t\right\}$. For convenience of calculation, a continuous linguistic term set is defined as $S = \{s_i | i \in [0, 2u]\}$.

The operational rules of two linguistic variables are: $s_i \oplus s_j = s_{i+j}$ and $\tau \cdot s_i = s_{\tau i}$; while the comparison method of them is that: $\forall s_i, s_j \in S$, if $i > j$, then $s_i > s_j$; if $i = j$, then $s_i = s_j$.

Definition 2 (*[23,24]*). If a function $G(s_i)$ satisfies the following conditions, this function $G(s_i)$ is a linguistic scale function: (1) There is a map from s_i to a certain real number r_i $(i \in [0, 2u])$, which belongs to zero to one; (2) The value of $G(s_i)$ monotonically increases with the increase of s_i value.

Wang et al. [23] defined three types of linguistic scale function $G(s_i)$ as follows:

(1) $G_1(s_i) = \frac{i}{2u}$ $(i \in [0, 2u])$;

(2) $G_2(s_i) = \begin{cases} \frac{\delta^u - \delta^{u-i}}{2\delta^u - 2} & (i = 0, 1, 2, \ldots, u) \\ \frac{\delta^u + \delta^{i-u} - 2}{2\delta^u - 2} & (i = u+1, u+2, \ldots, 2u) \end{cases}$ (Generally, $\delta = \sqrt[(2u+1)]{9}$);

(3) $G_3(s_i) = \begin{cases} \frac{u^\varepsilon - (u-i)^\varepsilon}{2u^\varepsilon} & (i = 0, 1, 2, \ldots, u) \\ \frac{u^\xi + (i-u)^\xi}{2u^\xi} & (i = u+1, u+2, \ldots, 2u) \end{cases}$ (Generally, $\varepsilon = \xi = 0.88$ [25]).

Definition 3 (*[26]*). If $S = s_i | i \in [0, 2u]$ is a linguistic term set, $s_{T_a}, s_{I_a}, s_{F_a} \in S$, then $a = (s_{T_a}, s_{I_a}, s_{F_a})$ is a LNN (linguistic neutrosophic number), where s_{T_a} is the linguistic true membership degree, s_{I_a} is the linguistic hesitant membership degree and s_{F_a} is the linguistic false membership degree.

Given two LNNs $b = (s_{T_b}, s_{I_b}, s_{F_b})$ and $c = (s_{T_c}, s_{I_c}, s_{F_c})$, the operational rules between them are:

(1) $b \oplus c = (s_{T_b}, s_{I_b}, s_{F_b}) \oplus (s_{T_c}, s_{I_c}, s_{F_c}) = (s_{T_b + T_c - \frac{T_b T_c}{2u}}, s_{\frac{I_b I_c}{2u}}, s_{\frac{F_b F_c}{2u}})$;

(2) $b \otimes c = (s_{T_b}, s_{I_b}, s_{F_b}) \otimes (s_{T_c}, s_{I_c}, s_{F_c}) = (s_{\frac{T_b T_c}{2u}}, s_{I_b + I_c - \frac{I_b I_c}{2u}}, s_{F_b + F_c - \frac{F_b F_c}{2u}})$;

(3) $\sigma \cdot b = \sigma \cdot (s_{T_b}, s_{I_b}, s_{F_b}) = (s_{2u - 2u(1 - \frac{T_b}{2u})^\sigma}, s_{2u(\frac{I_b}{2u})^\sigma}, s_{2u(\frac{F_b}{2u})^\sigma})$ $(\sigma > 0)$;

(4) $b^\sigma = (s_{T_b}, s_{I_b}, s_{F_b})^\sigma = (s_{2u(\frac{T_b}{2u})^\sigma}, s_{2u - 2u(1 - \frac{I_b}{2u})^\sigma}, s_{2u - 2u(1 - \frac{F_b}{2u})^\sigma})$ $(\sigma > 0)$.

The comparison method for two arbitrary LNNs $b = (s_{T_b}, s_{I_b}, s_{F_b})$ and $c = (s_{T_c}, s_{I_c}, s_{F_c})$, which is based on the score function $B(b) = \frac{4u + T_b - I_b - F_b}{6u}$ and the accuracy function $A(b) = \frac{T_b - F_b}{2u}$, is shown as follows:

(1) If $B(b) > B(c)$, then $b > c$;

(2) If $B(b) = B(c)$ and $A(b) > A(c)$, then $b > c$;

(3) If $B(b) = B(c)$ and $A(b) = A(c)$, then $b \approx c$.

Definition 4 (*[26]*). Let $a_i = (s_{T_{ai}}, s_{I_{ai}}, s_{F_{ai}})(i = 1, 2, \ldots, n)$ be a set of LNNs, and the weight vector is $(\upsilon_1, \upsilon_2, \ldots, \upsilon_n)^T$, where $0 \leq \upsilon_1, \upsilon_2, \ldots, \upsilon_n \leq 1$ and $\upsilon_1 + \upsilon_2 + \cdots + \upsilon_n = 1$, then the linguistic neutrosophic weighted arithmetic average (*LNWAA*) operator is defined as

$$LNWAA(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} \upsilon_i a_i$$
$$= (s_{2u - 2u\prod_{i=1}^{n}(1 - \frac{T_{ai}}{2u})^{\upsilon_i}}, s_{2u\prod_{i=1}^{n}(\frac{I_{ai}}{2u})^{\upsilon_i}}, s_{2u\prod_{i=1}^{n}(\frac{F_{ai}}{2u})^{\upsilon_i}}). \quad (1)$$

The *LNWAA* operator can be degenerated to the linguistic neutrosophic arithmetic average (*LNAA*) operator when $\upsilon_1 = \upsilon_2 = \cdots = \upsilon_n = 1/n$.

Definition 5 (*[26]*). Assume $a_i = (s_{T_{ai}}, s_{I_{ai}}, s_{F_{ai}})$ $(i = 1, 2, \ldots, n)$ is a group of LNNs, and the weight vector is $(\upsilon_1, \upsilon_2, \ldots, \upsilon_n)^T$, where $0 \leq \upsilon_1, \upsilon_2, \ldots, \upsilon_n \leq 1$ and $\upsilon_1 + \upsilon_2 + \cdots + \upsilon_n = 1$, then the linguistic neutrosophic weighted geometric average (*LNWGA*) operator can be defined as

$$LNWGA(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} a_i^{\sigma_i}$$
$$= (s_{2u\prod_{i=1}^{n}(\frac{T_{ai}}{2u})^{\upsilon_i}}, s_{2u-2u\prod_{i=1}^{n}(1-\frac{I_{ai}}{2u})^{\upsilon_i}}, s_{2u-2u\prod_{i=1}^{n}(1-\frac{F_{ai}}{2u})^{\upsilon_i}}). \quad (2)$$

If all the weight values are the same with each other, then *LNWGA* is degenerated to the linguistic neutrosophic geometric average (*LNGA*) operator.

3. Likelihood of linguistic neutrosophic numbers

The limitation of the comparison method mentioned in Section 2 is that both the score and accuracy functions are defined based on subscripts of linguistic values. In this case, the differences under distinct semantics are not considered. Besides, the degree that a certain LNN is prior to another one cannot be calculated with these functions. Thus, a new comparison rule of LNNs is established in this section.

Definition 6. Suppose $a = (s_{T_a}, s_{I_a}, s_{F_a})$ and $b = (s_{T_b}, s_{I_b}, s_{F_b})$ are two arbitrary LNNs, $G(s_i)$ is a linguistic scale function defined in Definition 2, then the likelihood of two LNNs $a = (s_{T_a}, s_{I_a}, s_{F_a})$ and $b = (s_{T_b}, s_{I_b}, s_{F_b})$ can be denoted as

$$L(a \geq b) = \frac{1}{3}(L^T(s_{T_a}, s_{T_b}) + L^I(s_{T_a}, s_{T_b}) + L^F(s_{T_a}, s_{T_b})) \quad (3)$$

where $L^T(s_{T_a}, s_{T_b}) = \begin{cases} \frac{G(s_{T_a})}{G(s_{T_a})+G(s_{T_b})}, & G(s_{T_a}) \neq 0 \text{ or } G(s_{T_b}) \neq 0 \\ 0.5. & G(s_{T_a}) = G(s_{T_b}) = 0 \end{cases}$ is the likelihood of two linguistic truth membership degrees (namely, s_{T_a} and s_{T_b}),

$L^I(s_{I_a}, s_{I_b}) = \begin{cases} 1 - \frac{G(s_{I_a})}{G(s_{I_a})+G(s_{I_b})}, & G(s_{I_a}) \neq 0 \text{ or } G(s_{I_b}) \neq 0 \\ 0.5. & G(s_{I_a}) = G(s_{I_b}) = 0 \end{cases}$ is the likelihood of two linguistic indeterminacy membership degrees (namely, s_{I_a} and s_{I_b}), and

$L^F(s_{F_a}, s_{F_b}) = \begin{cases} 1 - \frac{G(s_{F_a})}{G(s_{F_a})+G(s_{F_b})}, & G(s_{F_a}) \neq 0 \text{ or } G(s_{F_b}) \neq 0 \\ 0.5. & G(s_{F_a}) = G(s_{F_b}) = 0 \end{cases}$ is the likelihood of two linguistic falsity membership degrees (namely, s_{F_a} and s_{F_b}).

It is clear that for any two LNNs $a = (s_{T_a}, s_{I_a}, s_{F_a})$ and $b = (s_{T_b}, s_{I_b}, s_{F_b})$, if one LNN $a = (s_{T_a}, s_{I_a}, s_{F_a})$ has a bigger linguistic truth membership degree s_{T_a}, a smaller linguistic indeterminacy membership degree s_{I_a} and a smaller falsity membership degree s_{F_a}, it has a larger likelihood $L(a \geq b)$.

Example 1. Given two LNNs $a = (s_1, s_2, s_3)$ and $b = (s_3, s_6, s_2)$, the linguistic scale function is $G(s_i) = G_1(s_i) = \frac{i}{8}$ $(i \in [0, 8])$, then $L^T(s_{T_a}, s_{T_b}) = 0.25$, $L^I(s_{I_a}, s_{I_b}) = 0.75$, $L^F(s_{F_a}, s_{F_b}) = 0.4$, and $L(a \geq b) = \frac{1}{3}(L^T(s_{T_a}, s_{T_b}) + L^I(s_{I_a}, s_{I_b}) + L^F(s_{F_a}, s_{F_b})) \approx 0.467$.

Definition 7. Assume $a = (s_{T_a}, s_{I_a}, s_{F_a})$ and $b = (s_{T_b}, s_{I_b}, s_{F_b})$ are two LNNs, then the followings are true:

(1) When $L(a \geq b) < 0.5$, $a \prec b$, that is to say, a is inferior to b;

(2) When $L(a \geq b) = 0.5$, $a \sim b$, that is to say, a is indifferent to b;

(3) When $L(a \geq b) > 0.5$, $a \succ b$, that is to say, a is superior to b.

Example 2. Let two LNNs $a = (s_1, s_2, s_3)$ and $b = (s_3, s_6, s_2)$ be the same with Example 1. Since $L(a \geq b) \approx 0.467 < 0.5$, then a is inferior to b, denoted as $a \prec b$.

An obvious strength of the likelihood defined above is that it can compute the priority degree between two arbitrary LNNs. Furthermore, when various linguistic scale functions are adopted, the values of likelihood of two LNNs are changed. Then the elasticity of likelihood can be heightened.

Property 1. *If $a = (s_{T_a}, s_{I_a}, s_{F_a})$ and $b = (s_{T_b}, s_{I_b}, s_{F_b})$ are two LNNs, and the likelihood of $a = (s_{T_a}, s_{I_a}, s_{F_a})$ and $b = (s_{T_b}, s_{I_b}, s_{F_b})$ is $L(a \geq b)$, then the following properties are satisfied:*

(1) $L(a \geq b) \in [0, 1]$;

(2) $L(a \geq a) = 0.5$;

(3) $L(a \geq b) + L(b \geq a) = 1$;

(4) If $L(a \geq b) = L(b \geq a)$, then $L(a \geq b) = L(b \geq a) = 0.5$;

(5) Given another LNN $c = (s_{T_c}, s_{I_c}, s_{F_c})$, if $L(a \geq b) \geq 0.5$ and $L(b \geq c) \geq 0.5$, then $L(a \geq c) \geq 0.5$.

Proof. (1) If $G(s_{T_a}) = G(s_{T_b}) = 0$, then $L^T(s_{T_a}, s_{T_b}) = 0.5 \in [0, 1]$; If $G(s_{T_a}) \neq 0$ or $G(s_{T_b}) \neq 0$, since $0 \leq G(s_{T_a}), G(s_{T_b}) \leq 1 \Rightarrow G(s_{T_a}) \leq G(s_{T_a}) + G(s_{T_b}) \Rightarrow 0 \leq \frac{G(s_{T_a})}{G(s_{T_a})+G(s_{T_b})} \leq 1$, then $0 \leq L^T(s_{T_a}, s_{T_b}) \leq 1$.

Similarly, $0 \leq L^I(s_{I_a}, s_{I_b}) \leq 1$ and $0 \leq L^F(s_{F_a}, s_{F_b}) \leq 1$. As $0 \leq L^T(s_{T_a}, s_{T_b}) + L^I(s_{I_a}, s_{I_b}) + L^F(s_{F_a}, s_{F_b}) \leq 3$, then $0 \leq L(a \geq b) = \frac{1}{3}(L^T(s_{T_a}, s_{T_b}) + L^I(s_{I_a}, s_{I_b}) + L^F(s_{F_a}, s_{F_b})) \leq 1$.

(2) If $G(s_{T_a}) = 0$, then $L^T(s_{T_a}, s_{T_a}) = 0.5$; If $G(s_{T_a}) \neq 0$, then $L^T(s_{T_a}, s_{T_a}) = \frac{G(s_{T_a})}{G(s_{T_a})+G(s_{T_a})} = 0.5$.

Similarly, $L^I(s_{I_a}, s_{I_a}) = 0.5$ and $L^F(s_{F_a}, s_{F_b}) = 0.5$. As $L^T(s_{T_a}, s_{T_b}) + L^I(s_{I_a}, s_{I_b}) + L^F(s_{F_a}, s_{F_b}) = 1.5$, then $L(a \geq b) = \frac{1}{3}(L^T(s_{T_a}, s_{T_b}) + L^I(s_{I_a}, s_{I_b}) + L^F(s_{F_a}, s_{F_b})) = 0.5$.

(3) If $G(s_{T_a}) = G(s_{T_b}) = 0 \Rightarrow L^T(s_{T_a}, s_{T_b}) = L^T(s_{T_b}, s_{T_a}) = 0.5$, then $L^T(s_{T_a}, s_{T_b}) + L^T(s_{T_b}, s_{T_a}) = 1$; If $G(s_{T_a}) \neq 0$ or $G(s_{T_b}) \neq 0$, then $L^T(s_{T_a}, s_{T_b}) + L^T(s_{T_b}, s_{T_a}) = \frac{G(s_{T_a})}{G(s_{T_a})+G(s_{T_a})} + \frac{G(s_{T_b})}{G(s_{T_b})+G(s_{T_a})} = 1$.

Similarly, $L^I(s_{I_a}, s_{I_b}) + L^I(s_{I_b}, s_{I_a}) = 1$ and $L^F(s_{F_a}, s_{F_b}) + L^F(s_{F_b}, s_{F_a}) = 1$.

As $L^T(s_{T_a}, s_{T_b}) + L^T(s_{T_b}, s_{T_a}) + L^I(s_{I_a}, s_{I_b}) + L^I(s_{I_b}, s_{I_a}) + L^F(s_{F_a}, s_{F_b}) + L^F(s_{F_b}, s_{F_a}) = 3$, then

$L(a \geq b) + L(b \geq a) = \frac{1}{3}(L^T(s_{T_a}, s_{T_b}) + L^I(s_{I_a}, s_{I_b}) + L^F(s_{F_a}, s_{F_b})) + \frac{1}{3}(L^T(s_{T_b}, s_{T_a}) + L^I(s_{I_b}, s_{I_a}) + L^F(s_{F_b}, s_{F_a})) = 1$.

(4) If $L(a \geq b) = L(b \geq a)$, based on property (3) $L(a \geq b) + L(b \geq a) = 1$, then $L(a \geq b) = L(b \geq a) = 0.5$.

(5) If $L(a \geq b) \geq 0.5$, based on Definition 7, $a \succ b$; Similarly, if $L(b \geq c) \geq 0.5$, then $b \succ c$. As $a \succ b$ and $b \succ c \Rightarrow a \succ c$, then $L(a \geq c) \geq 0.5$.

4. A likelihood-based MABAC method

4.1. Description

With respect to a group decision making problem under linguistic neutrosophic environment, a discrete set of alternatives $d_1, d_2, \ldots, d_n$ is required to be ranked, so that the optimal alternative can be identified. k decision makers $f_1, f_2, \ldots, f_k$ evaluate these alternatives under m criteria $e_1, e_2, \ldots, e_m$. The importance degree vector of experts is $\alpha_1, \alpha_2, \ldots, \alpha_k$, where $\sum_{l=1}^{k} \alpha_l = 1$ and $\alpha_l \in [0, 1]$ $(l = 1, 2, \ldots, k)$. The weight values of all criteria β_j $(j = 1, 2, \ldots, m)$ (where $\sum_{j=1}^{m} \beta_j = 1$ and $\beta_j \in [0, 1]$) is completely unknown.

Then, an extended MABAC method based on likelihood is proposed in the following, and the framework of this approach is shown in Fig. 1.

4.2. Stage 1: Obtain the normalized decision making matrix

Step 1: Establish the original evaluation matrix.

All experts discuss and make evaluations, and then provide a linguistic neutrosophic evaluation matrix as follows:

$$H=(h_{ij})_{n\times m}=\begin{array}{c} \\ d_1\\ d_2\\ \vdots\\ d_n\end{array}\begin{array}{c}\begin{array}{cccc} e_1 & e_2 & \cdots & e_m\end{array}\\ \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1m}\\ h_{21} & h_{22} & \cdots & h_{2m}\\ \vdots & \vdots & & \vdots\\ h_{n1} & h_{n2} & \cdots & h_{nm}\end{bmatrix}\end{array},$$

where $h_{ij}=(h_{T_{ij}},h_{I_{ij}},h_{F_{ij}})$ is a LNN, $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$.

Step 2: Normalize the original evaluation matrix.

Normalize the decision making matrix $H=(h_{ij})_{n\times m}$ into H^* as follows [27].

$$H^*=(h^*_{ij})_{n\times m}=\begin{array}{c} \\ d_1\\ d_2\\ \vdots\\ d_n\end{array}\begin{array}{c}\begin{array}{cccc} e_1 & e_2 & \cdots & e_m\end{array}\\ \begin{bmatrix} h^*_{11} & h^*_{12} & \cdots & h^*_{1m}\\ h^*_{21} & h^*_{22} & \cdots & h^*_{2m}\\ \vdots & \vdots & & \vdots\\ h^*_{n1} & h^*_{n2} & \cdots & h^*_{nm}\end{bmatrix}\end{array},$$

where $h^*_{ij}=(h^*_{T_{ij}},h^*_{I_{ij}},h^*_{F_{ij}})$ is still a LNN, if e_j is a benefit criterion, $h^*_{ij}=(h^*_{T_{ij}},h^*_{I_{ij}},h^*_{F_{ij}})=(h_{T_{ij}},h_{I_{ij}},h_{F_{ij}})=h_{ij}$; and if e_j is a cost criterion, $h^*_{ij}=(h^*_{T_{ij}},h^*_{I_{ij}},h^*_{F_{ij}})=(h_{2t-T_{ij}},h_{2t-I_{ij}},h_{2t-F_{ij}})$.

4.3. Stage 2: Compute the weighted decision making matrix

Step 3: Determine the objective weights of criteria.

Calculate the average values $\overline{h^*_i}$ of assessment information in each row of the normalized matrix H^* by using the *LNAA* operators:

$$\overline{h^*_i}=\frac{1}{m}\sum_{j=1}^{m}h^*_{ij}(i=1,2,\ldots,n). \tag{4}$$

Compute the mean relational degree R_{ij} with

$$R_{ij}=\frac{\min_i|L(h^*_{ij}\geq\overline{h^*_i})-0.5|+\lambda\cdot\max_i|L(h^*_{ij}\geq\overline{h^*_i})-0.5|}{|L(h^*_{ij}\geq\overline{h^*_i})-0.5|+\lambda\cdot\max_i|L(h^*_{ij}\geq\overline{h^*_i})-0.5|} \tag{5}$$

where $L(h^*_{ij}\geq\overline{h^*_i})$ is the likelihood between h^*_{ij} and $\overline{h^*_i}$ $(i=1,2,\ldots,n,j=1,2,\ldots,m)$, which can be calculated by Eq. (3), and $\lambda\in[0,1]$ is an identification coefficient.

Then the objective weight $\beta_j^{(1)}$ of each criterion can be obtained as follows:

$$\beta_j^{(1)}=\frac{1-\frac{1}{n}\sum_{i=1}^{n}R_{ij}}{m-\frac{1}{n}\sum_{j=1}^{m}\sum_{i=1}^{n}R_{ij}}(j=1,2,\ldots,m). \tag{6}$$

The idea of this weight determination approach comes from the gray system theory. That is to say, if the evaluation information under a certain criterion matches with the average value more than other criteria, it means this criterion provides more information for experts, then this criterion has a greater weight [28]. So a modified gray system approach based on likelihood is suggested, the mean relational degree R_{ij} is also called the gray mean relational degree, and generally let the parameter $\lambda=0.5$. The advantage of the gray system method is that it can do well with a small sample and poor information.

Step 4: Determine the subjective weights of criteria.

Each decision maker offers a subjective criteria weight vector according to his experience or preference, then a weight matrix can be obtained as follows:

$$W=\begin{array}{c} \\ f_1\\ f_2\\ \vdots\\ f_k\end{array}\begin{array}{c}\begin{array}{cccc} e_1 & e_2 & \cdots & e_m\end{array}\\ \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1m}\\ w_{21} & w_{22} & \cdots & w_{2m}\\ \vdots & \vdots & & \vdots\\ w_{k1} & w_{k2} & \cdots & w_{km}\end{bmatrix}\end{array}.$$

As the importance degree of expert f_l is α_l $(l=1,2,\ldots,k)$, then the collective subjective weight value of criterion e_j $(j=1,2,\ldots,m)$ is

$$\beta_j^{(2)}=\alpha_1\cdot w_{1j}+\alpha_2\cdot w_{2j}+\cdots+\alpha_k\cdot w_{kj} \tag{7}$$

Step 5: Derive the combined weights of criteria.

The combined weight value of each criterion can be derived by

$$\beta_j=\frac{\beta_j^{(1)}\cdot\beta_j^{(2)}}{\sum_{j=1}^{m}\beta_j^{(1)}\cdot\beta_j^{(2)}}(j=1,2,\ldots,m). \tag{8}$$

Step 6: Calculate the weighted evaluation matrix.

According to the operational rules defined in Definition 3, the weighted evaluation matrix can be computed as follows:

$$Q=(q_{ij})_{n\times m}=\begin{array}{c} \\ d_1\\ d_2\\ \vdots\\ d_n\end{array}\begin{array}{c}\begin{array}{cccc} e_1 & e_2 & \cdots & e_m\end{array}\\ \begin{bmatrix} \beta_1\cdot h^*_{11} & \beta_2\cdot h^*_{12} & \cdots & \beta_m\cdot h^*_{1m}\\ \beta_1\cdot h^*_{21} & \beta_2\cdot h^*_{22} & \cdots & \beta_m\cdot h^*_{2m}\\ \vdots & \vdots & & \vdots\\ \beta_1\cdot h^*_{n1} & \beta_2\cdot h^*_{n2} & \cdots & \beta_m\cdot h^*_{nm}\end{bmatrix}\end{array}.$$

4.4. Stage 3: Get the ranking order of alternatives

Step 7: Distinguish the border approximation area.

The border approximation area vector R is calculated based on Definition 4 as follows:

$$R=(r_1,r_2,\ldots,r_m)=(\frac{1}{n}\sum_{i=1}^{n}q_{i1},\frac{1}{n}\sum_{i=1}^{n}q_{i2},\ldots,\frac{1}{n}\sum_{i=1}^{n}q_{im}) \tag{9}$$

Step 8: Distinguish both the upper and lower approximation areas.

The overall preference degree based on likelihood can be obtained with

$$p_{ij}=L(q_{ij}\geq r_j)-L(r_j\geq q_{ij}) \tag{10}$$

When $p_{ij}>0$, alternative d_i pertains to the upper approximation area R^+; when $p_{ij}=0$, alternative d_i pertains to the broader approximation area R; and when $p_{ij}<0$, alternative d_i pertains to the lower approximation area R^-.

Step 9: Obtain the ranking result.

It is clear that the ideal alternative d_i^+ is included in the upper approximation area R^+, and the anti-ideal alternative d_i^- is included in the lower approximation area R^- (See Fig. 2). That is to say, when an alternative has more corresponding $p_{ij}>0$ $(j=1,2,\ldots,m)$, this alternative d_i is more ideal. Consequently, all alternatives can be ranked by

$$p_i=\sum_{j=1}^{m}p_{ij}(i=1,2,\ldots,n). \tag{11}$$

The greater the value of p_i, the better the alternative d_i. Thus, the optimal alternative d_i^* has the largest value of p_i.

5. Case study

In this section, a case is studied by adopting the presented likelihood-based MABAC method to assess roadway support schemes in Sanshandao gold mine.

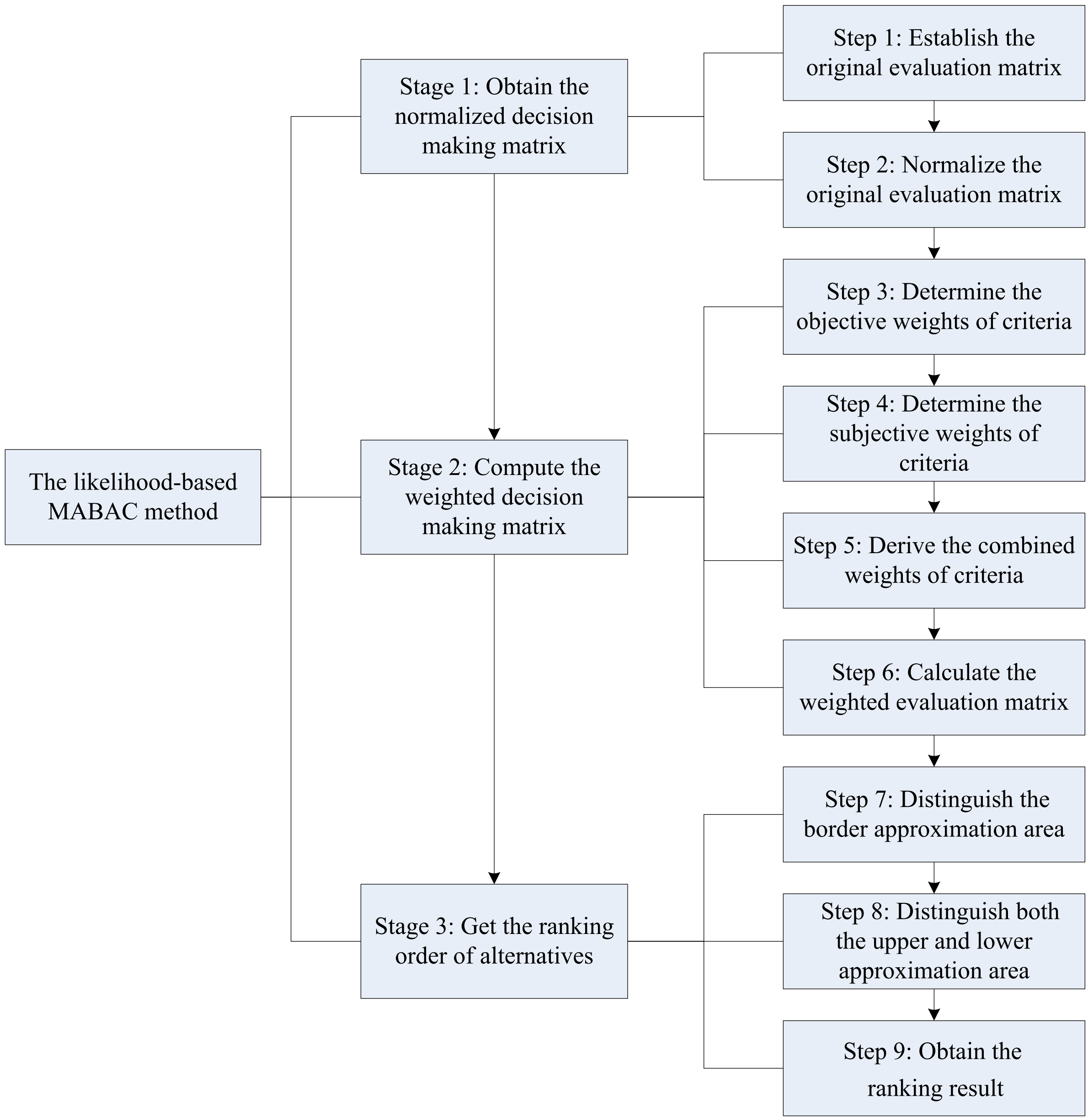

Fig. 1. Framework of the likelihood-based MABAC method.

5.1. Project profile

The Sanshandao gold mine, which is located in Sanshandao Town, Laizhou City, Shandong Province, China, is composed of Xinli, Xishan, Cangshang and Pinglidian sections. Among them, Xishan section is going into the stage of deep exploitation, and the mining depth exceeds one kilometer. With the increase of depth, the ground stress goes up largely and many engineering disasters, especially collapse, frequently happen. As a result, optimizing roadway support schemes is vital and urgent. A reasonable roadway support scheme can not only satisfy the service life, but also reduce the repair cost. It is a guarantee for mines to maintain the normal production. Accordingly, it is necessary for Sanshandao gold mine to select the optimal roadway support scheme.

5.2. Evaluation criteria system

The optimization of roadway support schemes is a complicated system. Many influence factors, such as the safety, economy and so on, should be taken into account. As the first step of the selection of roadway support schemes, to establish a reliable evaluation system is very necessary and important. According to the investigations of Chen et al. [29], Tang [30] and Zheng [31], four indexes, the safety performance, economic cost, technological feasibility and efficiency, are selected as primary evaluation criteria of roadway support schemes. Therefore, a comprehensive evaluation criteria system of roadway support schemes is constructed from these four aspects in this subsection, as shown in Table 1.

5.3. Selection process and result

According to the basic conditions of mining, four preliminary support schemes are determined. They are: Advance shotcrete and u-shaped steel (d_1), Mesh and bolt support (d_2), Advance shotcrete and mesh support (d_3), and Advance shotcrete and bolt support (d_4). Three experts f_1, f_2, f_3 in this field are invited to select the optimal scheme among these alternatives. The weight

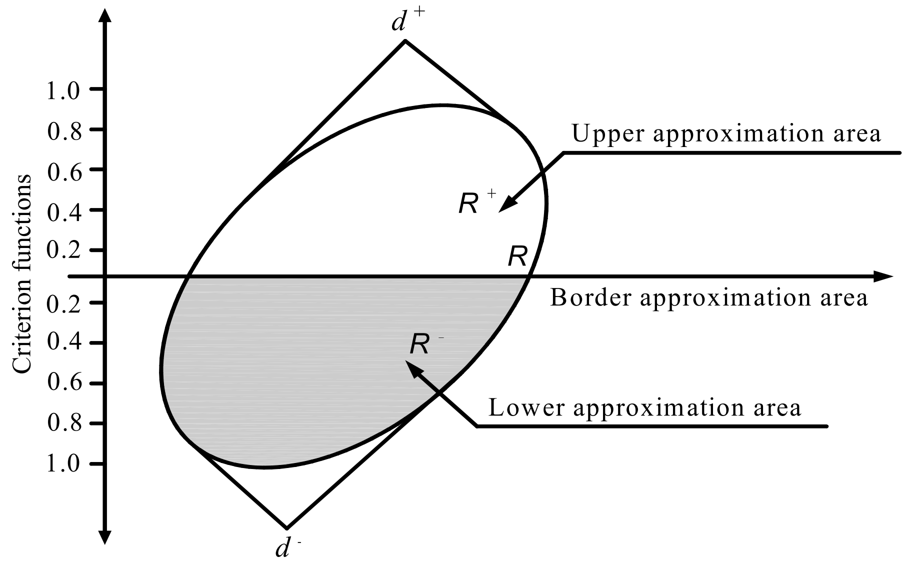

Fig. 2. Demonstration of the upper R^+, border R and lower R^- approximation areas.

Table 1
Evaluation criteria system of roadway support schemes [29–31].

Primary criteria	Descriptions
Safety performance	It indicates the safety performance in the process of support, which contains the degree of job security, effectiveness of emergency measures, success rates of similar programs and adaptability to the ore body. The better the safety performance, the better the scheme.
Economic cost	It indicates the economic cost of a support scheme, which contains the costs of materials, labors, machinery shift and maintenance. The lower the economic cost, the better the scheme.
Technological feasibility	It indicates the technological feasibility of a support scheme, which contains the degree of mechanization, engineering geology, difficulty of construction, familiarity of the scheme and impacts of surrounding environment. The higher the technological feasibility, the better the scheme.
Efficiency	It indicates the efficiency of a support scheme, which contains the construction period of supporting and construction period of auxiliary projects. The higher the efficiency, the better the scheme.

values of all decision makers are $\alpha_1 = 0.25$, $\alpha_2 = 0.35$ and $\alpha_3 = 0.40$, respectively. They can make evaluations with scores or linguistic phrases under four above-mentioned criteria (Safety performance (e_1), Economic costs (e_2), Technological feasibility (e_3) and Efficiency (e_4)). The conversion relationship between scores and linguistic terms is listed in Table 2.

Subsequently, the modified MABAC method based on likelihood suggested in Section 4 is used for experts to pick out the best roadway support scheme as follows.

Stage 1: Obtain the normalized decision making matrix.

Step 1: Establish the original evaluation matrix.

At first, three experts give their respective opinion, and the final evaluations are determined based on their importance degrees (namely, the weights of experts). Since the weight values of three decision makers are respectively $\alpha_1 = 0.25$, $\alpha_2 = 0.35$ and $\alpha_3 = 0.40$, then two conclusions can be obtained as follows: (1) If two of them make the same evaluation s_i, the final answer is s_i; (2) if an expert refuses to make an evaluation, the final answer is the same with the assessment value provided by the expert with larger weight. For example, if experts f_1 and f_2 think the truth membership degree of alternative d_1 under criterion e_1 is s_4 while expert f_3 holds the value as s_3, the final answer can be assigned as s_4; if expert f_1 does not provide an answer, f_2 thinks the hesitant membership degree of alternative d_1 under criterion e_1 is s_3 and expert f_3 holds the value as s_2, the final answer should be assigned as s_2 because the importance (weight value) of f_3 is bigger than that of f_2. It can be seen that the largest advantage of this mechanism is that it follows the principle that the minority is subordinate to the majority and the final evaluation values come from the original linguistic evaluation set (as shown in Table 2) so that each evaluation value can correspond to a certain linguistic phrase. After thorough investigations and full discussions, an original linguistic neutrosophic evaluation matrix H is provided by using this mechanism (See Table 3).

Step 2: Normalize the original evaluation matrix.

Since e_2 (Economic costs) is a cost criterion, whereas e_1 (Safety performance), e_3 (Technological feasibility) and e_4 (Efficiency) are benefit criteria, then the original decision making matrix can be normalized as follows (See Table 4).

Stage 2: Compute the weighted decision making matrix.

Step 3: Determine the objective weights of criteria.

The average values of assessment information in each row are: $\overline{h_1^*} = (s_{6.14}, s_{2.63}, s_{2.51})$, $\overline{h_2^*} = (s_{5.37}, s_{2.06}, s_{2.63})$, $\overline{h_3^*} = (s_{5.79}, s_{2.34}, s_{1.93})$ and $\overline{h_4^*} = (s_{5.79}, s_{2.21}, s_{2.91})$.

Suppose $G(s_i) = \frac{i}{2u} (i \in [0, 2u])$ and $\lambda = 0.5$, then the gray mean relational degrees and the objective weights of criteria are calculated as follows (See Table 5).

Step 4: Determine the subjective weights of criteria.

The subjective weight matrix W provided by experts is shown in Table 6.

Table 2
Conversion relationship between scores and linguistic terms.

Scores	0–15	16–25	26–35	36–45	46–55	56–65	66–75	76–85	86-100
Linguistic phrases	Very low	Low	Medium low	Slightly low	medium	Slightly high	Medium high	High	Very high
Representation	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8

Table 3
Original evaluation matrix H.

H	e_1	e_2	e_3	e_4
d_1	(s_4, s_2, s_1)	(s_1, s_2, s_3)	(s_5, s_4, s_2)	(s_7, s_1, s_4)
d_2	(s_6, s_3, s_4)	(s_2, s_5, s_4)	(s_4, s_1, s_3)	(s_5, s_2, s_1)
d_3	(s_5, s_2, s_1)	(s_4, s_3, s_1)	(s_6, s_3, s_1)	(s_7, s_1, s_2)
d_4	(s_7, s_1, s_3)	(s_3, s_4, s_2)	(s_4, s_2, s_4)	(s_6, s_3, s_1)

Table 4
Normalized evaluation matrix H^*.

H^*	e_1	e_2	e_3	e_4
d_1	(s_4, s_2, s_1)	(s_7, s_6, s_5)	(s_5, s_4, s_2)	(s_7, s_1, s_4)
d_2	(s_6, s_3, s_4)	(s_6, s_3, s_4)	(s_4, s_1, s_3)	(s_5, s_2, s_1)
d_3	(s_5, s_2, s_1)	(s_4, s_5, s_7)	(s_6, s_3, s_1)	(s_7, s_1, s_2)
d_4	(s_7, s_1, s_3)	(s_5, s_4, s_6)	(s_4, s_2, s_4)	(s_6, s_3, s_1)

Table 5
Gray mean relational degrees R_{ij} and objective criteria weights $\beta_j^{(1)}$.

R_{ij}	e_1	e_2	e_3	e_4
d_1	0.9445	0.7368	0.8206	1
d_2	0.9745	1	1	0.7861
d_3	1	0.5353	0.7834	0.7315
d_4	0.8046	0.7068	0.6412	0.8838
$\beta_j^{(1)}$	0.10	0.39	0.28	0.23

Table 6
Subjective weight matrix W.

W	e_1	e_2	e_3	e_4
f_1	0.15	0.25	0.33	0.27
f_2	0.05	0.42	0.21	0.32
f_3	0.21	0.34	0.19	0.26

Table 7
Weighted evaluation matrix Q.

Q	e_1	e_2	e_3	e_4
d_1	$(s_{0.27}, s_{7.46}, s_{7.21})$	$(s_{5.11}, s_{6.95}, s_{6.35})$	$(s_{1.62}, s_{6.82}, s_{5.82})$	$(s_{3.04}, s_{4.96}, s_{6.82})$
d_2	$(s_{0.54}, s_{7.62}, s_{7.73})$	$(s_{3.94}, s_{4.95}, s_{5.70})$	$(s_{1.18}, s_{4.96}, s_{6.38})$	$(s_{1.62}, s_{5.82}, s_{4.96})$
d_3	$(s_{0.38}, s_{7.47}, s_{7.21})$	$(s_{2.30}, s_{6.35}, s_{7.49})$	$(s_{2.18}, s_{6.38}, s_{4.96})$	$(s_{3.04}, s_{4.96}, s_{5.82})$
d_4	$(s_{0.79}, s_{7.21}, s_{7.62})$	$(s_{3.05}, s_{5.70}, s_{6.95})$	$(s_{1.18}, s_{5.82}, s_{6.82})$	$(s_{2.18}, s_{6.38}, s_{4.96})$

As the expert importance degree vector is $(0.250.350.40)^T$, then the collective subjective criteria weight values are: $\beta_1^{(2)} \approx 0.14$, $\beta_2^{(2)} \approx 0.35$, $\beta_3^{(2)} \approx 0.23$ and $\beta_4^{(2)} \approx 0.28$.

Step 5: Derive the combined weights of criteria.

Then, the combined criteria weight values are: $\beta_1 \approx 0.05$, $\beta_2 \approx 0.49$, $\beta_3 \approx 0.23$ and $\beta_4 \approx 0.23$.

Step 6: Calculate the weighted evaluation matrix.

Based on Definition 3, the weighted evaluation matrix Q is shown in Table 7.

Stage 3: Get the ranking order of alternatives.

Step 7: Distinguish the border approximation area.

According to Definition 4, the border approximation area vector R is

$$R = (r_1, r_2, r_3, r_4) = ((s_{0.50}, s_{7.44}, s_{7.44}), (s_{3.73}, s_{5.94}, s_{6.59}), (s_{2.05}, s_{5.72}, s_{5.72}), (s_{2.50}, s_{5.50}, s_{5.59})).$$

Step 8: Distinguish both the upper and lower approximation areas.

Table 8
Likelihood-based preference degrees p_{ij}.

p_{ij}	e_1	e_2	e_3	e_4
d_1	−0.0929	0.0317	−0.0687	0.0164
d_2	0.0019	0.0635	−0.0367	−0.0613
d_3	−0.0389	−0.1118	−0.0182	0.0429
d_4	0.0768	−0.0355	0.0706	−0.0277

Table 9
Rankings of alternatives with different values of λ under $G_1(s_i)$.

λ	Ranking of alternatives	λ	Ranking of alternatives
$\lambda = 0.0$	$d_4 \succ d_2 \succ d_1 \succ d_3$	$\lambda = 0.6$	$d_4 \succ d_2 \succ d_1 \succ d_3$
$\lambda = 0.1$	$d_4 \succ d_2 \succ d_1 \succ d_3$	$\lambda = 0.7$	$d_4 \succ d_2 \succ d_1 \succ d_3$
$\lambda = 0.2$	$d_4 \succ d_2 \succ d_1 \succ d_3$	$\lambda = 0.8$	$d_4 \succ d_2 \succ d_1 \succ d_3$
$\lambda = 0.3$	$d_4 \succ d_2 \succ d_1 \succ d_3$	$\lambda = 0.9$	$d_4 \succ d_2 \succ d_1 \succ d_3$
$\lambda = 0.4$	$d_4 \succ d_2 \succ d_1 \succ d_3$	$\lambda = 1.0$	$d_4 \succ d_2 \succ d_1 \succ d_3$
$\lambda = 0.5$	$d_4 \succ d_2 \succ d_1 \succ d_3$		

All the likelihood-based preference degrees p_{ij} are shown in Table 8.

Then, the upper approximation area R^+ contains: p_{12}, p_{14}, p_{21}, p_{22}, p_{34}, p_{41} and p_{43}, while the lower approximation area R^- contains: p_{11}, p_{13}, p_{23}, p_{24}, p_{31}, p_{32}, p_{33}, p_{42} and p_{44}.

Step 9: Obtain the ranking result.

Based on Eq. (11), $p_1 = -0.1135$, $p_2 = -0.0326$, $p_3 = -0.1261$ and $p_4 = 0.0843$. Since $p_4 > p_2 > p_1 > p_3$, then $d_4 \succ d_2 \succ d_1 \succ d_3$, and the optimal alternative is d_4.

Consequently, decision makers regard d_4 (Advance shotcrete and bolt support) as the ideal roadway support scheme. The selected optimal roadway support scheme has been successfully applied in Sanshandao gold mine and excellent performance has been obtained.

6. Analyses and discussions

6.1. Sensitive analysis and discussion

Considering that the rankings of alternatives may be affected by the value of λ and the type of linguistic scale functions, the sensitivity of the proposed method is analyzed and discussed in this subsection.

(1) The rankings of roadway support schemes with different λ values under the first type of linguistic function $G_1(s_i)$ are listed in Table 9. It is clear that same ranking order $d_4 \succ d_2 \succ d_1 \succ d_3$ is obtained no matter what the value of λ is.

The values of p for all alternatives with the likelihood-based MABAC method under $G_1(s_i)$ are shown in Fig. 3. As we can see, the roadway support scheme d_4 always has the largest values of p among all options under $G_1(s_i)$. The alternative d_2 comes second with around -0.04 of the value of p. The lowest figures on the picture are for scheme d_3, with figures below -0.12. Furthermore, there is not a great deal of difference between the p values of support schemes d_1 and d_3.

In order to show the impacts of different values of λ on the ranking results more intuitively, the changes of p values of each roadway support scheme under $G_1(s_i)$ are portrayed in Fig. 4. As can be seen from this diagram, when the value of λ alters from 0.0 to 1.0 with 0.1 increments, the p values of all schemes are diverse. The variation tendency of alternative d_1 stays roughly the same with that of d_3. Likewise, the trend of d_2 is similar to

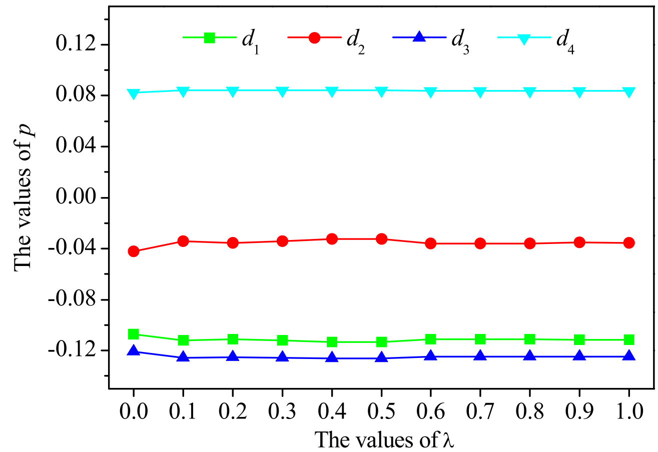

Fig. 3. Values of p for all alternatives with the likelihood-based MABAC method under $G_1(s_i)$.

Table 10
Rankings of alternatives with different values of λ under $G_2(s_i)$.

λ	Ranking of alternatives	λ	Ranking of alternatives
$\lambda = 0.0$	$d_2 \succ d_4 \succ d_3 \succ d_1$	$\lambda = 0.6$	$d_2 \succ d_4 \succ d_3 \succ d_1$
$\lambda = 0.1$	$d_2 \succ d_4 \succ d_3 \succ d_1$	$\lambda = 0.7$	$d_2 \succ d_4 \succ d_3 \succ d_1$
$\lambda = 0.2$	$d_2 \succ d_4 \succ d_3 \succ d_1$	$\lambda = 0.8$	$d_2 \succ d_4 \succ d_3 \succ d_1$
$\lambda = 0.3$	$d_2 \succ d_4 \succ d_3 \succ d_1$	$\lambda = 0.9$	$d_2 \succ d_4 \succ d_3 \succ d_1$
$\lambda = 0.4$	$d_2 \succ d_4 \succ d_3 \succ d_1$	$\lambda = 1.0$	$d_2 \succ d_4 \succ d_3 \succ d_1$
$\lambda = 0.5$	$d_2 \succ d_4 \succ d_3 \succ d_1$		

Table 11
Ranking of alternatives with different values of λ under $G_3(s_i)$.

λ	Ranking of alternatives	λ	Ranking of alternatives
$\lambda = 0.0$	$d_4 \succ d_2 \succ d_3 \succ d_1$	$\lambda = 0.6$	$d_4 \succ d_2 \succ d_3 \succ d_1$
$\lambda = 0.1$	$d_4 \succ d_2 \succ d_3 \succ d_1$	$\lambda = 0.7$	$d_4 \succ d_2 \succ d_3 \succ d_1$
$\lambda = 0.2$	$d_4 \succ d_2 \succ d_3 \succ d_1$	$\lambda = 0.8$	$d_4 \succ d_2 \succ d_3 \succ d_1$
$\lambda = 0.3$	$d_4 \succ d_2 \succ d_3 \succ d_1$	$\lambda = 0.9$	$d_4 \succ d_2 \succ d_3 \succ d_1$
$\lambda = 0.4$	$d_4 \succ d_2 \succ d_3 \succ d_1$	$\lambda = 1.0$	$d_4 \succ d_2 \succ d_3 \succ d_1$
$\lambda = 0.5$	$d_4 \succ d_2 \succ d_3 \succ d_1$		

that of d_4. When $\lambda = 0.5$, both the values of p for schemes d_1 and d_3 drop to their lowest points, while those of p for d_2 and d_4 reach the peaks. The equal influence of $\min_i |L(h_{ij}^* \geq \overline{h_i^*}) - 0.5|$ and $\max_i |L(h_{ij}^* \geq \overline{h_i^*}) - 0.5|$ is a possible cause. Moreover, a sudden change occurs in all four sub-graphs when the λ value varies from 0.0 to 0.1. The reason for this phenomenon may be that the effect of $\max_i |L(h_{ij}^* \geq \overline{h_i^*}) - 0.5|$ is not recognized when $\lambda = 0.0$.

(2) Table 10 gives information about the ranking results of alternatives with various λ values under the second type of linguistic function $G_2(s_i)$. From this table, we can see that in spite of how the value of λ changes, the ranking is still $d_2 \succ d_4 \succ d_3 \succ d_1$. Fig. 5 portrays the values of p for all alternatives with the likelihood-based MABAC method under $G_2(s_i)$. According to Fig. 5, the p values of alternative d_2 is the biggest among the four alternatives, with figure reaching 0.23. The roadway support scheme d_4 has the second largest values of p. By contrast, scheme d_1 records the lowest figures constantly

For the same reason, Fig. 6 depicts the variation of p values with the growth of λ values under $G_2(s_i)$. Different trends can be seen if we look at the specific figures of each scheme. The values of p for alternative d_1 fluctuate between −0.331 and −0.323 in Fig. 6(A). Rising trends are seen in the values of p for scheme d_2 in Fig. 6(B) and d_4 in Fig. 6(D), respectively. The greatest increase of p values is in plan d_2, rising from 0.18 to 0.23. However, there is a modestly decline of approximately 0.04 in the values of p for alternative d_3 in Fig. 6(C).

(3) The ranking orders of schemes with distinct values of λ under the last type of linguistic function $G_3(s_i)$ are itemized in Table 11. It is noticeable that no matter what value is assigned to λ, the rankings ($d_4 \succ d_2 \succ d_3 \succ d_1$) are the same with each other.

The values of p for all alternatives with the likelihood-based MABAC method under $G_3(s_i)$ are rendered in Fig. 7. From Fig. 7, we can see that alternative d_4 has the highest levels of the values of p. The next most ideal schemes are support schemes d_2 and d_3, while the p values of d_2 are slightly larger than those of d_3. On the other hand, alternative d_1 is the least ideal scheme, with figures below −0.7.

Similarly, the figures of p for each alternative with different p values under $G_3(s_i)$ are described in Fig. 8. The variation tendency of each alternative is dissimilar. Dropping trends can be seen for both alternatives d_2 and d_3, although there is a notable exception to this trend when $\lambda = 0.6$ in Fig. 8(B). Fig. 8(D) indicates that there is a steady increase in the values of p for alternative d_4. The trend in Fig. 8(A) is noticeably different from those described above. Great changes take place in the values of p for scheme d_1.

Overall, with the increase of λ values, the values of p for each alternative changed while the ranking results of schemes under a particular linguistic scale function stayed the same. It demonstrated that the proposed method has satisfactory robustness. On the other hand, when a certain linguistic scale function altered to another one (that is to say, the semantic varied), the ranking orders were dissimilar. Thus, the flexibility of this approach was displayed as well.

6.2. Comparative analysis and discussion

For the sake of confirming the availability of the presented approach, theoretical comparisons, numerical comparisons and advantages analyses are respectively drawn in this subsection.

Part 1: Theoretical comparisons

In this part, theoretical comparisons between the proposed approach and other related techniques are made in terms of the information expression, decision making method and criteria weights determination model.

(1) Since Zadeh [32] put forward the concept of linguistic terms, numerous methods with linguistic variables or their extensions have been suggested [33–36]. Nevertheless, not more than two of the three degrees (the membership degree, hesitant degree and non-membership degree) are considered in most of these extensions. Even though the simplified neutrosophic linguistic numbers contains these three degrees, they are all

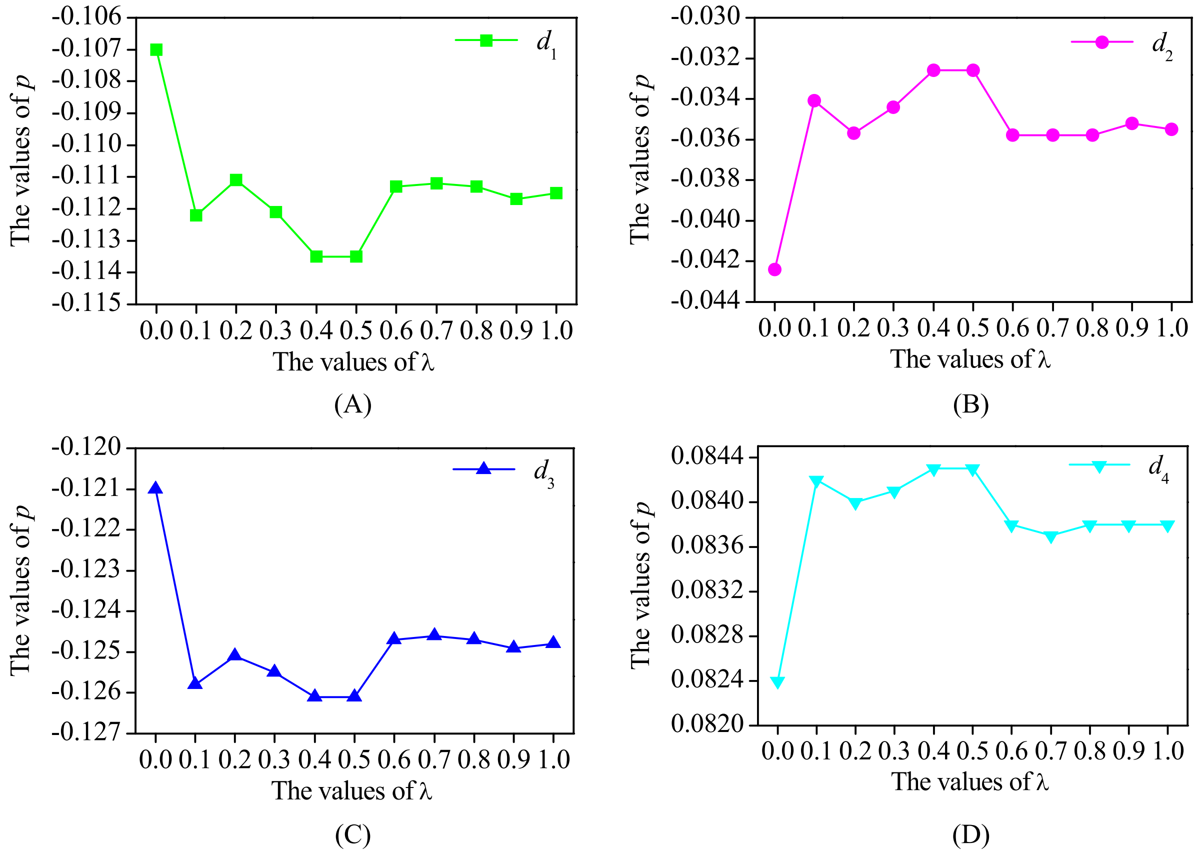

Fig. 4. Values of p for each alternative with the likelihood-based MABAC method under $G_1(s_i)$.

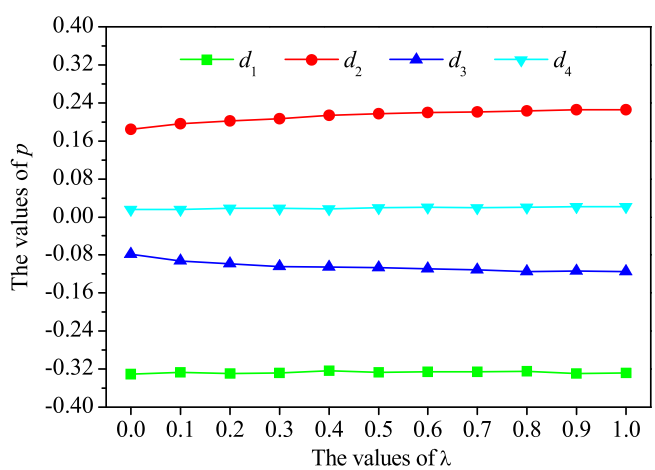

Fig. 5. Values of p for all alternatives with the likelihood-based MABAC method under $G_2(s_i)$.

quantitative crisp numbers [37,38]. On the contrary, this paper chooses LNNs to express assessment information. In a LNN, three linguistic values are used to signify the truth-membership degree, hesitancy-membership degree and falsity-membership degree directly and respectively.

(2) To the best of our knowledge, the decision making problems under linguistic neutrosophic environment addressed in extant literature are relatively rare. Related methods, like the TOPSIS (technique for order preference by similarity to ideal solution) [39], extended MULTIMOORA (multi-objective optimization by ratio analysis plus the full multiplicative form) [40], and approaches based on cosine similarity measures [41] and various aggregation operators [26,42–44], have been studied. However, some of these approaches have relative complex calculation process, which may hamper their further development and applications. On the other hand, a simple but significant method, the MABAC method, is not collaborated with LNNs to resolve optimization issues. Besides, all of aforementioned methods did not take into consideration the impacts of evaluation values on the ranking orders under different semantics. By contrast, this paper introduces a modified MABAC method based on the likelihood of LNNs. The likelihood is a new comparison method to compare two arbitrary LNNs. Moreover, the situations and results under three distinct linguistic scale functions are discussed.

(3) So far, plenty of methods have been studied to determine the weights of criteria. Some subjective weight determination

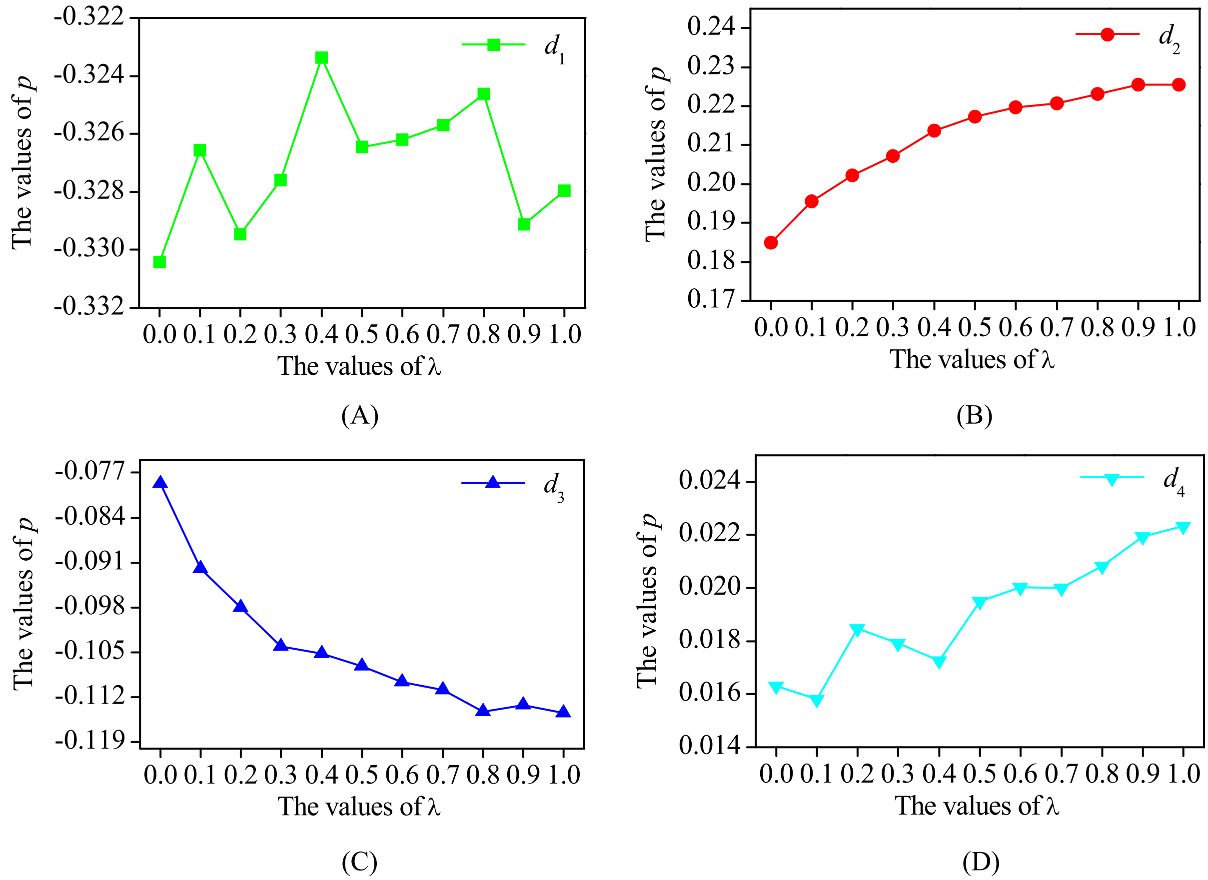

Fig. 6. Values of p for each alternative with the likelihood-based MABAC method under $G_2(s_i)$.

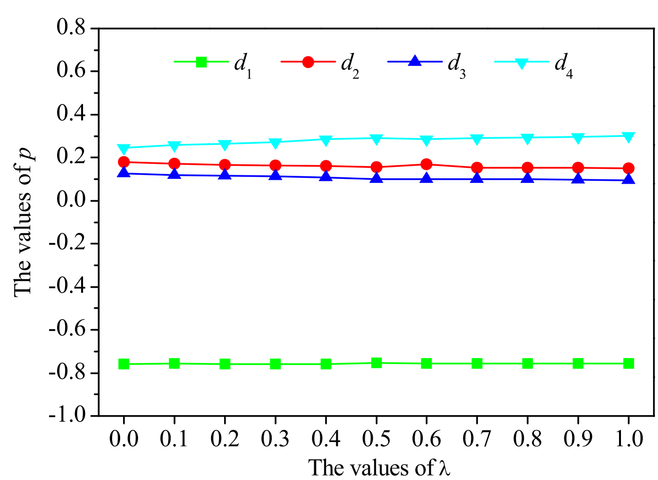

Fig. 7. Values of p for all alternatives with the likelihood-based MABAC method under $G_3(s_i)$.

ways, such as the direct ratings approach [45], AHP (analytic hierarchy process) method [46] and so on , only consider the decision makers' experience or preference, instead of unbiased facts. Some objective weight computation techniques, such as the mathematical programming approach [47], entropy weight model [48,49] and maximizing deviation method [50], are mainly based on certain impersonal principles. Compare with these methods, the method advised in this paper is a combined weight calculation model. The trait of this model is that not only the attitudes of experts, but also factual information are taken into account. Furthermore, unlike other linear combination means, a nonlinear integrated weight model on the basis of multiplier effects is constructed to obtain the comprehensive weights of evaluation criteria.

Part 2: Numerical comparisons

In this part, numerical comparisons between our method and other existent approaches are conducted.

At first, the ranking orders by using different methods are listed in Table 12.

Then, an aggregation method in literature [51] is adopted to find out the best ranking order. The first step is to compute the numbers of times a roadway support scheme is assigned

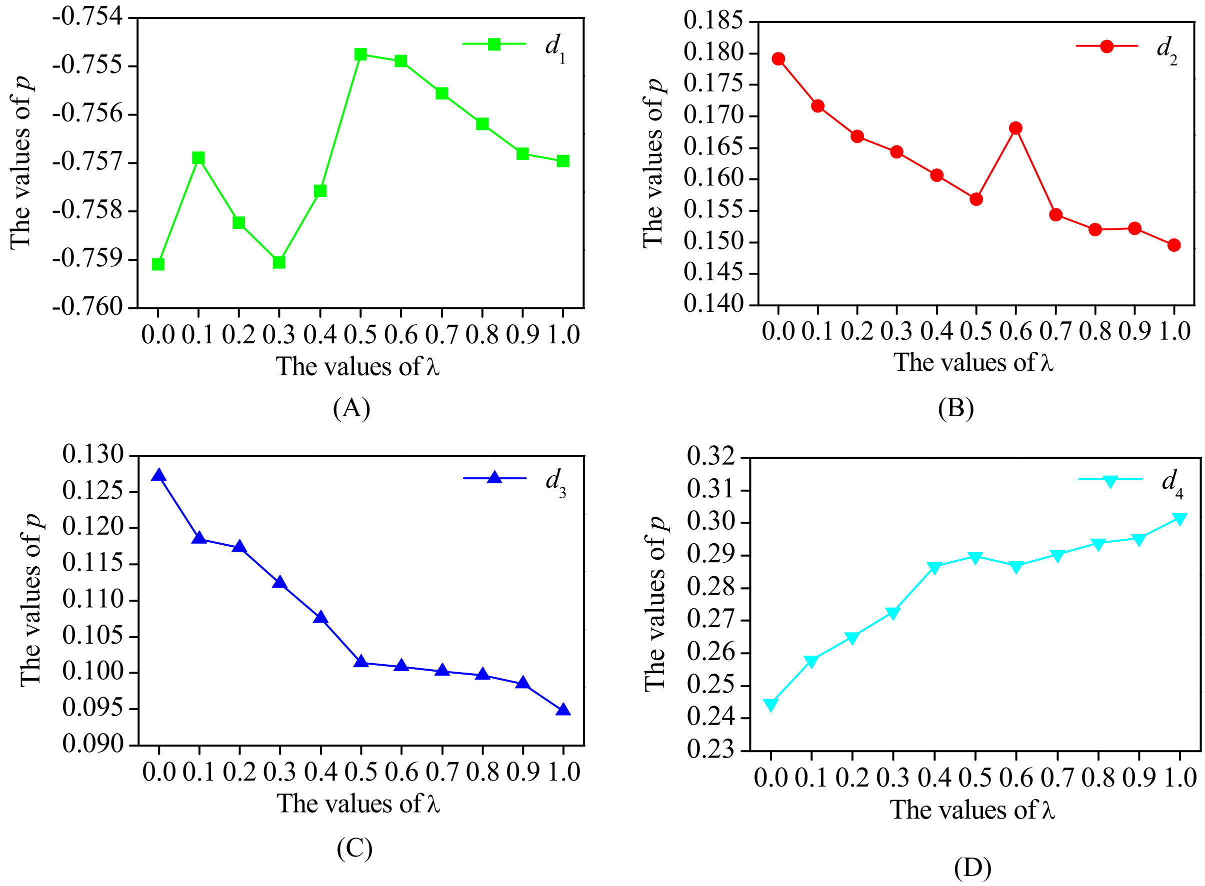

Fig. 8. Values of p for each alternative with the likelihood-based MABAC method under $G_3(s_i)$.

Table 12
Ranking results under distinct approaches.

Approaches	Ranking orders	The best alternatives
Extended TOPSIS [39]	$d_4 \succ d_1 \succ d_2 \succ d_3$	d_4
Extended MULTIMOORA [40]	$d_4 \succ d_2 \succ d_1 \succ d_3$	d_4
LNNWHA ($k = 2$) operator [42]	$d_2 \succ d_1 \succ d_4 \succ d_3$	d_2
LNNWAA operator [26]	$d_4 \succ d_3 \succ d_1 \succ d_2$	d_4
LNNWGA operator [26]	$d_3 \succ d_4 \succ d_1 \succ d_2$	d_3
The presented approach	$d_4 \succ d_2 \succ d_1 \succ d_3$	d_4

Table 13
Numbers of times a roadway support scheme is assigned to diverse ranks.

Support schemes	Ranks			
	1	2	3	4
d_1		2	4	
d_2	1	2	1	2
d_3	1	1		4
d_4	4	1	1	

to diverse ranks, as shown in Table 13. Take scheme d_1 as an example, it has twice a ranking of 2 and four times a ranking of 3. Then, the second step is to calculate the smoothing of each scheme assignment over ranks, as exhibited in Table 14. For each ranking, the previous column in Table 14 is added to the column of considered rank in Table 13.

Finally, on the basis of Table 14, a linear programming model is constructed as Eq. (12). By solving this linear programming model, the optimal ranking order is attained as $d_4 \succ d_2 \succ d_1 \succ d_3$.

Table 14
Smoothing of roadway support schemes assignment over ranks (C_{ig}).

Support schemes	Ranks			
	1	2	3	4
d_1	0	2	6	6
d_2	1	3	4	6
d_3	1	2	2	6
d_4	4	5	6	6

$$Max\ E = \sum_{i=1}^{4}\sum_{g=1}^{4}\left(C_{ig} \cdot \frac{4^2}{g} \cdot D_{ig}\right)$$

$$s.t. \begin{cases} \sum_{i=1}^{4} D_{ig} = 1, & g = 1, 2, 3, 4 \\ \sum_{g=1}^{4} D_{ig} = 1, & i = 1, 2, 3, 4 \\ D_{ig} = 0 \text{ or } 1, & \forall i, g \end{cases} \quad . \tag{12}$$

For clarity, all ranking orders by using different methods are described in Fig. 9. It can be seen that the optimal ranking order is the same with the ranking results by using the presented method in this paper, which demonstrates that our method performs well in addressing the selection of roadway support schemes.

Part 3: Advantages analyses

In general, the features and advantages of the method recommended in this paper are recapitulated as follows.

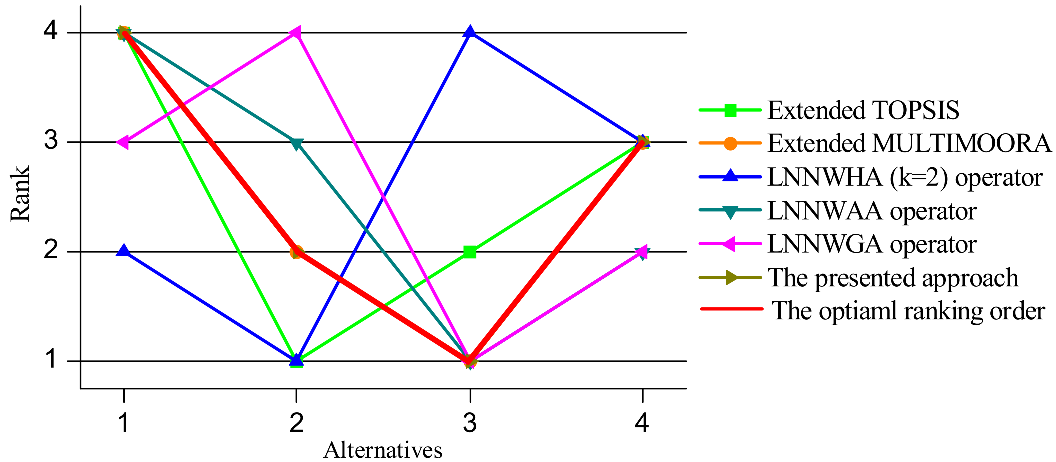

Fig. 9. Ranking orders under different approaches.

(1) All original evaluation information is transferred into LNNs. As the consistent linguistic values, indeterminate linguistic values and inconsistent linguistic values are included simultaneously, the qualitative assessment information can be conveyed more substantially and thoroughly.

(2) The MABAC method is extended in conjunction with LNNs to pick out the most ideal roadway support scheme. The calculation cost of this approach is much less, and it is convenient and readily comprehensible. Compared with the conventional MABAC method, the likelihood of LNNs takes the place of distance measures. The value of likelihood between two LNNs may change under varied semantics (namely linguistic scale functions). Hence, the final ranking result may be more exact and persuasive to some extent.

(3) A comprehensive model is constructed to compute the criteria weight values. The highlights of this model are threefold. Firstly, the combined model does not ignore both the objective facts and subjective preference of experts. Secondly, the objective weights are calculated by combining the gray system theory with likelihood of LNNs. Finally, unlike determined by a group of decision makers straightforwardly, the subjective weights are computed by aggregating several weight vectors offered by each decision maker and the corresponding weights of specialists.

7. Conclusions

A support scheme in line with the actual geological conditions of roadway is beneficial for mine enterprises to improve the economic benefit reasonably. In this paper, a likelihood-based MABAC method was proposed to select the most suitable roadway support scheme. LNNs were utilized to describe evaluation information and the likelihood of LNNs was defined to overcome the flaws of the existing comparison rule. Then the distance formula in typical MABAC method was replaced by likelihood to enhance the extensibility of MABAC. A combined weight model based on likelihood and gray system theory was also built to calculate the collective criteria weight values. Finally, the modified MABAC method was adopted to select the optimal roadway support scheme in a mine. The feasibility and effectiveness of this proposed method were verified by the case, sensitive analysis and comparative analysis. However, an imperfection of this approach is that the weight values of experts are allotted directly and artificially. In the future, the technique of determining decision makers' weights can be further studied.

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Conflict of interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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